Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.1-Sine/76-4.1.3.1-a+b-sin-^m-c+d-sin-^n-A+B-sin-

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [358]. This is test number [76].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (358)	0.00 (0)
Mathematica	93.02 (333)	6.98 (25)
Maple	82.68 (296)	17.32 (62)
Giac	80.45 (288)	19.55 (70)
Fricas	76.82 (275)	23.18 (83)
Mupad	49.72 (178)	50.28 (180)
Maxima	37.15 (133)	62.85 (225)
Sympy	28.49 (102)	71.51 (256)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

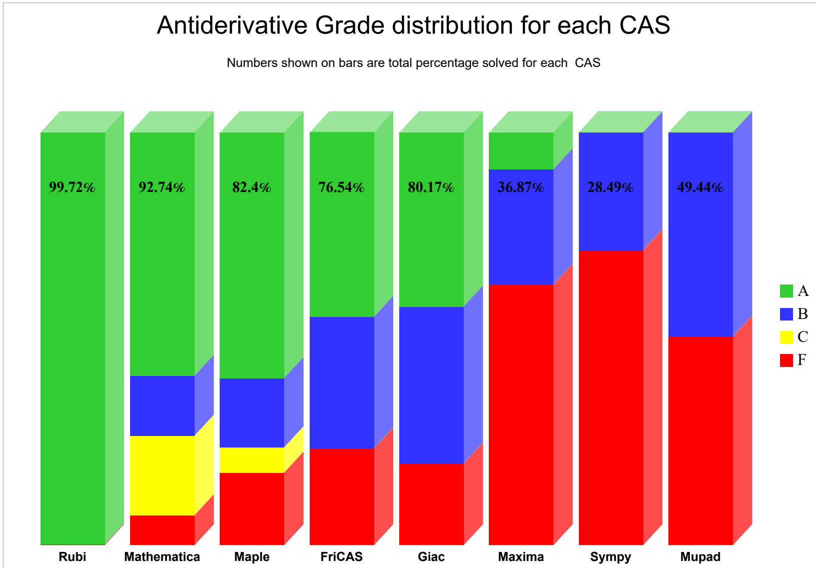
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

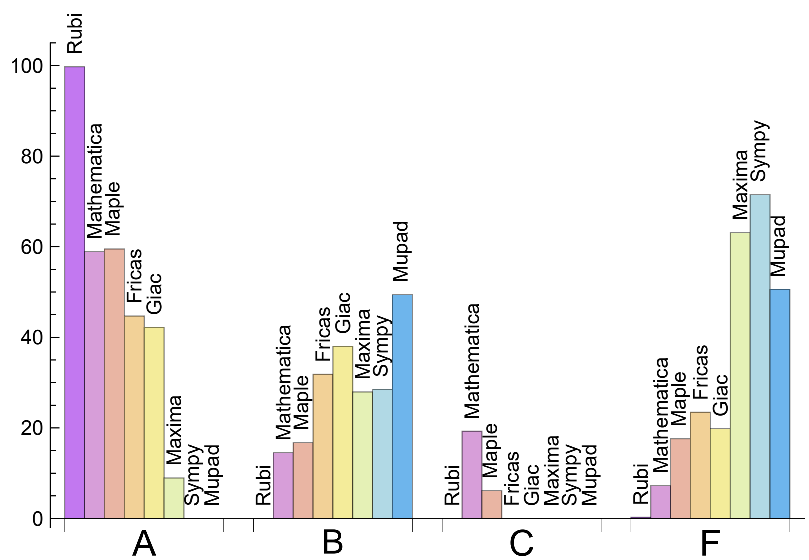
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.721	0.000	0.000	0.279
Maple	59.497	16.760	6.145	17.598
Mathematica	58.939	14.525	19.274	7.263
Fricas	44.693	31.844	0.000	23.464
Giac	42.179	37.989	0.000	19.832
Maxima	8.939	27.933	0.000	63.128
Mupad	0.000	49.441	0.000	50.559
Sympy	0.000	28.492	0.000	71.508

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	25	96.00	4.00	0.00
Maple	62	100.00	0.00	0.00
Giac	70	77.14	12.86	10.00
Fricas	83	98.80	1.20	0.00
Mupad	180	0.00	100.00	0.00
Maxima	225	86.67	4.44	8.89
Sympy	256	41.80	57.81	0.39

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.30
Maxima	0.50
Fricas	0.51
Giac	1.84
Mathematica	6.30
Maple	7.88
Sympy	15.22
Mupad	15.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	179.48	1.00	156.00	1.00
Mathematica	321.07	1.72	204.00	1.33
Fricas	527.80	2.59	234.00	1.77
Giac	578.75	8.93	276.50	1.72
Maxima	782.45	5.48	506.00	4.15
Mupad	833.52	4.61	298.50	2.09
Sympy	2842.08	19.47	1616.50	14.42
Maple	10659.20	15.38	166.50	1.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

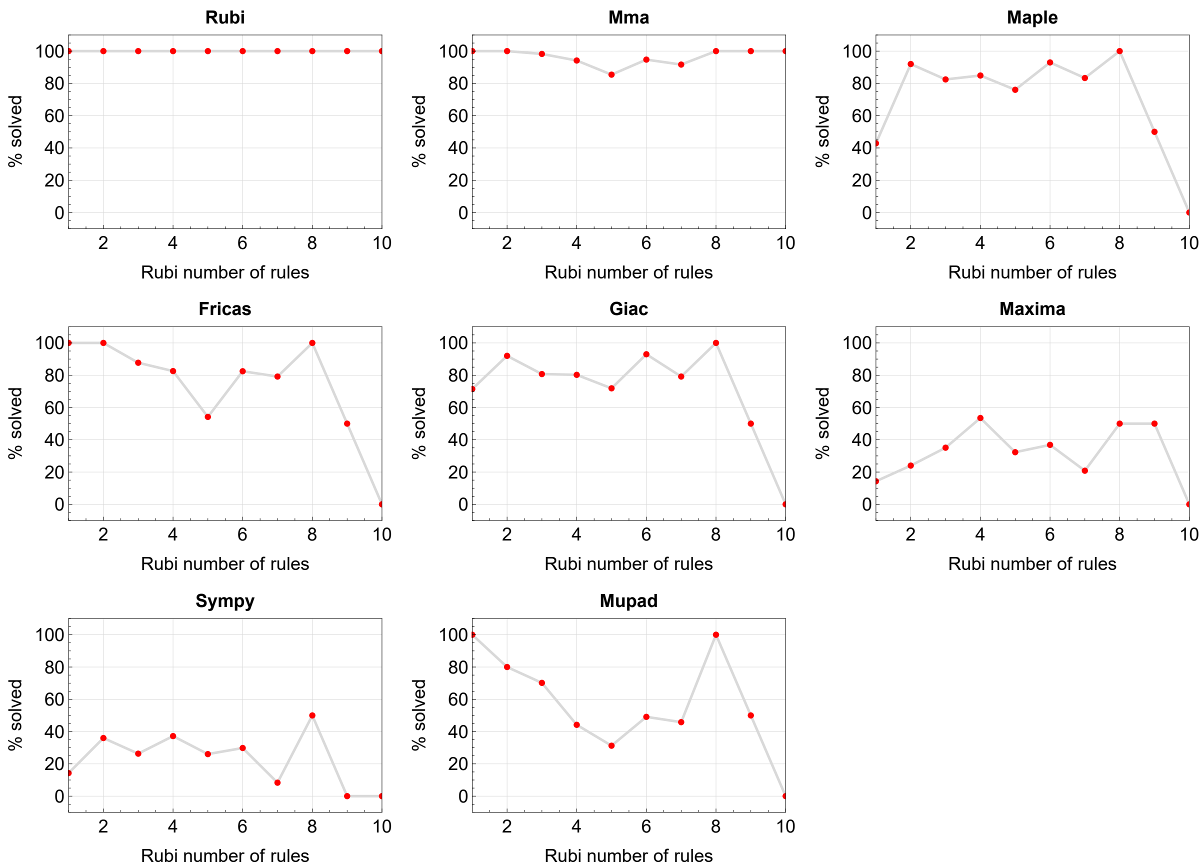


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

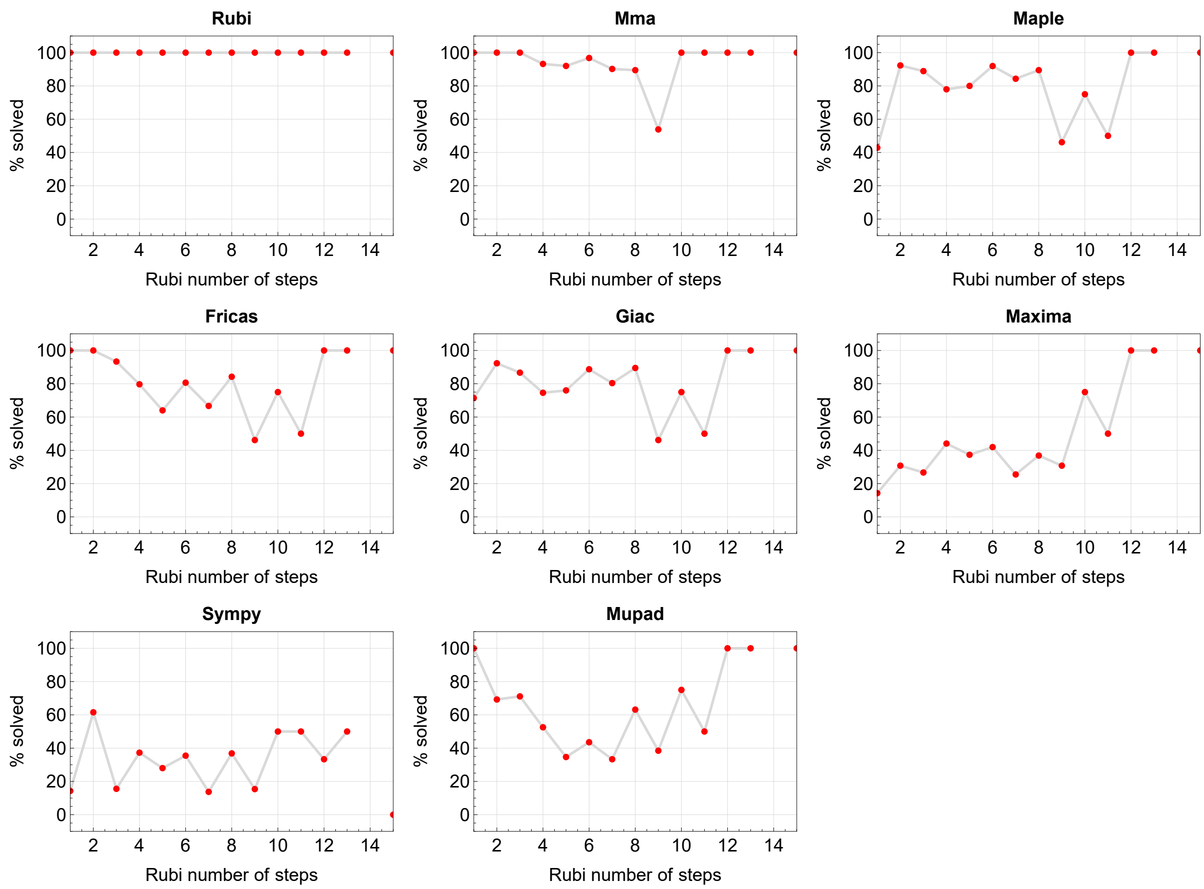


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

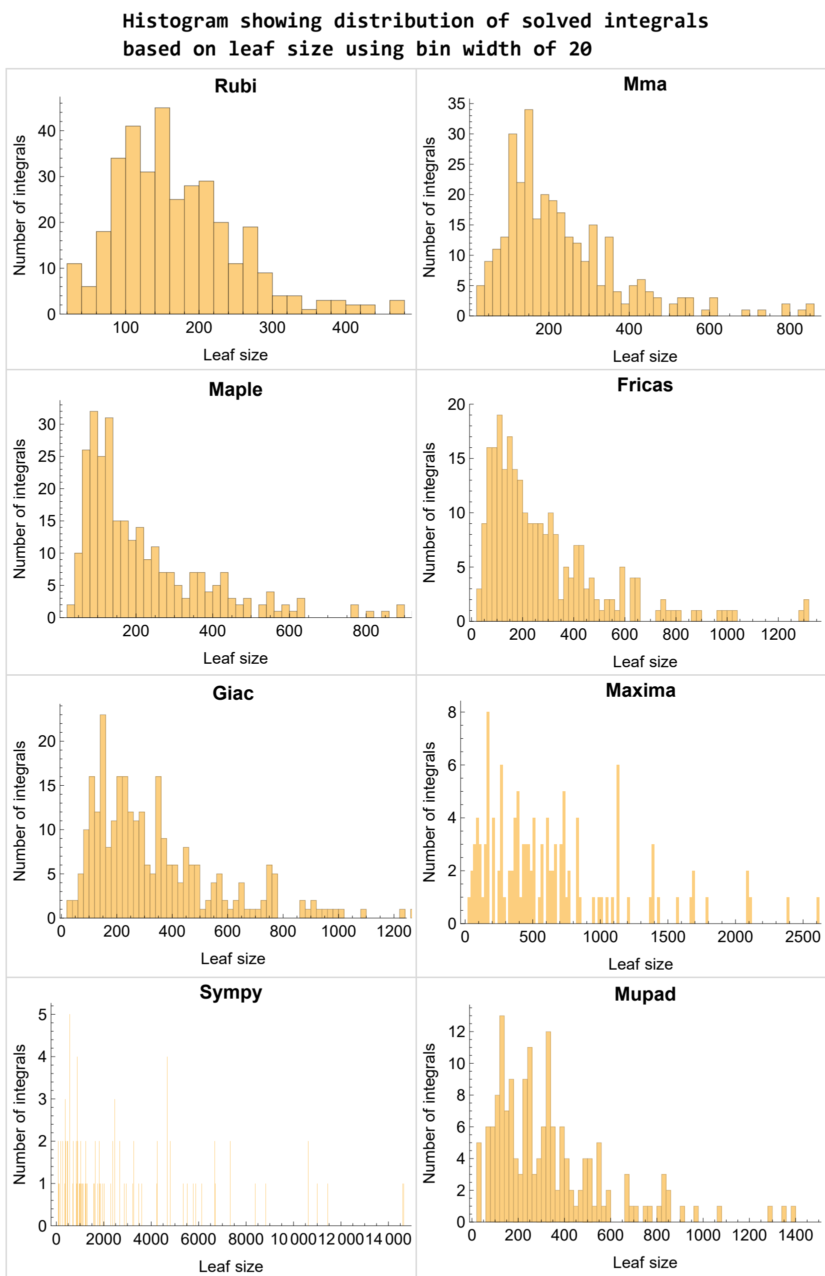


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

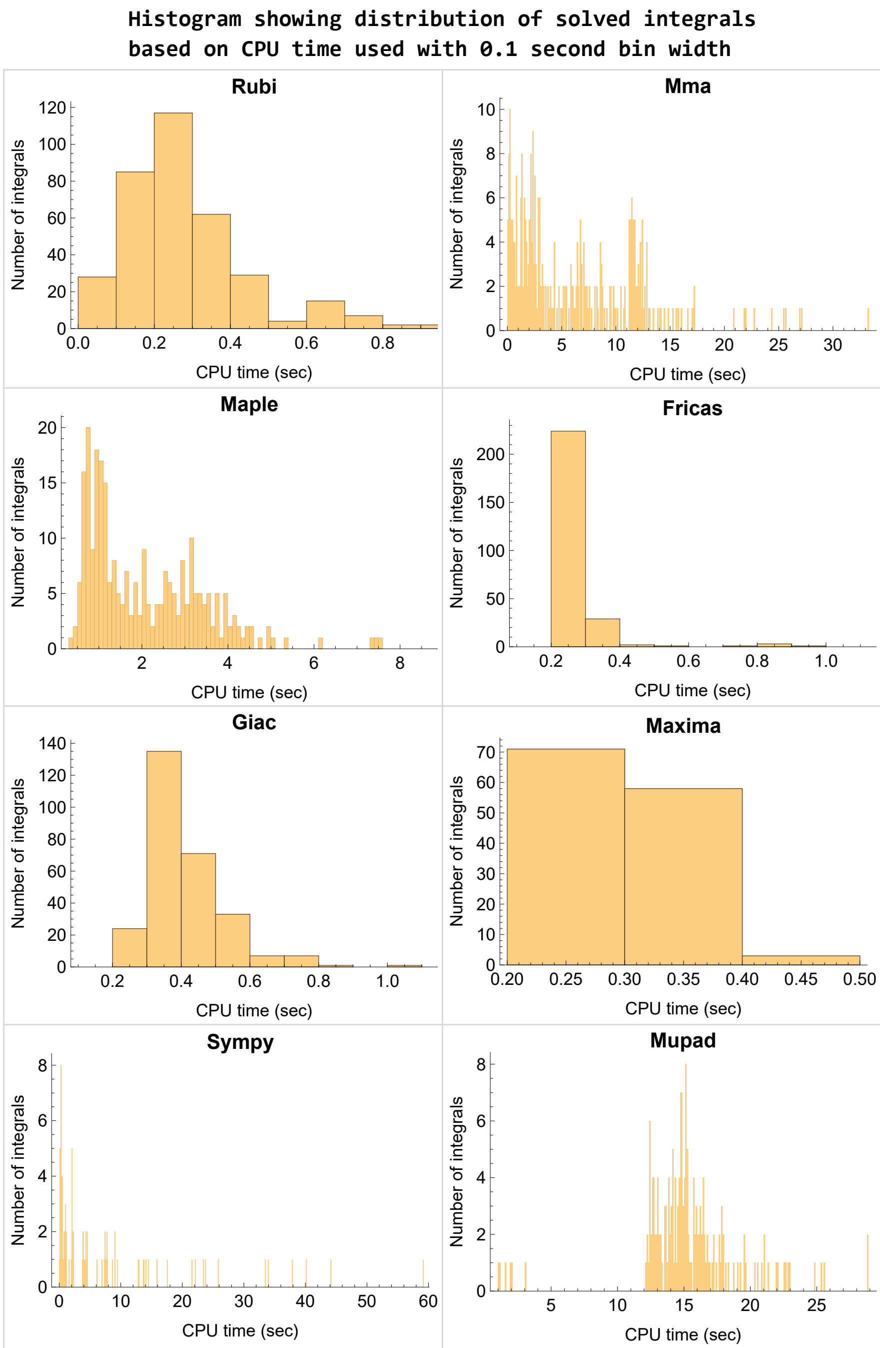


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

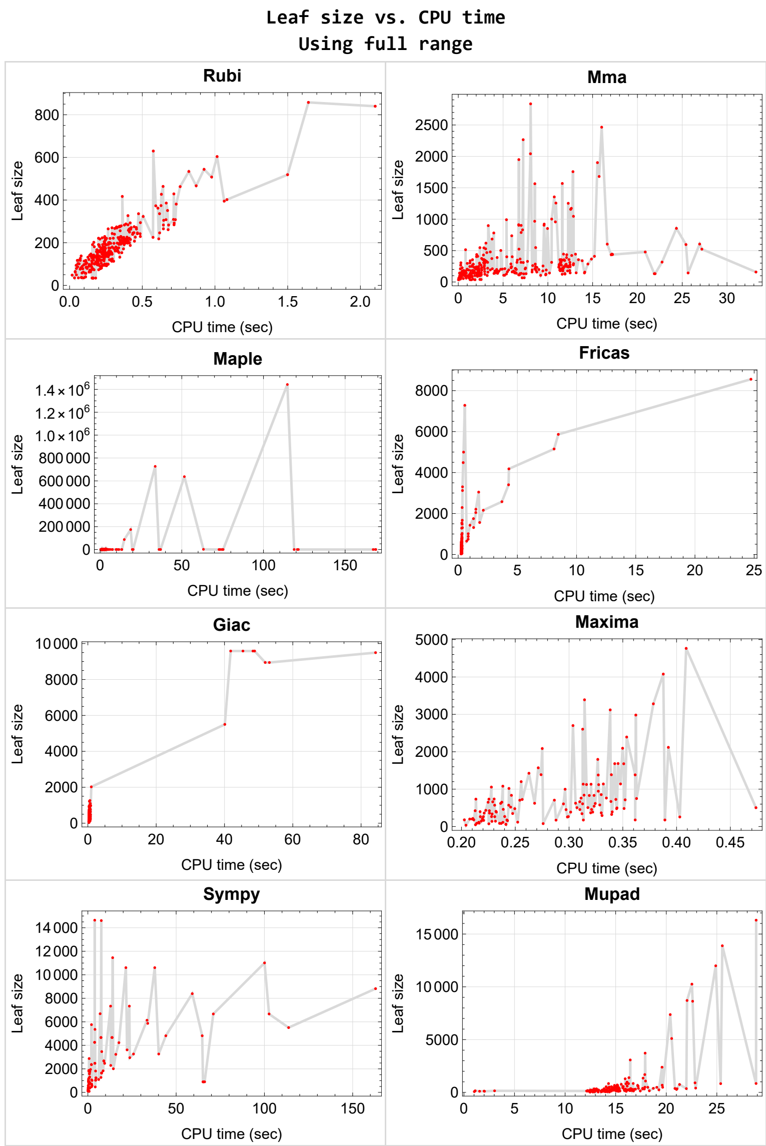


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{358\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{7, 8, 11, 196, 197, 306, 320, 326, 327, 334, 335, 337, 353, 354, 355, 356, 357\}$

Maple $\{353, 354, 355, 356, 357\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

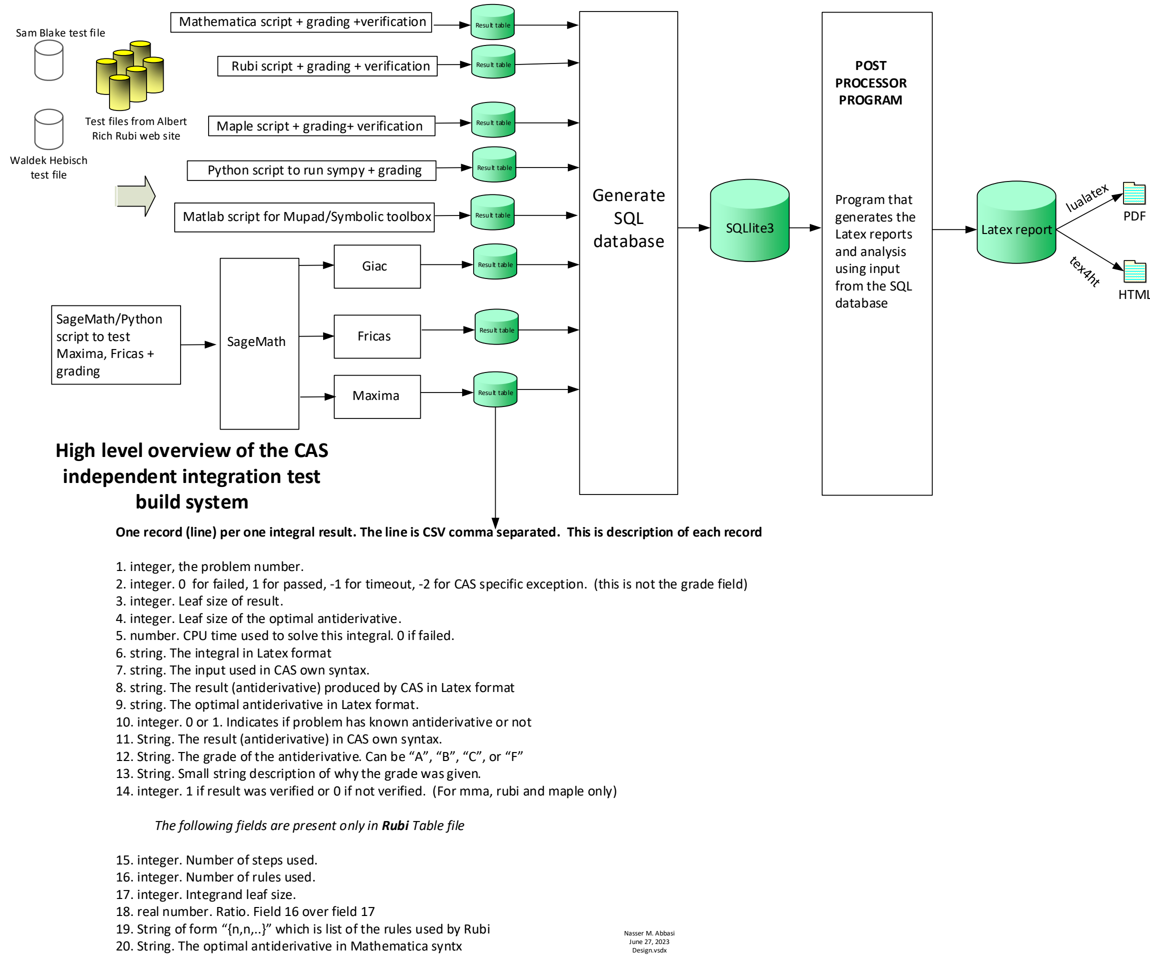
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 85, 86, 87, 88, 91, 92, 99, 101, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 236, 238, 239, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 301, 302, 303, 332, 334, 350, 351, 352 }

B grade { 8, 11, 14, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 84, 89, 90, 98, 100, 170, 171, 172, 232, 233, 234, 240, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 335, 353, 354, 355, 356, 357 }

C grade { 9, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 196, 197, 198, 199, 214, 215, 235, 237, 248, 249, 250, 290, 291, 292, 297, 298, 299, 300, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 336, 337, 338 }

F normal fail { 12, 13, 195, 200, 201, 202, 216, 217, 328, 329, 330, 331, 333, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { 93 }

F(-2) exception fail { }

Maple

A grade { 16, 17, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 52, 53, 54, 55, 56, 60, 61, 62, 63, 70, 71, 72, 73, 74, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 169, 171, 172, 173, 177, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 193, 194, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 352 }

B grade { 48, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 135, 143, 153, 165, 168, 170, 174, 175, 176, 178, 179, 184, 192, 250, 257, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 353, 354, 355, 356, 357 }

C grade { 22, 25, 37, 51, 57, 58, 59, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 79, 80, 237, 239, 282 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351 }

B grade { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 357 }

F(-1) timedout fail { 353 }

F(-2) exception fail { }

Maxima

A grade { 17, 18, 20, 30, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261 }

B grade { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 128, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 321, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 121, 122, 129, 130, 173, 313, 319, 320, 326, 327 }

F(-2) exception fail { 16, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 352 }

Giac

A grade { 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 77, 81, 82, 83, 84, 89, 112, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 152, 153, 155, 156, 159, 163, 164, 166, 167, 168, 169, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 192, 193, 224, 225, 226, 227, 228, 230, 231, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 268, 269, 274, 275, 276, 277, 281, 282, 287, 288, 289, 290, 291, 294, 295, 296, 301, 303, 309 }

B grade { 14, 15, 16, 21, 34, 35, 36, 37, 48, 49, 50, 51, 55, 61, 67, 68, 76, 78, 79, 80, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 141, 146, 149, 150, 151, 157, 158, 160, 161, 162, 165, 170, 171, 178, 184, 218, 219, 220, 221, 222, 223, 229, 232, 234, 250, 256, 257, 262, 263, 264, 265, 267, 270, 271, 272, 273, 278, 279, 280, 283, 284, 285, 286, 292, 293, 297, 298, 299, 300, 302, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 195, 196, 197, 198, 199, 200, 201, 202, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 10, 11, 203, 204, 208, 334, 335, 350, 351 }

F(-2) exception fail { 154, 177, 185, 194, 209, 210, 316 }

Mupad

A grade { }

B grade { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 111, 118, 119, 125, 126, 127, 131, 132, 133, 134, 138, 139, 140, 141, 142, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 191, 205, 206, 207, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 350, 351, 352 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 129, 130, 135, 136, 137, 143, 144, 145, 146, 153, 154, 155, 156, 157, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 218, 219, 220, 221, 224, 225, 226, 227, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 251, 252, 253, 254, 258, 259, 260, 261, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 100, 101, 102, 110, 111, 112, 113, 119, 120, 133, 134, 135, 136, 137, 142, 143, 144, 145, 175, 176, 177, 178, 179, 182, 183, 184, 185, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 203, 204, 207, 208, 209, 211, 212, 213, 214, 216, 217, 222, 223, 228, 229, 230, 231, 240, 241, 242, 243, 286, 287, 288, 289, 293, 294, 295, 296, 302, 303, 307, 308, 309, 310, 315, 316, 317, 324, 333, 334, 335, 336, 337, 338, 343, 344, 345, 353, 354, 355, 356 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 16, 81, 87, 88, 89, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 180, 181, 186, 187, 188, 189, 193, 194, 202, 205, 206, 210, 215, 232, 233, 234, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 300, 301, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 339, 340, 341, 342, 346, 347, 348, 349, 350, 351, 352, 357 }

F(-2) exception fail { 358 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	373	373	248	0	0	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	1.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	204	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	0.978	0.000	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	145	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.664	0.000	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	159	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	0.502	0.000	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	220	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	2.578	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	288	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	5.871	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	336	336	596	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	25.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	478	0	0	0	0	0	0
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	20.872	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	409	0	0	0	0	0	0
N.S.	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	2.167	0.000	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	250	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	12.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	523	0	0	0	0	0	0
N.S.	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	27.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	41	0	9496	61
N.S.	1	1.00	2.89	0.00	0.00	1.11	0.00	256.65	1.65
time (sec)	N/A	0.081	5.069	0.000	0.000	0.288	0.000	84.241	12.925

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	41	0	5502	38
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	157.20	1.09
time (sec)	N/A	0.061	1.446	0.000	0.000	0.267	0.000	40.100	14.390

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	147	212	0	804	0	356	3718
N.S.	1	1.00	0.96	1.39	0.00	5.25	0.00	2.33	24.30
time (sec)	N/A	0.263	1.969	0.903	0.000	0.332	0.000	0.661	17.913

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	141	119	336	123	853	178	454
N.S.	1	1.00	0.77	0.65	1.85	0.68	4.69	0.98	2.49
time (sec)	N/A	0.206	1.354	2.016	0.228	0.262	0.442	0.313	15.295

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	123	98	200	102	486	140	389
N.S.	1	1.00	0.87	0.69	1.41	0.72	3.42	0.99	2.74
time (sec)	N/A	0.166	1.279	1.497	0.210	0.264	0.299	0.319	14.194

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	105	105	78	179	82	396	110	345
N.S.	1	1.08	1.08	0.80	1.85	0.85	4.08	1.13	3.56
time (sec)	N/A	0.117	0.804	1.261	0.211	0.263	0.204	0.318	13.867

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	50	73	43	138	55	122
N.S.	1	1.00	0.98	1.02	1.49	0.88	2.82	1.12	2.49
time (sec)	N/A	0.047	0.248	0.742	0.225	0.255	0.125	0.295	14.573

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	107	67	265	116	828	117	111
N.S.	1	1.00	1.91	1.20	4.73	2.07	14.79	2.09	1.98
time (sec)	N/A	0.112	5.167	0.592	0.299	0.266	1.027	0.306	12.744

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	160	80	456	162	700	87	132
N.S.	1	1.00	2.22	1.11	6.33	2.25	9.72	1.21	1.83
time (sec)	N/A	0.149	6.011	0.642	0.321	0.260	2.034	0.321	12.125

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	147	94	737	183	1035	131	172
N.S.	1	1.00	1.41	0.90	7.09	1.76	9.95	1.26	1.65
time (sec)	N/A	0.156	6.277	0.750	0.230	0.271	4.342	0.327	12.455

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	174	130	1080	251	1831	176	228
N.S.	1	1.00	1.23	0.92	7.61	1.77	12.89	1.24	1.61
time (sec)	N/A	0.190	6.165	0.914	0.238	0.250	8.618	0.359	12.495

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	200	166	1425	305	3232	251	310
N.S.	1	1.00	1.14	0.94	8.10	1.73	18.36	1.43	1.76
time (sec)	N/A	0.215	6.468	1.039	0.263	0.256	15.825	0.375	13.045

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	219	159	571	158	1586	270	661
N.S.	1	1.00	0.96	0.69	2.49	0.69	6.93	1.18	2.89
time (sec)	N/A	0.242	3.280	3.059	0.231	0.275	0.872	0.381	14.881

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	163	137	460	135	1210	237	553
N.S.	1	1.00	0.86	0.72	2.43	0.71	6.40	1.25	2.93
time (sec)	N/A	0.204	5.482	2.321	0.239	0.284	0.616	0.381	15.146

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	137	119	360	114	910	202	542
N.S.	1	1.00	0.93	0.81	2.45	0.78	6.19	1.37	3.69
time (sec)	N/A	0.146	7.029	2.126	0.226	0.279	0.441	0.357	14.575

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	54	78	164	75	372	113	238
N.S.	1	1.00	0.61	0.88	1.84	0.84	4.18	1.27	2.67
time (sec)	N/A	0.087	0.113	1.165	0.221	0.275	0.278	0.333	14.882

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	115	82	179	77	396	107	339
N.S.	1	1.00	1.17	0.84	1.83	0.79	4.04	1.09	3.46
time (sec)	N/A	0.101	0.696	1.186	0.203	0.257	0.201	0.319	13.880

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	191	92	624	177	2365	156	244
N.S.	1	1.00	1.63	0.79	5.33	1.51	20.21	1.33	2.09
time (sec)	N/A	0.192	9.819	0.602	0.306	0.274	1.977	0.334	15.250

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	238	107	839	237	2474	129	246
N.S.	1	1.00	2.18	0.98	7.70	2.17	22.70	1.18	2.26
time (sec)	N/A	0.192	11.230	0.769	0.314	0.269	3.927	0.320	15.255

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	278	126	1139	277	1647	151	233
N.S.	1	1.00	2.48	1.12	10.17	2.47	14.71	1.35	2.08
time (sec)	N/A	0.187	11.275	0.769	0.332	0.260	7.776	0.322	15.853

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	191	133	1571	263	2008	217	269
N.S.	1	1.00	2.55	1.77	20.95	3.51	26.77	2.89	3.59
time (sec)	N/A	0.155	8.251	1.024	0.271	0.260	14.429	0.343	14.132

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	261	171	2087	335	3262	285	331
N.S.	1	1.00	2.27	1.49	18.15	2.91	28.37	2.48	2.88
time (sec)	N/A	0.196	8.308	0.987	0.275	0.260	25.838	0.354	15.177

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	285	219	2604	407	4816	353	423
N.S.	1	1.00	1.83	1.40	16.69	2.61	30.87	2.26	2.71
time (sec)	N/A	0.230	8.592	1.491	0.313	0.254	44.161	0.356	15.724

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	313	248	3120	475	6669	421	500
N.S.	1	1.00	1.59	1.26	15.84	2.41	33.85	2.14	2.54
time (sec)	N/A	0.283	10.419	1.562	0.338	0.273	71.059	0.403	14.894

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	255	189	661	181	1948	337	812
N.S.	1	1.00	0.96	0.71	2.49	0.68	7.35	1.27	3.06
time (sec)	N/A	0.265	9.459	4.209	0.229	0.292	1.514	0.376	16.014

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	232	164	617	158	1753	292	705
N.S.	1	1.00	1.05	0.74	2.78	0.71	7.90	1.32	3.18
time (sec)	N/A	0.244	8.776	2.642	0.324	0.301	1.143	0.370	15.153

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	209	149	571	137	1579	265	661
N.S.	1	1.00	1.15	0.82	3.15	0.76	8.72	1.46	3.65
time (sec)	N/A	0.170	7.467	2.940	0.244	0.278	0.874	0.356	15.200

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	64	102	264	92	682	155	325
N.S.	1	1.00	0.55	0.87	2.26	0.79	5.83	1.32	2.78
time (sec)	N/A	0.111	0.166	1.955	0.218	0.266	0.540	0.320	14.899

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	133	115	360	106	910	198	536
N.S.	1	1.00	0.96	0.83	2.61	0.77	6.59	1.43	3.88
time (sec)	N/A	0.147	6.886	2.080	0.221	0.273	0.452	0.321	14.056

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	141	97	200	100	486	140	390
N.S.	1	1.00	1.01	0.69	1.43	0.71	3.47	1.00	2.79
time (sec)	N/A	0.158	0.894	1.395	0.208	0.263	0.299	0.437	14.337

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	223	110	1139	218	4255	223	323
N.S.	1	1.00	1.43	0.71	7.30	1.40	27.28	1.43	2.07
time (sec)	N/A	0.222	11.644	0.655	0.348	0.260	3.789	0.384	14.853

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	280	166	1386	286	4665	222	341
N.S.	1	1.00	1.72	1.02	8.50	1.75	28.62	1.36	2.09
time (sec)	N/A	0.243	11.291	0.788	0.339	0.261	7.398	0.435	14.918

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	316	157	1685	337	4665	215	336
N.S.	1	1.00	2.07	1.03	11.01	2.20	30.49	1.41	2.20
time (sec)	N/A	0.247	11.412	0.918	0.346	0.262	13.602	0.440	14.757

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	356	177	2118	363	2951	202	316
N.S.	1	1.00	2.36	1.17	14.03	2.40	19.54	1.34	2.09
time (sec)	N/A	0.236	11.472	1.033	0.392	0.266	23.735	0.499	16.544

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	283	173	2701	331	3262	285	346
N.S.	1	1.00	3.68	2.25	35.08	4.30	42.36	3.70	4.49
time (sec)	N/A	0.162	12.042	1.164	0.304	0.263	40.133	0.502	14.760

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	313	211	3390	405	4816	353	408
N.S.	1	1.00	2.65	1.79	28.73	3.43	40.81	2.99	3.46
time (sec)	N/A	0.190	12.462	1.305	0.314	0.273	64.758	0.758	14.776

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	339	250	4078	475	6669	421	500
N.S.	1	1.00	2.17	1.60	26.14	3.04	42.75	2.70	3.21
time (sec)	N/A	0.240	13.834	1.690	0.388	0.266	102.643	0.432	14.357

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	365	298	4765	541	8821	489	577
N.S.	1	1.00	1.85	1.51	24.19	2.75	44.78	2.48	2.93
time (sec)	N/A	0.305	14.866	2.005	0.409	0.286	162.970	0.453	14.899

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	274	209	1796	261	6690	326	397
N.S.	1	1.00	1.44	1.10	9.45	1.37	35.21	1.72	2.09
time (sec)	N/A	0.255	12.479	0.808	0.327	0.275	6.941	0.323	15.293

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	220	110	1120	218	4255	224	319
N.S.	1	1.00	1.40	0.70	7.13	1.39	27.10	1.43	2.03
time (sec)	N/A	0.209	11.649	0.753	0.328	0.277	3.833	0.323	14.872

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	188	94	608	179	2365	157	241
N.S.	1	1.00	1.59	0.80	5.15	1.52	20.04	1.33	2.04
time (sec)	N/A	0.190	8.656	0.638	0.313	0.270	2.031	0.309	15.332

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	127	67	256	117	828	115	110
N.S.	1	1.00	2.23	1.18	4.49	2.05	14.53	2.02	1.93
time (sec)	N/A	0.108	5.372	0.602	0.403	0.261	1.063	0.463	13.592

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	35	28	83	39	39
N.S.	1	1.00	1.00	1.20	1.00	0.80	2.37	1.11	1.11
time (sec)	N/A	0.097	0.022	0.529	0.204	0.239	0.717	0.333	12.487

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	108	86	266	73	578	97	118
N.S.	1	1.00	1.71	1.37	4.22	1.16	9.17	1.54	1.87
time (sec)	N/A	0.144	1.279	0.713	0.228	0.256	2.234	0.351	12.206

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	157	111	423	107	1236	167	178
N.S.	1	1.00	1.54	1.09	4.15	1.05	12.12	1.64	1.75
time (sec)	N/A	0.197	1.640	0.997	0.226	0.249	4.554	0.319	12.237

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	240	136	619	141	2468	223	239
N.S.	1	1.00	1.69	0.96	4.36	0.99	17.38	1.57	1.68
time (sec)	N/A	0.235	2.022	0.964	0.236	0.253	9.496	0.324	13.141

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	354	252	2982	370	10608	391	500
N.S.	1	1.00	1.48	1.05	12.42	1.54	44.20	1.63	2.08
time (sec)	N/A	0.309	11.800	1.368	0.362	0.276	21.523	0.367	15.165

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	311	193	2094	322	7337	349	414
N.S.	1	1.00	1.73	1.07	11.63	1.79	40.76	1.94	2.30
time (sec)	N/A	0.241	11.466	1.124	0.350	0.290	12.857	0.331	15.039

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	274	162	1378	291	4665	222	336
N.S.	1	1.00	1.69	1.00	8.51	1.80	28.80	1.37	2.07
time (sec)	N/A	0.230	11.303	0.940	0.327	0.268	7.416	0.315	14.789

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	234	107	833	242	2474	130	242
N.S.	1	1.00	2.17	0.99	7.71	2.24	22.91	1.20	2.24
time (sec)	N/A	0.190	11.225	0.739	0.323	0.260	3.979	0.329	14.634

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	156	81	452	166	702	87	133
N.S.	1	1.00	2.17	1.12	6.28	2.31	9.75	1.21	1.85
time (sec)	N/A	0.141	6.303	0.687	0.297	0.263	2.094	0.340	12.677

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	110	86	265	69	578	97	117
N.S.	1	1.00	1.77	1.39	4.27	1.11	9.32	1.56	1.89
time (sec)	N/A	0.134	1.266	0.664	0.220	0.249	2.225	0.333	12.714

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	53	70	47	41	469	82	82
N.S.	1	1.00	0.85	1.13	0.76	0.66	7.56	1.32	1.32
time (sec)	N/A	0.095	0.087	0.720	0.213	0.251	2.043	0.325	12.461

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	237	136	651	116	2674	221	183
N.S.	1	1.00	2.55	1.46	7.00	1.25	28.75	2.38	1.97
time (sec)	N/A	0.149	2.003	1.235	0.238	0.253	9.027	0.341	12.537

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	285	161	835	151	4228	277	197
N.S.	1	1.00	2.11	1.19	6.19	1.12	31.32	2.05	1.46
time (sec)	N/A	0.197	2.395	1.780	0.246	0.255	17.577	0.462	12.666

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	329	184	998	187	5868	333	337
N.S.	1	1.00	1.88	1.05	5.70	1.07	33.53	1.90	1.93
time (sec)	N/A	0.246	3.147	2.380	0.296	0.253	33.910	0.529	13.105

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	388	240	3282	429	10608	357	501
N.S.	1	1.00	1.60	0.99	13.51	1.77	43.65	1.47	2.06
time (sec)	N/A	0.293	12.081	1.412	0.378	0.273	37.877	0.507	15.073

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	348	201	2394	392	7337	290	419
N.S.	1	1.00	1.73	1.00	11.91	1.95	36.50	1.44	2.08
time (sec)	N/A	0.275	11.630	1.102	0.354	0.269	23.489	0.509	15.723

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	308	157	1679	338	4665	215	333
N.S.	1	1.00	2.01	1.03	10.97	2.21	30.49	1.41	2.18
time (sec)	N/A	0.229	11.419	1.007	0.351	0.269	13.774	0.449	15.367

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	272	127	1134	279	1647	151	230
N.S.	1	1.00	2.47	1.15	10.31	2.54	14.97	1.37	2.09
time (sec)	N/A	0.181	11.252	0.783	0.313	0.280	7.716	0.437	16.202

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	139	92	733	191	1035	130	172
N.S.	1	1.00	1.35	0.89	7.12	1.85	10.05	1.26	1.67
time (sec)	N/A	0.154	6.473	0.762	0.213	0.261	4.419	0.655	13.632

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	156	111	423	106	1236	165	178
N.S.	1	1.00	1.53	1.09	4.15	1.04	12.12	1.62	1.75
time (sec)	N/A	0.172	1.684	0.933	0.213	0.259	4.536	0.328	12.985

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	237	136	650	108	2674	221	183
N.S.	1	1.00	2.63	1.51	7.22	1.20	29.71	2.46	2.03
time (sec)	N/A	0.148	1.991	1.174	0.225	0.268	9.003	0.466	13.083

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	83	60	56	1098	126	126
N.S.	1	1.00	0.77	0.99	0.71	0.67	13.07	1.50	1.50
time (sec)	N/A	0.114	0.138	1.166	0.241	0.266	6.104	0.353	15.425

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	325	186	1019	150	6135	333	217
N.S.	1	1.00	2.69	1.54	8.42	1.24	50.70	2.75	1.79
time (sec)	N/A	0.160	2.893	2.396	0.244	0.261	33.492	0.378	13.098

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	373	211	1201	185	8396	389	231
N.S.	1	1.00	2.30	1.30	7.41	1.14	51.83	2.40	1.43
time (sec)	N/A	0.202	3.746	3.192	0.255	0.261	59.132	0.366	13.689

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	401	236	1387	221	11011	445	474
N.S.	1	1.00	1.96	1.15	6.77	1.08	53.71	2.17	2.31
time (sec)	N/A	0.238	5.412	4.121	0.274	0.280	100.070	0.382	14.671

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	149	119	0	287	0	296	0
N.S.	1	1.00	0.75	0.60	0.00	1.45	0.00	1.49	0.00
time (sec)	N/A	0.362	4.164	7.458	0.000	0.273	0.000	0.457	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	123	103	0	243	0	262	0
N.S.	1	1.00	0.78	0.66	0.00	1.55	0.00	1.67	0.00
time (sec)	N/A	0.278	2.966	7.356	0.000	0.257	0.000	0.495	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	104	81	0	184	0	194	0
N.S.	1	1.00	0.90	0.70	0.00	1.59	0.00	1.67	0.00
time (sec)	N/A	0.211	2.451	1.601	0.000	0.272	0.000	0.374	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	191	63	0	130	0	118	0
N.S.	1	1.00	2.62	0.86	0.00	1.78	0.00	1.62	0.00
time (sec)	N/A	0.169	1.153	1.698	0.000	0.254	0.000	0.383	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	166	159	0	254	0	300	0
N.S.	1	1.00	1.36	1.30	0.00	2.08	0.00	2.46	0.00
time (sec)	N/A	0.236	2.941	1.879	0.000	0.281	0.000	0.330	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	157	227	0	318	0	390	0
N.S.	1	1.00	1.37	1.97	0.00	2.77	0.00	3.39	0.00
time (sec)	N/A	0.237	3.312	1.687	0.000	0.268	0.000	0.362	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	199	267	0	394	0	419	0
N.S.	1	1.00	1.58	2.12	0.00	3.13	0.00	3.33	0.00
time (sec)	N/A	0.237	4.284	2.642	0.000	0.273	0.000	0.379	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	217	352	0	490	0	628	0
N.S.	1	1.00	1.33	2.16	0.00	3.01	0.00	3.85	0.00
time (sec)	N/A	0.271	6.370	2.974	0.000	0.282	0.000	0.448	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	1355	121	0	358	0	373	0
N.S.	1	1.00	6.45	0.58	0.00	1.70	0.00	1.78	0.00
time (sec)	N/A	0.389	10.716	36.968	0.000	0.272	0.000	0.526	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	1173	105	0	313	0	340	0
N.S.	1	1.00	7.02	0.63	0.00	1.87	0.00	2.04	0.00
time (sec)	N/A	0.312	12.620	36.990	0.000	0.279	0.000	0.620	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	106	83	0	228	0	238	0
N.S.	1	1.00	0.88	0.69	0.00	1.90	0.00	1.98	0.00
time (sec)	N/A	0.274	9.196	2.052	0.000	0.264	0.000	0.651	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	193	0	200	0
N.S.	1	1.00	1.10	0.80	0.00	2.38	0.00	2.47	0.00
time (sec)	N/A	0.219	0.822	1.367	0.000	0.269	0.000	0.540	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F	B	F	B	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	0	197	0	310	0	480	0
N.S.	1	1.00	0.00	1.22	0.00	1.93	0.00	2.98	0.00
time (sec)	N/A	0.314	0.000	2.360	0.000	0.285	0.000	0.478	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	355	282	0	385	0	566	0
N.S.	1	1.00	2.02	1.60	0.00	2.19	0.00	3.22	0.00
time (sec)	N/A	0.338	11.348	2.579	0.000	0.275	0.000	0.520	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	344	386	0	449	0	601	0
N.S.	1	1.00	1.97	2.21	0.00	2.57	0.00	3.43	0.00
time (sec)	N/A	0.328	11.511	3.296	0.000	0.283	0.000	0.675	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	342	354	0	521	0	658	0
N.S.	1	1.00	1.95	2.02	0.00	2.98	0.00	3.76	0.00
time (sec)	N/A	0.328	11.883	3.256	0.000	0.271	0.000	0.738	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	357	440	0	654	0	745	0
N.S.	1	1.00	1.61	1.98	0.00	2.95	0.00	3.36	0.00
time (sec)	N/A	0.361	12.355	3.725	0.000	0.287	0.000	0.538	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	1569	121	0	405	0	458	0
N.S.	1	1.00	7.47	0.58	0.00	1.93	0.00	2.18	0.00
time (sec)	N/A	0.378	11.611	167.160	0.000	0.280	0.000	0.561	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	143	105	0	334	0	372	0
N.S.	1	1.00	0.89	0.65	0.00	2.07	0.00	2.31	0.00
time (sec)	N/A	0.332	10.180	168.774	0.000	0.275	0.000	0.591	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	1157	83	0	287	0	294	0
N.S.	1	1.00	9.33	0.67	0.00	2.31	0.00	2.37	0.00
time (sec)	N/A	0.294	12.547	2.448	0.000	0.277	0.000	0.488	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	232	0	253	0
N.S.	1	1.00	1.10	0.80	0.00	2.86	0.00	3.12	0.00
time (sec)	N/A	0.216	1.127	2.439	0.000	0.268	0.000	0.443	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	193	233	0	353	0	647	0
N.S.	1	1.00	0.96	1.16	0.00	1.76	0.00	3.24	0.00
time (sec)	N/A	0.384	11.554	2.968	0.000	0.279	0.000	0.389	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	444	354	0	430	0	730	0
N.S.	1	1.00	2.04	1.62	0.00	1.97	0.00	3.35	0.00
time (sec)	N/A	0.393	11.747	3.461	0.000	0.289	0.000	0.429	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	434	434	0	505	0	778	0
N.S.	1	1.00	1.93	1.93	0.00	2.24	0.00	3.46	0.00
time (sec)	N/A	0.373	12.106	3.984	0.000	0.285	0.000	0.449	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	422	524	0	554	0	719	0
N.S.	1	1.00	1.94	2.41	0.00	2.55	0.00	3.31	0.00
time (sec)	N/A	0.374	12.664	4.362	0.000	0.287	0.000	0.522	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	355	432	0	633	0	928	0
N.S.	1	1.00	1.64	1.99	0.00	2.92	0.00	4.28	0.00
time (sec)	N/A	0.386	13.414	4.518	0.000	0.296	0.000	0.492	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	409	526	0	760	0	990	0
N.S.	1	1.00	1.54	1.98	0.00	2.86	0.00	3.72	0.00
time (sec)	N/A	0.414	15.203	5.053	0.000	0.290	0.000	0.562	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	157	111	478	115	0	777	0
N.S.	1	1.00	0.78	0.56	2.39	0.58	0.00	3.88	0.00
time (sec)	N/A	0.268	14.124	3.194	0.344	0.275	0.000	0.515	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	134	95	386	95	0	573	0
N.S.	1	1.00	0.84	0.60	2.43	0.60	0.00	3.60	0.00
time (sec)	N/A	0.241	11.898	3.150	0.322	0.254	0.000	0.443	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	113	73	294	67	0	353	0
N.S.	1	1.00	0.96	0.62	2.49	0.57	0.00	2.99	0.00
time (sec)	N/A	0.221	9.755	0.826	0.331	0.262	0.000	0.420	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	174	44	0	175	128
N.S.	1	1.00	0.60	0.73	2.38	0.60	0.00	2.40	1.75
time (sec)	N/A	0.200	1.632	0.648	0.313	0.246	0.000	0.351	15.139

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	140	130	0	162	0	148	0
N.S.	1	1.00	1.54	1.43	0.00	1.78	0.00	1.63	0.00
time (sec)	N/A	0.208	1.353	0.875	0.000	0.281	0.000	0.348	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	284	225	0	231	0	405	0
N.S.	1	1.00	2.09	1.65	0.00	1.70	0.00	2.98	0.00
time (sec)	N/A	0.244	1.682	0.832	0.000	0.264	0.000	0.358	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	404	350	0	282	0	504	0
N.S.	1	1.00	2.24	1.94	0.00	1.57	0.00	2.80	0.00
time (sec)	N/A	0.293	2.038	1.014	0.000	0.270	0.000	0.618	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	762	153	0	978	0
N.S.	1	1.00	0.73	0.59	3.15	0.63	0.00	4.04	0.00
time (sec)	N/A	0.485	12.204	20.203	0.335	0.272	0.000	0.836	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	159	121	670	133	0	774	0
N.S.	1	1.00	0.79	0.60	3.33	0.66	0.00	3.85	0.00
time (sec)	N/A	0.399	8.236	20.181	0.339	0.263	0.000	0.747	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	130	105	577	110	0	293	0
N.S.	1	1.00	0.84	0.68	3.75	0.71	0.00	1.90	0.00
time (sec)	N/A	0.339	7.308	19.675	0.329	0.273	0.000	0.611	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	113	81	482	80	0	283	492
N.S.	1	1.00	0.98	0.70	4.19	0.70	0.00	2.46	4.28
time (sec)	N/A	0.287	6.740	0.796	0.351	0.251	0.000	0.545	17.888

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	63	343	60	0	223	137
N.S.	1	1.00	1.12	0.81	4.40	0.77	0.00	2.86	1.76
time (sec)	N/A	0.211	1.445	0.667	0.312	0.269	0.000	0.736	17.432

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	176	168	0	217	0	259	0
N.S.	1	1.00	1.30	1.24	0.00	1.61	0.00	1.92	0.00
time (sec)	N/A	0.250	1.764	0.855	0.000	0.280	0.000	0.410	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	300	280	0	206	0	499	0
N.S.	1	1.00	1.71	1.60	0.00	1.18	0.00	2.85	0.00
time (sec)	N/A	0.280	2.367	0.918	0.000	0.271	0.000	0.446	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	430	426	0	279	0	750	0
N.S.	1	1.00	1.91	1.89	0.00	1.24	0.00	3.33	0.00
time (sec)	N/A	0.346	2.807	0.918	0.000	0.279	0.000	0.574	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	945	166	0	395	0
N.S.	1	1.00	0.73	0.59	3.90	0.69	0.00	1.63	0.00
time (sec)	N/A	0.467	9.439	120.732	0.327	0.282	0.000	0.556	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	158	121	854	148	0	774	0
N.S.	1	1.00	0.76	0.58	4.09	0.71	0.00	3.70	0.00
time (sec)	N/A	0.403	8.766	121.349	0.330	0.265	0.000	0.560	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	132	105	761	125	0	449	904
N.S.	1	1.00	0.82	0.66	4.76	0.78	0.00	2.81	5.65
time (sec)	N/A	0.328	7.568	118.777	0.311	0.264	0.000	0.511	22.845

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	113	83	663	95	0	373	683
N.S.	1	1.00	0.93	0.69	5.48	0.79	0.00	3.08	5.64
time (sec)	N/A	0.284	6.828	0.687	0.310	0.262	0.000	0.449	19.599

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	89	65	505	77	0	368	479
N.S.	1	1.00	1.05	0.76	5.94	0.91	0.00	4.33	5.64
time (sec)	N/A	0.218	2.143	0.643	0.307	0.265	0.000	0.413	18.177

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	204	200	0	262	0	450	0
N.S.	1	1.00	1.17	1.15	0.00	1.51	0.00	2.59	0.00
time (sec)	N/A	0.303	2.003	0.781	0.000	0.272	0.000	0.456	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	357	308	0	277	0	650	0
N.S.	1	1.00	1.59	1.38	0.00	1.24	0.00	2.90	0.00
time (sec)	N/A	0.334	2.860	1.094	0.000	0.282	0.000	0.464	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	479	410	0	238	0	901	0
N.S.	1	1.00	1.86	1.59	0.00	0.92	0.00	3.49	0.00
time (sec)	N/A	0.405	3.571	0.990	0.000	0.288	0.000	0.561	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	118	131	0	140	0	150	173
N.S.	1	1.00	1.26	1.39	0.00	1.49	0.00	1.60	1.84
time (sec)	N/A	0.235	2.390	3.704	0.000	0.273	0.000	0.369	16.389

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	102	95	0	121	0	150	149
N.S.	1	1.00	1.09	1.01	0.00	1.29	0.00	1.60	1.59
time (sec)	N/A	0.241	2.106	3.587	0.000	0.273	0.000	0.380	15.011

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	71	0	92	0	144	122
N.S.	1	1.00	0.89	0.76	0.00	0.98	0.00	1.53	1.30
time (sec)	N/A	0.246	1.551	3.129	0.000	0.283	0.000	0.385	1.999

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	57	49	0	61	0	140	75
N.S.	1	1.00	0.62	0.53	0.00	0.66	0.00	1.52	0.82
time (sec)	N/A	0.224	0.096	2.884	0.000	0.266	0.000	0.363	1.030

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	120	300	175	0	0	141	0
N.S.	1	1.00	1.20	3.00	1.75	0.00	0.00	1.41	0.00
time (sec)	N/A	0.242	2.364	2.526	0.389	0.000	0.000	0.377	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	147	139	0	0	0	112	0
N.S.	1	1.00	1.48	1.40	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.266	2.570	2.761	0.000	0.000	0.000	0.375	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	101	71	0	87	0	112	0
N.S.	1	1.00	1.10	0.77	0.00	0.95	0.00	1.22	0.00
time (sec)	N/A	0.254	2.384	3.413	0.000	0.276	0.000	0.450	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	103	103	0	106	0	112	153
N.S.	1	1.00	1.10	1.10	0.00	1.13	0.00	1.19	1.63
time (sec)	N/A	0.248	3.226	3.555	0.000	0.266	0.000	0.433	18.328

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	205	135	0	148	0	247	323
N.S.	1	1.00	1.40	0.92	0.00	1.01	0.00	1.69	2.21
time (sec)	N/A	0.260	4.913	3.311	0.000	0.294	0.000	0.500	17.672

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	172	132	0	126	0	247	174
N.S.	1	1.00	1.18	0.90	0.00	0.86	0.00	1.69	1.19
time (sec)	N/A	0.257	4.539	3.198	0.000	0.284	0.000	0.497	16.597

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	99	80	0	83	0	237	103
N.S.	1	1.00	0.74	0.60	0.00	0.62	0.00	1.77	0.77
time (sec)	N/A	0.266	1.358	2.838	0.000	0.270	0.000	0.497	2.030

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	81	71	0	87	0	144	122
N.S.	1	1.00	0.84	0.74	0.00	0.91	0.00	1.50	1.27
time (sec)	N/A	0.233	1.394	3.026	0.000	0.267	0.000	0.431	14.136

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	136	424	0	0	0	243	0
N.S.	1	1.00	0.94	2.92	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.257	3.985	2.665	0.000	0.000	0.000	0.745	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	210	237	366	0	0	225	0
N.S.	1	1.00	1.33	1.50	2.32	0.00	0.00	1.42	0.00
time (sec)	N/A	0.273	8.556	2.676	0.330	0.000	0.000	0.398	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	198	216	0	0	0	205	0
N.S.	1	1.00	1.33	1.45	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.280	9.000	2.941	0.000	0.000	0.000	0.405	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	125	104	0	125	0	183	0
N.S.	1	1.00	1.30	1.08	0.00	1.30	0.00	1.91	0.00
time (sec)	N/A	0.194	9.781	3.348	0.000	0.270	0.000	0.415	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	123	127	0	134	0	183	245
N.S.	1	1.00	0.84	0.87	0.00	0.92	0.00	1.25	1.68
time (sec)	N/A	0.281	11.599	3.974	0.000	0.272	0.000	0.419	19.273

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	126	188	0	155	0	183	279
N.S.	1	1.00	0.82	1.22	0.00	1.01	0.00	1.19	1.81
time (sec)	N/A	0.263	11.913	4.242	0.000	0.282	0.000	0.407	21.015

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	223	200	0	160	0	355	383
N.S.	1	1.00	1.13	1.01	0.00	0.81	0.00	1.79	1.93
time (sec)	N/A	0.328	7.513	73.458	0.000	0.308	0.000	0.492	18.721

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	116	112	0	117	0	355	131
N.S.	1	1.00	0.64	0.62	0.00	0.65	0.00	1.97	0.73
time (sec)	N/A	0.318	2.561	72.761	0.000	0.274	0.000	0.436	16.763

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	165	132	0	119	0	341	174
N.S.	1	1.00	1.16	0.93	0.00	0.84	0.00	2.40	1.23
time (sec)	N/A	0.268	4.371	3.614	0.000	0.291	0.000	0.437	16.439

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	102	95	0	117	0	150	149
N.S.	1	1.00	1.06	0.99	0.00	1.22	0.00	1.56	1.55
time (sec)	N/A	0.235	1.861	3.399	0.000	0.274	0.000	0.427	3.030

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	177	493	0	0	0	303	0
N.S.	1	1.00	0.92	2.55	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	0.352	11.680	3.761	0.000	0.000	0.000	0.467	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	231	262	0	0	0	0	0
N.S.	1	1.00	1.10	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	11.768	3.568	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	207	368	506	0	0	297	0
N.S.	1	1.00	0.98	1.74	2.39	0.00	0.00	1.40	0.00
time (sec)	N/A	0.372	11.412	3.536	0.318	0.000	0.000	0.419	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	204	320	0	0	0	276	0
N.S.	1	1.00	1.04	1.63	0.00	0.00	0.00	1.41	0.00
time (sec)	N/A	0.352	11.384	4.024	0.000	0.000	0.000	0.421	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	145	150	0	165	0	268	0
N.S.	1	1.00	1.51	1.56	0.00	1.72	0.00	2.79	0.00
time (sec)	N/A	0.194	12.490	3.760	0.000	0.271	0.000	0.402	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	190	0	182	0	268	341
N.S.	1	1.00	1.00	1.30	0.00	1.25	0.00	1.84	2.34
time (sec)	N/A	0.269	13.159	3.471	0.000	0.275	0.000	0.439	21.066

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	144	217	0	196	0	268	357
N.S.	1	1.00	0.73	1.11	0.00	1.00	0.00	1.37	1.82
time (sec)	N/A	0.350	14.085	4.175	0.000	0.304	0.000	0.523	21.986

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	269	253	0	183	0	453	482
N.S.	1	1.00	1.08	1.01	0.00	0.73	0.00	1.81	1.93
time (sec)	N/A	0.412	11.512	74.961	0.000	0.303	0.000	0.549	19.642

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	138	140	0	134	0	453	384
N.S.	1	1.00	0.61	0.62	0.00	0.59	0.00	2.00	1.70
time (sec)	N/A	0.395	4.392	74.577	0.000	0.289	0.000	0.542	17.249

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	232	200	0	150	0	453	383
N.S.	1	1.00	1.21	1.04	0.00	0.78	0.00	2.36	1.99
time (sec)	N/A	0.340	5.897	75.322	0.000	0.295	0.000	0.563	17.662

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	212	135	0	142	0	247	321
N.S.	1	1.00	1.49	0.95	0.00	1.00	0.00	1.74	2.26
time (sec)	N/A	0.270	5.193	3.974	0.000	0.280	0.000	0.520	17.835

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	121	131	0	139	0	150	173
N.S.	1	1.00	1.26	1.36	0.00	1.45	0.00	1.56	1.80
time (sec)	N/A	0.245	2.107	2.992	0.000	0.278	0.000	0.431	15.871

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	183	549	0	0	0	469	0
N.S.	1	1.00	0.77	2.30	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	0.433	12.437	3.302	0.000	0.000	0.000	0.785	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	292	293	0	0	0	392	0
N.S.	1	1.00	1.08	1.08	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.459	12.864	3.754	0.000	0.000	0.000	0.464	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	251	406	0	0	0	413	0
N.S.	1	1.00	0.95	1.54	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	0.420	12.245	3.846	0.000	0.000	0.000	0.434	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	244	545	749	0	0	359	0
N.S.	1	1.00	0.92	2.06	2.84	0.00	0.00	1.36	0.00
time (sec)	N/A	0.460	12.492	4.484	0.363	0.000	0.000	0.457	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	238	409	0	0	0	338	0
N.S.	1	1.00	0.96	1.66	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.443	12.285	4.477	0.000	0.000	0.000	0.431	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	434	190	0	199	0	341	0
N.S.	1	1.00	4.52	1.98	0.00	2.07	0.00	3.55	0.00
time (sec)	N/A	0.201	17.068	4.772	0.000	0.297	0.000	0.418	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	442	206	0	214	0	341	406
N.S.	1	1.00	3.03	1.41	0.00	1.47	0.00	2.34	2.78
time (sec)	N/A	0.293	17.127	4.937	0.000	0.289	0.000	0.550	22.906

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	442	283	0	234	0	341	827
N.S.	1	1.00	2.19	1.40	0.00	1.16	0.00	1.69	4.09
time (sec)	N/A	0.370	17.202	4.968	0.000	0.300	0.000	0.506	25.378

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	436	309	0	243	0	341	841
N.S.	1	1.00	1.77	1.26	0.00	0.99	0.00	1.39	3.42
time (sec)	N/A	0.467	17.215	5.352	0.000	0.307	0.000	0.537	28.876

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	185	495	0	0	0	302	0
N.S.	1	1.00	0.94	2.51	0.00	0.00	0.00	1.53	0.00
time (sec)	N/A	0.342	11.561	4.173	0.000	0.000	0.000	0.462	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	426	0	0	0	247	0
N.S.	1	1.00	1.00	2.92	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.275	6.421	4.076	0.000	0.000	0.000	0.378	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	119	306	176	0	0	143	0
N.S.	1	1.00	1.24	3.19	1.83	0.00	0.00	1.49	0.00
time (sec)	N/A	0.252	2.144	3.391	0.362	0.000	0.000	0.385	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	83	131	0	0	0	0	0
N.S.	1	1.00	0.73	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.860	3.136	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	191	465	0	337	0	204	0
N.S.	1	1.00	1.85	4.51	0.00	3.27	0.00	1.98	0.00
time (sec)	N/A	0.197	2.334	3.121	0.000	0.293	0.000	0.357	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	222	773	0	424	0	229	0
N.S.	1	1.00	1.45	5.05	0.00	2.77	0.00	1.50	0.00
time (sec)	N/A	0.267	2.251	3.637	0.000	0.315	0.000	0.372	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	271	293	0	0	0	394	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.424	12.839	2.967	0.000	0.000	0.000	0.397	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	212	262	0	0	0	253	0
N.S.	1	1.00	1.01	1.25	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.347	11.711	3.234	0.000	0.000	0.000	0.363	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	190	238	367	0	0	226	0
N.S.	1	1.00	1.19	1.50	2.31	0.00	0.00	1.42	0.00
time (sec)	N/A	0.283	11.327	2.781	0.320	0.000	0.000	0.359	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	143	141	0	0	0	114	0
N.S.	1	1.00	1.43	1.41	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.256	2.366	2.536	0.000	0.000	0.000	0.354	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	186	460	0	329	0	183	0
N.S.	1	1.00	1.81	4.47	0.00	3.19	0.00	1.78	0.00
time (sec)	N/A	0.193	2.211	2.685	0.000	0.287	0.000	0.389	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	142	111	0	272	0	0	0
N.S.	1	1.00	0.95	0.74	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.268	1.755	2.787	0.000	0.335	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	306	361	0	437	0	281	0
N.S.	1	1.00	1.41	1.66	0.00	2.01	0.00	1.29	0.00
time (sec)	N/A	0.340	2.829	3.135	0.000	0.329	0.000	0.419	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	286	445	0	0	0	453	0
N.S.	1	1.00	0.89	1.38	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.506	14.471	3.908	0.000	0.000	0.000	0.456	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	243	407	0	0	0	414	0
N.S.	1	1.00	0.92	1.55	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	0.409	12.192	3.134	0.000	0.000	0.000	0.457	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	199	368	504	0	0	298	0
N.S.	1	1.00	0.94	1.74	2.39	0.00	0.00	1.41	0.00
time (sec)	N/A	0.349	11.414	3.069	0.474	0.000	0.000	0.479	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	179	214	0	0	0	206	0
N.S.	1	1.00	1.20	1.44	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.277	11.379	3.248	0.000	0.000	0.000	0.456	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	99	70	0	85	0	111	156
N.S.	1	1.00	1.05	0.74	0.00	0.90	0.00	1.18	1.66
time (sec)	N/A	0.247	2.136	2.547	0.000	0.272	0.000	0.466	15.092

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	214	772	0	408	0	211	0
N.S.	1	1.00	1.42	5.11	0.00	2.70	0.00	1.40	0.00
time (sec)	N/A	0.259	2.265	2.938	0.000	0.329	0.000	0.514	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	305	361	0	419	0	276	0
N.S.	1	1.00	1.47	1.74	0.00	2.01	0.00	1.33	0.00
time (sec)	N/A	0.350	2.986	2.717	0.000	0.311	0.000	0.503	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	192	136	0	306	0	0	0
N.S.	1	1.00	0.78	0.56	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.436	3.577	3.035	0.000	0.327	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	604	0	0	0	0	0	0
N.S.	1	1.00	4.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	26.946	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	443	0	0	0	0	0	0
N.S.	1	1.00	3.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	13.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	468	0	0	0	0	0	0
N.S.	1	1.00	3.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	3.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	101	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.304	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	132	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	21.883	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	132	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	21.974	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	271	0	725	562	0	0	749
N.S.	1	1.00	0.99	0.00	2.64	2.04	0.00	0.00	2.72
time (sec)	N/A	0.347	11.975	0.000	0.344	0.309	0.000	0.000	21.314

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	192	174	0	498	313	0	0	480
N.S.	1	1.16	1.05	0.00	3.00	1.89	0.00	0.00	2.89
time (sec)	N/A	0.248	4.090	0.000	0.345	0.299	0.000	0.000	19.073

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	323	165	0	0	105
N.S.	1	1.00	1.12	0.00	3.11	1.59	0.00	0.00	1.01
time (sec)	N/A	0.218	1.391	0.000	0.340	0.267	0.000	0.000	1.549

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	132	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	144	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	25.657	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	159	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	33.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	183	0	0	205	0	0	368
N.S.	1	1.00	0.69	0.00	0.00	0.77	0.00	0.00	1.38
time (sec)	N/A	0.311	0.608	0.000	0.000	0.277	0.000	0.000	20.859

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	141	0	0	137	0	0	239
N.S.	1	1.00	0.74	0.00	0.00	0.72	0.00	0.00	1.25
time (sec)	N/A	0.227	0.416	0.000	0.000	0.280	0.000	0.000	15.166

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	92	0	0	89	0	0	134
N.S.	1	1.00	0.81	0.00	0.00	0.78	0.00	0.00	1.18
time (sec)	N/A	0.152	0.327	0.000	0.000	0.277	0.000	0.000	14.180

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	310	0	0	0	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	7.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	402	0	0	0	0	0	0
N.S.	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	6.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	63	63	0	78	898	9586	64
N.S.	1	1.00	1.85	1.85	0.00	2.29	26.41	281.94	1.88
time (sec)	N/A	0.179	1.208	11.497	0.000	0.283	65.207	48.339	14.177

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	67	62	0	77	898	9587	61
N.S.	1	1.00	1.97	1.82	0.00	2.26	26.41	281.97	1.79
time (sec)	N/A	0.155	7.361	13.155	0.000	0.261	65.370	48.817	13.980

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	66	62	0	78	898	9586	61
N.S.	1	1.00	2.00	1.88	0.00	2.36	27.21	290.48	1.85
time (sec)	N/A	0.161	7.131	13.326	0.000	0.274	65.469	45.410	13.618

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	65	0	77	898	9587	64
N.S.	1	1.00	1.74	1.86	0.00	2.20	25.66	273.91	1.83
time (sec)	N/A	0.160	0.893	11.232	0.000	0.260	66.132	41.811	13.557

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	0	36	0	8947	36
N.S.	1	1.00	1.00	1.03	0.00	1.00	0.00	248.53	1.00
time (sec)	N/A	0.084	1.866	9.819	0.000	0.263	0.000	53.146	12.739

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	0	37	0	8948	37
N.S.	1	1.00	1.00	1.03	0.00	1.00	0.00	241.84	1.00
time (sec)	N/A	0.083	1.313	9.799	0.000	0.258	0.000	51.920	12.444

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	87	90	157	105	440	131	300
N.S.	1	1.00	0.62	0.64	1.12	0.75	3.14	0.94	2.14
time (sec)	N/A	0.141	0.370	2.191	0.243	0.254	0.481	0.305	14.600

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	77	79	138	91	359	113	256
N.S.	1	1.00	0.64	0.65	1.14	0.75	2.97	0.93	2.12
time (sec)	N/A	0.133	0.240	1.627	0.228	0.250	0.348	0.291	14.479

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	106	57	112	77	267	77	292
N.S.	1	1.00	1.10	0.59	1.17	0.80	2.78	0.80	3.04
time (sec)	N/A	0.097	0.723	1.359	0.239	0.250	0.249	0.287	14.730

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	57	86	63	196	77	250
N.S.	1	1.00	0.66	0.70	1.05	0.77	2.39	0.94	3.05
time (sec)	N/A	0.077	0.555	1.298	0.214	0.253	0.173	0.282	14.513

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	74	58	85	92	0	107	212
N.S.	1	1.00	0.97	0.76	1.12	1.21	0.00	1.41	2.79
time (sec)	N/A	0.073	0.242	0.962	0.235	0.262	0.000	0.296	12.643

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	80	83	111	0	153	226
N.S.	1	1.00	0.97	1.01	1.05	1.41	0.00	1.94	2.86
time (sec)	N/A	0.121	0.272	0.965	0.237	0.265	0.000	0.304	13.313

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	142	78	90	152	0	137	220
N.S.	1	1.00	1.82	1.00	1.15	1.95	0.00	1.76	2.82
time (sec)	N/A	0.091	0.135	1.210	0.216	0.300	0.000	0.308	13.776

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	141	99	117	175	0	150	245
N.S.	1	1.00	1.81	1.27	1.50	2.24	0.00	1.92	3.14
time (sec)	N/A	0.091	0.542	1.194	0.252	0.286	0.000	0.308	13.972

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	210	114	145	166	0	174	244
N.S.	1	1.00	2.44	1.33	1.69	1.93	0.00	2.02	2.84
time (sec)	N/A	0.115	0.233	1.284	0.218	0.285	0.000	0.322	13.202

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	268	121	175	201	0	174	244
N.S.	1	1.00	2.55	1.15	1.67	1.91	0.00	1.66	2.32
time (sec)	N/A	0.178	0.268	1.849	0.242	0.258	0.000	0.325	12.872

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	306	125	207	240	0	242	340
N.S.	1	1.00	2.35	0.96	1.59	1.85	0.00	1.86	2.62
time (sec)	N/A	0.152	0.290	1.886	0.219	0.281	0.000	0.320	12.866

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	211	154	715	248	3614	156	326
N.S.	1	1.00	1.64	1.19	5.54	1.92	28.02	1.21	2.53
time (sec)	N/A	0.166	3.802	1.044	0.352	0.264	22.200	0.319	16.441

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	155	115	543	225	2290	113	261
N.S.	1	1.00	1.50	1.12	5.27	2.18	22.23	1.10	2.53
time (sec)	N/A	0.158	2.187	0.934	0.323	0.269	12.943	0.312	16.239

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	112	85	392	204	1268	93	178
N.S.	1	1.00	1.26	0.96	4.40	2.29	14.25	1.04	2.00
time (sec)	N/A	0.133	2.119	0.660	0.340	0.269	7.409	0.316	14.394

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	107	62	348	156	461	63	110
N.S.	1	1.00	1.30	0.76	4.24	1.90	5.62	0.77	1.34
time (sec)	N/A	0.118	2.525	0.584	0.248	0.258	4.171	0.305	12.425

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	92	62	387	154	573	79	134
N.S.	1	1.00	1.59	1.07	6.67	2.66	9.88	1.36	2.31
time (sec)	N/A	0.086	1.549	0.558	0.232	0.247	2.323	0.303	12.336

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	313	95	433	310	0	99	199
N.S.	1	1.00	3.19	0.97	4.42	3.16	0.00	1.01	2.03
time (sec)	N/A	0.134	6.548	0.931	0.223	0.258	0.000	0.313	14.769

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	167	120	519	406	0	146	210
N.S.	1	1.00	1.48	1.06	4.59	3.59	0.00	1.29	1.86
time (sec)	N/A	0.303	2.327	1.125	0.246	0.276	0.000	0.321	15.918

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	245	148	622	498	0	180	288
N.S.	1	1.00	1.78	1.07	4.51	3.61	0.00	1.30	2.09
time (sec)	N/A	0.172	2.896	1.091	0.268	0.283	0.000	0.327	16.644

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	348	174	706	594	0	213	314
N.S.	1	1.00	2.27	1.14	4.61	3.88	0.00	1.39	2.05
time (sec)	N/A	0.226	6.595	1.170	0.286	0.269	0.000	0.324	15.980

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	267	244	406	239	996	309	830
N.S.	1	1.00	0.82	0.75	1.24	0.73	3.05	0.94	2.54
time (sec)	N/A	0.402	2.542	1.827	0.232	0.272	0.340	0.304	16.272

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	185	172	264	160	571	194	547
N.S.	1	1.00	0.87	0.81	1.24	0.75	2.68	0.91	2.57
time (sec)	N/A	0.246	1.636	1.125	0.231	0.269	0.228	0.301	15.703

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	104	94	143	84	277	98	134
N.S.	1	1.00	0.94	0.85	1.29	0.76	2.50	0.88	1.21
time (sec)	N/A	0.112	0.630	0.883	0.232	0.251	0.136	0.289	12.652

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	44	57	43	94	46	100
N.S.	1	1.00	0.94	0.92	1.19	0.90	1.96	0.96	2.08
time (sec)	N/A	0.018	0.143	0.438	0.241	0.258	0.095	0.281	12.550

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	196	120	0	292	5508	137	3074
N.S.	1	1.00	2.00	1.22	0.00	2.98	56.20	1.40	31.37
time (sec)	N/A	0.188	1.040	0.719	0.000	0.276	113.708	0.296	16.441

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	217	174	0	655	0	197	5102
N.S.	1	1.00	1.75	1.40	0.00	5.28	0.00	1.59	41.15
time (sec)	N/A	0.239	1.902	0.837	0.000	0.294	0.000	0.298	20.534

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	345	424	0	967	0	570	554
N.S.	1	1.00	1.96	2.41	0.00	5.49	0.00	3.24	3.15
time (sec)	N/A	0.269	2.991	1.100	0.000	0.304	0.000	0.327	15.752

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	464	437	324	724	364	1865	468	1291
N.S.	1	1.00	0.94	0.70	1.56	0.78	4.02	1.01	2.78
time (sec)	N/A	0.643	2.056	2.532	0.256	0.294	0.517	0.326	15.939

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	296	222	478	245	1129	306	765
N.S.	1	1.00	0.88	0.66	1.42	0.73	3.36	0.91	2.28
time (sec)	N/A	0.474	0.998	1.771	0.250	0.274	0.349	0.294	15.552

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	144	135	268	144	571	168	492
N.S.	1	1.00	0.87	0.81	1.61	0.87	3.44	1.01	2.96
time (sec)	N/A	0.189	0.426	1.158	0.240	0.261	0.229	0.284	14.991

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	106	70	114	70	199	85	91
N.S.	1	1.00	1.13	0.74	1.21	0.74	2.12	0.90	0.97
time (sec)	N/A	0.045	0.223	0.750	0.216	0.270	0.136	0.280	12.760

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	177	235	0	452	0	305	7371
N.S.	1	1.00	1.04	1.37	0.00	2.64	0.00	1.78	43.11
time (sec)	N/A	0.371	0.451	0.737	0.000	0.292	0.000	0.294	20.392

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	192	251	0	731	0	480	8706
N.S.	1	1.00	0.97	1.27	0.00	3.69	0.00	2.42	43.97
time (sec)	N/A	0.400	5.631	1.040	0.000	0.318	0.000	0.302	22.055

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	226	431	0	1483	0	678	8632
N.S.	1	1.00	1.05	2.00	0.00	6.90	0.00	3.15	40.15
time (sec)	N/A	0.419	5.895	1.020	0.000	0.331	0.000	0.328	22.607

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	604	528	384	1056	432	2878	559	1395
N.S.	1	1.00	0.87	0.64	1.75	0.72	4.76	0.93	2.31
time (sec)	N/A	1.014	3.053	2.718	0.228	0.298	0.716	0.335	16.311

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	355	268	704	299	1804	374	976
N.S.	1	1.00	0.77	0.58	1.52	0.65	3.90	0.81	2.11
time (sec)	N/A	0.760	1.589	2.096	0.254	0.288	0.504	0.316	16.123

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	156	167	398	178	960	212	550
N.S.	1	1.00	0.78	0.83	1.98	0.89	4.78	1.05	2.74
time (sec)	N/A	0.240	0.598	1.658	0.217	0.266	0.330	0.298	14.932

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	117	120	87	171	93	371	112	330
N.S.	1	0.92	0.94	0.69	1.35	0.73	2.92	0.88	2.60
time (sec)	N/A	0.074	0.344	1.086	0.288	0.268	0.199	0.279	14.608

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	233	378	0	627	0	597	10256
N.S.	1	1.00	0.95	1.54	0.00	2.55	0.00	2.43	41.69
time (sec)	N/A	0.619	1.764	1.041	0.000	0.300	0.000	0.307	22.535

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	244	406	0	1027	0	571	11993
N.S.	1	1.00	0.86	1.43	0.00	3.63	0.00	2.02	42.38
time (sec)	N/A	0.645	5.947	1.671	0.000	0.333	0.000	0.309	24.891

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	830	586	0	1670	0	953	13891
N.S.	1	1.00	2.72	1.92	0.00	5.48	0.00	3.12	45.54
time (sec)	N/A	0.647	7.176	2.056	0.000	0.343	0.000	0.345	25.541

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	788	337	1124	470	14644	460	839
N.S.	1	1.00	3.58	1.53	5.11	2.14	66.56	2.09	3.81
time (sec)	N/A	0.254	7.016	0.865	0.316	0.279	3.875	0.291	14.293

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	141	200	193	606	303	5763	214	297
N.S.	1	0.99	1.40	1.35	4.24	2.12	40.30	1.50	2.08
time (sec)	N/A	0.138	6.462	0.629	0.295	0.260	2.041	0.286	16.958

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	126	81	256	154	1307	151	122
N.S.	1	1.00	1.88	1.21	3.82	2.30	19.51	2.25	1.82
time (sec)	N/A	0.138	0.441	0.652	0.299	0.257	1.042	0.288	13.555

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	79	42	78	66	109	38	35
N.S.	1	1.00	2.26	1.20	2.23	1.89	3.11	1.09	1.00
time (sec)	N/A	0.034	0.134	0.346	0.276	0.258	0.557	0.294	13.269

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	148	94	0	595	0	110	154
N.S.	1	1.00	1.47	0.93	0.00	5.89	0.00	1.09	1.52
time (sec)	N/A	0.113	4.393	0.768	0.000	0.278	0.000	0.294	13.833

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	209	197	0	1538	0	425	437
N.S.	1	1.00	1.15	1.09	0.00	8.50	0.00	2.35	2.41
time (sec)	N/A	0.230	2.931	1.110	0.000	0.305	0.000	0.335	15.169

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	313	482	0	3303	0	727	1076
N.S.	1	1.00	1.11	1.70	0.00	11.67	0.00	2.57	3.80
time (sec)	N/A	0.377	3.278	2.488	0.000	0.366	0.000	0.382	17.950

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	547	340	1382	584	14612	472	663
N.S.	1	1.00	2.40	1.49	6.06	2.56	64.09	2.07	2.91
time (sec)	N/A	0.354	2.414	1.052	0.361	0.271	7.600	0.319	17.167

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	338	193	831	375	5358	264	365
N.S.	1	1.00	2.56	1.46	6.30	2.84	40.59	2.00	2.77
time (sec)	N/A	0.345	1.328	0.794	0.319	0.266	4.014	0.311	16.492

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	180	110	454	208	1062	133	94
N.S.	1	1.00	2.12	1.29	5.34	2.45	12.49	1.56	1.11
time (sec)	N/A	0.153	1.400	0.582	0.310	0.267	2.109	0.307	14.075

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	60	214	117	372	64	97
N.S.	1	1.00	0.66	0.92	3.29	1.80	5.72	0.98	1.49
time (sec)	N/A	0.039	0.041	0.472	0.211	0.249	1.121	0.290	13.836

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	229	159	0	1285	0	249	302
N.S.	1	1.00	1.51	1.05	0.00	8.45	0.00	1.64	1.99
time (sec)	N/A	0.290	2.632	1.104	0.000	0.296	0.000	0.310	14.739

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	313	263	0	3123	0	411	844
N.S.	1	1.00	1.14	0.96	0.00	11.36	0.00	1.49	3.07
time (sec)	N/A	0.444	7.005	1.867	0.000	0.363	0.000	0.337	16.731

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	1257	547	0	4997	0	911	1686
N.S.	1	1.00	3.26	1.42	0.00	12.95	0.00	2.36	4.37
time (sec)	N/A	0.665	10.899	4.518	0.000	0.453	0.000	0.371	17.877

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	366	349	1682	649	11456	567	593
N.S.	1	1.00	1.63	1.55	7.48	2.88	50.92	2.52	2.64
time (sec)	N/A	0.574	6.898	1.072	0.342	0.272	14.033	0.317	16.137

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	514	240	1132	432	3468	362	286
N.S.	1	1.00	3.13	1.46	6.90	2.63	21.15	2.21	1.74
time (sec)	N/A	0.330	0.812	0.734	0.320	0.261	7.853	0.312	16.842

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	176	139	733	271	1819	210	245
N.S.	1	1.00	1.39	1.09	5.77	2.13	14.32	1.65	1.93
time (sec)	N/A	0.157	1.718	0.655	0.226	0.251	4.366	0.315	14.016

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	95	387	190	1015	122	150
N.S.	1	1.00	0.62	0.93	3.79	1.86	9.95	1.20	1.47
time (sec)	N/A	0.058	0.058	0.565	0.302	0.239	2.371	0.291	13.020

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	502	252	0	2292	0	553	591
N.S.	1	1.00	2.19	1.10	0.00	10.01	0.00	2.41	2.58
time (sec)	N/A	0.487	4.747	1.516	0.000	0.333	0.000	0.317	17.222

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	1253	355	0	4486	0	743	1349
N.S.	1	1.00	3.29	0.93	0.00	11.77	0.00	1.95	3.54
time (sec)	N/A	0.733	12.275	2.897	0.000	0.421	0.000	0.347	17.708

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	548	639	0	7283	0	1224	2387
N.S.	1	1.00	1.08	1.26	0.00	14.34	0.00	2.41	4.70
time (sec)	N/A	0.977	8.637	6.115	0.000	0.559	0.000	0.433	19.582

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	305	242	0	467	0	550	0
N.S.	1	1.00	1.19	0.95	0.00	1.82	0.00	2.15	0.00
time (sec)	N/A	0.348	1.298	2.190	0.000	0.271	0.000	0.397	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	176	161	0	306	0	348	0
N.S.	1	1.00	0.92	0.84	0.00	1.59	0.00	1.81	0.00
time (sec)	N/A	0.246	0.841	1.763	0.000	0.277	0.000	0.345	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	117	102	0	175	0	186	0
N.S.	1	1.00	0.99	0.86	0.00	1.48	0.00	1.58	0.00
time (sec)	N/A	0.165	0.740	1.540	0.000	0.267	0.000	0.303	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	85	0	85	0
N.S.	1	1.00	1.32	0.94	0.00	1.37	0.00	1.37	0.00
time (sec)	N/A	0.039	0.147	0.931	0.000	0.272	0.000	0.291	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	651	0	125	0
N.S.	1	1.00	9.03	1.39	0.00	6.51	0.00	1.25	0.00
time (sec)	N/A	0.171	6.754	0.696	0.000	0.715	0.000	0.309	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	1012	0	212	0
N.S.	1	1.00	7.15	2.17	0.00	8.03	0.00	1.68	0.00
time (sec)	N/A	0.187	7.018	0.838	0.000	0.848	0.000	0.318	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	967	628	0	1750	0	422	0
N.S.	1	1.00	5.04	3.27	0.00	9.11	0.00	2.20	0.00
time (sec)	N/A	0.268	8.527	1.091	0.000	1.291	0.000	0.339	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	390	312	0	637	0	755	0
N.S.	1	1.00	1.04	0.83	0.00	1.70	0.00	2.02	0.00
time (sec)	N/A	0.629	4.337	2.475	0.000	0.296	0.000	0.454	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	267	207	0	430	0	497	0
N.S.	1	1.00	0.91	0.70	0.00	1.46	0.00	1.69	0.00
time (sec)	N/A	0.487	2.915	2.044	0.000	0.280	0.000	0.384	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	144	150	0	257	0	285	0
N.S.	1	1.00	0.87	0.91	0.00	1.56	0.00	1.73	0.00
time (sec)	N/A	0.234	2.348	1.821	0.000	0.269	0.000	0.326	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	137	0	139	0
N.S.	1	1.00	1.00	0.76	0.00	1.36	0.00	1.38	0.00
time (sec)	N/A	0.066	1.087	1.217	0.000	0.255	0.000	0.318	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	898	292	0	880	0	274	0
N.S.	1	1.00	5.87	1.91	0.00	5.75	0.00	1.79	0.00
time (sec)	N/A	0.359	3.347	1.058	0.000	0.848	0.000	0.309	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	922	592	0	1428	0	364	0
N.S.	1	1.00	4.83	3.10	0.00	7.48	0.00	1.91	0.00
time (sec)	N/A	0.375	9.590	0.967	0.000	0.996	0.000	0.327	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	957	896	0	2208	0	624	0
N.S.	1	1.00	4.33	4.05	0.00	9.99	0.00	2.82	0.00
time (sec)	N/A	0.410	10.840	1.344	0.000	1.494	0.000	0.356	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1565	374	0	863	0	1014	0
N.S.	1	1.00	2.93	0.70	0.00	1.62	0.00	1.90	0.00
time (sec)	N/A	0.820	8.546	168.510	0.000	0.308	0.000	0.553	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	328	257	0	593	0	684	0
N.S.	1	1.00	0.76	0.60	0.00	1.38	0.00	1.59	0.00
time (sec)	N/A	0.717	7.726	36.059	0.000	0.298	0.000	0.424	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	202	152	0	361	0	404	0
N.S.	1	1.00	0.95	0.72	0.00	1.70	0.00	1.91	0.00
time (sec)	N/A	0.246	4.732	7.572	0.000	0.269	0.000	0.360	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	191	0	202	0
N.S.	1	1.00	0.86	0.72	0.00	1.38	0.00	1.46	0.00
time (sec)	N/A	0.080	2.562	1.995	0.000	0.264	0.000	0.306	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	992	543	0	1314	0	523	0
N.S.	1	1.00	4.55	2.49	0.00	6.03	0.00	2.40	0.00
time (sec)	N/A	0.613	5.378	2.937	0.000	1.302	0.000	0.333	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	1002	932	0	2046	0	597	0
N.S.	1	1.00	3.78	3.52	0.00	7.72	0.00	2.25	0.00
time (sec)	N/A	0.645	10.427	10.175	0.000	1.487	0.000	0.361	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	1046	1587	0	3046	0	895	0
N.S.	1	1.00	3.40	5.15	0.00	9.89	0.00	2.91	0.00
time (sec)	N/A	0.663	12.857	63.259	0.000	1.731	0.000	0.408	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	375	560	0	629	0	555	0
N.S.	1	1.00	1.32	1.97	0.00	2.21	0.00	1.95	0.00
time (sec)	N/A	0.718	2.608	3.556	0.000	0.295	0.000	0.407	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	246	396	0	448	0	363	0
N.S.	1	1.00	1.23	1.98	0.00	2.24	0.00	1.82	0.00
time (sec)	N/A	0.412	2.263	2.617	0.000	0.272	0.000	0.350	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	135	232	0	303	0	221	0
N.S.	1	1.00	1.04	1.78	0.00	2.33	0.00	1.70	0.00
time (sec)	N/A	0.192	1.516	2.187	0.000	0.276	0.000	0.320	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	210	0	143	151
N.S.	1	1.00	1.34	1.62	0.00	2.66	0.00	1.81	1.91
time (sec)	N/A	0.047	0.281	2.230	0.000	0.261	0.000	0.293	1.114

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	619	199	0	744	0	252	0
N.S.	1	1.00	4.55	1.46	0.00	5.47	0.00	1.85	0.00
time (sec)	N/A	0.207	3.085	0.750	0.000	0.842	0.000	0.335	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	736	899	0	2159	0	492	0
N.S.	1	1.00	3.56	4.34	0.00	10.43	0.00	2.38	0.00
time (sec)	N/A	0.446	5.967	1.004	0.000	2.122	0.000	0.377	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	852	2275	0	4180	0	876	0
N.S.	1	1.00	2.76	7.36	0.00	13.53	0.00	2.83	0.00
time (sec)	N/A	0.715	9.961	1.589	0.000	4.283	0.000	0.479	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	684	817	0	784	0	650	0
N.S.	1	1.00	2.42	2.89	0.00	2.77	0.00	2.30	0.00
time (sec)	N/A	0.681	3.772	3.181	0.000	0.292	0.000	0.405	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	357	612	0	584	0	461	0
N.S.	1	1.00	1.76	3.01	0.00	2.88	0.00	2.27	0.00
time (sec)	N/A	0.391	2.731	2.592	0.000	0.275	0.000	0.347	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	246	389	0	407	0	0	0
N.S.	1	1.00	1.85	2.92	0.00	3.06	0.00	0.00	0.00
time (sec)	N/A	0.182	1.292	2.284	0.000	0.308	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	293	0	184	0
N.S.	1	1.00	1.72	2.02	0.00	3.37	0.00	2.11	0.00
time (sec)	N/A	0.056	0.325	1.464	0.000	0.270	0.000	0.310	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	781	624	0	1561	0	471	0
N.S.	1	1.00	4.18	3.34	0.00	8.35	0.00	2.52	0.00
time (sec)	N/A	0.417	3.963	0.984	0.000	1.814	0.000	0.357	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	904	2049	0	3403	0	866	0
N.S.	1	1.00	3.10	7.02	0.00	11.65	0.00	2.97	0.00
time (sec)	N/A	0.719	9.573	1.398	0.000	4.255	0.000	0.451	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	1757	4707	0	5864	0	1263	0
N.S.	1	1.00	4.37	11.71	0.00	14.59	0.00	3.14	0.00
time (sec)	N/A	1.082	12.801	2.070	0.000	8.457	0.000	0.598	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	523	1157	0	980	0	751	0
N.S.	1	1.00	1.70	3.76	0.00	3.18	0.00	2.44	0.00
time (sec)	N/A	0.728	6.775	3.994	0.000	0.302	0.000	0.554	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	544	852	0	744	0	530	0
N.S.	1	1.00	2.48	3.89	0.00	3.40	0.00	2.42	0.00
time (sec)	N/A	0.395	2.855	3.220	0.000	0.291	0.000	0.429	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	267	449	0	536	0	364	0
N.S.	1	1.00	1.77	2.97	0.00	3.55	0.00	2.41	0.00
time (sec)	N/A	0.192	1.577	2.586	0.000	0.273	0.000	0.353	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	392	0	236	0
N.S.	1	1.00	1.80	2.21	0.00	3.11	0.00	1.87	0.00
time (sec)	N/A	0.073	0.440	1.912	0.000	0.269	0.000	0.320	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	912	1418	0	2577	0	760	0
N.S.	1	1.00	3.49	5.43	0.00	9.87	0.00	2.91	0.00
time (sec)	N/A	0.678	6.737	1.319	0.000	3.693	0.000	0.425	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	1680	4092	0	5151	0	1099	0
N.S.	1	1.00	4.25	10.36	0.00	13.04	0.00	2.78	0.00
time (sec)	N/A	1.064	15.726	2.024	0.000	8.100	0.000	0.707	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	2465	7322	0	8555	0	2024	0
N.S.	1	1.00	4.75	14.11	0.00	16.48	0.00	3.90	0.00
time (sec)	N/A	1.499	16.006	3.448	0.000	24.729	0.000	1.001	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	427	245	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	11.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	244	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	8.626	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	603	0	0	0	0	0	0
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	16.631	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	351	854	0	0	0	0	0	0
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.674	24.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	199	198	317	0	0	0	0	0	0
N.S.	1	0.99	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	22.750	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	101	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.165	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	467	467	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.871	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	270	270	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	56	0	0	98
N.S.	1	1.00	1.00	0.00	0.00	1.44	0.00	0.00	2.51
time (sec)	N/A	0.109	2.555	0.000	0.000	0.317	0.000	0.000	16.044

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	57	0	0	99
N.S.	1	1.00	1.00	0.00	0.00	1.42	0.00	0.00	2.48
time (sec)	N/A	0.110	2.415	0.000	0.000	0.323	0.000	0.000	15.287

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	188	374	0	1308	0	749	16312
N.S.	1	1.00	0.94	1.88	0.00	6.57	0.00	3.76	81.97
time (sec)	N/A	0.386	2.631	1.453	0.000	0.356	0.000	0.497	28.885

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	840	840	2042	1442707	0	0	0	0	0
N.S.	1	1.00	2.43	1717.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.102	8.056	114.640	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	630	630	1901	637252	0	0	0	0	0
N.S.	1	1.00	3.02	1011.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.576	15.536	51.578	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	417	417	1949	87094	0	0	0	0	0
N.S.	1	1.00	4.67	208.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	6.761	14.801	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	544	544	2266	174462	0	0	0	0	0
N.S.	1	1.00	4.17	320.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.925	7.243	18.711	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	858	858	2837	726985	0	0	0	0	0
N.S.	1	1.00	3.31	847.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.642	8.074	33.723	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	37	0	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.06
time (sec)	N/A	0.054	14.642	1.000	29.145	0.543	0.000	3.788	18.233

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [11] had the largest ratio of [.2857000000000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	33	0.152
2	A	6	5	1.00	33	0.152
3	A	5	4	1.00	31	0.129
4	A	4	3	1.00	33	0.091
5	A	5	3	1.00	33	0.091
6	A	6	3	1.00	33	0.091
7	A	6	5	1.00	35	0.143
8	A	5	5	1.00	35	0.143
9	A	4	4	1.00	35	0.114
10	A	9	9	1.00	35	0.257
11	A	10	10	1.00	35	0.286
12	A	9	5	1.00	33	0.152
13	A	4	3	1.00	34	0.088
14	A	1	1	1.00	43	0.023
15	A	1	1	1.00	37	0.027
16	A	6	6	1.00	31	0.194
17	A	7	6	1.00	34	0.176
18	A	6	6	1.00	34	0.176
19	A	5	5	1.08	34	0.147
20	A	4	4	1.00	32	0.125
21	A	4	3	1.00	34	0.088
22	A	4	4	1.00	34	0.118
23	A	4	4	1.00	34	0.118
24	A	5	5	1.00	34	0.147

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	6	5	1.00	34	0.147
26	A	8	6	1.00	36	0.167
27	A	7	6	1.00	36	0.167
28	A	6	5	1.00	36	0.139
29	A	5	4	1.00	36	0.111
30	A	5	5	1.00	34	0.147
31	A	5	5	1.00	36	0.139
32	A	5	5	1.00	36	0.139
33	A	5	4	1.00	36	0.111
34	A	3	3	1.00	36	0.083
35	A	4	4	1.00	36	0.111
36	A	5	4	1.00	36	0.111
37	A	6	4	1.00	36	0.111
38	A	9	6	1.00	36	0.167
39	A	8	6	1.00	36	0.167
40	A	7	5	1.00	36	0.139
41	A	6	4	1.00	36	0.111
42	A	6	5	1.00	36	0.139
43	A	6	6	1.00	34	0.176
44	A	6	6	1.00	36	0.167
45	A	6	6	1.00	36	0.167
46	A	6	5	1.00	36	0.139
47	A	6	4	1.00	36	0.111
48	A	3	3	1.00	36	0.083
49	A	4	4	1.00	36	0.111
50	A	5	4	1.00	36	0.111
51	A	6	4	1.00	36	0.111
52	A	7	7	1.00	36	0.194
53	A	6	6	1.00	36	0.167
54	A	5	5	1.00	36	0.139
55	A	4	3	1.00	34	0.088
56	A	4	4	1.00	36	0.111
57	A	4	4	1.00	36	0.111
58	A	5	5	1.00	36	0.139
59	A	6	5	1.00	36	0.139

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	8	7	1.00	36	0.194
61	A	7	6	1.00	36	0.167
62	A	6	6	1.00	36	0.167
63	A	5	5	1.00	36	0.139
64	A	4	4	1.00	34	0.118
65	A	4	4	1.00	36	0.111
66	A	4	3	1.00	36	0.083
67	A	4	3	1.00	36	0.083
68	A	5	4	1.00	36	0.111
69	A	6	4	1.00	36	0.111
70	A	8	6	1.00	36	0.167
71	A	7	6	1.00	36	0.167
72	A	6	5	1.00	36	0.139
73	A	5	4	1.00	36	0.111
74	A	4	4	1.00	34	0.118
75	A	5	5	1.00	36	0.139
76	A	4	3	1.00	36	0.083
77	A	4	3	1.00	36	0.083
78	A	4	3	1.00	36	0.083
79	A	5	4	1.00	36	0.111
80	A	6	4	1.00	36	0.111
81	A	6	4	1.00	36	0.111
82	A	5	4	1.00	36	0.111
83	A	4	4	1.00	36	0.111
84	A	3	3	1.00	36	0.083
85	A	5	5	1.00	36	0.139
86	A	5	5	1.00	36	0.139
87	A	5	5	1.00	36	0.139
88	A	6	6	1.00	36	0.167
89	A	6	4	1.00	38	0.105
90	A	5	4	1.00	38	0.105
91	A	4	4	1.00	38	0.105
92	A	3	3	1.00	38	0.079
93	A	6	5	1.00	38	0.132
94	A	6	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	6	1.00	38	0.158
96	A	6	5	1.00	38	0.132
97	A	7	6	1.00	38	0.158
98	A	6	4	1.00	38	0.105
99	A	5	4	1.00	38	0.105
100	A	4	4	1.00	38	0.105
101	A	3	3	1.00	38	0.079
102	A	7	5	1.00	38	0.132
103	A	7	5	1.00	38	0.132
104	A	7	6	1.00	38	0.158
105	A	7	6	1.00	38	0.158
106	A	7	5	1.00	38	0.132
107	A	8	6	1.00	38	0.158
108	A	6	4	1.00	38	0.105
109	A	5	4	1.00	38	0.105
110	A	4	4	1.00	38	0.105
111	A	3	3	1.00	38	0.079
112	A	4	4	1.00	38	0.105
113	A	5	5	1.00	38	0.132
114	A	6	6	1.00	38	0.158
115	A	7	4	1.00	38	0.105
116	A	6	4	1.00	38	0.105
117	A	5	4	1.00	38	0.105
118	A	4	4	1.00	38	0.105
119	A	3	3	1.00	38	0.079
120	A	5	5	1.00	38	0.132
121	A	6	6	1.00	38	0.158
122	A	7	7	1.00	38	0.184
123	A	7	4	1.00	38	0.105
124	A	6	4	1.00	38	0.105
125	A	5	4	1.00	38	0.105
126	A	4	4	1.00	38	0.105
127	A	3	3	1.00	38	0.079
128	A	6	5	1.00	38	0.132
129	A	7	7	1.00	38	0.184

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	8	7	1.00	38	0.184
131	A	3	2	1.00	40	0.050
132	A	3	2	1.00	40	0.050
133	A	3	2	1.00	40	0.050
134	A	3	2	1.00	40	0.050
135	A	5	5	1.00	40	0.125
136	A	5	5	1.00	40	0.125
137	A	3	2	1.00	40	0.050
138	A	3	2	1.00	40	0.050
139	A	3	3	1.00	40	0.075
140	A	3	3	1.00	40	0.075
141	A	3	3	1.00	40	0.075
142	A	3	2	1.00	40	0.050
143	A	5	5	1.00	40	0.125
144	A	5	5	1.00	40	0.125
145	A	5	5	1.00	40	0.125
146	A	2	2	1.00	40	0.050
147	A	3	3	1.00	40	0.075
148	A	3	3	1.00	40	0.075
149	A	4	3	1.00	40	0.075
150	A	4	3	1.00	40	0.075
151	A	3	3	1.00	40	0.075
152	A	3	2	1.00	40	0.050
153	A	6	5	1.00	40	0.125
154	A	6	5	1.00	40	0.125
155	A	6	6	1.00	40	0.150
156	A	6	5	1.00	40	0.125
157	A	2	2	1.00	40	0.050
158	A	3	3	1.00	40	0.075
159	A	4	3	1.00	40	0.075
160	A	5	3	1.00	40	0.075
161	A	5	3	1.00	40	0.075
162	A	4	3	1.00	40	0.075
163	A	3	3	1.00	40	0.075
164	A	3	2	1.00	40	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	7	5	1.00	40	0.125
166	A	7	5	1.00	40	0.125
167	A	7	6	1.00	40	0.150
168	A	7	6	1.00	40	0.150
169	A	7	5	1.00	40	0.125
170	A	2	2	1.00	40	0.050
171	A	3	3	1.00	40	0.075
172	A	4	3	1.00	40	0.075
173	A	5	3	1.00	40	0.075
174	A	6	5	1.00	40	0.125
175	A	5	5	1.00	40	0.125
176	A	5	5	1.00	40	0.125
177	A	7	4	1.00	40	0.100
178	A	3	3	1.00	40	0.075
179	A	4	4	1.00	40	0.100
180	A	7	5	1.00	40	0.125
181	A	6	5	1.00	40	0.125
182	A	5	5	1.00	40	0.125
183	A	5	5	1.00	40	0.125
184	A	3	3	1.00	40	0.075
185	A	4	4	1.00	40	0.100
186	A	5	4	1.00	40	0.100
187	A	8	6	1.00	40	0.150
188	A	7	6	1.00	40	0.150
189	A	6	6	1.00	40	0.150
190	A	5	5	1.00	40	0.125
191	A	3	2	1.00	40	0.050
192	A	4	4	1.00	40	0.100
193	A	5	4	1.00	40	0.100
194	A	6	4	1.00	40	0.100
195	A	5	5	1.00	36	0.139
196	A	5	5	1.00	36	0.139
197	A	5	5	1.00	36	0.139
198	A	5	5	1.00	34	0.147
199	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	5	5	1.00	36	0.139
201	A	5	5	1.00	36	0.139
202	A	5	5	1.00	36	0.139
203	A	4	4	1.00	38	0.105
204	A	4	4	1.00	38	0.105
205	A	4	3	1.00	38	0.079
206	A	3	3	1.16	38	0.079
207	A	3	2	1.00	38	0.053
208	A	4	4	1.00	38	0.105
209	A	4	4	1.00	38	0.105
210	A	4	4	1.00	38	0.105
211	A	4	3	1.00	40	0.075
212	A	3	3	1.00	40	0.075
213	A	2	2	1.00	40	0.050
214	A	5	5	1.00	40	0.125
215	A	5	5	1.00	38	0.132
216	A	5	5	1.00	40	0.125
217	A	5	5	1.00	40	0.125
218	A	2	2	1.00	46	0.043
219	A	2	2	1.00	45	0.044
220	A	2	2	1.00	44	0.045
221	A	2	2	1.00	43	0.047
222	A	1	1	1.00	47	0.021
223	A	1	1	1.00	46	0.022
224	A	13	4	1.00	32	0.125
225	A	12	4	1.00	32	0.125
226	A	10	5	1.00	30	0.167
227	A	5	5	1.00	24	0.208
228	A	7	5	1.00	30	0.167
229	A	9	7	1.00	32	0.219
230	A	7	6	1.00	32	0.188
231	A	7	4	1.00	32	0.125
232	A	10	5	1.00	32	0.156
233	A	12	6	1.00	32	0.188
234	A	12	4	1.00	32	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
235	A	11	6	1.00	32	0.188
236	A	9	4	1.00	32	0.125
237	A	8	3	1.00	32	0.094
238	A	8	3	1.00	30	0.100
239	A	3	3	1.00	24	0.125
240	A	9	4	1.00	30	0.133
241	A	15	9	1.00	32	0.281
242	A	13	7	1.00	32	0.219
243	A	15	7	1.00	32	0.219
244	A	5	4	1.00	33	0.121
245	A	4	4	1.00	33	0.121
246	A	3	3	1.00	31	0.097
247	A	1	1	1.00	21	0.048
248	A	6	6	1.00	33	0.182
249	A	6	6	1.00	33	0.182
250	A	7	7	1.00	33	0.212
251	A	6	5	1.00	35	0.143
252	A	5	5	1.00	35	0.143
253	A	4	4	1.00	33	0.121
254	A	2	2	1.00	23	0.087
255	A	7	7	1.00	35	0.200
256	A	7	7	1.00	35	0.200
257	A	7	7	1.00	35	0.200
258	A	7	5	1.00	35	0.143
259	A	6	5	1.00	35	0.143
260	A	10	8	1.00	33	0.242
261	A	8	6	0.92	23	0.261
262	A	8	7	1.00	35	0.200
263	A	8	8	1.00	35	0.229
264	A	8	7	1.00	35	0.200
265	A	3	3	1.00	35	0.086
266	A	2	2	0.99	35	0.057
267	A	4	4	1.00	33	0.121
268	A	2	2	1.00	23	0.087
269	A	5	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	6	6	1.00	35	0.171
271	A	7	6	1.00	35	0.171
272	A	3	2	1.00	35	0.057
273	A	5	5	1.00	35	0.143
274	A	4	4	1.00	33	0.121
275	A	2	2	1.00	23	0.087
276	A	6	5	1.00	35	0.143
277	A	7	6	1.00	35	0.171
278	A	8	6	1.00	35	0.171
279	A	6	5	1.00	35	0.143
280	A	5	5	1.00	35	0.143
281	A	4	4	1.00	33	0.121
282	A	3	3	1.00	23	0.130
283	A	7	5	1.00	35	0.143
284	A	8	6	1.00	35	0.171
285	A	9	6	1.00	35	0.171
286	A	5	5	1.00	37	0.135
287	A	4	4	1.00	37	0.108
288	A	4	4	1.00	35	0.114
289	A	2	2	1.00	25	0.080
290	A	3	3	1.00	37	0.081
291	A	3	3	1.00	37	0.081
292	A	4	4	1.00	37	0.108
293	A	6	6	1.00	37	0.162
294	A	5	5	1.00	37	0.135
295	A	5	5	1.00	35	0.143
296	A	3	3	1.00	25	0.120
297	A	4	4	1.00	37	0.108
298	A	4	4	1.00	37	0.108
299	A	4	4	1.00	37	0.108
300	A	7	6	1.00	37	0.162
301	A	6	5	1.00	37	0.135
302	A	6	5	1.00	35	0.143
303	A	4	3	1.00	25	0.120
304	A	5	4	1.00	37	0.108

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	5	5	1.00	37	0.135
306	A	5	4	1.00	37	0.108
307	A	7	6	1.00	37	0.162
308	A	6	6	1.00	37	0.162
309	A	5	5	1.00	35	0.143
310	A	3	3	1.00	25	0.120
311	A	5	5	1.00	37	0.135
312	A	6	6	1.00	37	0.162
313	A	7	6	1.00	37	0.162
314	A	7	7	1.00	37	0.189
315	A	6	6	1.00	37	0.162
316	A	5	5	1.00	35	0.143
317	A	3	3	1.00	25	0.120
318	A	6	6	1.00	37	0.162
319	A	7	7	1.00	37	0.189
320	A	8	7	1.00	37	0.189
321	A	7	6	1.00	37	0.162
322	A	6	6	1.00	37	0.162
323	A	5	5	1.00	35	0.143
324	A	4	4	1.00	25	0.160
325	A	7	6	1.00	37	0.162
326	A	8	7	1.00	37	0.189
327	A	9	7	1.00	37	0.189
328	A	7	4	1.00	35	0.114
329	A	8	6	1.00	33	0.182
330	A	7	5	1.00	35	0.143
331	A	7	4	1.00	35	0.114
332	A	11	7	1.00	37	0.189
333	A	4	4	1.00	37	0.108
334	A	7	7	1.00	37	0.189
335	A	7	4	1.00	37	0.108
336	A	6	6	1.00	35	0.171
337	A	5	5	0.99	33	0.152
338	A	3	3	1.00	23	0.130
339	A	6	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	7	7	1.00	35	0.200
341	A	8	7	1.00	35	0.200
342	A	9	5	1.00	37	0.135
343	A	9	5	1.00	37	0.135
344	A	9	5	1.00	37	0.135
345	A	9	5	1.00	37	0.135
346	A	9	5	1.00	35	0.143
347	A	7	6	1.00	39	0.154
348	A	4	4	1.00	36	0.111
349	A	4	4	1.00	40	0.100
350	A	1	1	1.00	55	0.018
351	A	1	1	1.00	51	0.020
352	A	6	6	1.00	35	0.171
353	A	7	7	1.00	39	0.180
354	A	5	5	1.00	39	0.128
355	A	3	3	1.00	39	0.077
356	A	4	4	1.00	39	0.103
357	A	5	5	1.00	39	0.128
358	N/A	0	0	1.00	35	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 373

$$\begin{aligned}
&\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx \\
&= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)} \\
&\quad+\frac{a^3(B(15+19n+4n^2)+A(20+21n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{df(1+n)(2+n)(4+n)\sqrt{\cos^2(e+fx)}} \\
&\quad+\frac{a^3(B(9+4n)+A(11+4n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{}}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}} \\
&\quad-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)} \\
&\quad-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}
\end{aligned}$$

```
[Out] -a^3*(B*(2*n^2+14*n+27)+A*(2*n^2+15*n+28))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/
d/f/(4+n)/(n^2+5*n+6)-a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^
2/d/f/(4+n)-(A*(4+n)+B*(6+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^3+a^3*sin(
f*x+e))/d/f/(3+n)/(4+n)+a^3*(B*(4*n^2+19*n+15)+A*(4*n^2+21*n+20))*cos(f*x+e
)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)
/d/f/(4+n)/(n^2+3*n+2)/(cos(f*x+e)^2)^(1/2)+a^3*(B*(9+4*n)+A*(11+4*n))*cos(
f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n
)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3055, 3047, 3102, 2827, 2722}

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \frac{a^3(A(4n+11)+B(4n+9))\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx\right.}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a^3(A(4n^2+21n+20)+B(4n^2+19n+15))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+}{2}\right.}{df(n+1)(n+2)(n+4)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{a^3(A(2n^2+15n+28)+B(2n^2+14n+27))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)(n+3)(n+4)}$$
$$-\frac{(A(n+4)+B(n+6))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(d\sin(e+fx))^{n+1}}{df(n+3)(n+4)}$$
$$-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^3*(B*(27 + 14*n + 2*n^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[
e + f*x])^(1 + n))/(d*f*(2 + n)*(3 + n)*(4 + n))) + (a^3*(B*(15 + 19*n + 4*
n^2) + A*(20 + 21*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)
*(4 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e +
f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e
+ f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2]) - (a*B*Cos[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*(4 + n)) - ((A
*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^3 + a^3*Sin[
e + f*x]))/(d*f*(3 + n)*(4 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)} + \frac{\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(ad(B(1+n)+A(4+n))+ad(A(4+n)+B(6+n))\sin(e+fx)}{d(4+n)}$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)} - \frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)} + \frac{\int (d\sin(e+fx))^n(a+a\sin(e+fx))\left(a^2d^2(2A(8+6n+n^2)+B(9+11n+2n^2))+a^2d^2(B(27\right.}{d^2(3+n)(4+n)}$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)}$$
$$-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}$$
$$+\frac{\int(d\sin(e+fx))^n\left(a^3d^2(2A(8+6n+n^2)+B(9+11n+2n^2))+(a^3d^2(2A(8+6n+n^2)+B(9\right.}{}$$

$$= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)}$$
$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)}$$
$$-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}$$
$$+\frac{\int(d\sin(e+fx))^n\left(a^3d^3(3+n)\left(B(15+19n+4n^2)+A(20+21n+4n^2)\right)+a^3d^3(2+n)(4+n\right.}{d^3(2+n)(3+n)(4+n)}$$

$$= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)}$$
$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)}$$
$$-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}$$
$$+\frac{\left(a^3(B(9+4n)+A(11+4n))\right)\int(d\sin(e+fx))^{1+n}\,dx}{d(3+n)}$$
$$+\frac{\left(a^3(B(15+19n+4n^2)+A(20+21n+4n^2))\right)\int(d\sin(e+fx))^n\,dx}{(2+n)(4+n)}$$

$$= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)}$$
$$+\frac{a^3(B(15+19n+4n^2)+A(20+21n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2\right.}{df(1+n)(2+n)(4+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a^3(B(9+4n)+A(11+4n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)}$$
$$-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.66

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \frac{a^3\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(3A+B)\,\text{Hype}}{}\right.\right.}{}$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((3*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4 + n)
/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (5 + n)
/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n))))))/(f*Sqrt[Cos[e + f
*x]^2])
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^3 (A+B\sin(fx+e))\,dx$$

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*sin(
f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\int (d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,dx$$

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3, x)
```

3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 277

$$\begin{aligned}
&\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx \\
&= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} \\
&\quad+\frac{a^2(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}} \\
&\quad+\frac{a^2(2A(3+n)+B(5+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}} \\
&\quad-\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)}
\end{aligned}$$

```
[Out] -a^2*(A*(3+n)+B*(4+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)/(3+n)-B*co
s(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^2+a^2*sin(f*x+e))/d/f/(3+n)+a^2*(2*B*(1+n)
+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)
*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a^2*(2*A*(3+n)+B
*(5+2*n))*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*si
n(f*x+e))^(2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3055, 3047, 3102, 2827, 2722}

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a^2(A(2n+3)+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)(n+2)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{a^2(A(n+3)+B(n+4))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)(n+3)}$$
$$-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*
(2 + n)*(3 + n))) + (a^2*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeom
etric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n
))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) + B*(5 +
2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x
]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2])
 - (B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)} + \frac{\int (d\sin(e+fx))^n(a+a\sin(e+fx))(ad(B(1+n)+A(3+n))+ad(A(3+n)+B(4+n))\sin(e+fx}{d(3+n)}$$

$$= -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)} + \frac{\int (d\sin(e+fx))^n\left(a^2d(B(1+n)+A(3+n))+(a^2d(B(1+n)+A(3+n))+a^2d(A(3+n)+\right.}{d(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} - \frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)} + \frac{\int (d\sin(e+fx))^n\left(a^2d^2(3+n)(2B(1+n)+A(3+2n))+a^2d^2(2+n)(2A(3+n)+B(5+2n\right.}{d^2(2+n)(3+n)}$$

$$
\begin{aligned}
&= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} \\
&\quad - \frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)} \\
&\quad + \frac{\left(a^2(2B(1+n)+A(3+2n))\right)\int(d\sin(e+fx))^n\,dx}{2+n} \\
&\quad + \frac{\left(a^2(2A(3+n)+B(5+2n))\right)\int(d\sin(e+fx))^{1+n}\,dx}{d(3+n)} \\
&= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} \\
&\quad + \frac{a^2(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e-}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}} \\
&\quad + \frac{a^2(2A(3+n)+B(5+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e-}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}} \\
&\quad - \frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.74

$$
\int(d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx
$$

$$
= \frac{a^2\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(2A+B)\,\text{Hype}}{}\right.\right.}{}
$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((2*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*(((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Sin[e +
 f*x]^2]*Sin[e + f*x])/(4 + n)))))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^2 (A+B\sin(fx+e))\,dx$$

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="fricas")
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2\,dx$$

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2, x)
```

3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{a(B(1+n)+A(2+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}} + \frac{a(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)+a*(B*(1+n)+A*(2+n))*cos(f*x+
e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n
)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a*(A+B)*cos(f*x+e)*hypergeom([1/2, 1
+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(cos(f*x+e
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used

= {3047, 3102, 2827, 2722}

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx)\right)}{d^2f(n+2)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)(n+2)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + (a*(B*(1 + n
) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)
/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*Sqrt[Cos[e + f
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (d\sin(e+fx))^n \left(aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)\right)\,dx\\
&= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)}\\
&\quad+\frac{\int (d\sin(e+fx))^n(ad(B(1+n)+A(2+n))+a(A+B)d(2+n)\sin(e+fx))\,dx}{d(2+n)}\\
&= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)}+\frac{(a(A+B))\int (d\sin(e+fx))^{1+n}\,dx}{d}\\
&\quad+\frac{(a(B(1+n)+A(2+n)))\int (d\sin(e+fx))^n\,dx}{2+n}\\
&= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)}\\
&\quad+\frac{a(B(1+n)+A(2+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}}\\
&\quad+\frac{a(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{d^2f(2+n)\sqrt{\cos^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$=\frac{a\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left((B(1+n)+A(2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(\right.\right.}{f(1+n)(2-}$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((B*(1 + n) + A*(2 + n))*Hy
pergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] - (1 + n)*(B*Sqr
t[Cos[e + f*x]^2] - (A + B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Si
n[e + f*x]^2]*Sin[e + f*x])))/(f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2])
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))(A+B\sin(fx+e))\,dx$$

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
fricas")
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$=\int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))\,dx$$

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)), x)
```

3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 202

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(B-An+Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{adf(1+n)\sqrt{\cos^2(e+fx)}} + \frac{(A-B)(1+n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{ad^2f(2+n)\sqrt{\cos^2(e+fx)}} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))}$$

```
[Out] (A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))+(-A*n+B*n+B)*cos
(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))
^(1+n)/a/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*(1+n)*cos(f*x+e)*hypergeom([1
/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a/d^2/f/(2+n)/(co
s(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used

$= \{3057, 2827, 2722\}$

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx)\right)}{ad^2f(n+2)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(-An+Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{adf(n+1)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(a*d^2*f*(2 + n)*Sqrt[Cos[e
 + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*S
in[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} \\
&\quad + \frac{\int (d\sin(e+fx))^n(ad(B-An+Bn)+a(A-B)d(1+n)\sin(e+fx))\,dx}{a^2d} \\
&= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{((A-B)(1+n))\int (d\sin(e+fx))^{1+n}\,dx}{ad} \\
&\quad + \frac{(B-An+Bn)\int (d\sin(e+fx))^n\,dx}{a} \\
&= \frac{(B-An+Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{adf(1+n)\sqrt{\cos^2(e+fx)}} \\
&\quad + \frac{(A-B)(1+n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{ad^2f(2+n)\sqrt{\cos^2(e+fx)}} \\
&\quad + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$
\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx
$$

$$
= \frac{(d\sin(e+fx))^n\left(\frac{(B-An+Bn)\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\frac{(A-B)(1+n)\sqrt{\cos^2(e+fx)}\,\text{Hypergeometr}\ldots}{2\ldots}\right.}{af}
$$

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((d*Sin[e + f*x])^n*(((B - A*n + B*n)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + ((A - B)*(1 + n)*Sq
rt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x
]^2]*Sin[e + f*x])/(2 + n) + ((A - B)*Cos[e + f*x]^2)/(1 + Sin[e + f*x]))*T
an[e + f*x])/(a*f)
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{a+a\sin(fx+e)}dx$$

[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)

Giac [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(d\sin(e+fx))^n\,(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$

[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)),x)

[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)), x)

3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 279

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{n(A-2An+2B(1+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+}}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1+n)(B+2A(1-n)+2Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+}{3a^2d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2}$$

```
[Out] 1/3*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+sin(f*x+
e))+1/3*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^2-1/3*n*
(A-2*A*n+2*B*(1+n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f
*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+1/3*(1+n)*
(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e
)^2)*(d*sin(f*x+e))^(2+n)/a^2/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3057, 2827, 2722}

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{(n+1)(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+f\right.}{3a^2d^2f(n+2)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{n(-2An+A+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right.}{3a^2df(n+1)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3a^2df(\sin(e+fx)+1)}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a^2*d*f*(1 + n)
*Sqrt[Cos[e + f*x]^2]) + ((1 + n)*(B + 2*A*(1 - n) + 2*B*n)*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])
^(2 + n))/(3*a^2*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((B + 2*A*(1 - n) +
2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + Sin[e + f*x])
) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f
*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
```

```
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2} + \frac{\int \frac{(d\sin(e+fx))^n(ad(2A+B-An+Bn)+a(A-B)dn\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{3a^2d}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2} + \frac{\int(d\sin(e+fx))^n\left(-a^2d^2n(A-2An+2B(1+n))+a^2d^2(1+n)(2A(1-n)+B(1+2n))\sin(\cdots\right.}{3a^4d^2}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2} + \frac{((1+n)(B+2A(1-n)+2Bn))\int(d\sin(e+fx))^{1+n}\,dx}{3a^2d} - \frac{(n(A-2An+2B(1+n)))\int(d\sin(e+fx))^n\,dx}{3a^2}$$

$$= -\frac{n(A-2An+2B(1+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e-\cdots}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}} + \frac{(1+n)(B+2A(1-n)+2Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\,\text{s}\cdots}{3a^2d^2f(2+n)\sqrt{\cos^2(e+fx)}} + \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2}$$

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{(d\sin(e+fx))^n\left((A-B)\sin(2(e+fx)) - \frac{2(1+\sin(e+fx))\left((1+n)(2+n)(2A(-1+n)-B(1+2n))\cos^2(e+fx)+\sqrt{\cos^2(e+fx)}(1\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] ((d*Sin[e + f*x])^n*((A - B)*Sin[2*(e + f*x)] - (2*(1 + Sin[e + f*x])*((1 +
 n)*(2 + n)*(2*A*(-1 + n) - B*(1 + 2*n))*Cos[e + f*x]^2 + Sqrt[Cos[e + f*x]
^2]*(1 + Sin[e + f*x])*(n*(2 + n)*(A - 2*A*n + 2*B*(1 + n))*Hypergeometric2
F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] + (1 + n)^2*(2*A*(-1 + n) - B
*(1 + 2*n))*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Si
n[e + f*x]))*Tan[e + f*x])/((1 + n)*(2 + n))))/(6*a^2*f*(1 + Sin[e + f*x])^
2)
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^n\,(A+B\sin(fx+e))}{(a+a\sin(fx+e))^2}dx$$

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

Giac [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^2}\,dx$$

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2, x)
```

3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 362

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{n(B(3-n-4n^2)+A(2-9n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1-n)(1+n)(7A+3B-4An+4Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin}{15a^3d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3}$$
$$+\frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2}$$
$$+\frac{(1-n)(7A+3B-4An+4Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15df\,(a^3+a^3\sin(e+fx))}$$

```
[Out] 1/5*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^3+1/15*(A*(5
-2*n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a/d/f/(a+a*sin(f*x+e))^2+1/15*
(1-n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a^3+a^3*s
in(f*x+e))-1/15*n*(B*(-4*n^2-n+3)+A*(4*n^2-9*n+2))*cos(f*x+e)*hypergeom([1/
2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^3/d/f/(1+n)/
(cos(f*x+e)^2)^(1/2)+1/15*(1-n)*(1+n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*hyp
ergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^3/d^2/
f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3057, 2827, 2722}

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$=\frac{(1-n)(n+1)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2}\right.}{15a^3d^2f(n+2)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{n(A(4n^2-9n+2)+B(-4n^2-n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+}{2}\right.}{15a^3df(n+1)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{15df\,(a^3\sin(e+fx)+a^3)}$$
$$+\frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{n+1}}{15adf(a\sin(e+fx)+a)^2}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))
/(a^3*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((1 - n)*(1 + n)*(7*A + 3*B - 4*A
*n + 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e
 + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(15*a^3*d^2*f*(2 + n)*Sqrt[Cos[e + f*x
]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e
 + f*x])^3) + ((A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))
/(15*a*d*f*(a + a*Sin[e + f*x])^2) + ((1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*C
os[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*d*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} \\
&\quad + \frac{\int \frac{(d\sin(e+fx))^n(ad(4A+B-An+Bn)-a(A-B)d(1-n)\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{5a^2d} \\
&= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} \\
&\quad + \frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2} \\
&\quad + \frac{\int \frac{(d\sin(e+fx))^n\left(a^2d^2\left(B\left(3+n-2n^2\right)+A\left(7-6n+2n^2\right)\right)+a^2d^2n(A(5-2n)+2Bn)\sin(e+fx)\right)}{a+a\sin(e+fx)}\,dx}{15a^4d^2} \\
&= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} \\
&\quad + \frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2} \\
&\quad + \frac{(1-n)(7A+3B-4An+4Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15df\left(a^3+a^3\sin(e+fx)\right)} \\
&\quad + \frac{\int (d\sin(e+fx))^n\left(-a^3d^3n\left(B\left(3-n-4n^2\right)+A\left(2-9n+4n^2\right)\right)+a^3d^3(1-n)(1+n)(7A+3B\right.}{15a^6d^3} \\
&= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} \\
&\quad + \frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2} \\
&\quad + \frac{(1-n)(7A+3B-4An+4Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15df\left(a^3+a^3\sin(e+fx)\right)} \\
&\quad + \frac{((1-n)(1+n)(7A+3B-4An+4Bn))\int (d\sin(e+fx))^{1+n}\,dx}{15a^3d} \\
&\quad - \frac{\left(n\left(B\left(3-n-4n^2\right)+A\left(2-9n+4n^2\right)\right)\right)\int (d\sin(e+fx))^n\,dx}{15a^3}
\end{aligned}$$

$$= -\frac{n(B(3-n-4n^2)+A(2-9n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+f\right.}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1-n)(1+n)(7A+3B-4An+4Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+\right.}{15a^3d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3}$$
$$+\frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2}$$
$$+\frac{(1-n)(7A+3B-4An+4Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15df\,(a^3+a^3\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.80

$$\int\frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$=\frac{(d\sin(e+fx))^n\left(\frac{3}{2}a^5(A-B)\sin(2(e+fx))+\frac{a^5(1+\sin(e+fx))\left((1+n)(2+n)(A(5-2n)+2Bn)\cos^2(e+fx)-(1+\sin(e+fx))\right.}{}\right.}{}$$

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] ((d*Sin[e + f*x])^n*((3*a^5*(A - B)*Sin[2*(e + f*x)])/2 + (a^5*(1 + Sin[e +
 f*x])*((1 + n)*(2 + n)*(A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]^2 - (1 + Sin[e +
 f*x])*(-((-1 + n)*(1 + n)*(2 + n)*(A*(-7 + 4*n) - B*(3 + 4*n))*Cos[e + f*x
]^2) + Sqrt[Cos[e + f*x]^2]*(1 + Sin[e + f*x])*(n*(2 + n)*(A*(2 - 9*n + 4*n
^2) - B*(-3 + n + 4*n^2))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[
e + f*x]^2] - (-1 + n)*(1 + n)^2*(A*(-7 + 4*n) - B*(3 + 4*n))*Hypergeometri
c2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])))*Tan[e + f*x
])/((1 + n)*(2 + n))))/(15*a^8*f*(1 + Sin[e + f*x])^3)
```

Maple [F]

$$\int\frac{(d\sin(fx+e))^n(A+B\sin(fx+e))}{(a+a\sin(fx+e))^3}dx$$

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 - 4
*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

Giac [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3, x)
```

3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 336

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx =$$
$$-\frac{2a^3(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx)\,\text{Hypergeometric2F1}\,(}{f(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)}$$
$$-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)}$$

```
[Out] -2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(3/2)/d/f/(7+2*n)-2
*a^3*(2*B*(16*n^3+104*n^2+203*n+115)+A*(32*n^3+224*n^2+478*n+301))*cos(f*x+
e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*
n)/(7+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a^3*(2*B*(4*n^2+23*n+35)
+A*(8*n^2+50*n+77))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(5+2*n)/(7+
2*n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(5+n)+A*(7+2*n))*cos(f*x+e)*(d*sin(f
*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)/(7+2*n)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3055, 3060, 2855, 69, 67}

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2a^3(A(32n^3+224n^2+478n+301)+2B(16n^3+104n^2+203n+115))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(}{f(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}} - \frac{2a^3(A(8n^2+50n+77)+2B(4n^2+23n+35))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}} - \frac{2a^2(A(2n+7)+2B(n+5))\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)(2n+7)} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^3*(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 +
32*n^3))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*
Sin[e + f*x])^n)/(f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a
*Sin[e + f*x]]) - (2*a^3*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*
Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*
(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n)*(7 + 2*n)
) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2)
)/(d*f*(7 + 2*n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)} + \frac{2\int(d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}\left(\frac{1}{2}ad\left(2B(1+n)+2A\left(\frac{7}{2}+n\right)\right)+\frac{1}{2}ad(2B(5+n)+A(7+2n)}{d(7+2n)}$$

$$= -\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)} + \frac{4\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\left(\frac{1}{4}a^2d^2(2B(15+19n+4n^2)+A(49+42n+8n^2))+\frac{1}{4}a^2}{d^2(5+2n)(7+2n)}$$

$$= -\frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)} + \frac{(a^2(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3)))\int(d\sin(e+fx))^n\sqrt{a}}{(3+2n)(5+2n)(7+2n)}$$

$$= -\frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)} + \frac{(a^4(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx))\operatorname{Subst}\left(\int\right.}{f(3+2n)(5+2n)(7+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)} + \frac{(a^4(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx)\sin^{-n}(e+}{f(3+2n)(5+2n)(7+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+}}$$

$$= -\frac{2a^3(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx)\operatorname{Hypergeo}}{f(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin}} - \frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)}$$

Mathematica [A] (warning: unable to verify)

Time = 25.42 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.77

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{5/2}\tan\left(\frac{1}{2}(e+fx\right.}{}$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((5*A +
 2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2]
)/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 + n)/2, 9
/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((5*(3*
A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e + f*x)/2]
^2])/(4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1[9/2 + n,
(5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + Tan[(e + f*x)/2]*(((1
1*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -Tan[(e + f*x)
/2]^2])/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 + n)/2, (9 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(7 + n)))))))))/(f*Sqrt[Sec[(e +
 f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]^n)
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^{\frac{5}{2}} (A+B\sin(fx+e))\,dx$$

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e
))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}\,dx$$

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

3.8 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 229

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2a^2(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)}$$

```
[Out] -2*a^2*(2*B*(4*n^2+13*n+9)+A*(8*n^2+30*n+25))*cos(f*x+e)*hypergeom([1/2, -n
],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*n)/(sin(f*x+e)^n)/(a+
a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(3+n)+A*(5+2*n))*cos(f*x+e)*(d*sin(f*x+e))^(
1+n)/d/f/(3+2*n)/(5+2*n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(d*sin(f*x
+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {3055, 3060, 2855, 69, 67}

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2a^2(A(8n^2+30n+25)+2B(4n^2+13n+9))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2}}{f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}} - \frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^2*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2))*Cos[e + f*x]*Hyper
geometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*
n)*(5 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(3 + n)
 + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 +
2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)} \\
&\quad+\frac{2\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}ad\left(2B(1+n)+2A\left(\frac{5}{2}+n\right)\right)+\frac{1}{2}ad(2B(3+n)+A(5+2n))\right.}{d(5+2n)} \\
&= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)} \\
&\quad+\frac{(a(2B(9+13n+4n^2)+A(25+30n+8n^2)))\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx}{(3+2n)(5+2n)} \\
&= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)} \\
&\quad+\frac{(a^3(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx))\operatorname{Subst}\left(\int\frac{(dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f(3+2n)(5+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)} + \frac{(a^3(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\,\text{Subst}}{f(3+2n)(5+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a^2(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin\right.}{f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 478 vs. 2(229) = 458.

Time = 20.87 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.09

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{3/2}\tan\left(\frac{1}{2}(e+f\right.}{}$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[
(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e + f*x
)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2, -Tan[
(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2
F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2])/(4 + n) + Tan[(e +
f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n)/2, -Tan
[(e + f*x)/2]^2])/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)/2, (8 + n
)/2, -Tan[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(6 + n)))))))/(f*Sqrt[Sec[(e +
f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]^n)
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^{\frac{3}{2}} (A+B\sin(fx+e))\,dx$$

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

Sympy [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}(d\sin(e+fx))^n(A+B\sin(e+fx))\,dx$$

[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)

[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)), x)

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^
n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

3.9 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 137

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2a(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a*(2*B*(1+n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e
))*(d*sin(f*x+e))^n/f/(3+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*c
os(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3060, 2855, 69, 67}

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2a(A(2n+3)+2B(n+1))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1\right.}{f(2n+3)\sqrt{a\sin(e+fx)+a}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3
/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[
a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(
3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}\\ &\quad+\left(A+\frac{2B(1+n)}{3+2n}\right)\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx\end{aligned}$$

$$= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2\left(A+\frac{2B(1+n)}{3+2n}\right)\cos(e+fx)\right)\text{Subst}\left(\int\frac{(dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2\left(A+\frac{2B(1+n)}{3+2n}\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int\frac{x^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a\left(A+\frac{2B(1+n)}{3+2n}\right)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\text{s}}{f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.99

$$\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{(1+i)2^{-2-n}e^{-\frac{3ie}{2}+ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n\left(\frac{2Be^{-\frac{1}{2}if(3+2n)x}\,\text{Hypergeometric2F1}(\frac{1}{4}(}{f(3+2n}\right.}{}$$

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),
x]
```

```
[Out] ((-1 - I)*2^(-2 - n)*E^(((-3*I)/2)*e + I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e +
f*x))))/E^(I*(e + f*x)))^n*((2*B*Hypergeometric2F1[(-3 - 2*n)/4, -n, (1 - 2
*n)/4, E^((2*I)*(e + f*x))])/(E^((I/2)*f*(3 + 2*n)*x)*f*(3 + 2*n)) + 2*E^(I
*e)*(((-I)*(2*A + B)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2
*I)*(e + f*x))])/(E^((I/2)*f*(1 + 2*n)*x)*(f + 2*f*n)) + (E^((I/2)*(2*e + f
*(1 - 2*n)*x))*(-((2*A + B)*(-3 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (
5 - 2*n)/4, E^((2*I)*(e + f*x))]) + I*B*E^(I*(e + f*x))*(-1 + 2*n)*Hypergeo
metric2F1[(3 - 2*n)/4, -n, (7 - 2*n)/4, E^((2*I)*(e + f*x))]))/(f*(-3 + 2*n
)*(-1 + 2*n))))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])])/((1 - E^((2*
I)*(e + f*x)))^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]^n)
```

Maple [F]

$$\int (d\sin(fx+e))^n \sqrt{a+a\sin(fx+e)}\,(A+B\sin(fx+e))\,dx$$

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)

Fricas [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

Sympy [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int \sqrt{a(\sin(e+fx)+1)}(d\sin(e+fx))^n(A+B\sin(e+fx))\,dx$$

[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)), x)

Maxima [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

Giac [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$=\int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,dx$$

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 152

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+}{f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -(A-B)*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d
*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*B*cos(f*x+e)*hyper
geom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*s
in(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3066, 2866, 2865, 2864, 129, 440, 2855, 69, 67}

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+}{f\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2
]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x
]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*
(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
```

```
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx}{a}$$

$$= \frac{\left((A-B)\sqrt{1+\sin(e+fx)}\right)\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(dx)^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} + \frac{\left(aB\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}} - \frac{((A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\,\text{Subst}\left(\int\frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx,x,1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}} - \frac{(2(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\,\text{Subst}\left(\int\frac{(1-x^2)^n}{2-x^2}\,dx,x,\sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(}{f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 12.00 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.64

$$\int\frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\sin^n(e+fx)(d\sin(e+fx))^n\left(-\sin^2(e+fx)\right)^{-n}\sqrt{a(1+\sin(e+fx))}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4(\right.}{}$$

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 +
 Sin[e + f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e
+ f*x])] - (A + B)*(1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2,
1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n)
)/(4*a*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e
+ f*x])^(-1))^n)
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{\sqrt{a+a\sin(fx+e)}}dx$$

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
x)
```

Sympy [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(d\sin(e+fx))^n (A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)
), x)
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2), x
)
```

3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 226

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}}$$
$$-\frac{(A-4An+B(3+4n))\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+}{4af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(A-B)(1+2n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))}{2af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 1/2*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A
-4*A*n+B*(3+4*n))*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-1/2*(A-
B)*(1+2*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e)
)^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3057, 3066, 2866, 2865, 2864, 129, 440, 2855, 69, 67}

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{(-4An+A+B(4n+3))\cos(e+fx)\sin^{-n}(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+\right.}{4af\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx\right.}{2af\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x])
^(3/2)) - ((A - 4*A*n + B*(3 + 4*n))*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e +
f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(4*a*f*Sin[e +
 f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*(1 + 2*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e
 + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(d\sin(e+fx))^n\left(ad(A+B-An+Bn)+\frac{1}{2}a(A-B)d(1+2n)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2d}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{((A-B)(1+2n))\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx}{4a^2} + \frac{\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+B-An+Bn)\right)\int\frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^3d}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+B-An+Bn)\right)\sqrt{1+\sin(e+fx)}\right)\int\frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{2a^3d\sqrt{a+a\sin(e+fx)}} + \frac{((A-B)(1+2n)\cos(e+fx))\text{Subst}\left(\int\frac{(dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{4f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+B-An+Bn)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(}\right.}{2a^3d\sqrt{a+a\sin(e+fx)}} + \frac{((A-B)(1+2n)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\,\text{Subst}\left(\int\frac{x^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right.}{4f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin}{2af\sqrt{a+a\sin(e+fx)}} - \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+B-An+Bn)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^{n}}{2a^3df\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin}{2af\sqrt{a+a\sin(e+fx)}} - \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+B-An+Bn)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^{n}}{a^3df\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A+3B-4An+4Bn)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{4af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)(1+2n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin}{2af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 523 vs. $2(226) = 452$.

Time = 27.19 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.31

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sec(e+fx)(d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(a\,\text{AppellF1}\right.\right.}{}$$

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1/
2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*
(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Si
n[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Si
n[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/
2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e +
 f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)) + A*((a^2*AppellF1
[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e +
```

```
f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Sin[e + f*x
])/(1 + Sin[e + f*x])]*(1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n,
-1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^
(-1))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{(a+a\sin(fx+e))^{\frac{3}{2}}}dx$$

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/
(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(d\sin(e+fx))^n (A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(
3/2), x)
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2), x
)
```

3.12 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 221

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx =$$
$$-\frac{2^{\frac{3}{2}+m}B\,\text{AppellF1}\left(\frac{1}{2},-n,-\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d}{f}$$
$$-\frac{2^{\frac{1}{2}+m}(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+f}{f}$$

```
[Out] -2^(3/2+m)*B*AppellF1(1/2,-n,-1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(sin
(f*x+e)^n)-2^(1/2+m)*(A-B)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*s
in(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x
+e))^m/f/(sin(f*x+e)^n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3066, 2866, 2865, 2864, 138}

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx =$$
$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\,\text{A}}{f}$$
$$-\frac{B2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\,\text{AppellF}}{f}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Si
n[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*Appell
F1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f
*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/(f*Sin[e + f*x]^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

```
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx + \frac{B\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{1+m}\,dx}{a}$$

$$= \left((A-B)(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx + \left(B(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^n(1+\sin(e+fx))^{1+m}\,dx$$

$$= \left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx + \left(B\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int \sin^n(e+fx)(1+\sin(e+fx))^{1+m}\,dx$$

$$= -\frac{\left((A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))\right.}{f\sqrt{1-\sin(e+fx)}} - \frac{\left(B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m\right)\text{Sub}}{f\sqrt{1-\sin(e+fx)}}$$

$$= -\frac{2^{\frac{3}{2}+m}B\,\text{AppellF1}\left(\frac{1}{2},-n,-\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e}{f} - \frac{2^{\frac{1}{2}+m}(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-}}{f}$$

Mathematica [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
[Out] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x
]
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^m (A+B\sin(fx+e))\,dx$$

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (a(\sin(e+fx)+1))^m (d\sin(e+fx))^n (A+B\sin(e+fx))\,dx$$

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n*(A + B*sin(e + f*x))
, x)
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm ="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,dx$$

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)

3.13 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 34, antiderivative size = 114

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \frac{\operatorname{AppellF1}\left(1+n,-\frac{1}{2},\frac{1}{2}-m,2+n,\sin(e+fx),-\sin(e+fx)\right)\sec(e+fx)(d\sin(e+fx))^{1+n}(1+\sin(e+}{df(1+n)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1+n,1/2-m,-1/2,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(d*sin(f*x+e
))^(1+n)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m/d/f/(1+
n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3087, 140, 138}

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}\operatorname{AppellF1}(n+}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e +
 f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e +
f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}\left(\int(dx)^n\sqrt{a-ax}(a+ax)^{-\frac{1}{2}+m}\,dx,x,\sin(e+}{f}$$

$$= \frac{\left(\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}\left(\int\sqrt{1-x}(dx)^n(a+ax)^{-\frac{1}{2}+m}\,dx,x,\mathrm{s}}{f\sqrt{1-\sin(e+fx)}}$$

$$= \frac{\left(\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a-a\sin(e+fx))(a+a\sin(e+fx))^m\right)\mathrm{Subst}\left(\int\sqrt{1-x}(dx}{f\sqrt{1-\sin(e+fx)}}$$

$$= \frac{\mathrm{AppellF1}\left(1+n,-\frac{1}{2},\frac{1}{2}-m,2+n,\sin(e+fx),-\sin(e+fx)\right)\sec(e+fx)(d\sin(e+fx))^{1+n}(1+}{df(1+n)\sqrt{1-\sin(e+fx)}}$$

Mathematica [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m, x
]
```

Maple [F]

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}dx$$

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int -(a\sin\left(fx+e\right)-a)(a\sin\left(fx+e\right)+a)^{m}(d\sin\left(fx+e\right))^{n}\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Sympy [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=-a\bigg(\int \left(-\left(d\sin\left(e+fx\right)\right)^{n}\left(a\sin\left(e+fx\right)+a\right)^{m}\right)\,dx$$
$$+\int \left(d\sin\left(e+fx\right)\right)^{n}\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)dx\bigg)$$

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m,x)
```

```
[Out] -a*(Integral(-(d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m, x) + Integral((d
*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int -(a\sin{(fx+e)}-a)(a\sin{(fx+e)}+a)^m(d\sin{(fx+e)})^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int -(a\sin{(fx+e)}-a)(a\sin{(fx+e)}+a)^m(d\sin{(fx+e)})^n\,dx$$

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(a-a\,\sin\left(e+f\,x\right)\right)\,dx$$

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)), x)
```

3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$

Optimal result

Integrand size = 43, antiderivative size = 37

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(c+dx))^{-2-n}}{d}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))^(-2-n)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {3053}

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

```
[In] Int[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c +
 d*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
```

```
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\text{integral} = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(c+dx))^{-2-n}}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(37) = 74$.

Time = 5.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx =$$
$$-\frac{2^n\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{4}(c+dx)\right)\left(-\sin\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{3}{4}(c+\right.\right.\right.}{d}$$

```
[In] Integrate[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*S
in[c + d*x]),x]
```

```
[Out] -((2^n*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/4]*(-Sin[(c + d*x)/4] + Sin[(3*(c + d*x))/4]))^n*(1 + Cos[c + d*x] - Sin[
c + d*x])*(a*(1 + Sin[c + d*x]))^(-2 - n))/d)
```

Maple [F]

$$\int (\sin^n(dx+c))\,(a+a\sin(dx+c))^{-2-n}\,(-1-n-(-2-n)\sin(dx+c))\,dx$$

```
[In] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= -\frac{(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\cos(dx+c)\sin(dx+c)}{d}$$

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \int (a(\sin{(c+dx)}+1))^{-n-2}\,(n\sin{(c+dx)}-n+2\sin{(c+dx)}-1)\sin^n{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x
)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(-n - 2)*(n*sin(c + d*x) - n + 2*sin(c + d
*x) - 1)*sin(c + d*x)**n, x)
```

Maxima [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \int ((n+2)\sin{(dx+c)}-n-1)(a\sin{(dx+c)}+a)^{-n-2}\sin{(dx+c)}^n\,dx$$

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="maxima")
```

```
[Out] integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin(
d*x + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9496 vs. $2(37) = 74$.

Time = 84.24 (sec) , antiderivative size = 9496, normalized size of antiderivative = 256.65

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="giac")
```

```
[Out] -8*(cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) +
4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) -
```

```
 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8
*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*e^(-n*log(
sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2
*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*
x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4
*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*ta
n(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*t
an(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)
^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*
x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 +
 1)) + n*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*
log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x +
c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8
*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
 + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2
)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 +
 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)
^2 + 1)))*tan(1/4*pi*n*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1
/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 +
8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(4*a*tan(1/2
*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) +
1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*ta
n(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x
+ 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n - pi*floor(d*x/pi + c/pi
+ 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2
 + 4*a*tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^3 - 2*e^(-n*log(sqrt(2
)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1
/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d
*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d
*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
+ c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan
(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2
```

```
*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2
*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) +
 n*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(sq
rt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*t
an(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1
/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(
d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2
*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/
2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2
 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1
)))*sin(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) +
4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) -
 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8
*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*tan(1/4*pi
*n*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/
2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/
2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) + 1/2*pi*sgn(2*a*tan(1
/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) -
 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan
(1/2*d*x + 1/2*c)) - 1/4*pi*n - pi*floor(d*x/pi + c/pi + 1/2) + 1/2*pi*sgn(
4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x +
 1/2*c)))*tan(1/2*d*x + 1/2*c)^3 - cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
 + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2
) + 5/8) + 1/4*pi*n*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2
+ 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x +
c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan
(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8)
 + 1/2*pi*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
```

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2*d*x + 1/2*c) + 2))*e^(-n*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2
*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/
2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 +
 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 +
 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c
)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/
2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*
x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x +
c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) + n*log(4*abs(tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/
2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2
 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(
d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)
^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)
^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1
/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*
x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)))*tan(1/4*pi*n*sgn(2*a*tan(1/2*d*x +
 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sg
n(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x
 + 1/2*c)) - 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2
*c)) + 1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2
 + 4*a*tan(1/2*d*x + 1/2*c)) + 1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*
tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*
x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4
*pi*n - pi*floor(d*x/pi + c/pi + 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x + 1/2*c)
^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x
+ 1/2*c) - cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x
 + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(
4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/
2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c)
 + 2) - 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*e^(
-n*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2
```

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+ 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4
*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c)
+ 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d
*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/
(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2
*c)^2 + 1)) + n*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1
)) - 2*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c
)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + a
bs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2
*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2
*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs
(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x +
 1/2*c)^2 + 1)))*tan(1/2*d*x + 1/2*c)^3 + 2*e^(-n*log(sqrt(2)*sqrt(abs(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
 + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2
)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x
+ 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*t
an(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*
tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) +
 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 +
8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x +
1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x
+ 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) + n*log(4*abs(ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(sqrt(2)*sqrt(abs(
4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/
2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c)
 + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 +
 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*
c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*
d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)))*sin(-1/2*pi
```

+ 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*tan(1/4*pi*n*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) + 1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n - pi*floor(d*x/pi + c/pi + 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)))*tan(1/2*d*x + 1/2*c) + cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*e^(-n*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) + n*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/

```
2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*
x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) +
2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/
2*d*x + 1/2*c)^2 + 1)))*tan(1/2*d*x + 1/2*c))/(d*tan(1/4*pi*n*sgn(2*a*tan(1
/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) -
 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan
(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d
*x + 1/2*c)) + 1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x +
1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) + 1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^
4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*ta
n(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c
)) - 1/4*pi*n - pi*floor(d*x/pi + c/pi + 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x
+ 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)))^2*tan(
1/2*d*x + 1/2*c)^4 + 2*d*tan(1/4*pi*n*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*
tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*
x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4
*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*
sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d
*x + 1/2*c)) + 1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/
2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8
*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n - pi*floor
(d*x/pi + c/pi + 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2
*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*t
an(1/2*d*x + 1/2*c)^4 + d*tan(1/4*pi*n*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a
*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d
*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/
4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n
*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*
d*x + 1/2*c)) + 1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1
/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 +
8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n - pi*floo
r(d*x/pi + c/pi + 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/
2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)))^2 + 2*d*tan(1/2*d*x + 1/2*c)^
2 + d)
```

Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$=-\frac{\sin\left(c+d\,x\right)^n\,\sin\left(2\,c+2\,d\,x\right)}{a^2\,d\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^n\,\left(2\sin\left(c+d\,x\right)^2+4\,\sin\left(c+d\,x\right)+2\right)}$$

```
[In] int(-(sin(c + d*x)^n*(n - sin(c + d*x)*(n + 2) + 1))/(a + a*sin(c + d*x))^(
n + 2),x)
```

```
[Out] -(sin(c + d*x)^n*sin(2*c + 2*d*x))/(a^2*d*(a*(sin(c + d*x) + 1))^n*(4*sin(c
 + d*x) + 2*sin(c + d*x)^2 + 2))
```

3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$

Optimal result

Integrand size = 37, antiderivative size = 35

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+dx))^m}{d}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(-1-m)*(a+a*sin(d*x+c))^m/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {3053}

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

```
[In] Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),x
]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
```

```
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\text{integral} = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+dx))^m}{d}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a(1+\sin(c+dx)))^m}{d}$$

```
[In] Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d
*x]),x]
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a*(1 + Sin[c + d*x]))^m)/d)
```

Maple [F]

$$\int \left(\sin^{-2-m}(dx+c)\right)(a+a\sin(dx+c))^m(1+m-m\sin(dx+c))\,dx$$

```
[In] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
[Out] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\cos(dx+c)\sin(dx+c)}{d}$$

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="fricas")
[Out] -(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2)*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= -\int \left(-(a\sin\left(c+dx\right)+a)^m \sin^{-m-2}\left(c+dx\right)\right)\,dx$$
$$-\int \left(-m(a\sin\left(c+dx\right)+a)^m \sin^{-m-2}\left(c+dx\right)\right)\,dx$$
$$-\int m(a\sin\left(c+dx\right)+a)^m \sin\left(c+dx\right)\sin^{-m-2}\left(c+dx\right)dx$$

```
[In] integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] -Integral(-(a*sin(c + d*x) + a)**m*sin(c + d*x)**(-m - 2), x) - Integral(-m
*(a*sin(c + d*x) + a)**m*sin(c + d*x)**(-m - 2), x) - Integral(m*(a*sin(c +
 d*x) + a)**m*sin(c + d*x)*sin(c + d*x)**(-m - 2), x)
```

Maxima [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= \int -(m\sin\left(dx+c\right)-m-1)(a\sin\left(dx+c\right)+a)^m \sin\left(dx+c\right)^{-m-2}\,dx$$

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m
 - 2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5502 vs. $2(35) = 70$.

Time = 40.10 (sec) , antiderivative size = 5502, normalized size of antiderivative = 157.20

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="giac")
```

```
[Out] -8*(cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2
```

```
*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)
*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(
d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)
^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*
c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(
a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x +
1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*ta
n(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2
*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d
*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/
2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m +
 pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^3 - 2*e^(m*log(sqrt(
2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
 + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d
*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 +
2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))
- m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4
*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*sin(2*pi*m*floor(
-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2
*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x
 + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*tan(-1/2*pi + 1/4*pi
*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/
2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/
2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
```

a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi +
 c/pi + 1/2))*tan(1/2*d*x + 1/2*c)^3 - cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/
8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^
2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*ta
n(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1
/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2
+ 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)))*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x
 + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*
sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d
*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1
/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)
^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn
(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/
2*d*x + 1/2*c) - cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(
1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
 + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2
) - 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*t
an(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1
/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/
2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1
/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*
c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + t
an(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c

```
)^2 + 1)))*tan(1/2*d*x + 1/2*c)^3 + 2*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*
tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan
(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*sin(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d
*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) +
 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
2*d*x + 1/2*c) + 2) - 1/4*pi*m)*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x
+ 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*s
gn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*
x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/
2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^
2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(
tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))*tan(1/2*d
*x + 1/2*c) + cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) +
4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) -
 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4
*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c)
+ 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(
1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 + 1)))*tan(1/2*d*x + 1/2*c))/(d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x
 + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*
```

```
sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d
*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1
/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)
^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn
(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/
2*d*x + 1/2*c)^4 + 2*d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^
4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*ta
n(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c
)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1
/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*t
an(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d
*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/
2*c)^2 + d*tan(1/2*d*x + 1/2*c)^4 + d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/
2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) -
2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(
1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*
x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1
/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2 +
 2*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\sin(2c+2dx)\,(a(\sin(c+dx)+1))^m}{2d\sin(c+dx)^{m+2}}$$

```
[In] int(((a + a*sin(c + d*x))^m*(m - m*sin(c + d*x) + 1))/sin(c + d*x)^(m + 2),
x)
```

```
[Out] -(sin(2*c + 2*d*x)*(a*(sin(c + d*x) + 1))^m)/(2*d*sin(c + d*x)^(m + 2))
```

3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2} dx = \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}f} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))}$$

```
[Out] (A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((b+a*tan(1
/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f-B*cos(f*x+e)/b^2/f+a^
2*(A*b-B*a)*cos(f*x+e)/b^2/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3067, 3102, 2814, 2739, 632, 210}

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2} dx = \frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))} - \frac{2a(-2a^3B+a^2Ab+3ab^2B-2Ab^3)\arctan\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f(a^2-b^2)^{3/2}} + \frac{x(Ab-2aB)}{b^3} - \frac{B\cos(e+fx)}{b^2f}$$

```
[In] Int[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f) - (
B*Cos[e + f*x])/(b^2*f) + (a^2*(A*b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} + \frac{\int \frac{ab(Ab-aB)+(a^2-b^2)(Ab-aB)\sin(e+fx)+b(a^2-b^2)B\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx}{b^2(a^2-b^2)}$$

$$= -\frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} + \frac{\int \frac{ab^2(Ab-aB)+b(a^2-b^2)(Ab-2aB)\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b^3(a^2-b^2)}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} - \frac{(a(a^2Ab-2Ab^3-2a^3B+3ab^2B))\int \frac{1}{a+b\sin(e+fx)}\,dx}{b^3(a^2-b^2)}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} - \frac{(2a(a^2Ab-2Ab^3-2a^3B+3ab^2B))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^3(a^2-b^2)f}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} + \frac{(4a(a^2Ab-2Ab^3-2a^3B+3ab^2B))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^3(a^2-b^2)f}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}f} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \frac{(Ab-2aB)(e+fx)+\frac{2a(-a^2Ab+2Ab^3+2a^3B-3ab^2B)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-bB\cos(e+fx)+\frac{a^2b(Ab-aB)\cos(e+\ldots}{(a-b)(a+b)(a+b\sin(e\ldots}}{b^3f}$$

```
[In] Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*B
*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Si
n[e + f*x])))/(b^3*f)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{2a\left(\frac{-\frac{b^2(Ab-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-Ba)}{a^2-b^2}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(A\,a^2b-2A\,b^3-2B\,a^3+3Ba\,b^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^3}+\frac{-\frac{2Bb}{1+\tan^2\left(\frac{fx}{2}+\ldots\right.}\ldots}{\ldots}}{f}$
default	$\frac{-\frac{2a\left(\frac{-\frac{b^2(Ab-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-Ba)}{a^2-b^2}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(A\,a^2b-2A\,b^3-2B\,a^3+3Ba\,b^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^3}+\frac{-\frac{2Bb}{1+\tan^2\left(\frac{fx}{2}+\ldots\right.}\ldots}{\ldots}}{f}$
risch	$\frac{xA}{b^2}-\frac{2xBa}{b^3}-\frac{B\,e^{i(fx+e)}}{2b^2f}-\frac{B\,e^{-i(fx+e)}}{2b^2f}+\frac{2ia^2(-Ab+Ba)\left(ib+a\,e^{i(fx+e)}\right)}{b^3(a^2-b^2)f\left(-ie^{2i(fx+e)}b+2a\,e^{i(fx+e)}+ib\right)}+\frac{a^3\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a\ldots}}{b}\ldots\right.}{\sqrt{-a^2+b^2}\,(a+b\ldots}$

```
[In] int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-2*a/b^3*((-b^2*(A*b-B*a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)-b*a*(A*b-B*a)/(
a^2-b^2))/(tan(1/2*f*x+1/2*e)^2*a+2*b*tan(1/2*f*x+1/2*e)+a)+(A*a^2*b-2*A*b^
3-2*B*a^3+3*B*a*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b
)/(a^2-b^2)^(1/2)))+2/b^3*(-B*b/(1+tan(1/2*f*x+1/2*e)^2)+(A*b-2*B*a)*arctan
(tan(1/2*f*x+1/2*e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(152) = 304$.

Time = 0.33 (sec) , antiderivative size = 804, normalized size of antiderivative = 5.25

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \Bigg[-\frac{2\,(2\,Ba^6 - Aa^5b - 4\,Ba^4b^2 + 2\,Aa^3b^3 + 2\,Ba^2b^4 - Aab^5)fx + (2\,Ba^5 - Aa^4b - 3\,Ba^3b^2 + 2\,Aa^2b^3 + (}{}$$
$$-\frac{(2\,Ba^6 - Aa^5b - 4\,Ba^4b^2 + 2\,Aa^3b^3 + 2\,Ba^2b^4 - Aab^5)fx + (2\,Ba^5 - Aa^4b - 3\,Ba^3b^2 + 2\,Aa^2b^3 + (2\,}{}$$

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a
*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A
*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
 e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 -
2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 +
 A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^
3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6
)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) +
 (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A
*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 +
2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e
))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x +
e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x
+ e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*
b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((
a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(152) = 304$.

Time = 0.66 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.33

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \frac{\frac{2\,(2\,Ba^4 - Aa^3b - 3\,Ba^2b^2 + 2\,Aab^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{a^2-b^2}} - \frac{2\left(Ba^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - Aab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2}{}}{}$$

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^
2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*b^2
*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(1/2*
f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*x + 1
```

```
/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) + 2*B*a
^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*
e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(a^2*b^2
- b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 3718, normalized size of antiderivative = 24.30

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((sin(e + f*x)^2*(A + B*sin(e + f*x)))/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(A*a^2*b - 2*B*a^3 + B*a*b^2))/(b^2*(a^2 - b^2)) - (2*tan(e/2 + (f*x)/2
)^3*(B*a^2 - A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(2*B*b^2 - 3*B
*a^2 + A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(A*a^2*b - 2*B*a^3
 + B*a*b^2))/(b^2*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*tan(e/
2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) + (log
(tan(e/2 + (f*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*f) - (log(tan(e/2 + (f*x)/
2) - 1i)*(A*b*1i - B*a*2i))/(b^3*f) - (a*atan(((a*(-(a + b)^3*(a - b)^3)^(1
/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2
*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/
(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*
a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 +
28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^
6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b
^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^
6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(
b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^
9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 +
7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^
3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^
4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b
^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b
^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((3
2*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^
4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 -
2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8
 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*
a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 +
 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(
1/2)*((32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/
(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^
```

4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2
*a^2*b^8 + a^4*b^6) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a
^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b
^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*
a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*
a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(4*B^3*a^8 + 2*A^3*a^3*b^5 - A^3*a^5*b
^3 - 6*B^3*a^6*b^2 - 8*A*B^2*a^7*b + 13*A*B^2*a^5*b^3 - 9*A^2*B*a^4*b^4 + 5
*A^2*B*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (64*tan(e/2 + (f*x)/2)*(16*B
^3*a^9 - 4*A^3*a^2*b^7 + 6*A^3*a^4*b^5 - 2*A^3*a^6*b^3 + 24*B^3*a^5*b^4 - 4
0*B^3*a^7*b^2 - 24*A*B^2*a^8*b - 40*A*B^2*a^4*b^5 + 64*A*B^2*a^6*b^3 + 22*A
^2*B*a^3*b^6 - 34*A^2*B*a^5*b^4 + 12*A^2*B*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^
4*b^6) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6
+ A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b
^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(
e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^
4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B
*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*
b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4
*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B
*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3
*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(
a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2
+ (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2
*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b
- 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 -
A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 +
2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (
a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*
b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B
*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x
)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*
a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 +
32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*
b^6) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B
*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2
+ (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B
*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*((32*(a^2*b^12 -
2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2
)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4
b^6))(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^
2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b -
3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 -
A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3

```
*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*2i)/(f*(b^9 - 3
*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

3.17 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal result

Integrand size = 34, antiderivative size = 182

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx &= \frac{7}{16}a(2A-B)c^4x+\frac{7a(2A-B)c^4\cos^3(e+fx)}{24f}+\frac{7a(2A-B)c^4\cos(e+fx)\sin(e+fx)}{16f}\\ &\quad-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}+\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f}\\ &\quad+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f}\end{aligned}$$

```
[Out] 7/16*a*(2*A-B)*c^4*x+7/24*a*(2*A-B)*c^4*cos(f*x+e)^3/f+7/16*a*(2*A-B)*c^4*c
os(f*x+e)*sin(f*x+e)/f-1/6*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^3/f+1/10*a*(
2*A-B)*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/40*a*(2*A-B)*cos(f*x+e)^3*(c
^4-c^4*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used

$= \{3046, 2939, 2757, 2748, 2715, 8\}$

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{7ac^4(2A-B)\cos^3(e+fx)}{24f} + \frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f} + \frac{7ac^4(2A-B)\sin(e+fx)\cos(e+fx)}{16f} + \frac{7}{16}ac^4x(2A-B) + \frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} - \frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a*(2*A - B)*c^4*x)/16 + (7*a*(2*A - B)*c^4*Cos[e + f*x]^3)/(24*f) + (7*a
*(2*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a*B*c*Cos[e + f*x]^3*(c
 - c*Sin[e + f*x])^3)/(6*f) + (a*(2*A - B)*Cos[e + f*x]^3*(c^2 - c^2*Sin[e
+ f*x])^2)/(10*f) + (7*a*(2*A - B)*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))
/(40*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} \\
&\quad +\frac{1}{2}(a(2A-B)c)\int \cos^2(e+fx)(c-c\sin(e+fx))^3\,dx \\
&= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} \\
&\quad +\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} \\
&\quad +\frac{1}{10}\left(7a(2A-B)c^2\right)\int \cos^2(e+fx)(c-c\sin(e+fx))^2\,dx \\
&= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} \\
&\quad +\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} \\
&\quad +\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f} \\
&\quad +\frac{1}{8}\left(7a(2A-B)c^3\right)\int \cos^2(e+fx)(c-c\sin(e+fx))\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} - \frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} \\
&\quad + \frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} \\
&\quad + \frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f} \\
&\quad + \frac{1}{8}\left(7a(2A-B)c^4\right)\int\cos^2(e+fx)\,dx \\
&= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} + \frac{7a(2A-B)c^4\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad - \frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} \\
&\quad + \frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} \\
&\quad + \frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f} + \frac{1}{16}\left(7a(2A-B)c^4\right)\int 1\,dx \\
&= \frac{7}{16}a(2A-B)c^4x + \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} \\
&\quad + \frac{7a(2A-B)c^4\cos(e+fx)\sin(e+fx)}{16f} - \frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} \\
&\quad + \frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} \\
&\quad + \frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$
\int(a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx
$$

$$
= \frac{ac^4\cos(e+fx)\left(272A-176B-\frac{210(2A-B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+15(2A+7B)\sin(e+fx)-32(7A-B)\sin\right.}{240f}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,
x]
```

```
[Out] (a*c^4*Cos[e + f*x]*(272*A - 176*B - (210*(2*A - B)*ArcSin[Sqrt[1 - Sin[e +
 f*x]]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 15*(2*A + 7*B)*Sin[e + f*x] - 32*(7
*A - B)*Sin[e + f*x]^2 + 10*(18*A - 17*B)*Sin[e + f*x]^3 - 48*(A - 3*B)*Sin
[e + f*x]^4 - 40*B*Sin[e + f*x]^5))/(240*f)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{c^4 a\left(\left(-\frac{65A}{3}+\frac{35B}{3}\right)\cos(3fx+3e)+(A-3B)\cos(5fx+5e)+\left(-20A-\frac{5B}{4}\right)\sin(2fx+2e)+\left(\frac{15A}{2}-\frac{35B}{4}\right)\sin(4fx+4e)+\frac{5B}{}\right.}{80f}$
risch	$\frac{7a\,c^4xA}{8}-\frac{7a\,c^4xB}{16}+\frac{7c^4a\cos(fx+e)A}{8f}-\frac{5c^4a\cos(fx+e)B}{8f}-\frac{B\,c^4a\sin(6fx+6e)}{192f}-\frac{c^4a\cos(5fx+5e)A}{80f}+\frac{3c^4a}{}$
parts	$-\frac{(-3A\,c^4a+B\,c^4a)\cos(fx+e)}{f}+\frac{(-3A\,c^4a+2B\,c^4a)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(A\,c^4a-}{}$
derivativedivides	$\frac{-\frac{A\,c^4a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-3A\,c^4a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^4a(2}{}}{}$
default	$\frac{-\frac{A\,c^4a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-3A\,c^4a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^4a(2}{}}{}$
norman	$\frac{(\frac{7}{8}A\,c^4a-\frac{7}{16}B\,c^4a)x+(\frac{7}{8}A\,c^4a-\frac{7}{16}B\,c^4a)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{21}{4}A\,c^4a-\frac{21}{8}B\,c^4a)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{21}{4}A\,c^4a-\frac{21}{8}B\,c}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/80*c^4*a*((-65/3*A+35/3*B)*cos(3*f*x+3*e)+(A-3*B)*cos(5*f*x+5*e)+(-20*A-
5/4*B)*sin(2*f*x+2*e)+(15/2*A-35/4*B)*sin(4*f*x+4*e)+5/12*B*sin(6*f*x+6*e)+
(-70*A+50*B)*cos(f*x+e)-70*f*x*A+35*f*x*B-272/3*A+176/3*B)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx =$$
$$-\frac{48\,(A-3\,B)ac^4\cos{(fx+e)}^5-320\,(A-B)ac^4\cos{(fx+e)}^3-105\,(2\,A-B)ac^4fx+5\,\big(8\,Bac^4\cos{(fx}}{240\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] -1/240*(48*(A - 3*B)*a*c^4*cos(f*x + e)^5 - 320*(A - B)*a*c^4*cos(f*x + e)^
3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*cos(f*x + e)^5 + 2*(18*A - 25*B)
*a*c^4*cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(163) = 326$.

Time = 0.44 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.69

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx$$
$$= \begin{cases} -\frac{9Aac^4x\sin^4(e+fx)}{8} - \frac{9Aac^4x\sin^2(e+fx)\cos^2(e+fx)}{4} + Aac^4x\sin^2(e+fx) - \frac{9Aac^4x\cos^4(e+fx)}{8} + Aac^4x\cos^2(e + \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c)^4 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)**
4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4*co
s(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos(e +
f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin(e + f
*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*cos(e +
 f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f*x)**6/1
6 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*x*sin(e +
 f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a*c**4*
x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)**2/2 + 5*B*
a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 - 3*B*a*c**4*x
*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 3*B*
a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)**3*cos(e +
f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)
/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a*c**4*sin(e + f*
x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 8*B
*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)**3/(3*f) - B*a*c**4
*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c
)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx = -\frac{64 \left(3 \cos\left(fx + e\right)^5 - 10 \cos\left(fx + e\right)^3 + 15 \cos\left(fx + e\right)\right) Aac^4 - 640 \left(\cos\left(fx + e\right)^3 - 3 \cos\left(fx + e\right)\right) A\ldots}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a*c^4
 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - sin(2*f*x +
 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a*c^4 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*
B*a*c^4 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2
*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^4 - 2880*
A*a*c^4*cos(f*x + e) + 960*B*a*c^4*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\begin{aligned} &\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx \\ &= -\frac{Bac^4 \sin\left(6\, fx + 6\, e\right)}{192\, f} + \frac{7}{16} \left(2\, Aac^4 - Bac^4\right) x - \frac{\left(Aac^4 - 3\, Bac^4\right) \cos\left(5\, fx + 5\, e\right)}{80\, f} \\ &\quad + \frac{\left(13\, Aac^4 - 7\, Bac^4\right) \cos\left(3\, fx + 3\, e\right)}{48\, f} + \frac{\left(7\, Aac^4 - 5\, Bac^4\right) \cos\left(fx + e\right)}{8\, f} \\ &\quad - \frac{\left(6\, Aac^4 - 7\, Bac^4\right) \sin\left(4\, fx + 4\, e\right)}{64\, f} + \frac{\left(16\, Aac^4 + Bac^4\right) \sin\left(2\, fx + 2\, e\right)}{64\, f} \end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/192*B*a*c^4*sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A*
a*c^4 - 3*B*a*c^4)*cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*cos(3
*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*cos(f*x + e)/f - 1/64*(6*A*a*c^
4 - 7*B*a*c^4)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*sin(2*f*x +
 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.30 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{A\,a\,c^4}{4}+\frac{7\,B\,a\,c^4}{8}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(6\,A\,a\,c^4-2\,B\,a\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(12\,A\,a\,c^4-4\,B\,a\,c^4\right)}{}$$
$$+\frac{7\,a\,c^4\,\mathrm{atan}\left(\frac{7\,a\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-B)}{8\left(\frac{7\,A\,a\,c^4}{4}-\frac{7\,B\,a\,c^4}{8}\right)}\right)(2\,A-B)}{8\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^10*(
6*A*a*c^4 - 2*B*a*c^4) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^4 - 4*B*a*c^4) - ta
n(e/2 + (f*x)/2)^11*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^8*(2
2*A*a*c^4 - 18*B*a*c^4) + tan(e/2 + (f*x)/2)^5*((13*A*a*c^4)/2 - (37*B*a*c^
4)/4) - tan(e/2 + (f*x)/2)^7*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) + tan(e/2 +
(f*x)/2)^2*((38*A*a*c^4)/5 - (34*B*a*c^4)/5) + tan(e/2 + (f*x)/2)^6*((68*A*
a*c^4)/3 - (44*B*a*c^4)/3) + tan(e/2 + (f*x)/2)^3*((27*A*a*c^4)/4 - (73*B*a
*c^4)/24) - tan(e/2 + (f*x)/2)^9*((27*A*a*c^4)/4 - (73*B*a*c^4)/24) + (34*A
*a*c^4)/15 - (22*B*a*c^4)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*
x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (
f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (7*a*c^4*atan((7*a*c^4*tan(e/2 +
 (f*x)/2)*(2*A - B))/(8*((7*A*a*c^4)/4 - (7*B*a*c^4)/8)))*(2*A - B))/(8*f)
```

3.18 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\begin{aligned}&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx\\&=\frac{1}{8}a(5A-2B)c^3x+\frac{a(5A-2B)c^3\cos^3(e+fx)}{12f}+\frac{a(5A-2B)c^3\cos(e+fx)\sin(e+fx)}{8f}\\&\quad-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}\end{aligned}$$

```
[Out] 1/8*a*(5*A-2*B)*c^3*x+1/12*a*(5*A-2*B)*c^3*cos(f*x+e)^3/f+1/8*a*(5*A-2*B)*c
^3*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^2/f+1/20
*a*(5*A-2*B)*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\begin{aligned}&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx\\&=\frac{ac^3(5A-2B)\cos^3(e+fx)}{12f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}\\&\quad+\frac{ac^3(5A-2B)\sin(e+fx)\cos(e+fx)}{8f}\\&\quad+\frac{1}{8}ac^3x(5A-2B)-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*(5*A - 2*B)*c^3*x)/8 + (a*(5*A - 2*B)*c^3*Cos[e + f*x]^3)/(12*f) + (a*(5
*A - 2*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(c -
 c*Sin[e + f*x])^2)/(5*f) + (a*(5*A - 2*B)*Cos[e + f*x]^3*(c^3 - c^3*Sin[e
+ f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx \\
&= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} \\
&\quad +\frac{1}{5}(a(5A-2B)c)\int \cos^2(e+fx)(c-c\sin(e+fx))^2\,dx \\
&= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} \\
&\quad +\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f} \\
&\quad +\frac{1}{4}\left(a(5A-2B)c^2\right)\int \cos^2(e+fx)(c-c\sin(e+fx))\,dx \\
&= \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f}-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} \\
&\quad +\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f} \\
&\quad +\frac{1}{4}\left(a(5A-2B)c^3\right)\int \cos^2(e+fx)\,dx \\
&= \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f}+\frac{a(5A-2B)c^3\cos(e+fx)\sin(e+fx)}{8f} \\
&\quad -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} \\
&\quad +\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}+\frac{1}{8}\left(a(5A-2B)c^3\right)\int 1\,dx \\
&= \frac{1}{8}a(5A-2B)c^3x+\frac{a(5A-2B)c^3\cos^3(e+fx)}{12f}+\frac{a(5A-2B)c^3\cos(e+fx)\sin(e+fx)}{8f} \\
&\quad -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{ac^3\cos(e+fx)\left(80A-56B-\frac{30(5A-2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+15(3A+2B)\sin(e+fx)+(-80A+32B)\right.}{120f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,
x]
[Out] (a*c^3*Cos[e + f*x]*(80*A - 56*B - (30*(5*A - 2*B)*ArcSin[Sqrt[1 - Sin[e +
f*x]]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 15*(3*A + 2*B)*Sin[e + f*x] + (-80*A
 + 32*B)*Sin[e + f*x]^2 + 30*(A - 2*B)*Sin[e + f*x]^3 + 24*B*Sin[e + f*x]^4
))/(120*f)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{\left(\left(\frac{2A}{3}-\frac{5B}{12}\right)\cos(3fx+3e)+\left(-\frac{A}{8}+\frac{B}{4}\right)\sin(4fx+4e)+\frac{\cos(5fx+5e)B}{20}+A\sin(2fx+2e)+\left(2A-\frac{3B}{2}\right)\cos(fx+e)+\frac{5fxA}{2}-fx\right.}{4f}$
risch	$\frac{5a\,c^3xA}{8}-\frac{a\,c^3xB}{4}+\frac{c^3a\cos(fx+e)A}{2f}-\frac{3c^3a\cos(fx+e)B}{8f}+\frac{Ba\,c^3\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)A\,c^3a}{32f}+\frac{\sin(4}{}$
parts	$-\frac{(-2A\,c^3a+B\,c^3a)\cos(fx+e)}{f}+\frac{(-A\,c^3a+2B\,c^3a)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+a\,c^3xA$
derivativedivides	$\frac{-A\,c^3a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A\cos(fx+e)a\,c^3+\frac{B\,c^3}{}}{}$
default	$\frac{-A\,c^3a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A\cos(fx+e)a\,c^3+\frac{B\,c^3}{}}{}$
norman	$\frac{(\frac{5}{8}A\,c^3a-\frac{1}{4}B\,c^3a)x+(\frac{5}{8}A\,c^3a-\frac{1}{4}B\,c^3a)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,c^3a-\frac{5}{2}B\,c^3a)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,c^3a-\frac{5}{2}B\,c^3}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

[Out] 1/4*((2/3*A-5/12*B)*cos(3*f*x+3*e)+(-1/8*A+1/4*B)*sin(4*f*x+4*e)+1/20*cos(5 *f*x+5*e)*B+A*sin(2*f*x+2*e)+(2*A-3/2*B)*cos(f*x+e)+5/2*f*x*A-f*x*B+8/3*A-2 8/15*B)*c^3*a/f

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3 dx$$
$$= \frac{24\,Bac^3\cos{(fx+e)}^5 + 80\,(A-B)ac^3\cos{(fx+e)}^3 + 15\,(5\,A-2\,B)ac^3fx - 15\,\big(2\,(A-2\,B)ac^3\cos{(fx+}}{120\,f}$$

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm ="fricas")

[Out] 1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5* A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a*c ^3*cos(f*x + e))*sin(f*x + e))/f

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(129) = 258$.

Time = 0.30 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.42

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3 dx$$
$$= \begin{cases} -\frac{3Aac^3x\sin^4{(e+fx)}}{8} - \frac{3Aac^3x\sin^2{(e+fx)}\cos^2{(e+fx)}}{4} - \frac{3Aac^3x\cos^4{(e+fx)}}{8} + Aac^3x + \frac{5Aac^3\sin^3{(e+fx)}\cos{(e+fx)}}{8f} - \frac{2Aac^3}{} \\ x(A + B\sin{(e)})\,(a\sin{(e)} + a)\,(-c\sin{(e)} + c)^3 \end{cases}$$

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)

[Out] Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*c os(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c**3 *sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e + f* x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e + f *x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**4/4 + 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e + f*x)** 2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*a*c**3* sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e + f*x)/(4 *f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*B*a* c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B *sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.41

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aac^3-15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)Aac^3 + \cdots}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 + 32
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*x +
2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*co
s(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= \frac{Bac^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{Aac^3\sin\left(2\,fx+2\,e\right)}{4\,f}\\ &\quad+\frac{1}{8}\left(5\,Aac^3-2\,Bac^3\right)x+\frac{\left(8\,Aac^3-5\,Bac^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}\\ &\quad+\frac{\left(4\,Aac^3-3\,Bac^3\right)\cos\left(fx+e\right)}{8\,f}-\frac{\left(Aac^3-2\,Bac^3\right)\sin\left(4\,fx+4\,e\right)}{32\,f}\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f + 1
/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*sin(
4*f*x + 4*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.74

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aac^3}{4}+\frac{Bac^3}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(4Aac^3-2Bac^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{7Aac^3}{2}-3Bac^3\right)-\mathrm{t}}{f\left(\tan\right.}$$
$$+\frac{ac^3\,\mathrm{atan}\left(\frac{ac^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(5A-2B)}{4\left(\frac{5Aac^3}{4}-\frac{Bac^3}{2}\right)}\right)(5A-2B)}{4f}$$
$$-\frac{ac^3(5A-2B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^8*(4
*A*a*c^3 - 2*B*a*c^3) + tan(e/2 + (f*x)/2)^3*((7*A*a*c^3)/2 - 3*B*a*c^3) -
tan(e/2 + (f*x)/2)^7*((7*A*a*c^3)/2 - 3*B*a*c^3) - tan(e/2 + (f*x)/2)^9*((3
*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 - 8*B*a*c^3) +
 tan(e/2 + (f*x)/2)^2*((8*A*a*c^3)/3 - (8*B*a*c^3)/3) + tan(e/2 + (f*x)/2)^
4*((16*A*a*c^3)/3 - (4*B*a*c^3)/3) + (4*A*a*c^3)/3 - (14*B*a*c^3)/15)/(f*(5
*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 +
 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) + (a*c^3*atan((a*c^3*
tan(e/2 + (f*x)/2)*(5*A - 2*B))/(4*((5*A*a*c^3)/4 - (B*a*c^3)/2)))*(5*A - 2
*B))/(4*f) - (a*c^3*(5*A - 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.19 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 34, antiderivative size = 97

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx &= \frac{1}{8}a(4A-B)c^2x+\frac{a(A-B)c^2\cos^3(e+fx)}{3f}\\ &+\frac{a(4A-B)c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{aBc^2\cos^3(e+fx)\sin(e+fx)}{4f}\end{aligned}$$

```
[Out] 1/8*a*(4*A-B)*c^2*x+1/3*a*(A-B)*c^2*cos(f*x+e)^3/f+1/8*a*(4*A-B)*c^2*cos(f*
x+e)*sin(f*x+e)/f+1/4*a*B*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3046, 2939, 2748, 2715, 8}

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx &= \frac{ac^2(4A-B)\cos^3(e+fx)}{12f}+\frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f}\\ &+\frac{1}{8}ac^2x(4A-B)-\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A - B)*c^2*x)/8 + (a*(4*A - B)*c^2*Cos[e + f*x]^3)/(12*f) + (a*(4*A -
 B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*Cos[e + f*x]^3*(c^2 - c^2*S
in[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$

$$= -\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}+\frac{1}{4}(a(4A-B)c)\int \cos^2(e+fx)(c-c\sin(e+fx))\,dx$$

$$
\begin{aligned}
&= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} - \frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f} \\
&\quad + \frac{1}{4}\left(a(4A-B)c^2\right)\int \cos^2(e+fx)\,dx \\
&= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} + \frac{a(4A-B)c^2\cos(e+fx)\sin(e+fx)}{8f} \\
&\quad - \frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f} + \frac{1}{8}\left(a(4A-B)c^2\right)\int 1\,dx \\
&= \frac{1}{8}a(4A-B)c^2x + \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} \\
&\quad + \frac{a(4A-B)c^2\cos(e+fx)\sin(e+fx)}{8f} - \frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$
\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx
$$

$$
= \frac{ac^2\cos(e+fx)\left(8A-8B-\frac{6(4A-B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+3(4A+B)\sin(e+fx)-8(A-B)\sin^2(e+fx}{24f}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (a*c^2*Cos[e + f*x]*(8*A - 8*B - (6*(4*A - B)*ArcSin[Sqrt[1 - Sin[e + f*x]]
/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 3*(4*A + B)*Sin[e + f*x] - 8*(A - B)*Sin[
e + f*x]^2 - 6*B*Sin[e + f*x]^3))/(24*f)
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{c^{2}\left(\frac{(A-B)\cos(3fx+3e)}{3}+A\sin(2fx+2e)+\frac{B\sin(4fx+4e)}{8}+(A-B)\cos(fx+e)+2fxA-\frac{fxB}{2}+\frac{4A}{3}-\frac{4B}{3}\right)a}{4f}$
risch	$\frac{a\,c^{2}xA}{2}-\frac{a\,c^{2}xB}{8}+\frac{c^{2}a\cos(fx+e)A}{4f}-\frac{c^{2}a\cos(fx+e)B}{4f}+\frac{B\,c^{2}a\sin(4fx+4e)}{32f}+\frac{c^{2}a\cos(3fx+3e)A}{12f}-\frac{c^{2}a\cos(3}{12}$
parts	$\frac{(-A\,c^{2}a-B\,c^{2}a)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-A\,c^{2}a+B\,c^{2}a)\cos(fx+e)}{f}-\frac{(A\,c^{2}a-B\,c^{2}a)(2+\sin^{2}(fx+e))\cos}{3f}$
derivativedivides	$\frac{-\frac{A\,c^{2}a\left(2+\sin^{2}(fx+e)\right)\cos(fx+e)}{3}-A\,c^{2}a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\cos(fx+e)a\,c^{2}+B\,c^{2}a\left(-\frac{\left(\sin^{3}(fx+e)+\frac{3\sin(f}{2}\right.}{4}\right.}{}$
default	$\frac{-\frac{A\,c^{2}a\left(2+\sin^{2}(fx+e)\right)\cos(fx+e)}{3}-A\,c^{2}a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\cos(fx+e)a\,c^{2}+B\,c^{2}a\left(-\frac{\left(\sin^{3}(fx+e)+\frac{3\sin(f}{2}\right.}{4}\right.}{}$
norman	$\frac{(\frac{1}{2}A\,c^{2}a-\frac{1}{8}B\,c^{2}a)x+(2A\,c^{2}a-\frac{1}{2}B\,c^{2}a)x\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2A\,c^{2}a-\frac{1}{2}B\,c^{2}a)x\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3A\,c^{2}a-\frac{3}{4}B\,c^{2}a)x\left(\tan\right.}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*c^2*(1/3*(A-B)*cos(3*f*x+3*e)+A*sin(2*f*x+2*e)+1/8*B*sin(4*f*x+4*e)+(A-
B)*cos(f*x+e)+2*f*x*A-1/2*f*x*B+4/3*A-4/3*B)*a/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\frac{8\,(A-B)ac^2\cos{(fx+e)}^3+3\,(4\,A-B)ac^2fx+3\,\left(2\,Bac^2\cos{(fx+e)}^3+(4\,A-B)ac^2\cos{(fx+e)}\right)\sin}{24\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^2
*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.08

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \begin{cases} -\frac{Aac^2x\sin^2(e+fx)}{2} - \frac{Aac^2x\cos^2(e+fx)}{2} + Aac^2x - \frac{Aac^2\sin^2(e+fx)\cos(e+fx)}{f} + \frac{Aac^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aac^2\cos^3(e}{3f} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c)^2 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 + A
*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*x)
*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e + f
*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x)**4/
8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e + f*x)*c
os(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a*c**
2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(89) = 178$.

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \frac{32\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aac^2 - 24\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aac^2 + 96\,(fx+e)Aac^2 - 32}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin(
2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos(f
*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*cos(f*
x + e) - 96*B*a*c^2*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{Bac^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{Aac^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,Aac^2-Bac^2\right)x + \frac{\left(Aac^2-Bac^2\right)\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{\left(Aac^2-Bac^2\right)\cos\left(fx+e\right)}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*A
*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(A*
a*c^2 - B*a*c^2)*cos(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.56

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a\,c^2+\frac{B\,a\,c^2}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,A\,a\,c^2-2\,B\,a\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(2\,A\,a\,c^2-2\,B\,a\,c^2\right)+\mathrm{t}\cdots}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cdots\right.} + \frac{a\,c^2\,\mathrm{atan}\left(\frac{a\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,A-B\right)}{4\,\left(A\,a\,c^2-\frac{B\,a\,c^2}{4}\right)}\right)\,\left(4\,A-B\right)}{4\,f}-\frac{a\,c^2\,\left(4\,A-B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^4*(2*A*a*c
^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2 +
(f*x)/2)^2*((2*A*a*c^2)/3 - (2*B*a*c^2)/3) - tan(e/2 + (f*x)/2)^7*(A*a*c^2
+ (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^3*(A*a*c^2 - (7*B*a*c^2)/4) - tan(e/2 +
 (f*x)/2)^5*(A*a*c^2 - (7*B*a*c^2)/4) + (2*A*a*c^2)/3 - (2*B*a*c^2)/3)/(f*(
4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 +
tan(e/2 + (f*x)/2)^8 + 1)) + (a*c^2*atan((a*c^2*tan(e/2 + (f*x)/2)*(4*A - B
))/(4*(A*a*c^2 - (B*a*c^2)/4)))*(4*A - B))/(4*f) - (a*c^2*(4*A - B)*(atan(t
an(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.20 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*A*c*x-1/3*a*B*c*cos(f*x+e)^3/f+1/2*a*A*c*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3046, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{aAc\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*A*c*x)/2 - (a*B*c*Cos[e + f*x]^3)/(3*f) + (a*A*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))\,dx\\
&= -\frac{aBc\cos^3(e+fx)}{3f}+(aAc)\int \cos^2(e+fx)\,dx\\
&= -\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}(aAc)\int 1\,dx\\
&= \frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{ac(3B\cos(e+fx)+B\cos(3(e+fx))-3A(-2e+2fx+\sin(2(e+fx))))}{12f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
[Out] -1/12*(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Sin
[2*(e + f*x)])))/f
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{ac(6fxA+3A\sin(2fx+2e)-3\cos(fx+e)B-\cos(3fx+3e)B-4B)}{12f}$
risch	$\frac{aAcx}{2}-\frac{Bac\cos(fx+e)}{4f}-\frac{Bac\cos(3fx+3e)}{12f}+\frac{Aac\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{\frac{Bac\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
default	$\frac{\frac{Bac\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
parts	$aAcx-\frac{Bac\cos(fx+e)}{f}-\frac{Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{Bac(2+\sin^2(fx+e))\cos(fx+e)}{3f}$
norman	$\frac{\frac{Aac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2Bac}{3f}-\frac{2Bac\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{aAcx}{2}-\frac{Aac\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3aAcx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3aAcx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/12*a*c*(6*f*x*A+3*A*sin(2*f*x+2*e)-3*cos(f*x+e)*B-cos(3*f*x+3*e)*B-4*B)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{2\,Bac\cos\left(fx+e\right)^3-3\,Aacfx-3\,Aac\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x +
 e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(46) = 92$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.82

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{Aacx\sin^2(e+fx)}{2} - \frac{Aacx\cos^2(e+fx)}{2} + Aacx + \frac{Aac\sin(e+fx)\cos(e+fx)}{2f} + \frac{Bac\sin^2(e+fx)\cos(e+fx)}{f} + \frac{2Bac\cos^3(e+fx)}{3f} - \\ x(A+B\sin(e))(a\sin(e)+a)(-c\sin(e)+c) \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*x
 + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e + f*
x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$
$$-\frac{3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aac - 12\,(fx+e)Aac + 4\,\left(\cos(fx+e)^3 - 3\,\cos(fx+e)\right)Bac + 12\,Ba}{12\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(c
os(f*x + e)^3 - 3*cos(f*x + e))*B*a*c + 12*B*a*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{1}{2}\,Aacx - \frac{Bac\cos(3\,fx+3\,e)}{12\,f} - \frac{Bac\cos(fx+e)}{4\,f} + \frac{Aac\sin(2\,fx+2\,e)}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1/
4*A*a*c*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx=\frac{A\,a\,c\,x}{2}$$
$$-\frac{A\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(\frac{a\,c\,(12\,B-9\,A\,(e+f\,x))}{6}+\frac{3\,A\,a\,c\,(e+f\,x)}{2}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-A\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a\,c\,(4\,B-3\,A}{6}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^3}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

```
[Out] (A*a*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((a*c*(12*B - 9*A*(e + f*x)))/6 + (3*A*
a*c*(e + f*x))/2) + (a*c*(4*B - 3*A*(e + f*x)))/6 - A*a*c*tan(e/2 + (f*x)/2
) + (A*a*c*(e + f*x))/2 + A*a*c*tan(e/2 + (f*x)/2)^5)/(f*(tan(e/2 + (f*x)/2
)^2 + 1)^3)
```

3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 56

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{a(A+2B)x}{c}+\frac{aB\cos(e+fx)}{cf}+\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}$$

```
[Out] -a*(A+2*B)*x/c+a*B*cos(f*x+e)/c/f+2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3046, 2936, 2718}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax(A+2B)}{c}+\frac{aB\cos(e+fx)}{cf}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((a*(A + 2*B)*x)/c) + (a*B*Cos[e + f*x])/(c*f) + (2*a*(A + B)*Cos[e + f*x]
)/(f*(c - c*Sin[e + f*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} + \frac{a\int(-Ac-2Bc-Bc\sin(e+fx))\,dx}{c^2} \\
&= -\frac{a(A+2B)x}{c} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} - \frac{(aB)\int\sin(e+fx)\,dx}{c} \\
&= -\frac{a(A+2B)x}{c} + \frac{aB\cos(e+fx)}{cf} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{a\cos(e+fx)\left(-2(A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)\sqrt{1-\sin(e+fx)}+\sqrt{1+\sin(e+fx)}(-2A-3B+B\sin(e+fx))\right)}{cf(-1+\sin(e+fx))\sqrt{1+\sin(e+fx)}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a*Cos[e + f*x]*(-2*(A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]*Sqrt[1
 - Sin[e + f*x]] + Sqrt[1 + Sin[e + f*x]]*(-2*A - 3*B + B*Sin[e + f*x])))/(
c*f*(-1 + Sin[e + f*x])*Sqrt[1 + Sin[e + f*x]])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{2a\left(-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$
default	$\frac{2a\left(-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$
parallelrisch	$\frac{2\left(\frac{B\cos(2fx+2e)}{4}+\left(-\frac{1}{2}fxA-fxB+A+\frac{3}{2}B\right)\cos(fx+e)+(A+B)\sin(fx+e)+A+\frac{5B}{4}\right)a}{cf\cos(fx+e)}$
risch	$-\frac{axA}{c}-\frac{2axB}{c}+\frac{Ba\,e^{i(fx+e)}}{2cf}+\frac{Ba\,e^{-i(fx+e)}}{2cf}+\frac{4aA}{fc\left(e^{i(fx+e)}-i\right)}+\frac{4aB}{fc\left(e^{i(fx+e)}-i\right)}$
norman	$\frac{\frac{a(A+2B)x}{c}+\frac{a(A+2B)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4aA+4Ba}{cf}-\frac{2Ba\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2Ba\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{(4aA+2Ba)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-}{\left(1+\tan^2\left(\frac{fx}{2}\right.\right.}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*a/c*(-(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)+B/(1+tan(1/2*f*x+1/2*e)^2)-(A+2*
B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$
$$-\frac{(A+2\,B)afx-Ba\cos\left(fx+e\right)^2-2\,(A+B)a+((A+2\,B)afx-(2\,A+3\,B)a)\cos\left(fx+e\right)-((A+}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x - (
2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A + B
)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(48) = 96$.

Time = 1.03 (sec) , antiderivative size = 828, normalized size of antiderivative = 14.79

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(
e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)**2
/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
- c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 +
f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2)**3
- c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e/2 + f
*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
 c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 +
 f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*
f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2
+ c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f
*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*f*
x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
 - c*f) - 4*B*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2
+ f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*tan(e/2 + f*x/2)/(c*f*tan
(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) -
6*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*
x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(57) = 114$.

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.73

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{2\left(Ba\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+Aa\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)\cdots}}\cdots\right)\right)}{f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c -
 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.09

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = -\frac{\frac{(Aa+2\,Ba)(fx+e)}{c} + \frac{2\left(2\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Aa+3\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)c}}{f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(1
/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f*x
 + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f
```

Mupad [B] (verification not implemented)

Time = 12.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \frac{(4\,A\,a+4\,B\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,a+6\,B\,a}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)} - \frac{A\,a\,f\,x+2\,B\,a\,f\,x}{c\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x)),x)
```

```
[Out] (4*A*a + 6*B*a + tan(e/2 + (f*x)/2)^2*(4*A*a + 4*B*a) - 2*B*a*tan(e/2 + (f*
x)/2))/(f*(c - c*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2 - c*tan(e/2 +
(f*x)/2)^3)) - (A*a*f*x + 2*B*a*f*x)/(c*f)
```

3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{aBx}{c^2} - \frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))} + \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] a*B*x/c^2-1/3*a*(A+7*B)*cos(f*x+e)/c^2/f/(1-sin(f*x+e))+2/3*a*(A+B)*cos(f*x
+e)/f/(c-c*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3046, 2936, 2814, 2727}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))} + \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2} + \frac{aBx}{c^2}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/c^2 - (a*(A + 7*B)*Cos[e + f*x])/(3*c^2*f*(1 - Sin[e + f*x])) + (2*
a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{a\int \frac{-Ac-4Bc-3Bc\sin(e+fx)}{c-c\sin(e+fx)}\,dx}{3c^2} \\
&= \frac{aBx}{c^2}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}-\frac{(a(A+7B))\int \frac{1}{c-c\sin(e+fx)}\,dx}{3c} \\
&= \frac{aBx}{c^2}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}-\frac{a(A+7B)\cos(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.

Time = 6.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.22

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{a\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A+3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)+14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e\right.\right.}{6c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] -1/6*(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e +
 (3*f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B
*Sin[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/(c^
2*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
risch	$\frac{aBx}{c^2}-\frac{2\left(3Aa\,e^{2i(fx+e)}-12iBa\,e^{i(fx+e)}+9Ba\,e^{2i(fx+e)}-aA-7Ba\right)}{3\left(e^{i(fx+e)}-i\right)^3 f\,c^2}$
derivativedivides	$\frac{2a\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}\right)}{f\,c^2}$
default	$\frac{2a\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}\right)}{f\,c^2}$
parallelrisch	$-\frac{2\left(-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(\frac{3}{2}fxB+A-B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B\left(-\frac{3fx}{2}+4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{fxB}{2}+\frac{A}{3}-\frac{5B}{3}\right)a}{f\,c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
norman	$\frac{\frac{axB\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{2aA-10Ba}{3cf}-\frac{axB}{c}-\frac{16Ba\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8Ba\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf}-\frac{(2aA-2Ba)\left(\tan^6\left(\right.\right.}{cf}}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] a*B*x/c^2-2/3*(3*A*a*exp(2*I*(f*x+e))-12*I*B*a*exp(I*(f*x+e))+9*B*a*exp(2*I
*(f*x+e))-a*A-7*B*a)/(exp(I*(f*x+e))-I)^3/f/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.25

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2} dx = -\frac{6\,Bafx - (3\,Bafx + (A + 7\,B)a)\cos(fx + e)^2 + 2\,(A + B)a + (3\,Bafx + (A - 5\,B)a)\cos(fx + e) - (\cdots}{3\,(c^2f\cos(fx + e)^2 - c^2f\cos(fx + e) - 2\,c^2f + (c^2f\cos(\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a +
(3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*a*
f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*
f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(65) = 130$.

Time = 2.03 (sec) , antiderivative size = 700, normalized size of antiderivative = 9.72

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2} dx = \begin{cases} -\frac{6Aa\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{2Aa}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} \cdots \\ \frac{x(A+B\sin(e))(a\sin(e)+a)}{(-c\sin(e)+c)^2} \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*A*a/(3*
c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/
2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 +
f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*B*a*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)
**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3*c**2*f*tan(e/2 +
 f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c
**2*f) + 6*B*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f
```

```
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*B*a*tan(e
/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 +
9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 10*B*a/(3*c**2*f*tan(e/2 + f*x/2)**
3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), N
e(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 6.33

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx$$
$$= \frac{2\left(Ba\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 4}{c^2 - \frac{3c^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right) - \frac{Aa\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2\right)}{c^2 - \frac{3c^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right.}{3f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a*(3
*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx$$
$$= \frac{\frac{3(fx+e)Ba}{c^2} - \frac{2\left(3Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 3Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 12Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + Aa - 5Ba\right)}{c^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}}{3f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2*
f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2*f
*x + 1/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{B\,a\,x}{c^2} - \frac{\left(\frac{a\,(6\,A-6\,B+9\,B\,(e+f\,x))}{3}-3\,B\,a\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\left(\frac{a\,(24\,B-9\,B\,(e+f\,x))}{3}+3\,B\,a\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\frac{f}{2}\right.}{c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (B*a*x)/c^2 - ((a*(2*A - 10*B + 3*B*(e + f*x)))/3 + tan(e/2 + (f*x)/2)^2*((
a*(6*A - 6*B + 9*B*(e + f*x)))/3 - 3*B*a*(e + f*x)) + tan(e/2 + (f*x)/2)*((
a*(24*B - 9*B*(e + f*x)))/3 + 3*B*a*(e + f*x)) - B*a*(e + f*x))/(c^2*f*(tan
(e/2 + (f*x)/2) - 1)^3)
```

3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3} dx$

Optimal result

Integrand size = 34, antiderivative size = 104

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx=\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}-\frac{a(A+11B)c\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] 2/5*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^3-1/15*a*(A+11*B)*c*cos(f*x+e)/f/
(c^2-c^2*sin(f*x+e))^2-1/15*a*(A-4*B)*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3046, 2936, 2829, 2727}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx=-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}-\frac{ac(A+11B)\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) - (a*(A + 11*B)*c*C
os[e + f*x])/(15*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(A - 4*B)*Cos[e + f*x])
/(15*f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}+\frac{a\int \frac{-Ac-6Bc-5Bc\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{5c^2} \\
&= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}-\frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2}-\frac{(a(A-4B))\int \frac{1}{c-c\sin(e+fx)}\,dx}{15c^2} \\
&= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}-\frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2}-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{a\left(15(A-B)\cos\left(e+\frac{fx}{2}\right)-5(A-B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)+25B\sin\left(\frac{fx}{2}\right)+15B\sin\left(2e+\frac{3fx}{2}\right)+\right.}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(f
*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*x
)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(5A+B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}+\frac{4A}{15}-\frac{B}{15}\right)a}{f\,c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$
derivativedivides	$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2B+6A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^3}$
default	$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2B+6A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^3}$
risch	$\frac{-\frac{10Ba\,e^{2i(fx+e)}}{3}-2iBa\,e^{3i(fx+e)}+\frac{2iBa\,e^{i(fx+e)}}{3}+2Ba\,e^{4i(fx+e)}-\frac{2iAa\,e^{i(fx+e)}}{3}-\frac{2Aa\,e^{2i(fx+e)}}{3}-\frac{2aA}{15}+\frac{8Ba}{15}+2iAa\,e^{3i}}{\left(e^{i(fx+e)}-i\right)^5 f\,c^3}$
norman	$\frac{-\frac{8aA-2Ba}{15cf}-\frac{2aA\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{10(aA-Ba)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2(aA-Ba)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(2aA-2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}-\frac{2(11}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2c^2\left(t\right.}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(A*tan(1/2*f*x+1/2*e)^4+(-A+B)*tan(1/2*f*x+1/2*e)^3+1/3*(5*A+B)*tan(1/2*
f*x+1/2*e)^2+1/3*(-A+B)*tan(1/2*f*x+1/2*e)+4/15*A-1/15*B)*a/f/c^3/(tan(1/2*
f*x+1/2*e)-1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{(A-4\,B)a\cos\left(fx+e\right)^3-(2\,A+7\,B)a\cos\left(fx+e\right)^2+3\,(A+B)a\cos\left(fx+e\right)+6\,(A+B)a+\big((A-}{15\,\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A + B
)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*a*
cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*c
os(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*
c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(92) = 184.

Time = 4.34 (sec) , antiderivative size = 1035, normalized size of antiderivative = 9.95

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*
c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*ta
n(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan(e/
2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**
4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c*
*3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c**3*f*
tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 1
5*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a/(15*c**3*f
*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
 f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75
*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*t
```

```
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e
/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)*
*4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f*x/2)/(15*c**3*f*ta
n(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*
x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*
c**3*f) + 2*B*a/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e
) + a)/(-c*sin(e) + c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(101) = 202$.

Time = 0.23 (sec) , antiderivative size = 737, normalized size of antiderivative = 7.09

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 3*A*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/
(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3
*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)
/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*a*(5*sin(f*x + e)/(cos
(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{15\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}$$

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm ="giac")

[Out] -2/15*(15*A*a*tan(1/2*f*x + 1/2*e)^4 - 15*A*a*tan(1/2*f*x + 1/2*e)^3 + 15*B *a*tan(1/2*f*x + 1/2*e)^3 + 25*A*a*tan(1/2*f*x + 1/2*e)^2 + 5*B*a*tan(1/2*f *x + 1/2*e)^2 - 5*A*a*tan(1/2*f*x + 1/2*e) + 5*B*a*tan(1/2*f*x + 1/2*e) + 4 *A*a - B*a)/(c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{11\,A\,a\,\cos(e+f\,x)}{2}-\frac{B\,a}{4}-\frac{41\,A\,a}{4}+\frac{B\,a\,\cos(e+f\,x)}{2}+5\,A\,a\,\sin\left(e+f\,x\right)-5\,B\,a\,\sin\left(e+f\,x\right)+\frac{3\,A}{}\right.}{15\,c^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\mathrm{co}}{}\right.}$$

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^3,x)

[Out] (2*cos(e/2 + (f*x)/2)*((11*A*a*cos(e + f*x))/2 - (B*a)/4 - (41*A*a)/4 + (B* a*cos(e + f*x))/2 + 5*A*a*sin(e + f*x) - 5*B*a*sin(e + f*x) + (3*A*a*cos(2* e + 2*f*x))/4 + (3*B*a*cos(2*e + 2*f*x))/4 - (5*A*a*sin(2*e + 2*f*x))/4 + (5*B*a*sin(2*e + 2*f*x))/4))/(15*c^3*f*((5*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f *x)/2))/4 - (5*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4))

3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2} - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] 2/7*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^4-1/35*a*(A+15*B)*cos(f*x+e)/c/f/
(c-c*sin(f*x+e))^3-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^2-1/
105*a*(2*A-5*B)*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3046, 2936, 2829, 2729, 2727}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = -\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)} - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} + \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) - (a*(A + 15*B)*Cos
[e + f*x])/(35*c*f*(c - c*Sin[e + f*x])^3) - (a*(2*A - 5*B)*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^
4 - c^4*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}+\frac{a\int \frac{-Ac-8Bc-7Bc\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{7c^2} \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}-\frac{(a(2A-5B))\int \frac{1}{(c-c\sin(e+fx))^2}\,dx}{35c^2} \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} \\
&\quad -\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{(a(2A-5B))\int \frac{1}{c-c\sin(e+fx)}\,dx}{105c^3} \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} \\
&\quad -\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx
= \frac{a\left(35(4A-B)\cos\left(e+\frac{fx}{2}\right)-42A\cos\left(e+\frac{3fx}{2}\right)+2A\cos\left(3e+\frac{7fx}{2}\right)-5B\cos\left(3e+\frac{7fx}{2}\right)+70A\sin\left(\frac{fx}{2}\right)+\cdots\right.}{420c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\cdots\right.}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e +
 (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f*x
)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin[2*
e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^7)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(B-2A)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(13A-B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{2(-5A+2B)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{13A\left(\tan^{2}\left(\frac{fx}{2}\right.\right.}{5}\right.}{f\,c^{4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}$
risch	$-\frac{2ia\left(140iA\,e^{4i(fx+e)}-35iB\,e^{4i(fx+e)}+105B\,e^{5i(fx+e)}-42iA\,e^{2i(fx+e)}-70A\,e^{3i(fx+e)}-140B\,e^{3i(fx+e)}+2iA-14A\,e^{i(\right.}{105f\,c^{4}\left(e^{i(fx+e)}-i\right)^{7}}$
derivativedivides	$\frac{2a\left(-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{48A+48B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^{4}}$
default	$\frac{2a\left(-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{48A+48B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^{4}}$
norman	$\frac{\frac{(4aA-2Ba)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{46aA-10Ba}{105cf}-\frac{2aA\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(16aA-10Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cf}+\frac{2(22aA-10Ba)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(A*tan(1/2*f*x+1/2*e)^6+(B-2*A)*tan(1/2*f*x+1/2*e)^5+1/3*(13*A-B)*tan(1/
2*f*x+1/2*e)^4+2/3*(-5*A+2*B)*tan(1/2*f*x+1/2*e)^3+13/5*A*tan(1/2*f*x+1/2*e
)^2+1/3*(-8/5*A+B)*tan(1/2*f*x+1/2*e)+23/105*A-1/21*B)*a/f/c^4/(tan(1/2*f*x
+1/2*e)-1)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.77

$$\int\frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$=\frac{(2A-5B)a\cos(fx+e)^4+4(2A-5B)a\cos(fx+e)^3-3(3A+10B)a\cos(fx+e)^2+15(A+B)a\,c}{105\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+4c\right.}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(3
*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a - ((
2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B)*a
*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f
```

```
*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f +
 (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*
c^4*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1831 vs. $2(124) = 248$.

Time = 8.62 (sec) , antiderivative size = 1831, normalized size of antiderivative = 12.89

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*A*a*tan(e
/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 910*A*a*tan(e/2 + f*x/2)**4/(105*c**4*
f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/
2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x
/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 10
5*c**4*f) + 700*A*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 546*A*a*tan(e
/2 + f*x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 112*A*a*tan(e/2 + f*x/2)/(105*c**4*f*t
an(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
 f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)
**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c
**4*f) - 46*A*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/
2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c*
*4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 210*B*a*tan(e/2 + f*x/2)**5/(105*c**4
*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e
/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*
x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 1
05*c**4*f) + 70*B*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
```

```
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 280*B*a*tan(e
/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 70*B*a*tan(e/2 + f*x/2)/(105*c**4*f*ta
n(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)*
*3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c*
*4*f) + 10*B*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e)
+ a)/(-c*sin(e) + c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(138) = 276$.

Time = 0.24 (sec) , antiderivative size = 1080, normalized size of antiderivative = 7.61

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] 2/105*(A*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + B*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(
c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7) - 3*A*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos
(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*
```

```
x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*B*a*(14*sin(f*
x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.24

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{2\left(105\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+105\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,Aa\tan\left(\frac{1}{2}\,fx\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -2/105*(105*A*a*tan(1/2*f*x + 1/2*e)^6 - 210*A*a*tan(1/2*f*x + 1/2*e)^5 + 1
05*B*a*tan(1/2*f*x + 1/2*e)^5 + 455*A*a*tan(1/2*f*x + 1/2*e)^4 - 35*B*a*tan
(1/2*f*x + 1/2*e)^4 - 350*A*a*tan(1/2*f*x + 1/2*e)^3 + 140*B*a*tan(1/2*f*x
+ 1/2*e)^3 + 273*A*a*tan(1/2*f*x + 1/2*e)^2 - 56*A*a*tan(1/2*f*x + 1/2*e) +
 35*B*a*tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.61

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{15\,B\,a}{4}-\frac{171\,A\,a}{2}+\frac{353\,A\,a\,\cos(e+f\,x)}{8}+\frac{5\,B\,a\,\cos(e+f\,x)}{4}+\frac{595\,A\,a\,\sin(e+f\,x)}{8}-35\,B\,a\,\sin\left(e+f\,x\right)\right.}{105\,c^4\,f\,\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\frac{21}{}\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^4,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((15*B*a)/4 - (171*A*a)/2 + (353*A*a*cos(e + f*x))/8
 + (5*B*a*cos(e + f*x))/4 + (595*A*a*sin(e + f*x))/8 - 35*B*a*sin(e + f*x)
+ (43*A*a*cos(2*e + 2*f*x))/2 - (25*A*a*cos(3*e + 3*f*x))/8 - (5*B*a*cos(2*
e + 2*f*x))/4 + (5*B*a*cos(3*e + 3*f*x))/4 - (77*A*a*sin(2*e + 2*f*x))/4 -
(21*A*a*sin(3*e + 3*f*x))/8 + (35*B*a*sin(2*e + 2*f*x))/4))/(105*c^4*f*((35
*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3
*f*x)/2))/8 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos(
(7*e)/2 - pi/4 + (7*f*x)/2))/8))
```

3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)c\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3} - \frac{2a(A-2B)c\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2} - \frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}$$

```
[Out] 2/9*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^5-1/63*a*(A+19*B)*cos(f*x+e)/c/f/
(c-c*sin(f*x+e))^4-1/105*a*(A-2*B)*c*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^3-2/
315*a*(A-2*B)*c*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))^2-2/315*a*(A-2*B)*cos(f*x
+e)/f/(c^5-c^5*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used

= {3046, 2936, 2829, 2729, 2727}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)} - \frac{2ac(A-2B)\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2} - \frac{ac(A-2B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} + \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(9*f*(c - c*Sin[e + f*x])^5) - (a*(A + 19*B)*Cos
[e + f*x])/(63*c*f*(c - c*Sin[e + f*x])^4) - (a*(A - 2*B)*c*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a*(A - 2*B)*c*Cos[e + f*x])/(315*f*(
c^3 - c^3*Sin[e + f*x])^2) - (2*a*(A - 2*B)*Cos[e + f*x])/(315*f*(c^5 - c^5
*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
```

```
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} + \frac{a\int \frac{-Ac-10Bc-9Bc\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{9c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{(a(A-2B))\int \frac{1}{(c-c\sin(e+fx))^3}\,dx}{21c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)\cos(e+fx)}{105c^2f(c-c\sin(e+fx))^3} - \frac{(2a(A-2B))\int \frac{1}{(c-c\sin(e+fx))^2}\,dx}{105c^3}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)\cos(e+fx)}{105c^2f(c-c\sin(e+fx))^3} - \frac{2a(A-2B)\cos(e+fx)}{315c^3f(c-c\sin(e+fx))^2} - \frac{(2a(A-2B))\int \frac{1}{c-c\sin(e+fx)}\,dx}{315c^4}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)\cos(e+fx)}{105c^2f(c-c\sin(e+fx))^3} - \frac{2a(A-2B)\cos(e+fx)}{315c^3f(c-c\sin(e+fx))^2} - \frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 6.47 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.14

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a\left(315A\cos\left(e+\frac{fx}{2}\right)-42(2A+B)\cos\left(e+\frac{3fx}{2}\right)+9A\cos\left(3e+\frac{7fx}{2}\right)-18B\cos\left(3e+\frac{7fx}{2}\right)+189A\sin\left(\frac{fx}{2}\right.\right.}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\right.\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] (a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e
+ (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin[(
f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*Sin
[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))/(12
60*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{4\left(-42iBa\,e^{3i(fx+e)}+210Ba\,e^{6i(fx+e)}+9iAa\,e^{i(fx+e)}-2Ba-18iBa\,e^{i(fx+e)}+aA-84iAa\,e^{3i(fx+e)}-36Aa\,e^{2i(fx+e)}+\right.}{315\left(e^{i(fx+e)}-i\right)^9 f\,c^5}$
parallelrisch	$-\frac{2a\left(A\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3A+B)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{25A}{3}-B\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-11A+3B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(61A}{}\right.}{f\,c^5\left(\tan\left(\frac{f}{}\right.\right.}$
derivativedivides	$\frac{2a\left(-\frac{128A+128B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{296A+248B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32A+32B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{f\,c^5}$
default	$\frac{2a\left(-\frac{128A+128B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{296A+248B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32A+32B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{f\,c^5}$
norman	$\frac{(34aA-10Ba)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{116aA-22Ba}{315cf}-\frac{2aA\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2(3aA-Ba)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2(31aA-3Ba)\left(\tan^{10}\left(\frac{fx}{2}+\right.\right.}{3cf}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*(-42*I*B*a*exp(3*I*(f*x+e))+210*B*a*exp(6*I*(f*x+e))+9*I*A*a*exp(I*(
f*x+e))-2*B*a-18*I*B*a*exp(I*(f*x+e))+a*A-84*I*A*a*exp(3*I*(f*x+e))-36*A*a*
```

```
exp(2*I*(f*x+e))+72*B*a*exp(2*I*(f*x+e))+315*I*A*a*exp(5*I*(f*x+e))-189*A*a
*exp(4*I*(f*x+e))-252*B*a*exp(4*I*(f*x+e)))/(exp(I*(f*x+e))-I)^9/f/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.73

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{2\,(A-2\,B)a\cos{(fx+e)}^5-8\,(A-2\,B)a\cos{(fx+e)}^4-25\,(A-2\,B)a\cos{(fx+e)}^3+5\,(4\,A+13\,B}{315\,\left(c^5f\cos{(fx+e)}^5+5\,c^5f\cos{(fx+e)}^4-8\,c^5f\cos{(fx+}\right.}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="fricas")
```

```
[Out] -1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(A
 - 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*c
os(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B)*a
*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x + e)
 - 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)
^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e
) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*co
s(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3232 vs. $2(160) = 320$.

Time = 15.82 (sec) , antiderivative size = 3232, normalized size of antiderivative = 18.36

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*A*a*tan
(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*
x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)*
*6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 +
```

```
26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*
c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*A*a*tan(e/2 + f*x/2)**6/(315*c
**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*
tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e
/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 +
f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2)
 - 315*c**5*f) + 6930*A*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)*
*9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 2
6460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*
c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*
f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 7686*A
*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f
*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)
**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 +
 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 4494*A*a*tan(e/2 + f*x/2)**3/
(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c
**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f
*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(
e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 +
f*x/2) - 315*c**5*f) - 2286*A*a*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f
*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)*
*7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 -
39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340
*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) +
414*A*a*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(
e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 +
 f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/
2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2
 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 116*A*a/(315*c**5*f*tan(e/2
 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x
/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**
5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 1
1340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f
) - 630*B*a*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5
*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*ta
n(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2
 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*
x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 630*B*a*tan(e/2 + f*
x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 +
 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 3969
0*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**
5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*ta
n(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan
(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 +
```

```
f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/
2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3
 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c*
*5*f) + 882*B*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*
c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*
f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan
(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
+ f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1218*B*a*tan(e/2
 + f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) + 162*B*a*tan(e/2 + f*x/2)**2/(315*c**5*f
*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e
/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 +
f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 31
5*c**5*f) - 198*B*a*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835
*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5
*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*ta
n(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 22*B*a/(315*c**
5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*ta
n(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2
 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*
x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) -
 315*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**
5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(171) = 342$.

Time = 0.26 (sec) , antiderivative size = 1425, normalized size of antiderivative = 8.10

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="maxima")
```

```
[Out] -2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
```

```
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 5*A*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14
7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*
c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a*(45*sin(f*x + e)/(cos(f*x + e
) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x +
e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*
c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*B*a
*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^
5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$

$$-\frac{2\left(315\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+315\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,Aa\tan\left(\frac{1}{2}\,fx\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="giac")
```

```
[Out] -2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 + 3
15*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a*t
an(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/2*f
*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x + 1/2
*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)^3 +
1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207*A*a*t
an(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)/(c^5*f
*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1357\,A\,a}{4}-\frac{461\,B\,a}{16}-\frac{635\,A\,a\,\cos(e+f\,x)}{4}+\frac{5\,B\,a\,\cos(e+f\,x)}{2}-\frac{1575\,A\,a\,\sin(e+f\,x)}{4}+\frac{945\,B\,a\,\sin(e+f\,x)}{8}-62\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((1357*A*a)/4 - (461*B*a)/16 - (635*A*a*cos(e + f*x))
/4 + (5*B*a*cos(e + f*x))/2 - (1575*A*a*sin(e + f*x))/4 + (945*B*a*sin(e +
f*x))/8 - (625*A*a*cos(2*e + 2*f*x))/4 + (121*A*a*cos(3*e + 3*f*x))/4 + (7*
A*a*cos(4*e + 4*f*x))/2 + (95*B*a*cos(2*e + 2*f*x))/4 - 8*B*a*cos(3*e + 3*f
*x) - (7*B*a*cos(4*e + 4*f*x))/16 + (399*A*a*sin(2*e + 2*f*x))/4 + (141*A*a
*sin(3*e + 3*f*x))/4 - (15*A*a*sin(4*e + 4*f*x))/4 - (231*B*a*sin(2*e + 2*f
*x))/8 - (39*B*a*sin(3*e + 3*f*x))/8 + (15*B*a*sin(4*e + 4*f*x))/16))/(315*
c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 -
 pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4 + (9*
2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2 + pi/4 +
 (9*f*x)/2))/16))
```

3.26 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal result

Integrand size = 36, antiderivative size = 229

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx\\
&=\frac{9}{128}a^2(8A-3B)c^5x+\frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f}\\
&\quad+\frac{9a^2(8A-3B)c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{3a^2(8A-3B)c^5\cos^3(e+fx)\sin(e+fx)}{64f}\\
&\quad+\frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f}\\
&\quad-\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}\\
&\quad+\frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f}
\end{aligned}$$

```
[Out] 9/128*a^2*(8*A-3*B)*c^5*x+3/80*a^2*(8*A-3*B)*c^5*cos(f*x+e)^5/f+9/128*a^2*(
8*A-3*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+3/64*a^2*(8*A-3*B)*c^5*cos(f*x+e)^3*si
n(f*x+e)/f+1/56*a^2*(8*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^2/f-1/8*a^2
*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^3/f+3/112*a^2*(8*A-3*B)*cos(f*x+e)^5*(
c^5-c^5*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{3a^2c^5(8A-3B)\cos^5(e+fx)}{80f}+\frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f} \\
&+\frac{3a^2c^5(8A-3B)\sin(e+fx)\cos^3(e+fx)}{64f}+\frac{9a^2c^5(8A-3B)\sin(e+fx)\cos(e+fx)}{128f} \\
&+\frac{9}{128}a^2c^5x(8A-3B)+\frac{a^2c^3(8A-3B)\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&-\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}
\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (9*a^2*(8*A - 3*B)*c^5*x)/128 + (3*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]^5)/(80*
f) + (9*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (3*a^2*(8*
A - 3*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64*f) + (a^2*(8*A - 3*B)*c^3*Cos
[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(56*f) - (a^2*B*c^2*Cos[e + f*x]^5*(c -
 c*Sin[e + f*x])^3)/(8*f) + (3*a^2*(8*A - 3*B)*Cos[e + f*x]^5*(c^5 - c^5*Si
n[e + f*x]))/(112*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= -\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad +\frac{1}{8}\left(a^2(8A-3B)c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^3\,dx \\
&= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad -\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad +\frac{1}{56}\left(9a^2(8A-3B)c^3\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^2\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad - \frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad + \frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f} \\
&\quad + \frac{1}{16}\left(3a^2(8A-3B)c^4\right)\int \cos^4(e+fx)(c-c\sin(e+fx))\,dx \\
&= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad - \frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad + \frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f} \\
&\quad + \frac{1}{16}\left(3a^2(8A-3B)c^5\right)\int \cos^4(e+fx)\,dx \\
&= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{3a^2(8A-3B)c^5\cos^3(e+fx)\sin(e+fx)}{64f} \\
&\quad + \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad - \frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad + \frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f} \\
&\quad + \frac{1}{64}\left(9a^2(8A-3B)c^5\right)\int \cos^2(e+fx)\,dx \\
&= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{9a^2(8A-3B)c^5\cos(e+fx)\sin(e+fx)}{128f} \\
&\quad + \frac{3a^2(8A-3B)c^5\cos^3(e+fx)\sin(e+fx)}{64f} \\
&\quad + \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad - \frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad + \frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f} + \frac{1}{128}\left(9a^2(8A-3B)c^5\right)\int 1\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9}{128}a^2(8A-3B)c^5x + \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} \\
&\quad + \frac{9a^2(8A-3B)c^5\cos(e+fx)\sin(e+fx)}{128f} \\
&\quad + \frac{3a^2(8A-3B)c^5\cos^3(e+fx)\sin(e+fx)}{64f} \\
&\quad + \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad - \frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad + \frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$
\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx
$$

$$
= \frac{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5(2520(8A-3B)(e+fx)+560(27A-17B)\cos(e+fx)+560(13}{}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x)
+ 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 112*
(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A - 3
*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*Sin[6
*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{11c^5\left(\frac{5(13A-7B)\cos(3fx+3e)}{11}+\left(A-\frac{B}{11}\right)\cos(5fx+5e)+\frac{5(-A+3B)\cos(7fx+7e)}{77}+\frac{5(19A-3B)\sin(2fx+2e)}{11}+\frac{5\left(-A+\frac{7B}{2}\right)\sin(}{11}\right.}{320f}$
risch	$\frac{9a^2c^5xA}{16}-\frac{27a^2c^5xB}{128}+\frac{27c^5a^2\cos(fx+e)A}{64f}-\frac{17c^5a^2\cos(fx+e)B}{64f}-\frac{B\,a^2c^5\sin(8fx+8e)}{1024f}-\frac{c^5a^2\cos(7fx+7e}{448f}$
parts	$\frac{(-5A\,a^2c^5+5B\,a^2c^5)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(-3A\,a^2c^5+B\,a^2c^5)\cos(fx+e)}{f}-\frac{(-A}{}$
derivativedivides	$\frac{A\,a^2c^5(fx+e)+\frac{A\,a^2c^5\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+3A\,a^2c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx-\right.}{4}\right.}{}\right.}{}$
default	$\frac{A\,a^2c^5(fx+e)+\frac{A\,a^2c^5\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+3A\,a^2c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx-\right.}{4}\right.}{}\right.}{}$
norman	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 11/320*c^5*(5/11*(13*A-7*B)*cos(3*f*x+3*e)+(A-1/11*B)*cos(5*f*x+5*e)+5/77*(
-A+3*B)*cos(7*f*x+7*e)+5/11*(19*A-3*B)*sin(2*f*x+2*e)+5/11*(-A+7/2*B)*sin(4
*f*x+4*e)+5/11*(-A+B)*sin(6*f*x+6*e)-5/176*B*sin(8*f*x+8*e)+5/11*(27*A-17*B
)*cos(f*x+e)+180/11*f*x*A-135/22*f*x*B+1472/77*A-832/77*B)*a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx =$$
$$-\frac{640\,(A-3\,B)a^2c^5\cos{(fx+e)}^7-3584\,(A-B)a^2c^5\cos{(fx+e)}^5-315\,(8\,A-3\,B)a^2c^5fx+35\,(16\,B}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f*
x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7 +
8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x + e)^
3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(218) = 436$.

Time = 0.87 (sec) , antiderivative size = 1586, normalized size of antiderivative = 6.93

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^5\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*x*co
s(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*cos(e
 + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f -
 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*sin(e +
 f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5
*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**5*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*A*a**2*c**
5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)
**5/(16*f) + 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*c**
5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*x)**7/(35*f) +
 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e + f*x)**3/(3*f
) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e + f*x)**8/128 - 3
5*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a**2*c**5*x*sin(e
+ f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 15*B*
a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a**2*c**5*x*sin(e + f
*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 15*B*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 15*B*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 - 3*B*a**2*c**5*x*sin(e + f*x)**2/2 - 35*B*a**2*c**5*x*
cos(e + f*x)**8/128 - 5*B*a**2*c**5*x*cos(e + f*x)**6/16 + 15*B*a**2*c**5*x
*cos(e + f*x)**4/8 - 3*B*a**2*c**5*x*cos(e + f*x)**2/2 + 93*B*a**2*c**5*sin
(e + f*x)**7*cos(e + f*x)/(128*f) - 3*B*a**2*c**5*sin(e + f*x)**6*cos(e + f
*x)/f + 511*B*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 11*B*a**2
*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 6*B*a**2*c**5*sin(e + f*x)**4*c
os(e + f*x)**3/f + 5*B*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*B*a**
2*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**2*c**5*sin(e + f*x)
**3*cos(e + f*x)**3/(6*f) - 25*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*
f) - 24*B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 20*B*a**2*c**5*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*c**5*sin(e + f*x)**2*cos(e +
 f*x)/f + 35*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 5*B*a**2*c*
*5*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 15*B*a**2*c**5*sin(e + f*x)*cos(e
+ f*x)**3/(8*f) + 3*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) - 48*B*a**2
```

```
*c**5*cos(e + f*x)**7/(35*f) + 8*B*a**2*c**5*cos(e + f*x)**5/(3*f) - 2*B*a*
*2*c**5*cos(e + f*x)**3/(3*f) - B*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(218) = 436.

Time = 0.23 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.49

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^5 \, dx =$$
$$-\frac{3072\left(5\cos\left(fx+e\right)^7 - 21\cos\left(fx+e\right)^5 + 35\cos\left(fx+e\right)^3 - 35\cos\left(fx+e\right)\right)Aa^2c^5 - 7168\left(3\cos\left(fx\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2
*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*
sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*
c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f*x + e)
^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^2*
c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*s
in(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^
5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.18

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=-\frac{Ba^2c^5\sin(8fx+8e)}{1024f}+\frac{9}{128}\left(8Aa^2c^5-3Ba^2c^5\right)x-\frac{(Aa^2c^5-3Ba^2c^5)\cos(7fx+7e)}{448f}$$
$$+\frac{(11Aa^2c^5-Ba^2c^5)\cos(5fx+5e)}{320f}+\frac{(13Aa^2c^5-7Ba^2c^5)\cos(3fx+3e)}{64f}$$
$$+\frac{(27Aa^2c^5-17Ba^2c^5)\cos(fx+e)}{64f}-\frac{(Aa^2c^5-Ba^2c^5)\sin(6fx+6e)}{64f}$$
$$-\frac{(2Aa^2c^5-7Ba^2c^5)\sin(4fx+4e)}{128f}+\frac{(19Aa^2c^5-3Ba^2c^5)\sin(2fx+2e)}{64f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x
- 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^5
- B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(3*f
*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*(A*a
^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*c^5)*
sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.89

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}(6Aa^2c^5-2Ba^2c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(30Aa^2c^5-10Ba^2c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(22Aa^2c^5}{}$$
$$+\frac{9a^2c^5\,\mathrm{atan}\left(\frac{9a^2c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8A-3B)}{64\left(\frac{9Aa^2c^5}{8}-\frac{27Ba^2c^5}{64}\right)}\right)(8A-3B)}{64f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^14*(6*A*a^2*c^5 - 2*B*a^2*c^5) + tan(e/2 + (f*x)/2)^10*
(30*A*a^2*c^5 - 10*B*a^2*c^5) + tan(e/2 + (f*x)/2)^12*(22*A*a^2*c^5 - 18*B*
a^2*c^5) + tan(e/2 + (f*x)/2)^8*(46*A*a^2*c^5 - 26*B*a^2*c^5) + tan(e/2 + (
f*x)/2)^4*((74*A*a^2*c^5)/5 - (14*B*a^2*c^5)/5) - tan(e/2 + (f*x)/2)^15*((7
*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^2*((158*A*a^2*c^5)/
35 - (138*B*a^2*c^5)/35) + tan(e/2 + (f*x)/2)^6*((218*A*a^2*c^5)/5 - (158*B
```

```
*a^2*c^5)/5) + tan(e/2 + (f*x)/2)^3*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64)
 - tan(e/2 + (f*x)/2)^13*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) + tan(e/2
+ (f*x)/2)^5*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^1
1*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^7*((13*A*a^2
*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^9*((13*A*a^2*c^5)/8 - (9
19*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64
) + (46*A*a^2*c^5)/35 - (26*B*a^2*c^5)/35)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*
tan(e/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 +
56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^
14 + tan(e/2 + (f*x)/2)^16 + 1)) + (9*a^2*c^5*atan((9*a^2*c^5*tan(e/2 + (f*
x)/2)*(8*A - 3*B))/(64*((9*A*a^2*c^5)/8 - (27*B*a^2*c^5)/64)))*(8*A - 3*B))
/(64*f)
```

3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal result

Integrand size = 36, antiderivative size = 189

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\
&= \frac{1}{16}a^2(7A-2B)c^4x+\frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} \\
&\quad+\frac{a^2(7A-2B)c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^2(7A-2B)c^4\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad-\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} \\
&\quad+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}
\end{aligned}$$

```
[Out] 1/16*a^2*(7*A-2*B)*c^4*x+1/30*a^2*(7*A-2*B)*c^4*cos(f*x+e)^5/f+1/16*a^2*(7*
A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(7*A-2*B)*c^4*cos(f*x+e)^3*sin(
f*x+e)/f-1/7*a^2*B*cos(f*x+e)^5*(c^2-c^2*sin(f*x+e))^2/f+1/42*a^2*(7*A-2*B)
*cos(f*x+e)^5*(c^4-c^4*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{a^2c^4(7A-2B)\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)\cos^5(e+fx)\,(c^4-c^4\sin(e+fx))}{42f}$$
$$+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos(e+fx)}{16f}$$
$$+\frac{1}{16}a^2c^4x(7A-2B)-\frac{a^2B\cos^5(e+fx)\,(c^2-c^2\sin(e+fx))^2}{7f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(7*A - 2*B)*c^4*x)/16 + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^5)/(30*f) +
(a^2*(7*A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(7*A - 2*B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^2 - c^2*S
in[e + f*x])^2)/(7*f) + (a^2*(7*A - 2*B)*Cos[e + f*x]^5*(c^4 - c^4*Sin[e +
f*x]))/(42*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\
&= -\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f}\\
&\quad+\frac{1}{7}\left(a^2(7A-2B)c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^2\,dx\\
&= -\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f}\\
&\quad+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}\\
&\quad+\frac{1}{6}\left(a^2(7A-2B)c^3\right)\int \cos^4(e+fx)(c-c\sin(e+fx))\,dx\\
&= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f}-\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f}\\
&\quad+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}+\frac{1}{6}\left(a^2(7A-2B)c^4\right)\int \cos^4(e\\
&\quad+fx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} + \frac{a^2(7A-2B)c^4\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad - \frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} \\
&\quad + \frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f} \\
&\quad + \frac{1}{8}\left(a^2(7A-2B)c^4\right)\int \cos^2(e+fx)\,dx \\
&= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} + \frac{a^2(7A-2B)c^4\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad + \frac{a^2(7A-2B)c^4\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad - \frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} \\
&\quad + \frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f} + \frac{1}{16}\left(a^2(7A-2B)c^4\right)\int 1\,dx \\
&= \frac{1}{16}a^2(7A-2B)c^4x + \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} \\
&\quad + \frac{a^2(7A-2B)c^4\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad + \frac{a^2(7A-2B)c^4\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad - \frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} \\
&\quad + \frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$
\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx
= \frac{a^2c^4(2940Ae-840Be+2940Afx-840Bfx+105(16A-11B)\cos(e+fx)+105(8A-5B)\cos(3(e+
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*C
os[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] - 6
3*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] - 21
0*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)] - 35
*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)]))/(6720*f)
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{\left(5\left(A-\frac{5B}{8}\right)\cos(3fx+3e)+\left(A-\frac{3B}{8}\right)\cos(5fx+5e)+\frac{5\left(\frac{17A}{2}-B\right)\sin(2fx+2e)}{4}+\frac{5\left(\frac{A}{2}+B\right)\sin(4fx+4e)}{4}+\frac{5\left(-\frac{A}{2}+B\right)\sin(6fx+6e}{12}\right.}{40f}$
risch	$\frac{7a^2c^4xA}{16}-\frac{a^2c^4xB}{8}+\frac{a^2c^4\cos(fx+e)A}{4f}-\frac{11a^2c^4\cos(fx+e)B}{64f}+\frac{B\,a^2c^4\cos(7fx+7e)}{448f}-\frac{\sin(6fx+6e)A\,a^2c^4}{192f}+$
parts	$-\frac{(-2A\,a^2c^4-B\,a^2c^4)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)}{5f}-\frac{(-2A\,a^2c^4+B\,a^2c^4)\cos(fx+e)}{f}+\frac{(-A\,a^2c^4-}{}$
derivativedivides	$\frac{A\,a^2c^4(fx+e)+\frac{B\,a^2c^4(2+\sin^2(fx+e))\cos(fx+e)}{3}+2A\,a^2c^4\cos(fx+e)-2B\,a^2c^4\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\,a^2c^4\left(\right.}{}$
default	$\frac{A\,a^2c^4(fx+e)+\frac{B\,a^2c^4(2+\sin^2(fx+e))\cos(fx+e)}{3}+2A\,a^2c^4\cos(fx+e)-2B\,a^2c^4\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\,a^2c^4\left(\right.}{}$
norman	$\frac{(\frac{7}{16}A\,a^2c^4-\frac{1}{8}B\,a^2c^4)x+(\frac{7}{16}A\,a^2c^4-\frac{1}{8}B\,a^2c^4)x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{49}{16}A\,a^2c^4-\frac{7}{8}B\,a^2c^4)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{49}{16}A\,a^2c^4-}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/40*(5*(A-5/8*B)*cos(3*f*x+3*e)+(A-3/8*B)*cos(5*f*x+5*e)+5/4*(17/2*A-B)*si
n(2*f*x+2*e)+5/4*(1/2*A+B)*sin(4*f*x+4*e)+5/12*(-1/2*A+B)*sin(6*f*x+6*e)+5/
56*cos(7*f*x+7*e)*B+5*(2*A-11/8*B)*cos(f*x+e)+35/2*f*x*A-5*f*x*B+16*A-72/7*
B)*c^4*a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{240\,Ba^2c^4\cos(fx+e)^7+672\,(A-B)a^2c^4\cos(fx+e)^5+105\,(7\,A-2\,B)a^2c^4fx-35\,\big(8\,(A-2\,B)a^2c^4\,\mathrm{co}}{1680\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5 +
 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B)*a^2*c^4*cos(f*x + e)^5 - 2*(
7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. $2(172) = 344$.

Time = 0.62 (sec) , antiderivative size = 1210, normalized size of antiderivative = 6.40

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*c*
*4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*cos(e
+ f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e + f*
x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f
) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8
*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a**2*c**4*s
in(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/
(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**4*sin
(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(15*f) - 8*A*
a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f - 5*B*a**2*c
**4*x*sin(e + f*x)**6/8 - 15*B*a**2*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/
8 + 3*B*a**2*c**4*x*sin(e + f*x)**4/2 - 15*B*a**2*c**4*x*sin(e + f*x)**2*co
s(e + f*x)**4/8 + 3*B*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2 - B*a**2*
c**4*x*sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8 + 3*B*a**2*c**4*
x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a**2*c**4*sin(e + f
*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(8*f) -
 2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B*a**2*c**4*sin(e + f*x)
**4*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) -
5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(2*f) - 8*B*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**5/(5*f) + 4*B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3
/(3*f) + B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e +
 f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(2
*f) + B*a**2*c**4*sin(e + f*x)*cos(e + f*x)/f - 16*B*a**2*c**4*cos(e + f*x)
**7/(35*f) + 8*B*a**2*c**4*cos(e + f*x)**5/(15*f) + 2*B*a**2*c**4*cos(e + f
*x)**3/(3*f) - B*a**2*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(179) = 358.

Time = 0.24 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.43

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \frac{896\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)Aa^2c^4+8960\left(\cos(fx+e)^3-3\cos(fx+e)\right)A}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*
c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x +
2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2*c^
4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^4 -
 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^2*c^4
 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f
*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x
 + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4 - 70*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 + 13440*A*a^2
*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.25

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx &= \frac{Ba^2c^4\cos(7\,fx+7\,e)}{448\,f}+\frac{1}{16}\,(7\,Aa^2c^4-2\,Ba^2c^4)x \\ &+ \frac{(8\,Aa^2c^4-3\,Ba^2c^4)\cos(5\,fx+5\,e)}{320\,f}+\frac{(8\,Aa^2c^4-5\,Ba^2c^4)\cos(3\,fx+3\,e)}{64\,f} \\ &+ \frac{(16\,Aa^2c^4-11\,Ba^2c^4)\cos(fx+e)}{64\,f}-\frac{(Aa^2c^4-2\,Ba^2c^4)\sin(6\,fx+6\,e)}{192\,f} \\ &+ \frac{(Aa^2c^4+2\,Ba^2c^4)\sin(4\,fx+4\,e)}{64\,f}+\frac{(17\,Aa^2c^4-2\,Ba^2c^4)\sin(2\,fx+2\,e)}{64\,f}\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x + 1
/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 - 5
*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(f*x
 + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*a^2*
c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c^4)*s
in(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.93

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(4\,A\,a^2\,c^4-2\,B\,a^2\,c^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}(12\,A\,a^2\,c^4-2\,B\,a^2\,c^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(8\,A\,a^2\,c^4-}{}$$
$$+\frac{a^2\,c^4\,\mathrm{atan}\left(\frac{a^2\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(7\,A-2\,B)}{8\left(\frac{7\,A\,a^2\,c^4}{8}-\frac{B\,a^2\,c^4}{4}\right)}\right)(7\,A-2\,B)}{8\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^12*(4*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^8*(
12*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^10*(8*A*a^2*c^4 - 8*B*a^2*
c^4) + tan(e/2 + (f*x)/2)^2*((8*A*a^2*c^4)/5 - (8*B*a^2*c^4)/5) - tan(e/2 +
 (f*x)/2)^13*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + tan(e/2 + (f*x)/2)^6*(16*A
*a^2*c^4 - 16*B*a^2*c^4) + tan(e/2 + (f*x)/2)^3*((29*A*a^2*c^4)/6 - (11*B*a
^2*c^4)/3) - tan(e/2 + (f*x)/2)^11*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)/3) +
tan(e/2 + (f*x)/2)^4*((44*A*a^2*c^4)/5 - (14*B*a^2*c^4)/5) + tan(e/2 + (f*x
)/2)^5*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) - tan(e/2 + (f*x)/2)^9*((23*
A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) + tan(e/2 + (f*x)/2)*((9*A*a^2*c^4)/8 +
(B*a^2*c^4)/4) + (4*A*a^2*c^4)/5 - (18*B*a^2*c^4)/35)/(f*(7*tan(e/2 + (f*x)
/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f
*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (
f*x)/2)^14 + 1)) + (a^2*c^4*atan((a^2*c^4*tan(e/2 + (f*x)/2)*(7*A - 2*B))/(
8*((7*A*a^2*c^4)/8 - (B*a^2*c^4)/4)))*(7*A - 2*B))/(8*f)
```

3.28 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 147

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{1}{16}a^2(6A-B)c^3x+\frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{a^2(6A-B)c^3\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}$$

```
[Out] 1/16*a^2*(6*A-B)*c^3*x+1/30*a^2*(6*A-B)*c^3*cos(f*x+e)^5/f+1/16*a^2*(6*A-B)
*c^3*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(6*A-B)*c^3*cos(f*x+e)^3*sin(f*x+e)/f
-1/6*a^2*B*cos(f*x+e)^5*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{a^2c^3(6A-B)\cos^5(e+fx)}{30f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos^3(e+fx)}{24f} + \frac{a^2c^3(6A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2c^3x(6A-B) - \frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A - B)*c^3*x)/16 + (a^2*(6*A - B)*c^3*Cos[e + f*x]^5)/(30*f) + (a^2
*(6*A - B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(6*A - B)*c^3*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^3 - c^3*Sin[e + f
*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$

$$
\begin{aligned}
&= -\frac{a^2 B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f} \\
&\quad+\frac{1}{6}\left(a^2(6A-B)c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))\,dx \\
&= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}-\frac{a^2 B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f} \\
&\quad+\frac{1}{6}\left(a^2(6A-B)c^3\right)\int \cos^4(e+fx)\,dx \\
&= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad-\frac{a^2 B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}+\frac{1}{8}\left(a^2(6A-B)c^3\right)\int \cos^2(e+fx)\,dx \\
&= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad+\frac{a^2(6A-B)c^3\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad-\frac{a^2 B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}+\frac{1}{16}\left(a^2(6A-B)c^3\right)\int 1\,dx \\
&= \frac{1}{16}a^2(6A-B)c^3x+\frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad+\frac{a^2(6A-B)c^3\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a^2 B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= \frac{a^2c^3(360Ae-60Be+360Afx-60Bfx+120(A-B)\cos(e+fx)+60(A-B)\cos(3(e+fx))+12A\,\mathrm{co}}{}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*x
] + 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{c^3\left(5(A-B)\cos(3fx+3e)+(A-B)\cos(5fx+5e)+5\left(4A-\frac{B}{4}\right)\sin(2fx+2e)+\frac{5\left(A+\frac{B}{2}\right)\sin(4fx+4e)}{2}+\frac{5B\sin(6fx+6e)}{12}+10(\right.}{80f}$
risch	$\frac{3a^2c^3xA}{8}-\frac{a^2c^3xB}{16}+\frac{c^3a^2\cos(fx+e)A}{8f}-\frac{c^3a^2\cos(fx+e)B}{8f}+\frac{B\,a^2c^3\sin(6fx+6e)}{192f}+\frac{c^3a^2\cos(5fx+5e)A}{80f}-$
parts	$\frac{(-2A\,a^2c^3-B\,a^2c^3)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-A\,a^2c^3+B\,a^2c^3)\cos(fx+e)}{f}-\frac{(-A\,a^2c^3+B\,a^2c^3)\left(\frac{8}{3}+\mathrm{si}\right.}{}$
derivativedivides	$\frac{\frac{A\,a^2c^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+A\,a^2c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,a^2c^3}{}}{}$
default	$\frac{\frac{A\,a^2c^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+A\,a^2c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,a^2c^3}{}}{}$
norman	$\frac{(\frac{3}{8}A\,a^2c^3-\frac{1}{16}B\,a^2c^3)x+(\frac{3}{8}A\,a^2c^3-\frac{1}{16}B\,a^2c^3)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}A\,a^2c^3-\frac{3}{8}B\,a^2c^3)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}A\,a^2c^3-}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/80*c^3*(5*(A-B)*cos(3*f*x+3*e)+(A-B)*cos(5*f*x+5*e)+5*(4*A-1/4*B)*sin(2*f
*x+2*e)+5/2*(A+1/2*B)*sin(4*f*x+4*e)+5/12*B*sin(6*f*x+6*e)+10*(A-B)*cos(f*x
+e)+30*f*x*A-5*f*x*B+16*A-16*B)*a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{48\,(A-B)a^2c^3\cos\left(fx+e\right)^5+15\,(6\,A-B)a^2c^3fx+5\,\left(8\,Ba^2c^3\cos\left(fx+e\right)^5+2\,(6\,A-B)a^2c^3\cos\left(fx\right.\right.}{240\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8*
B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A - B)
*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(128) = 256$.

Time = 0.44 (sec) , antiderivative size = 910, normalized size of antiderivative = 6.19

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \begin{cases} \frac{3Aa^2c^3x\sin^4(e+fx)}{8} + \frac{3Aa^2c^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^3x\sin^2(e+fx) + \frac{3Aa^2c^3x\cos^4(e+fx)}{8} - Aa^2c^3x\cos^2(e - \\ x(A + B\sin(e))(a\sin(e) + a)^2(-c\sin(e) + c)^3 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*cos
(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*c**
3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e + f*
x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a**2*c
**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3*cos(e
+ f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**3*cos(e
+ f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15*B*a**2*c*
*3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c**3*x*cos(e
+ f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3*x*cos(e + f*
x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**2*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c**
3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*sin(e + f*x)**2*cos
(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*
c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**3*cos(e + f
*x)**3/(3*f) - B*a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(138) = 276$.

Time = 0.23 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.45

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{64\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)Aa^2c^3+640\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c^
3 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^3 - 640*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
 + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^2*c^3 + 960*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{Ba^2c^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(6\,Aa^2c^3-Ba^2c^3\right)x+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(5\,fx+5\,e\right)}{80\,f}$$
$$+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(3\,fx+3\,e\right)}{16\,f}+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(fx+e\right)}{8\,f}$$
$$+\frac{\left(2\,Aa^2c^3+Ba^2c^3\right)\sin\left(4\,fx+4\,e\right)}{64\,f}+\frac{\left(16\,Aa^2c^3-Ba^2c^3\right)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/192*B*a^2*c^3*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^2*c^3 - B*a^2*c^3)*x + 1/8
0*(A*a^2*c^3 - B*a^2*c^3)*cos(5*f*x + 5*e)/f + 1/16*(A*a^2*c^3 - B*a^2*c^3)
*cos(3*f*x + 3*e)/f + 1/8*(A*a^2*c^3 - B*a^2*c^3)*cos(f*x + e)/f + 1/64*(2*
A*a^2*c^3 + B*a^2*c^3)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^2*c^3 - B*a^2*c^3)
*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.69

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(4\,A\,a^2\,c^3-4\,B\,a^2\,c^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(2\,A\,a^2\,c^3-2\,B\,a^2\,c^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(4\,A\,a^2\,c^3-4\,B\right.}{}$$
$$+\frac{a^2\,c^3\,\mathrm{atan}\left(\frac{a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,A-B\right)}{8\,\left(\frac{3\,A\,a^2\,c^3}{4}-\frac{B\,a^2\,c^3}{8}\right)}\right)\,\left(6\,A-B\right)}{8\,f}$$
$$-\frac{a^2\,c^3\,\left(6\,A-B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{8\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^4*(4*A*a^2*c^3 - 4*B*a^2*c^3) + tan(e/2 + (f*x)/2)^8*(2
*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c^3 - 4*B*a^2*c^3
) + tan(e/2 + (f*x)/2)^10*(2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^
2*((2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5) + tan(e/2 + (f*x)/2)^5*((A*a^2*c^3)/2
 + (13*B*a^2*c^3)/4) - tan(e/2 + (f*x)/2)^7*((A*a^2*c^3)/2 + (13*B*a^2*c^3)
/4) - tan(e/2 + (f*x)/2)^11*((5*A*a^2*c^3)/4 + (B*a^2*c^3)/8) + tan(e/2 + (
f*x)/2)^3*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) - tan(e/2 + (f*x)/2)^9*((7*
A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) + tan(e/2 + (f*x)/2)*((5*A*a^2*c^3)/4 + (
B*a^2*c^3)/8) + (2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5)/(f*(6*tan(e/2 + (f*x)/2)
^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)
/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*c^3*at
an((a^2*c^3*tan(e/2 + (f*x)/2)*(6*A - B))/(8*((3*A*a^2*c^3)/4 - (B*a^2*c^3)
/8)))*(6*A - B))/(8*f) - (a^2*c^3*(6*A - B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 3/8*a^2*A*c^2*x-1/5*a^2*B*c^2*cos(f*x+e)^5/f+3/8*a^2*A*c^2*cos(f*x+e)*sin(f
*x+e)/f+1/4*a^2*A*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{a^2Ac^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2Ac^2\sin(e+fx)\cos(e+fx)}{8f} + \frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*A*c^2*x)/8 - (a^2*B*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^2*A*c^2*Cos[e +
 f*x]*Sin[e + f*x])/(8*f) + (a^2*A*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))\,dx \\
&= -\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\left(a^2Ac^2\right)\int \cos^4(e+fx)\,dx \\
&= -\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}\left(3a^2Ac^2\right)\int \cos^2(e+fx)\,dx \\
&= -\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f} \\
&\quad+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{8}\left(3a^2Ac^2\right)\int 1\,dx \\
&= \frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f} \\
&\quad+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx = \frac{a^2c^2(-32B\cos^5(e + fx) + 5A(12(e + fx) + 8\sin(2(e + fx)) + \sin(4(e + fx))))}{160f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + S
in[4*(e + f*x)])))/(160*f)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{a^2c^2(60fxA+5\sin(4fx+4e)A+40A\sin(2fx+2e)-20\cos(fx+e)B-2\cos(5fx+5e)B-10\cos(3fx+3e)B-32B)}{160f}$
risch	$\frac{3a^2Ac^2x}{8}-\frac{Ba^2c^2\cos(fx+e)}{8f}-\frac{Ba^2c^2\cos(5fx+5e)}{80f}+\frac{Aa^2c^2\sin(4fx+4e)}{32f}-\frac{Ba^2c^2\cos(3fx+3e)}{16f}+\frac{Aa^2c^2\text{s}}{}$
derivativedivides	$\frac{Aa^2c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2Aa^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(\right.}{}}{f}$
default	$\frac{Aa^2c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2Aa^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(\right.}{}}{f}$
parts	$a^2Ac^2x+\frac{Aa^2c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{Ba^2c^2\cos(fx+e)}{f}-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(f\right.}{}$
norman	$-\frac{2Ba^2c^2}{5f}-\frac{4Ba^2c^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2Ba^2c^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2Ac^2x}{8}+\frac{5Aa^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{Aa^2c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{A}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/160*a^2*c^2*(60*f*x*A+5*sin(4*f*x+4*e)*A+40*A*sin(2*f*x+2*e)-20*cos(f*x+e
)*B-2*cos(5*f*x+5*e)*B-10*cos(3*f*x+3*e)*B-32*B)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = -\frac{8\,Ba^2c^2\cos\left(fx+e\right)^5-15\,Aa^2c^2fx-5\left(2\,Aa^2c^2\cos\left(fx+e\right)^3+3\,Aa^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(f
*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.18

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \begin{cases} \frac{3Aa^2c^2x\sin^4\left(e+fx\right)}{8}+\frac{3Aa^2c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-Aa^2c^2x\sin^2\left(e+fx\right)+\frac{3Aa^2c^2x\cos^4\left(e+fx\right)}{8}-Aa^2c^2x\cos^2\left(e-\right. \\ x(A+B\sin\left(e\right))\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*cos
(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**2*c
**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos(e +
 f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2*sin(
e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)**3/
(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2*cos(e
+ f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e
+ f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**2
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(81) = 162.

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.84

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{15\,(12\,fx+12\,e+\sin{(4\,fx+4\,e)}-8\,\sin{(2\,fx+2\,e)})Aa^2c^2-240\,(2\,fx+2\,e-\sin{(2\,fx+2\,e)})Aa^2c^2+}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^2
 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c^2
 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^2 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*cos(f*x + e
))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{3}{8}\,Aa^2c^2x-\frac{Ba^2c^2\cos{(5\,fx+5\,e)}}{80\,f}-\frac{Ba^2c^2\cos{(3\,fx+3\,e)}}{16\,f}-\frac{Ba^2c^2\cos{(fx+e)}}{8\,f}+\frac{Aa^2c^2\sin{(4\,fx+4\,e)}}{32\,f}+\frac{Aa^2c^2\sin{(2\,fx+2\,e)}}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3*
f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x + 4*
e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.67

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx=\frac{3\,A\,a^2\,c^2\,x}{8}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\left(\frac{a^2\,c^2\,(80\,B-75\,A\,(e+f\,x))}{40}+\frac{15\,A\,a^2\,c^2\,(e+f\,x)}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^2\,c^2\,(160\,B-150\,A\,(e+f\,x))}{40}+\frac{15\,A\,a^2\,c^2\,(e}{4}\right.}{f\,(}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (3*A*a^2*c^2*x)/8 - (tan(e/2 + (f*x)/2)^8*((a^2*c^2*(80*B - 75*A*(e + f*x))
)/40 + (15*A*a^2*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^2*(160*B
- 150*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/4) + (a^2*c^2*(16*B - 15*
A*(e + f*x)))/40 + (3*A*a^2*c^2*(e + f*x))/8 - (A*a^2*c^2*tan(e/2 + (f*x)/2
)^3)/2 + (A*a^2*c^2*tan(e/2 + (f*x)/2)^7)/2 + (5*A*a^2*c^2*tan(e/2 + (f*x)/
2)^9)/4 - (5*A*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f*(tan(e/2 + (f*x)/2)^2 + 1)
^5)
```

3.30 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 98

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{1}{8}a^2(4A+B)cx-\frac{a^2(4A+B)c\cos^3(e+fx)}{12f} + \frac{a^2(4A+B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f}$$

```
[Out] 1/8*a^2*(4*A+B)*c*x-1/12*a^2*(4*A+B)*c*cos(f*x+e)^3/f+1/8*a^2*(4*A+B)*c*cos
(f*x+e)*sin(f*x+e)/f-1/4*B*c*cos(f*x+e)^3*(a^2+a^2*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3046, 2939, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{a^2c(4A+B)\cos^3(e+fx)}{12f}+\frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f} + \frac{1}{8}a^2cx(4A+B)-\frac{Bc\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(4*A + B)*c*x)/8 - (a^2*(4*A + B)*c*Cos[e + f*x]^3)/(12*f) + (a^2*(4*A
 + B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (B*c*Cos[e + f*x]^3*(a^2 + a^2*S
in[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx\\
&= -\frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f}\\
&\quad+\frac{1}{4}(a(4A+B)c)\int \cos^2(e+fx)(a+a\sin(e+fx))\,dx
\end{aligned}$$

$$= -\frac{a^2(4A+B)c\cos^3(e+fx)}{12f} - \frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f} + \frac{1}{4}\left(a^2(4A+B)c\right)\int \cos^2(e+fx)\,dx$$

$$= -\frac{a^2(4A+B)c\cos^3(e+fx)}{12f} + \frac{a^2(4A+B)c\cos(e+fx)\sin(e+fx)}{8f} - \frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f} + \frac{1}{8}\left(a^2(4A+B)c\right)\int 1\,dx$$

$$= \frac{1}{8}a^2(4A+B)cx - \frac{a^2(4A+B)c\cos^3(e+fx)}{12f} + \frac{a^2(4A+B)c\cos(e+fx)\sin(e+fx)}{8f} - \frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{a^2c\cos(e+fx)\left(12(4A+B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(8A+8B+8(A+B)\cos(2(e+f$$

$$48f\sqrt{\cos^2(e+fx)}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] -1/48*(a^2*c*Cos[e + f*x]*(12*(4*A + B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(8*A + 8*B + 8*(A + B)*Cos[2*(e + f*x)] - 3*(8*A
 + B)*Sin[e + f*x] + 3*B*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{c\left(\frac{(-A-B)\cos(3fx+3e)}{3}+A\sin(2fx+2e)-\frac{B\sin(4fx+4e)}{8}+(-A-B)\cos(fx+e)+2fxA+\frac{fxB}{2}-\frac{4A}{3}-\frac{4B}{3}\right)a^2}{4f}$
risch	$\frac{a^2cxA}{2}+\frac{a^2cxB}{8}-\frac{a^2c\cos(fx+e)A}{4f}-\frac{a^2c\cos(fx+e)B}{4f}-\frac{B\,a^2c\sin(4fx+4e)}{32f}-\frac{a^2c\cos(3fx+3e)A}{12f}-\frac{a^2c\cos(3}{12}$
parts	$-\frac{(-A\,a^2c-B\,a^2c)\,(2+\sin^2(fx+e))\cos(fx+e)}{3f}+\frac{(-A\,a^2c+B\,a^2c)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(A\,a^2c+B\,a^2c)}{f}$
derivativedivides	$\frac{\frac{A\,a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-A\,a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-B\,a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3}{8}\right.}{}$
default	$\frac{\frac{A\,a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-A\,a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-B\,a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3}{8}\right.}{}$
norman	$\frac{(\frac{1}{2}A\,a^2c+\frac{1}{8}B\,a^2c)x+(2A\,a^2c+\frac{1}{2}B\,a^2c)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2A\,a^2c+\frac{1}{2}B\,a^2c)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3A\,a^2c+\frac{3}{4}B\,a^2c)x\left(\tan\right.}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*c*(1/3*(-A-B)*cos(3*f*x+3*e)+A*sin(2*f*x+2*e)-1/8*B*sin(4*f*x+4*e)+(-A-
B)*cos(f*x+e)+2*f*x*A+1/2*f*x*B-4/3*A-4/3*B)*a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{8\,(A+B)a^2c\cos{(fx+e)}^3-3\,(4\,A+B)a^2cfx+3\,\left(2\,Ba^2c\cos{(fx+e)}^3-(4\,A+B)a^2c\cos{(fx+e)}\right)\mathrm{s}}{24\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2*
c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(90) = 180$.

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.04

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\begin{cases} -\frac{Aa^2cx\sin^2(e+fx)}{2}-\frac{Aa^2cx\cos^2(e+fx)}{2}+Aa^2cx+\frac{Aa^2c\sin^2(e+fx)\cos(e+fx)}{f}+\frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{2Aa^2c\cos^3(e}{3f} \\ x(A+B\sin(e))(a\sin(e)+a)^2(-c\sin(e)+c) \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*x)
*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e + f
*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x)**4/
8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a**2*
c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$
$$-\frac{32\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aa^2c+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c-96\left(fx+e\right)Aa^2c+}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*cos(f
*x + e) + 96*B*a^2*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{Ba^2c\sin(4\,fx+4\,e)}{32\,f}+\frac{Aa^2c\sin(2\,fx+2\,e)}{4\,f}+\frac{1}{8}\,(4\,Aa^2c+Ba^2c)x - \frac{(Aa^2c+Ba^2c)\cos(3\,fx+3\,e)}{12\,f}-\frac{(Aa^2c+Ba^2c)\cos(fx+e)}{4\,f}$$

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm ="giac")

[Out] -1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4*A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*(A*a^2*c + B*a^2*c)*cos(f*x + e)/f

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.46

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{a^2\,c\,\mathrm{atan}\left(\frac{a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,A+B)}{4\,\left(A\,a^2\,c+\frac{B\,a^2\,c}{4}\right)}\right)\,(4\,A+B)}{4\,f}-\frac{a^2\,c\,(4\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(2\,A\,a^2\,c+2\,B\,a^2\,c)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a^2\,c-\frac{B\,a^2\,c}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(2\,A\,a^2\,c+2\,B\,a^2\,c)-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f}{2}\right.\right.}$$

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)

[Out] (a^2*c*atan((a^2*c*tan(e/2 + (f*x)/2)*(4*A + B))/(4*(A*a^2*c + (B*a^2*c)/4)))*(4*A + B))/(4*f) - (a^2*c*(4*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f) - (tan(e/2 + (f*x)/2)^4*(2*A*a^2*c + 2*B*a^2*c) - tan(e/2 + (f*x)/2)*(A*a^2*c - (B*a^2*c)/4) + tan(e/2 + (f*x)/2)^6*(2*A*a^2*c + 2*B*a^2*c) + tan(e/2 + (f*x)/2)^2*((2*A*a^2*c)/3 + (2*B*a^2*c)/3) + tan(e/2 + (f*x)/2)^7*(A*a^2*c - (B*a^2*c)/4) - tan(e/2 + (f*x)/2)^3*(A*a^2*c + (7*B*a^2*c)/4) + tan(e/2 + (f*x)/2)^5*(A*a^2*c + (7*B*a^2*c)/4) + (2*A*a^2*c)/3 + (2*B*a^2*c)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))

3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{3a^2(2A+3B)x}{2c}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}$$

```
[Out] -3/2*a^2*(2*A+3*B)*x/c+3/2*a^2*(2*A+3*B)*cos(f*x+e)/c/f+a^2*(A+B)*c^2*cos(f
*x+e)^5/f/(c-c*sin(f*x+e))^3+1/2*a^2*(2*A+3*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+
e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2758, 2761, 8}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a^2c^2(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{3a^2x(2A+3B)}{2c}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*(2*A + 3*B)*x)/(2*c) + (3*a^2*(2*A + 3*B)*Cos[e + f*x])/(2*c*f) + (
a^2*(A + B)*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) + (a^2*(2*A + 3*
B)*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\left(a^2(2A+3B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}\\
&\quad -\frac{1}{2}\left(3a^2(2A+3B)\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx\\
&= \frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}\\
&\quad +\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{\left(3a^2(2A+3B)\right)\int 1\,dx}{2c}\\
&= -\frac{3a^2(2A+3B)x}{2c}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}\\
&\quad +\frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 9.82 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.63

$$
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx
= \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A+3B)(e+fx)-4(A+\right.}{4cf\left(\cos\left(\right.\right.}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e + f
*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f
*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x) -
4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{4\left(\frac{(A+3B)\cos(2fx+2e)}{8}+\frac{B\sin(3fx+3e)}{32}+\frac{\left(-3fxA-\frac{9}{2}fxB+5A+7B\right)\cos(fx+e)}{4}+\left(A+\frac{33B}{32}\right)\sin(fx+e)+\frac{9A}{8}+\frac{11B}{8}\right)a^2}{cf\cos(fx+e)}$
derivativedivides	$\frac{2a^2\left(-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2}\right.}{fc}$
default	$\frac{2a^2\left(-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2}\right.}{fc}$
risch	$-\frac{3a^2xA}{c}-\frac{9a^2xB}{2c}+\frac{a^2e^{i(fx+e)}A}{2cf}+\frac{3a^2e^{i(fx+e)}B}{2cf}+\frac{a^2e^{-i(fx+e)}A}{2cf}+\frac{3a^2e^{-i(fx+e)}B}{2cf}+\frac{8a^2A}{fc\left(e^{i(fx+e)}-i\right)}+$
norman	$\frac{-\frac{2Aa^2+5Ba^2}{cf}+\frac{3a^2(2A+3B)x}{2c}-\frac{\left(2Aa^2+3Ba^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(4Aa^2+8Ba^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(6Aa^2+4Ba^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{cf}}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4*(1/8*(A+3*B)*cos(2*f*x+2*e)+1/32*B*sin(3*f*x+3*e)+1/4*(-3*f*x*A-9/2*f*x*B
+5*A+7*B)*cos(f*x+e)+(A+33/32*B)*sin(f*x+e)+9/8*A+11/8*B)*a^2/c/f/cos(f*x+e
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$=\frac{Ba^2\cos(fx+e)^3-3(2A+3B)a^2fx+2(A+3B)a^2\cos(fx+e)^2+8(A+B)a^2-(3(2A+3B)a^2fx}{2(cf\,\mathrm{co}}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*x
 + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos(f
*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^2*c
os(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x
+ e) + c*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. 2(104) = 208.

Time = 1.98 (sec) , antiderivative size = 2365, normalized size of antiderivative = 20.21

$$\int \frac{(a + a\sin(e + fx))^2 (A + B\sin(e + fx))}{c - c\sin(e + fx)}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*a**2*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2
*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/
2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x*tan(e/2 + f*x/2)**4/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 12*
A*a**2*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/
2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c
*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*A*a**2*f*x*tan
(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
- 2*c*f) + 6*A*a**2*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)
**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
 + f*x/2) - 2*c*f) - 16*A*a**2*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)*
*5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
+ f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*tan(e/2 + f*x/2)**
3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 +
f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 3
6*A*a**2*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f
*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*ta
n(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)
**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 20*A*a**2/(2*c*f*tan(e/2 +
 f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*
tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/
2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
- 2*c*f) + 9*B*a**2*f*x*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*f*x*tan(e/2 + f*x/2)**3/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*
B*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
```

```
tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 +
f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*t
an(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 9*B*a**2*f*x/(2*c*f*
tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*
tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f
*x/2) - 2*c*f) + 14*B*a**2*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 -
 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 42*B*a**2*tan(e/2 + f*x/2)**2/(
2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x
/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 10*B
*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)*
*4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
+ f*x/2) - 2*c*f) - 28*B*a**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*t
an(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-
c*sin(e) + c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 624, normalized size of antiderivative = 5.33

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$

$$-\frac{2\,Aa^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+4\,Ba^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\cdots}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\cdots}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -(2*A*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c) + 4*B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) +
1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + B*a^2*((sin(f*x + e
)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c -
 c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e
```

```
) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/c) + 4*A*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
- 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*a^2*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*a
^2/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{\frac{3\,(2\,Aa^2+3\,Ba^2)(fx+e)}{c}+\frac{16\,(Aa^2+Ba^2)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^2\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c}}{2\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(tan(1/2*f*
x + 1/2*e) - 1)) + 2*(B*a^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*tan(1/2*f*x +
1/2*e)^2 - 6*B*a^2*tan(1/2*f*x + 1/2*e)^2 - B*a^2*tan(1/2*f*x + 1/2*e) - 2*
A*a^2 - 6*B*a^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{10\,A\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,a^2+5\,B\,a^2\right)+14\,B\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(2\,A\,a^2+7\,B\,a^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(8\,A\right.}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right.}$$
$$-\frac{3\,a^2\,\mathrm{atan}\left(\frac{3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A+3\,B)}{6\,A\,a^2+9\,B\,a^2}\right)\,(2\,A+3\,B)}{c\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x)),x)
```

```
[Out] (10*A*a^2 - tan(e/2 + (f*x)/2)*(2*A*a^2 + 5*B*a^2) + 14*B*a^2 - tan(e/2 + (
f*x)/2)^3*(2*A*a^2 + 7*B*a^2) + tan(e/2 + (f*x)/2)^4*(8*A*a^2 + 9*B*a^2) +
tan(e/2 + (f*x)/2)^2*(18*A*a^2 + 21*B*a^2))/(f*(c - c*tan(e/2 + (f*x)/2) +
2*c*tan(e/2 + (f*x)/2)^2 - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^
4 - c*tan(e/2 + (f*x)/2)^5)) - (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(2*A +
 3*B))/(6*A*a^2 + 9*B*a^2))*(2*A + 3*B))/(c*f)
```

3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 109

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{a^2(A+4B)x}{c^2}-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] a^2*(A+4*B)*x/c^2-a^2*(A+4*B)*cos(f*x+e)/c^2/f+1/3*a^2*(A+B)*c^2*cos(f*x+e)
^5/f/(c-c*sin(f*x+e))^4-2/3*a^2*(A+4*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2759, 2761, 8}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{a^2x(A+4B)}{c^2}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(A + 4*B)*x)/c^2 - (a^2*(A + 4*B)*Cos[e + f*x])/(c^2*f) + (a^2*(A + B)
*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - (2*a^2*(A + 4*B)*Cos[e
+ f*x]^3)/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$=\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{1}{3}\left(a^2(A+4B)c\right)\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$

$$=\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{\left(a^2(A+4B)\right)\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx}{c}$$

$$=-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{\left(a^2(A+4B)\right)\int 1\,dx}{c^2}$$

$$=\frac{a^2(A+4B)x}{c^2}-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 238 vs. $2(109)=218$.

Time = 11.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.18

$$\int\frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$=\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3(A+4B)(e+f\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A + B
)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*
Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4*(c - c*Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{2a^2\left(-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
default	$\frac{2a^2\left(-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
risch	$\frac{a^2xA}{c^2}+\frac{4a^2xB}{c^2}-\frac{B\,a^2e^{i(fx+e)}}{2c^2f}-\frac{B\,a^2e^{-i(fx+e)}}{2c^2f}-\frac{8\left(-3iA\,a^2e^{i(fx+e)}+3A\,a^2e^{2i(fx+e)}-9iB\,a^2e^{i(fx+e)}+6B\,a^2\right.}{3\left(e^{i(fx+e)}-i\right)^3f\,c^2}$
parallelrisch	$\frac{3\left(\left(4(-fx+2)B-fxA+\frac{4A}{3}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(\left(\frac{11}{6}+4fx\right)B+fxA+\frac{4A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+\left(fxA+4fxB-\frac{4}{3}A-\frac{14}{3}B\right)\sin\left(\frac{fx}{2}\right.\right.}{f\,c^2\left(-3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\sin\left(\frac{3fx}{2}+\frac{3e}{2}\right)\right.}$
norman	$\frac{\frac{8B\,a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{a^2(A+4B)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{8A\,a^2+38B\,a^2}{3cf}-\frac{a^2(A+4B)x}{c}-\frac{2\left(4A\,a^2+13B\,a^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2(4A}{}}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^2*(-B/(1+tan(1/2*f*x+1/2*e)^2)+(A+4*B)*arctan(tan(1/2*f*x+1/2*e))
-1/3*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1
)^2+4*B/(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{3\,Ba^2\cos\left(fx+e\right)^3+6\,(A+4\,B)a^2fx+4\,(A+B)a^2-\left(3\,(A+4\,B)a^2fx+(8\,A+23\,B)a^2\right)\cos\left(fx+\right.}{3\,(c^2}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A
+ 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x -
2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*x +
 e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(f*x
+ e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. 2(100) = 200.

Time = 3.93 (sec) , antiderivative size = 2474, normalized size of antiderivative = 22.70

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*A*a**2*f*x*t
an(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2
)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c*
*2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c*
*2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2
 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
3*c**2*f) - 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5
 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*A*a**2*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)
**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*A*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)*
*5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2
*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*A*a**2*
tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/
2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c
**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 24*A*a**2*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2/(3*c**2*f*tan(
e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)
**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f)
 + 12*B*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2
*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 36*B*a**2*f*x*tan(e/2
 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) + 48*B*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*t
an(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x
/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 48*B*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
```

```
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 36*B*a**2*f*x*tan(
e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) - 12*B*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*B*a**2*tan(
e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**
4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*
f*tan(e/2 + f*x/2) - 3*c**2*f) - 78*B*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*ta
n(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*
f) + 74*B*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 90*B*a**2*tan(e/2 + f*x
/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*
f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f) + 38*B*a**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan
(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(107) = 214$.

Time = 0.31 (sec) , antiderivative size = 839, normalized size of antiderivative = 7.70

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + A*a
^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e)/(cos(f*x
 + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2
```

```
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
 + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\frac{3\,(Aa^2+4\,Ba^2)(fx+e)}{c^2}-\frac{6\,Ba^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2}+\frac{8\left(3\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2+4\,Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*e)
 - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1/2*
e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.26

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{2\,a^2\,\mathrm{atan}\left(\frac{2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A+4\,B)}{2\,A\,a^2+8\,B\,a^2}\right)\,(A+4\,B)}{c^2\,f} - \frac{\frac{8\,A\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(8\,A\,a^2+30\,B\,a^2)+\frac{38\,B\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3(8\,A\,a^2+26\,B\,a^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{8\,A}{3}\right.}{f\,\left(-c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\,c\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (2*a^2*atan((2*a^2*tan(e/2 + (f*x)/2)*(A + 4*B))/(2*A*a^2 + 8*B*a^2))*(A +
4*B))/(c^2*f) - ((8*A*a^2)/3 - tan(e/2 + (f*x)/2)*(8*A*a^2 + 30*B*a^2) + (3
```

```
8*B*a^2)/3 - tan(e/2 + (f*x)/2)^3*(8*A*a^2 + 26*B*a^2) + tan(e/2 + (f*x)/2)
^2*((8*A*a^2)/3 + (74*B*a^2)/3) + 8*B*a^2*tan(e/2 + (f*x)/2)^4)/(f*(4*c^2*t
an(e/2 + (f*x)/2)^2 - 4*c^2*tan(e/2 + (f*x)/2)^3 + 3*c^2*tan(e/2 + (f*x)/2)
^4 - c^2*tan(e/2 + (f*x)/2)^5 + c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3} dx$

Optimal result

Integrand size = 36, antiderivative size = 112

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3} dx = -\frac{a^2Bx}{c^3}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] -a^2*B*x/c^3+1/5*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^2*B*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3+2*a^2*B*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2759, 8}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3} dx = \frac{a^2c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^2Bx}{c^3}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^2*B*x)/c^3) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x
])^5) - (2*a^2*B*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^2*B*Co
s[e + f*x])/(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-(a^2Bc)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{(a^2B)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{c} \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))}-\frac{(a^2B)\int 1\,dx}{c^3} \\
&= -\frac{a^2Bx}{c^3}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 278 vs. $2(112) = 224$.

Time = 11.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(3A+8B)\left(\cos\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 -
 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin[(e
 + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e +
 f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e +
f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4*(c - c*Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{2a^2\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\cdots\right.}{f\,c^3}$
default	$\frac{2a^2\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\cdots\right.}{f\,c^3}$
parallelrisch	$-\frac{2\left(\frac{B\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(-\frac{5}{2}fxB+A+B\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B(5fx-4)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-5fxB+2A+\frac{34}{3}B\right)\left(\tan^2\left(\cdots\right.\right.\right.}{f\,c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$
risch	$-\frac{a^2Bx}{c^3}+\frac{-4A\,a^2e^{2i(fx+e)}+2A\,a^2e^{4i(fx+e)}-\frac{100B\,a^2e^{2i(fx+e)}}{3}-24iB\,a^2e^{3i(fx+e)}+\frac{56iB\,a^2e^{i(fx+e)}}{3}+10B\,a^2e^{4i(fx+e)}\cdots}{\left(e^{i(fx+e)}-i\right)^5 f\,c^3}$
norman	$\frac{\frac{a^2xB}{c}+\frac{8B\,a^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{48B\,a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{6A\,a^2+46B\,a^2}{15cf}+\frac{40B\,a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}+\frac{64B\,a^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{112B\,a^2\cdots}{}}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^3*(-B*arctan(tan(1/2*f*x+1/2*e))-(A+B)/(tan(1/2*f*x+1/2*e)-1)-1/5
*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(24*A+16*B)/(tan(1/2*f*x+1/2*e)-1
)^3-1/4*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^4-4*A/(tan(1/2*f*x+1/2*e)-1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(111) = 222$.

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.47

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{60\,Ba^2fx-(15\,Ba^2fx-(3\,A+43\,B)a^2)\cos(fx+e)^3-12\,(A+B)a^2-(45\,Ba^2fx-(9\,A-11\,B)a^2)\cos}{15\,(c^3f\cos(fx+e)^3+3\,c}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12*
(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2
*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (15*
B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*B)*a
^2)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e
)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.

Time = 7.78 (sec) , antiderivative size = 1647, normalized size of antiderivative = 14.71

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 -
75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f
*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 60*A*a**2*
tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*
x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 +
 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 6*A*a**2/(15*c**3*f*tan(e/2 + f*
x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 -
150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
```

```
15*B*a**2*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*
f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2
 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*B*a**2*f*x*tan(
e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*B*a**2*f*x*tan(e/2 + f*x/2)**3/(
15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*
tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 +
f*x/2) - 15*c**3*f) + 150*B*a**2*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2
 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)*
*3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*
f) - 75*B*a**2*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*B*a**2*f*x/(1
5*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*t
an(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f
*x/2) - 15*c**3*f) - 30*B*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x
/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 1
50*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1
20*B*a**2*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*ta
n(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f
*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 340*B*a**2*tan(e/2 + f
*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 1
50*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) + 200*B*a**2*tan(e/2 + f*x/2)/(15*c**3*f*tan(
e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/
2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c*
*3*f) - 46*B*a**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/
2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 7
5*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin
(e) + a)**2/(-c*sin(e) + c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(111) = 222$.

Time = 0.33 (sec) , antiderivative size = 1139, normalized size of antiderivative = 10.17

$$\int \frac{(a + a\sin(e + fx))^2 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^3} \, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(B*a^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)
```

```
^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3
) + A*a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 6*A*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) -
3*B*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*A*a^2*(5*sin(f*x + e)
/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*
c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*a^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{15\,(fx+e)Ba^2}{c^3} + \frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+170\,Ba^2\tan\left(\frac{1}{2}\,f\right.\right.}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 15*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*tan(
```

```
1/2*f*x + 1/2*e)^2 + 170*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*tan(1/2*f
*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{B\,a^2\,x}{c^3} - \frac{\frac{a^2\,(6\,A+46\,B-15\,B\,(e+f\,x))}{15} - \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{a^2\,(120\,B-150\,B\,(e+f\,x))}{15} + 10\,B\,a^2\,(e+f\,x)\right) + \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^2\,(30\,A\cdots}{}\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] - (B*a^2*x)/c^3 - ((a^2*(6*A + 46*B - 15*B*(e + f*x)))/15 - tan(e/2 + (f*x)
/2)^3*((a^2*(120*B - 150*B*(e + f*x)))/15 + 10*B*a^2*(e + f*x)) + tan(e/2 +
 (f*x)/2)^4*((a^2*(30*A + 30*B - 75*B*(e + f*x)))/15 + 5*B*a^2*(e + f*x)) +
 tan(e/2 + (f*x)/2)^2*((a^2*(60*A + 340*B - 150*B*(e + f*x)))/15 + 10*B*a^2
*(e + f*x)) - tan(e/2 + (f*x)/2)*((a^2*(200*B - 75*B*(e + f*x)))/15 + 5*B*a
^2*(e + f*x)) + B*a^2*(e + f*x))/(c^3*f*(tan(e/2 + (f*x)/2) - 1)^5)
```

3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 75

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] 1/7*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*(A-6*B)*c*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 2750}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a^2c^2(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c(A-6B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*(A - 6
*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
```

```
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{1}{7}\left(a^2(A-6B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(75) = 150.

Time = 8.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =
$$

$$
-\frac{a^2\left(-35(A+4B)\cos\left(\frac{1}{2}(e+fx)\right)+7(2A+13B)\cos\left(\frac{3}{2}(e+fx)\right)+35B\cos\left(\frac{5}{2}(e+fx)\right)+A\cos\left(\frac{7}{2}(e+\right.}{}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] -1/140*(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f*
x))/2] + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*(e
 + f*x))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(3*(
e + f*x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7*B*S
in[(5*(e + f*x))/2]))/(c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.77

method	result
parallelrisch	$-\frac{2a^2\left(A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-A+B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A+B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2(-A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(13A+2}{}\right.}{f\,c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$
derivativedivides	$\frac{2a^2\left(-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128A+}{5\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,c^4}$
default	$\frac{2a^2\left(-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128A+}{5\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,c^4}$
risch	$-\frac{2\left(14A\,a^2\mathrm{e}^{2i(fx+e)}+A\,a^2+7iA\,a^2\mathrm{e}^{i(fx+e)}-7iB\,a^2\mathrm{e}^{i(fx+e)}-6B\,a^2+91B\,a^2\mathrm{e}^{2i(fx+e)}-70iA\,a^2\mathrm{e}^{3i(fx+e)}-35A\,a^2\mathrm{e}^{4i(\right.}{35\left(\mathrm{e}^{i(fx+e)}-i\right)^7 f}$
norman	$\frac{\frac{\left(10A\,a^2-10B\,a^2\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{12A\,a^2-2B\,a^2}{35cf}-\frac{2A\,a^2\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(A\,a^2-B\,a^2\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(2A\,a^2-2B\,a^2\right)}{5c}}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*a^2*(A*tan(1/2*f*x+1/2*e)^6+(-A+B)*tan(1/2*f*x+1/2*e)^5+(4*A+B)*tan(1/2*
f*x+1/2*e)^4+2*(-A+B)*tan(1/2*f*x+1/2*e)^3+1/5*(13*A+2*B)*tan(1/2*f*x+1/2*e
)^2+1/5*(-A+B)*tan(1/2*f*x+1/2*e)+6/35*A-1/35*B)/f/c^4/(tan(1/2*f*x+1/2*e)-
1)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(73) = 146$.

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.51

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{(A-6\,B)a^2\cos\left(fx+e\right)^4+(4\,A+11\,B)a^2\cos\left(fx+e\right)^3+(13\,A+27\,B)a^2\cos\left(fx+e\right)^2-10\,(A+B}{35\,\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (13
*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*a^
2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10*(A
 + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^
4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e)
+ 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*
x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(66) = 132$.

Time = 14.43 (sec) , antiderivative size = 2008, normalized size of antiderivative = 26.77

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*A*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 -
245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4
*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e
/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*A*a**2*tan(e
/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)
**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 12
25*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f
*tan(e/2 + f*x/2) - 35*c**4*f) - 280*A*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*
tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 +
 f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)
**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**
4*f) + 140*A*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*
c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*t
an(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 +
 f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*A*a**2*tan(e/2
+ f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6
 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*
c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*ta
n(e/2 + f*x/2) - 35*c**4*f) + 14*A*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2
 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)
**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 7
35*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) -
12*A*a**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 +
 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c*
```

```
*4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(
e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**5/(35*c**4*f*tan(e/
2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2
)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 -
735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) -
 70*B*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*
tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2
+ f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)
**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 140*B*a**2*tan(e/2 + f*x/2
)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*
c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*
tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 +
 f*x/2) - 35*c**4*f) - 28*B*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f
*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5
- 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c
**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 14*B
*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2
+ f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)
**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 24
5*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 2*B*a**2/(35*c**4*f*tan(e/2 + f*x/
2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1
225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4
*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f), Ne(f, 0)
), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(73) = 146$.

Time = 0.27 (sec) , antiderivative size = 1571, normalized size of antiderivative = 20.95

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(2*A*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/
(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7) + B*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(
```

```
f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7) - 3*A*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2
10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x +
 e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^
4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*A*a^2*
(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7) - 8*B*a^2*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 4
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 6*B*a^2*(7*
sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(73) = 146$.

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.89

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx =$$

$$-\frac{2\left(35\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^6 - 35\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^5 + 35\,Ba^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^5 + 140\,Aa^2\tan\left(\frac{1}{2}\,fx + \right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 +
35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2*f*x
 + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2*e) +
 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{109\,A\,a^2}{4}+\frac{11\,B\,a^2}{4}-\frac{27\,A\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{5\,A\,a^2\,\cos(3\,e+3\,f\,x)}{8}-\frac{13\,B\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{5\,B\,a^2\,\cos(3\,e+3 \cdots}{8}\right.}{35\,c^4\,f\,\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\cdots\right.}{8}\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((109*A*a^2)/4 + (11*B*a^2)/4 - (27*A*a^2*cos(2*e + 2
*f*x))/4 + (5*A*a^2*cos(3*e + 3*f*x))/8 - (13*B*a^2*cos(2*e + 2*f*x))/4 + (
5*B*a^2*cos(3*e + 3*f*x))/8 + (7*A*a^2*sin(2*e + 2*f*x))/2 + (7*A*a^2*sin(3
*e + 3*f*x))/8 - (7*B*a^2*sin(2*e + 2*f*x))/2 - (7*B*a^2*sin(3*e + 3*f*x))/
8 - (121*A*a^2*cos(e + f*x))/8 - (9*B*a^2*cos(e + f*x))/8 - (105*A*a^2*sin(
e + f*x))/8 + (105*B*a^2*sin(e + f*x))/8))/(35*c^4*f*((35*2^(1/2)*cos(e/2 +
 pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8 - (7*2
^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 - pi/4 + (
7*f*x)/2))/8))
```

3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5} dx$

Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5} dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} + \frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

```
[Out] 1/9*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+1/63*a^2*(2*A-7*B)*c*co
s(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/315*a^2*(2*A-7*B)*cos(f*x+e)^5/f/(c-c*sin
(f*x+e))^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 2750}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5} dx = \frac{a^2c^2(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5} + \frac{a^2c(2A-7B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (a^2*(2*A -
 7*B)*c*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (a^2*(2*A - 7*B)*Co
s[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{1}{9}\left(a^2(2A-7B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}\\
&\quad+\frac{1}{63}\left(a^2(2A-7B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx
\end{aligned}$$

$$=\frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(115) = 230.

Time = 8.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.27

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(315(2A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-63(4A+11\ldots\right.}{\ldots}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -1/2520*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(31
5*(2*A + 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 315
*B*Cos[(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f*x)
)/2] + 882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e + f
*x))/2] + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*B*Sin
[(5*(e + f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/2]))/(
c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(B-2A)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(22A+B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+(-8A+3B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(54A+B)(\tan\ldots}{5\ldots}\ldots\right.}{f c^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\ldots\right.\right.}$
risch	$\frac{2ia^2\left(420iA\,e^{6i(fx+e)}+105iB\,e^{6i(fx+e)}+315B\,e^{7i(fx+e)}-882iA\,e^{4i(fx+e)}-630A\,e^{5i(fx+e)}-63iB\,e^{4i(fx+e)}-945B\,e^{5i(fx\ldots}\right.}{315f c^5\left(e^{i(fx+\ldots}\right.}$
derivativedivides	$\frac{2a^2\left(-\frac{256A+256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{200A+104B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{544A+448B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{480A+448B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12\ldots}{2(\tan(\ldots}\ldots\right.}{f c^5}$
default	$\frac{2a^2\left(-\frac{256A+256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{200A+104B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{544A+448B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{480A+448B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12\ldots}{2(\tan(\ldots}\ldots\right.}{f c^5}$
norman	$\frac{\frac{(4A a^2-2B a^2)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(28A a^2-12B a^2)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{94A a^2-14B a^2}{315cf}-\frac{2A a^2\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(24A a^2-14B\ldots}{3\ldots}\ldots}{\ldots}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*(A*tan(1/2*f*x+1/2*e)^8+(B-2*A)*tan(1/2*f*x+1/2*e)^7+1/3*(22*A+B)*tan(1/
2*f*x+1/2*e)^6+(-8*A+3*B)*tan(1/2*f*x+1/2*e)^5+1/5*(54*A+B)*tan(1/2*f*x+1/2
*e)^4+1/5*(-26*A+11*B)*tan(1/2*f*x+1/2*e)^3+1/5*(118/7*A+B)*tan(1/2*f*x+1/2
*e)^2+1/5*(-12/7*A+B)*tan(1/2*f*x+1/2*e)+47/315*A-1/45*B)*a^2/f/c^5/(tan(1/
2*f*x+1/2*e)-1)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(112) = 224$.

Time = 0.26 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.91

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx$$
$$= \frac{(2A - 7B)a^2\cos(fx + e)^5 - 4(2A - 7B)a^2\cos(fx + e)^4 - 5(5A + 14B)a^2\cos(fx + e)^3 - 5(17A + }{315\left(c^5 f\cos(fx + e)^5 + 5c^5 f\cos(fx + e)^4 - 8c^5 f\cos}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 -
5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 + 70
*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x + e)
^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2 + 7
0*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e))/(c^5*f*cos(f*x
+ e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
 + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(102) = 204$.

Time = 25.84 (sec) , antiderivative size = 3262, normalized size of antiderivative = 28.37

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9
- 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 2646
```

0*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**
5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*t
an(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260*A*a*
*2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f
*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)
4 + 26460*c5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 +
 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*A*a**2*tan(e/2 + f*x/2)*
*6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 1134
0*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**
5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*t
an(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2
 + f*x/2) - 315*c**5*f) + 5040*A*a**2*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e
/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f
*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)
5 - 39690*c5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 -
 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5
*f) - 6804*A*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 3276*A*a**2*ta
n(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f
*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)
6 + 39690*c5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 +
 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835
*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 2124*A*a**2*tan(e/2 + f*x/2)**2/(3
15*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**
5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*t
an(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/
2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*
x/2) - 315*c**5*f) + 216*A*a**2*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/
2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7
 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 396
90*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c*
*5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 94*
A*a**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 +
11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690
*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5
*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan
(e/2 + f*x/2) - 315*c**5*f) - 630*B*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*ta
n(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2
+ f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x
/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**
3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c

```
**5*f) - 210*B*a**2*tan(e/2 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a**2*
tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 +
 f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/
2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4
 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 28
35*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2 + f*x/2)**4/(
315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c*
*5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*
tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e
/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f
*x/2) - 315*c**5*f) - 1386*B*a**2*tan(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 +
 f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2
)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5
- 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 113
40*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f)
- 126*B*a**2*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**
5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*t
an(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/
2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f
*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2
+ f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8
+ 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 396
90*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c*
*5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*t
an(e/2 + f*x/2) - 315*c**5*f) + 14*B*a**2/(315*c**5*f*tan(e/2 + f*x/2)**9 -
 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460
*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5
*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*ta
n(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f), Ne(f, 0)),
(x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2087 vs. $2(112) = 224$.

Time = 0.28 (sec) , antiderivative size = 2087, normalized size of antiderivative = 18.15

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

[Out] -2/315*(A*a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*A*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*A*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*s

```
in(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^
9) + 28*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) +
 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(
cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(112) = 224$.

Time = 0.35 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.48

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = -\frac{2\left(315\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8 - 630\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7 + 315\,Ba^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7 + 2310\,Aa^2\tan\left(\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/315*(315*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*tan(1/2*f*x + 1/2*e)^7
 + 315*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*tan(1/2*f*x + 1/2*e)^6 + 1
05*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 945*B
*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 63*B*a^2*
tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 1062*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*tan(1/2*f
*x + 1/2*e)^2 - 108*A*a^2*tan(1/2*f*x + 1/2*e) + 63*B*a^2*tan(1/2*f*x + 1/2
*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.88

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = -\frac{2\,\cos\left(\frac{e}{2} + \frac{f\,x}{2}\right)\left(\frac{265\,A\,a^2\,\cos(2\,e+2\,f\,x)}{2} - \frac{49\,B\,a^2}{8} - \frac{4967\,A\,a^2}{16} - \frac{89\,A\,a^2\,\cos(3\,e+3\,f\,x)}{4} - \frac{49\,A\,a^2\,\cos(4\,e+4\,f\,x)}{16} + \frac{35\,B\,a^2\,\mathrm{c}}{}\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^5,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((265*A*a^2*cos(2*e + 2*f*x))/2 - (49*B*a^2)/8 - (49
67*A*a^2)/16 - (89*A*a^2*cos(3*e + 3*f*x))/4 - (49*A*a^2*cos(4*e + 4*f*x))/
16 + (35*B*a^2*cos(2*e + 2*f*x))/4 - (7*B*a^2*cos(3*e + 3*f*x))/8 + (7*B*a^
2*cos(4*e + 4*f*x))/8 - (567*A*a^2*sin(2*e + 2*f*x))/8 - (243*A*a^2*sin(3*e
 + 3*f*x))/8 + (45*A*a^2*sin(4*e + 4*f*x))/16 + (63*B*a^2*sin(2*e + 2*f*x))
/2 + (63*B*a^2*sin(3*e + 3*f*x))/8 + (625*A*a^2*cos(e + f*x))/4 + (35*B*a^2
*cos(e + f*x))/8 + (2205*A*a^2*sin(e + f*x))/8 - (945*B*a^2*sin(e + f*x))/8
))/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((
3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))
/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2
+ pi/4 + (9*f*x)/2))/16))
```

3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6} dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6} dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}$$

```
[Out] 1/11*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/99*a^2*(3*A-8*B)*c*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/693*a^2*(3*A-8*B)*cos(f*x+e)^5/f/(c-c*si
n(f*x+e))^6+2/3465*a^2*(3*A-8*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6} dx = \frac{a^2c^2(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6} + \frac{a^2c(3A-8B)\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*(3*A
- 8*B)*c*Cos[e + f*x]^5)/(99*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(3*A - 8*B)
*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(3*A - 8*B)*Cos[e
+ f*x]^5)/(3465*c*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{1}{11}\left(a^2(3A-8B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx
\end{aligned}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{1}{99}\left(2a^2(3A-8B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6} + \frac{\left(2a^2(3A-8B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx}{693c}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}$$

Mathematica [A] (verified)

Time = 8.59 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.83

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(231(27A+28B)\cos\left(\frac{1}{2}(e+fx)\right)-2475(A+\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A
+ 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*Cos
[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))/2]
 + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(e +
f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310*B*Si
n[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f*x))/2
] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c^6*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{2a^2\left(A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3A+B)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(12A-\frac{B}{3}\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2\left(-10A+\frac{7B}{3}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\right.}{}$
derivativedivides	$\frac{2a^2\left(-\frac{1752A+1208B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{90A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2304A+2048B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{128A+}{11\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^6}$
default	$\frac{2a^2\left(-\frac{1752A+1208B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{90A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2304A+2048B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{128A+}{11\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^6}$
risch	$-\frac{4A\,a^2}{1155}-\frac{16iB\,a^2\mathrm{e}^{5i(fx+e)}}{5}-\frac{44iA\,a^2\mathrm{e}^{5i(fx+e)}}{5}+\frac{32B\,a^2}{3465}+\frac{32iB\,a^2\mathrm{e}^{i(fx+e)}}{315}-\frac{4iA\,a^2\mathrm{e}^{i(fx+e)}}{105}+\frac{8iB\,a^2\mathrm{e}^{7i(fx+e)}}{3}-\frac{32B\,a^2\mathrm{e}^{2i(fx}}{63}$
norman	$-\frac{912A\,a^2-122B\,a^2}{3465cf}-\frac{2A\,a^2\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(3A\,a^2-B\,a^2\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2\left(45A\,a^2-B\,a^2\right)\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2\left(87A\,a^2-\right.}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*a^2*(A*tan(1/2*f*x+1/2*e)^10+(-3*A+B)*tan(1/2*f*x+1/2*e)^9+(12*A-1/3*B)*
tan(1/2*f*x+1/2*e)^8+2*(-10*A+7/3*B)*tan(1/2*f*x+1/2*e)^7+2/5*(81*A-13/3*B)
*tan(1/2*f*x+1/2*e)^6+2/5*(-71*A+16*B)*tan(1/2*f*x+1/2*e)^5+4/7*(41*A-2*B)*
tan(1/2*f*x+1/2*e)^4+2/7*(-34*A+9*B)*tan(1/2*f*x+1/2*e)^3+1/21*(89*A+2/3*B)
*tan(1/2*f*x+1/2*e)^2+1/105*(-47*A+61/3*B)*tan(1/2*f*x+1/2*e)+152/1155*A-61
/3465*B)/f/c^6/(tan(1/2*f*x+1/2*e)-1)^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(152) = 304.

Time = 0.25 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.61

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = -\frac{2\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^6+12\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^5-25\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^4-35\,(6\,A}{3465\,\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(f\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e)
^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)^3
```

```
 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) + 126
0*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*cos(
f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*cos(f*
x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x + e))/(
c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 2
0*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) -
32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*
x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(
f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(141) = 282$.

Time = 44.16 (sec) , antiderivative size = 4816, normalized size of antiderivative = 30.87

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-6930*A*a**2*tan(e/2 + f*x/2)**10/(3465*c**6*f*tan(e/2 + f*x/2)*
*11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9
 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 -
 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 -
1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 19
0575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6
*f) + 20790*A*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 -
38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571
725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 16008
30*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 114345
0*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c
**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) -
83160*A*a**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*
c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c*
*6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**
6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6
*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*
tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 138600
*A*a**2*tan(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*
f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*
tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*t
an(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*ta
n(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e
/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 224532*A*a*
*2*tan(e/2 + f*x/2)**6/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan
```

(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e
/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/
2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2
 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 +
f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 196812*A*a**2*ta
n(e/2 + f*x/2)**5/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2
 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 +
f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f
*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*
x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2
)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 162360*A*a**2*tan(e/2
 + f*x/2)**4/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x
/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2
)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)
6 + 1600830*c6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)*
*4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2
 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 67320*A*a**2*tan(e/2 + f*x
/2)**3/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**1
0 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 +
 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 +
1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 5
71725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 3811
5*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 29370*A*a**2*tan(e/2 + f*x/2)**2
/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 19
0575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 11434
50*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 160083
0*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*
c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6
*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 3102*A*a**2*tan(e/2 + f*x/2)/(3465*c**
6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*
f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*
tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*t
an(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan
(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2
 + f*x/2) - 3465*c**6*f) - 912*A*a**2/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 3
8115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 5717
25*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 160083
0*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450
*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c*
*6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 6
930*B*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c*
*6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6
*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*
f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f
*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*ta

n(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 2310*B*a
2*tan(e/2 + f*x/2)8/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*ta
n(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(
e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e
/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/
2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 +
 f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 32340*B*a**2*ta
n(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2
 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 +
f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f
*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*
x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2
)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 12012*B*a**2*tan(e/2
 + f*x/2)**6/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/
2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)
8 + 1143450*c6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)*
*6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**
4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 +
 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 44352*B*a**2*tan(e/2 + f*x/
2)**5/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10
 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 +
1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1
600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 57
1725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115
*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 7920*B*a**2*tan(e/2 + f*x/2)**4/(
3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 1905
75*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450
*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*
c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c*
*6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f
*tan(e/2 + f*x/2) - 3465*c**6*f) - 17820*B*a**2*tan(e/2 + f*x/2)**3/(3465*c
6*f*tan(e/2 + f*x/2)11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**
6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*
f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f
*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*t
an(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e
/2 + f*x/2) - 3465*c**6*f) - 220*B*a**2*tan(e/2 + f*x/2)**2/(3465*c**6*f*ta
n(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(
e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/
2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2
 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 +
 f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x
/2) - 3465*c**6*f) - 1342*B*a**2*tan(e/2 + f*x/2)/(3465*c**6*f*tan(e/2 + f*
x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/
2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)

```
**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)*
*5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465
*c**6*f) + 122*B*a**2/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(
e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/
2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2
 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2
+ f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f
*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f), Ne(f, 0)), (x*(A +
 B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(152) = 304$.

Time = 0.31 (sec) , antiderivative size = 2604, normalized size of antiderivative = 16.69

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*A*a^2*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
```

 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^2*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*B*a^
2*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*A*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11) + 8*B*a^2*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(
cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 -
 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f

```
*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(152) = 304$.

Time = 0.36 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.26

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx =$$

$$-\frac{2\left(3465\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{10} - 10395\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^9 + 3465\,Ba^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^9 + 41580\,Aa^2\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/3465*(3465*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 10395*A*a^2*tan(1/2*f*x + 1/2
*e)^9 + 3465*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*tan(1/2*f*x + 1/2*e
)^8 - 1155*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*tan(1/2*f*x + 1/2*e)^
7 + 16170*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*tan(1/2*f*x + 1/2*e)^
6 - 6006*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*tan(1/2*f*x + 1/2*e)^5
+ 22176*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*tan(1/2*f*x + 1/2*e)^4 -
 3960*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 8
910*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 110
*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*tan(1/2*f*x + 1/2*e) + 671*B*a^2
*tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(tan(1/2*f*x + 1/2*e)
- 1)^11)
```

Mupad [B] (verification not implemented)

Time = 15.72 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.71

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx$$

$$= \frac{2\,\cos\left(\frac{e}{2} + \frac{f\,x}{2}\right)\,\left(\frac{38163\,A\,a^2}{8} - \frac{1283\,B\,a^2}{8} - \frac{11931\,A\,a^2\,\cos(2\,e+2\,f\,x)}{4} + \frac{9609\,A\,a^2\,\cos(3\,e+3\,f\,x)}{16} + \frac{1383\,A\,a^2\,\cos(4\,e+4\,f\,x)}{8} - 225\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^6,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((38163*A*a^2)/8 - (1283*B*a^2)/8 - (11931*A*a^2*cos(
2*e + 2*f*x))/4 + (9609*A*a^2*cos(3*e + 3*f*x))/16 + (1383*A*a^2*cos(4*e +
```

4*f*x))/8 - (225*A*a^2*cos(5*e + 5*f*x))/16 + (631*B*a^2*cos(2*e + 2*f*x))/
4 - (1583*B*a^2*cos(3*e + 3*f*x))/32 - (223*B*a^2*cos(4*e + 4*f*x))/8 + (45
*B*a^2*cos(5*e + 5*f*x))/32 + 1386*A*a^2*sin(2*e + 2*f*x) + (14949*A*a^2*si
n(3*e + 3*f*x))/16 - (561*A*a^2*sin(4*e + 4*f*x))/4 - (231*A*a^2*sin(5*e +
5*f*x))/16 - (3003*B*a^2*sin(2*e + 2*f*x))/8 - (4653*B*a^2*sin(3*e + 3*f*x)
)/32 + (209*B*a^2*sin(4*e + 4*f*x))/16 + (77*B*a^2*sin(5*e + 5*f*x))/32 - 2
091*A*a^2*cos(e + f*x) + (281*B*a^2*cos(e + f*x))/16 - (22869*A*a^2*sin(e +
f*x))/4 + (23331*B*a^2*sin(e + f*x))/16))/(3465*c^6*f*((231*2^(1/2)*cos(e/
2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/16
- (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2)*cos((7*e)/
2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32
- (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))

3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 197

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}\\&\quad+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}\end{aligned}$$

```
[Out] 1/13*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^9+1/143*a^2*(4*A-9*B)*c*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/429*a^2*(4*A-9*B)*cos(f*x+e)^5/f/(c-c*s
in(f*x+e))^7+2/3003*a^2*(4*A-9*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^6+2/150
15*a^2*(4*A-9*B)*cos(f*x+e)^5/c^2/f/(c-c*sin(f*x+e))^5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 2750}

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx &= \frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}\\&\quad+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}+\frac{a^2c(4A-9B)\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}\end{aligned}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + (a^2*(4*A
- 9*B)*c*Cos[e + f*x]^5)/(143*f*(c - c*Sin[e + f*x])^8) + (a^2*(4*A - 9*B)*
Cos[e + f*x]^5)/(429*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(4*A - 9*B)*Cos[e +
 f*x]^5)/(3003*c*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(4*A - 9*B)*Cos[e + f*x
]^5)/(15015*c^2*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{1}{13}\left(a^2(4A-9B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{1}{143}\left(3a^2(4A-9B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7} + \frac{\left(2a^2(4A-9B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx}{429c}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7} + \frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6} + \frac{\left(2a^2(4A-9B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx}{3003c^2}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7} + \frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6} + \frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}$$

Mathematica [A] (verified)

Time = 10.42 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.59

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(6006(8A+7B)\cos\left(\frac{1}{2}(e+fx)\right)-1716(11A\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -1/240240*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(
6006*(8*A + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/2]
 - 15015*B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*Cos[
(7*(e + f*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x))/2]
 + 54912*A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(3*(e
+ f*x))/2] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/2] - 8
580*B*Sin[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[(9*(e +
 f*x))/2] - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))/(c^7*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{4ia^2(4iA+2860iA\,e^{4i(fx+e)}+15015B\,e^{9i(fx+e)}+21021iB\,e^{8i(fx+e)}-48048A\,e^{7i(fx+e)}-312iA\,e^{2i(fx+e)}-42042B\,e^{7}}{}$
parallelrisch	$-\frac{2a^2\left(A\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A+B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(18A-B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-40A+7B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\right.}{}$
derivativedivides	$\frac{2a^2\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{256A+256B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{560A+208B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8320A+7680B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{12}{3(\tan(}\right.}{}$
default	$\frac{2a^2\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{256A+256B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{560A+208B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8320A+7680B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{12}{3(\tan(}\right.}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETURN
VERBOSE)
```

```
[Out] -4/15015*I*a^2*(4*I*A+2860*I*A*exp(4*I*(f*x+e))+15015*B*exp(9*I*(f*x+e))+21
021*I*B*exp(8*I*(f*x+e))-48048*A*exp(7*I*(f*x+e))-312*I*A*exp(2*I*(f*x+e))-
42042*B*exp(7*I*(f*x+e))-9*I*B+18876*A*exp(5*I*(f*x+e))-26598*I*B*exp(6*I*(
f*x+e))+32604*B*exp(5*I*(f*x+e))+702*I*B*exp(2*I*(f*x+e))+1144*A*exp(3*I*(f
*x+e))+24024*I*A*exp(8*I*(f*x+e))-2574*B*exp(3*I*(f*x+e))+8580*I*B*exp(4*I*
(f*x+e))-52*A*exp(I*(f*x+e))-54912*I*A*exp(6*I*(f*x+e))+117*B*exp(I*(f*x+e)
))/f/c^7/(exp(I*(f*x+e))-I)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(192) = 384.

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.41

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$
$$=\frac{2\,(4\,A-9\,B)a^2\cos{(fx+e)}^7-12\,(4\,A-9\,B)a^2\cos{(fx+e)}^6-49\,(4\,A-9\,B)a^2\cos{(fx+e)}^5+70\,(4\,A}{15015\,\left(c^7f\cos{(fx+e)}^7+\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] 1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e)
^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^4
```

```
+ 105*(7*A + 20*B)*a^2*cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*cos(f*x + e)^
2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a^2*c
os(f*x + e)^6 + 14*(4*A - 9*B)*a^2*cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2*cos(
f*x + e)^4 - 105*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 105*(3*A + 29*B)*a^2*cos(
f*x + e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2)*sin(f*x + e)
)/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5
- 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e
)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*co
s(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f
*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(178) = 356$.

Time = 71.06 (sec) , antiderivative size = 6669, normalized size of antiderivative = 33.85

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
[Out] Piecewise((-30030*A*a**2*tan(e/2 + f*x/2)**12/(15015*c**7*f*tan(e/2 + f*x/2
)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2
)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x
/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*
x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f
*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f
*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x
/2) - 15015*c**7*f) + 120120*A*a**2*tan(e/2 + f*x/2)**11/(15015*c**7*f*tan(
e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(
e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*ta
n(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*t
an(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*
tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*
tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*ta
n(e/2 + f*x/2) - 15015*c**7*f) - 540540*A*a**2*tan(e/2 + f*x/2)**10/(15015*
c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*
c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 1073572
5*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 257657
40*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324
305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294
290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 19519
5*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 1201200*A*a**2*tan(e/2 + f*x/2)
**9/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12
 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**1
```

0 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)*
*8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)
6 + 19324305*c7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2
)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)
2 + 195195*c7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 2348346*A*a**2*tan(e
/2 + f*x/2)**8/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 +
 f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2
+ f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/
2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e
/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(
e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e
/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 2930928*A
*a**2*tan(e/2 + f*x/2)**7/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*
f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7
*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c*
*7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c
7*f*tan(e/2 + f*x/2)6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*
c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c
7*f*tan(e/2 + f*x/2)2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f)
- 3119688*A*a**2*tan(e/2 + f*x/2)**6/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 1
95195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4
294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 -
19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 -
 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5
- 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3
- 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 150
15*c**7*f) + 2189616*A*a**2*tan(e/2 + f*x/2)**5/(15015*c**7*f*tan(e/2 + f*x
/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x
/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f
*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 +
f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 +
 f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 +
 f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f
*x/2) - 15015*c**7*f) - 1319890*A*a**2*tan(e/2 + f*x/2)**4/(15015*c**7*f*ta
n(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*ta
n(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*
tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f
*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*
f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*
f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*
tan(e/2 + f*x/2) - 15015*c**7*f) + 467896*A*a**2*tan(e/2 + f*x/2)**3/(15015
*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170
*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 107357
25*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765
740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 1932

```
4305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 429
4290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 1951
95*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 154908*A*a**2*tan(e/2 + f*x/2)
**2/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12
 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**1
0 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)*
*8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)
**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2
)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)
**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 15808*A*a**2*tan(e/2
 + f*x/2)/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/
2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x
/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f
*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 +
f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 +
 f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 +
f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 3526*A*a**2/(1
5015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 117
1170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10
735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 2
5765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 +
19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 +
 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 +
195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 30030*B*a**2*tan(e/2 + f*x
/2)**11/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)
**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2
)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x
/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*
x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f
*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*
x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 30030*B*a**2*tan
(e/2 + f*x/2)**10/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/
2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e
/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan
(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*ta
n(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*t
an(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*ta
n(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 210210
*B*a**2*tan(e/2 + f*x/2)**9/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**
7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c*
*7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*
c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740
*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 1073572
5*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170
*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f
```

) + 186186*B*a**2*tan(e/2 + f*x/2)**8/(15015*c**7*f*tan(e/2 + f*x/2)**13 -
195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 -
4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 -
19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7
- 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5
- 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3
- 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15
015*c**7*f) - 468468*B*a**2*tan(e/2 + f*x/2)**7/(15015*c**7*f*tan(e/2 + f*x
/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x
/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f
*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 +
f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 +
f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 +
f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f
*x/2) - 15015*c**7*f) + 262548*B*a**2*tan(e/2 + f*x/2)**6/(15015*c**7*f*tan
(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan
(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*t
an(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*
tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f
*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f
*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*t
an(e/2 + f*x/2) - 15015*c**7*f) - 362076*B*a**2*tan(e/2 + f*x/2)**5/(15015*
c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*
c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 1073572
5*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 257657
40*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324
305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294
290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 19519
5*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 94380*B*a**2*tan(e/2 + f*x/2)**
4/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 +
1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10
+ 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8
+ 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**
6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)*
*4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**
2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 91806*B*a**2*tan(e/2 +
f*x/2)**3/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x
/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*
x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 +
f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 +
f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2
+ f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 +
f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 3198*B*a**2*t
an(e/2 + f*x/2)**2/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e
/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(

```
e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*ta
n(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*t
an(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*
tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*t
an(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 5538*
B*a**2*tan(e/2 + f*x/2)/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*
tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f
*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7
*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**
7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c*
*7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**
7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) +
426*B*a**2/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x
/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*
x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 +
f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 +
 f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2
+ f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 +
 f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f), Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**7, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3120 vs. $2(192) = 384$.

Time = 0.34 (sec) , antiderivative size = 3120, normalized size of antiderivative = 15.84

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

```
[Out] -2/45045*(2*A*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
```

11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) + 4*B*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^2*(3796*sin(f*x + e)/(cos(f*x + e) + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 70*A*a^2*(611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(

```
f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^2*(611*sin(f*x + e)/(cos(f*x +
e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(co
s(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) +
 1)^13) - 462*B*a^2*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e)
+ 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^1
3/(cos(f*x + e) + 1)^13))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(192) = 384$.

Time = 0.40 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.14

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx =$$

$$-\frac{2\left(15015\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-60060\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+15015\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+27027\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x + 1
/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x +
 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*f*x
 + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1/2*f
*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan(1/2*
f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*tan(1/
2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a^2*tan(
1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A*a^2*tan
(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*A*a^2*tan
(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*A*a^2*tan(
1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*a^2*tan(1/2
*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 - 213*B*a^2)/(
c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.54

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = -\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{994249\,A\,a^2}{32}-\frac{63639\,B\,a^2}{32}-\frac{1609013\,A\,a^2\,\cos(2\,e+2\,f\,x)}{64}+\frac{85687\,A\,a^2\,\cos(3\,e+3\,f\,x)}{16}+\frac{79591\,A\,a^2\,\cos(4\,e+4}{32}\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^7,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((994249*A*a^2)/32 - (63639*B*a^2)/32 - (1609013*A*a
^2*cos(2*e + 2*f*x))/64 + (85687*A*a^2*cos(3*e + 3*f*x))/16 + (79591*A*a^2*
cos(4*e + 4*f*x))/32 - (5261*A*a^2*cos(5*e + 5*f*x))/16 - (1771*A*a^2*cos(6
*e + 6*f*x))/64 + (140553*B*a^2*cos(2*e + 2*f*x))/64 - (4431*B*a^2*cos(3*e
+ 3*f*x))/8 - (10161*B*a^2*cos(4*e + 4*f*x))/32 + 36*B*a^2*cos(5*e + 5*f*x)
 + (231*B*a^2*cos(6*e + 6*f*x))/64 + (636207*A*a^2*sin(2*e + 2*f*x))/64 + (
309309*A*a^2*sin(3*e + 3*f*x))/32 - (7007*A*a^2*sin(4*e + 4*f*x))/4 - (1238
9*A*a^2*sin(5*e + 5*f*x))/32 + (1755*A*a^2*sin(6*e + 6*f*x))/64 - (121407*B
*a^2*sin(2*e + 2*f*x))/64 - (39039*B*a^2*sin(3*e + 3*f*x))/32 + (3003*B*a^2
*sin(4*e + 4*f*x))/16 + (1599*B*a^2*sin(5*e + 5*f*x))/32 - (195*B*a^2*sin(6
*e + 6*f*x))/64 - (93221*A*a^2*cos(e + f*x))/8 + (3291*B*a^2*cos(e + f*x))/
8 - (704847*A*a^2*sin(e + f*x))/16 + (125697*B*a^2*sin(e + f*x))/16))/(1501
5*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*c
os(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2)
)/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((
9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)
/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

3.38 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$

Optimal result

Integrand size = 36, antiderivative size = 265

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx \\
&= \frac{11}{256}a^3(10A-3B)c^6x+\frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} \\
&\quad+\frac{11a^3(10A-3B)c^6\cos(e+fx)\sin(e+fx)}{256f} \\
&\quad+\frac{11a^3(10A-3B)c^6\cos^3(e+fx)\sin(e+fx)}{384f} \\
&\quad+\frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f}-\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} \\
&\quad+\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} \\
&\quad+\frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f}
\end{aligned}$$

```
[Out] 11/256*a^3*(10*A-3*B)*c^6*x+11/560*a^3*(10*A-3*B)*c^6*cos(f*x+e)^7/f+11/256
*a^3*(10*A-3*B)*c^6*cos(f*x+e)*sin(f*x+e)/f+11/384*a^3*(10*A-3*B)*c^6*cos(f
*x+e)^3*sin(f*x+e)/f+11/480*a^3*(10*A-3*B)*c^6*cos(f*x+e)^5*sin(f*x+e)/f-1/
10*a^3*B*cos(f*x+e)^7*(c^2-c^2*sin(f*x+e))^3/f+1/90*a^3*(10*A-3*B)*cos(f*x+
e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/720*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^6-c^6*s
in(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$$
$$=\frac{11a^3c^6(10A-3B)\cos^7(e+fx)}{560f}+\frac{11a^3(10A-3B)\cos^7(e+fx)\,(c^6-c^6\sin(e+fx))}{720f}$$
$$+\frac{11a^3c^6(10A-3B)\sin(e+fx)\cos^5(e+fx)}{480f}$$
$$+\frac{11a^3c^6(10A-3B)\sin(e+fx)\cos^3(e+fx)}{384f}$$
$$+\frac{11a^3c^6(10A-3B)\sin(e+fx)\cos(e+fx)}{256f}+\frac{11}{256}a^3c^6x(10A-3B)$$
$$+\frac{a^3(10A-3B)\cos^7(e+fx)\,(c^3-c^3\sin(e+fx))^2}{90f}$$
$$-\frac{a^3B\cos^7(e+fx)\,(c^2-c^2\sin(e+fx))^3}{10f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (11*a^3*(10*A - 3*B)*c^6*x)/256 + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^7)/
(560*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]*Sin[e + f*x])/(256*f) + (11
*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(384*f) + (11*a^3*(10*A
- 3*B)*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(480*f) - (a^3*B*Cos[e + f*x]^7*(c^
2 - c^2*Sin[e + f*x])^3)/(10*f) + (a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^3 - c
^3*Sin[e + f*x])^2)/(90*f) + (11*a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^6 - c^6
*Sin[e + f*x]))/(720*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} \\
&\quad +\frac{1}{10}\left(a^3(10A-3B)c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^3\,dx \\
&= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} \\
&\quad +\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} \\
&\quad +\frac{1}{90}\left(11a^3(10A-3B)c^4\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^2\,dx
\end{aligned}$$

$$= -\frac{a^3 B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f} + \frac{1}{80}\left(11a^3(10A-3B)c^5\right)\int\cos^6(e+fx)(c-c\sin(e+fx))\,dx$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} - \frac{a^3 B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f} + \frac{1}{80}\left(11a^3(10A-3B)c^6\right)\int\cos^6(e+fx)\,dx$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f} - \frac{a^3 B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f} + \frac{1}{96}\left(11a^3(10A-3B)c^6\right)\int\cos^4(e+fx)\,dx$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A-3B)c^6\cos^3(e+fx)\sin(e+fx)}{384f} + \frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f} - \frac{a^3 B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f} + \frac{1}{128}\left(11a^3(10A-3B)c^6\right)\int\cos^2(e+fx)\,dx$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A-3B)c^6\cos(e+fx)\sin(e+fx)}{256f} + \frac{11a^3(10A-3B)c^6\cos^3(e+fx)\sin(e+fx)}{384f} + \frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f} - \frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f} + \frac{1}{256}\left(11a^3(10A-3B)c^6\right)\int 1\,dx$$

$$= \frac{11}{256}a^3(10A-3B)c^6x + \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A-3B)c^6\cos(e+fx)\sin(e+fx)}{256f} + \frac{11a^3(10A-3B)c^6\cos^3(e+fx)\sin(e+fx)}{384f} + \frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f} - \frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f}$$

Mathematica [A] (verified)

Time = 9.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx = \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6(27720(10A-3B)(e+fx)+5040(33A-19B)\cos(e+fx)+336}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
6,x]
```

[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*x
) + 5040*(33*A - 19*B)*Cos[e + f*x] + 3360*(29*A - 15*B)*Cos[3*(e + f*x)] +
 10080*(3*A - B)*Cos[5*(e + f*x)] + 360*(9*A + 5*B)*Cos[7*(e + f*x)] - 280*
(A - 3*B)*Cos[9*(e + f*x)] + 1260*(144*A - 25*B)*Sin[2*(e + f*x)] + 2520*(6
*A + 7*B)*Sin[4*(e + f*x)] - 210*(32*A - 51*B)*Sin[6*(e + f*x)] - 315*(6*A
- 5*B)*Sin[8*(e + f*x)] - 126*B*Sin[10*(e + f*x)]))/(645120*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^12*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{3\left(\left(\frac{29A}{9}-\frac{5B}{3}\right)\cos(3fx+3e)+\left(A-\frac{B}{3}\right)\cos(5fx+5e)+\left(\frac{3A}{28}+\frac{5B}{84}\right)\cos(7fx+7e)+\left(-\frac{A}{108}+\frac{B}{36}\right)\cos(9fx+9e)+\left(6A-\frac{25B}{24}\right.\right.}{}$
risch	$\frac{55a^3c^6xA}{128}-\frac{33a^3c^6xB}{256}+\frac{33c^6a^3\cos(fx+e)A}{128f}-\frac{19c^6a^3\cos(fx+e)B}{128f}-\frac{B\,a^3c^6\sin(10fx+10e)}{5120f}-\frac{c^6a^3\cos(9fx}{2304f}$
parts	$-\frac{\left(-6A\,a^3c^6-6B\,a^3c^6\right)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}+\frac{\left(-6A\,a^3c^6+8B\,a^3c^6\right)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(}{}\right.}{4}\right.}{f}$
derivativedivides	$\frac{3A\,a^3c^6\cos(fx+e)-3B\,a^3c^6\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,a^3c^6\left(\frac{128}{35}+\sin^8(fx+e)+\frac{8\left(\sin^6(fx+e)\right)}{7}+\frac{48\left(\sin^4(fx+e)\right)}{35}\right.}{9}}{}$
default	$\frac{3A\,a^3c^6\cos(fx+e)-3B\,a^3c^6\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,a^3c^6\left(\frac{128}{35}+\sin^8(fx+e)+\frac{8\left(\sin^6(fx+e)\right)}{7}+\frac{48\left(\sin^4(fx+e)\right)}{35}\right.}{9}}{}$

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)

[Out] 3/64*((29/9*A-5/3*B)*cos(3*f*x+3*e)+(A-1/3*B)*cos(5*f*x+5*e)+(3/28*A+5/84*B
)*cos(7*f*x+7*e)+(-1/108*A+1/36*B)*cos(9*f*x+9*e)+(6*A-25/24*B)*sin(2*f*x+2
*e)+(1/2*A+7/12*B)*sin(4*f*x+4*e)+(-2/9*A+17/48*B)*sin(6*f*x+6*e)+(-1/16*A+
5/96*B)*sin(8*f*x+8*e)-1/240*B*sin(10*f*x+10*e)+(11/2*A-19/6*B)*cos(f*x+e)+
55/6*f*x*A-11/4*f*x*B+1856/189*A-320/63*B)*c^6*a^3/f

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx = -\frac{8960\,(A-3\,B)a^3c^6\cos\left(fx+e\right)^9-46080\,(A-B)a^3c^6\cos\left(fx+e\right)^7-3465\,(10\,A-3\,B)a^3c^6fx+21\,(3}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/80640*(8960*(A - 3*B)*a^3*c^6*cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*cos
(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*cos(f*x + e
)^9 + 48*(30*A - 41*B)*a^3*c^6*cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*cos
(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*cos(f*x + e)^3 - 165*(10*A - 3*B)*a^
3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. 2(252) = 504.

Time = 1.51 (sec) , antiderivative size = 1948, normalized size of antiderivative = 7.35

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(e
 + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A*a
**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e + f*
x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**3*c*
*6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x)**2*
cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105
*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6/2 - 9*
A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e + f*x)*
*8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) -
8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*c**6*sin(e
 + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)**5*cos(e +
 f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) + 6*A*a*
*3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e + f*x)**3*co
s(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**3/(3*f
) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A*a**3*c**6*sin(
```

```
e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e + f*x)**2*cos(e +
f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f + 105*A*a**3*c**6*
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(e + f*x)*cos(e + f
*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 128*A*a**
3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(e + f*x)**5/(5*f) - 16*
A*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*A*a**3*c**6*cos(e + f*x)/f + 63*B*a**
3*c**6*x*sin(e + f*x)**10/256 + 315*B*a**3*c**6*x*sin(e + f*x)**8*cos(e + f
*x)**2/256 + 315*B*a**3*c**6*x*sin(e + f*x)**6*cos(e + f*x)**4/128 - 15*B*a
**3*c**6*x*sin(e + f*x)**6/8 + 315*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*
x)**6/128 - 45*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*B*a**3*c
**6*x*sin(e + f*x)**4 + 315*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**8/2
56 - 45*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/8 + 6*B*a**3*c**6*x*s
in(e + f*x)**2*cos(e + f*x)**2 - 3*B*a**3*c**6*x*sin(e + f*x)**2/2 + 63*B*a
**3*c**6*x*cos(e + f*x)**10/256 - 15*B*a**3*c**6*x*cos(e + f*x)**6/8 + 3*B*
a**3*c**6*x*cos(e + f*x)**4 - 3*B*a**3*c**6*x*cos(e + f*x)**2/2 - 193*B*a**
3*c**6*sin(e + f*x)**9*cos(e + f*x)/(256*f) + 3*B*a**3*c**6*sin(e + f*x)**8
*cos(e + f*x)/f - 237*B*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)**3/(128*f) +
 8*B*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/f - 8*B*a**3*c**6*sin(e + f*
x)**6*cos(e + f*x)/f - 21*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**5/(10*f
) + 33*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 48*B*a**3*c**6*sin(
e + f*x)**4*cos(e + f*x)**5/(5*f) - 16*B*a**3*c**6*sin(e + f*x)**4*cos(e +
f*x)**3/f + 6*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f - 147*B*a**3*c**6*
sin(e + f*x)**3*cos(e + f*x)**7/(128*f) + 5*B*a**3*c**6*sin(e + f*x)**3*cos
(e + f*x)**3/f - 5*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/f + 192*B*a**3*
c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 64*B*a**3*c**6*sin(e + f*x)**
2*cos(e + f*x)**5/(5*f) + 8*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**3/f -
 63*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**9/(256*f) + 15*B*a**3*c**6*sin(e
 + f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
f + 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)/(2*f) + 128*B*a**3*c**6*cos(e +
 f*x)**9/(105*f) - 128*B*a**3*c**6*cos(e + f*x)**7/(35*f) + 16*B*a**3*c**6*
cos(e + f*x)**5/(5*f) - B*a**3*c**6*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*si
n(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(252) = 504.

Time = 0.23 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.49

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx =$$

$$-\frac{2048\left(35\cos\left(fx+e\right)^9-180\cos\left(fx+e\right)^7+378\cos\left(fx+e\right)^5-420\cos\left(fx+e\right)^3+315\cos\left(fx+e\right)\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -1/645120*(2048*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*A*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c^6 - 1720320*(cos(f*x +
e)^3 - 3*cos(f*x + e))*A*a^3*c^6 + 630*(128*sin(2*f*x + 2*e)^3 + 840*f*x +
840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A
*a^3*c^6 - 26880*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e)
 - 48*sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + sin(4*f*x + 4*e
) - 8*sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 6144*(35*c
os(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^5 - 420*cos(f*x + e)^
3 + 315*cos(f*x + e))*B*a^3*c^6 - 147456*(5*cos(f*x + e)^7 - 21*cos(f*x + e
)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^6 + 63*(32*sin(2*f*x +
2*e)^5 - 640*sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*sin(8*f*x + 8*e) -
 600*sin(4*f*x + 4*e) + 2560*sin(2*f*x + 2*e))*B*a^3*c^6 + 20160*(4*sin(2*f
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a
^3*c^6 - 161280*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a
^3*c^6 + 483840*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^6 - 1935360*A*a^3*
c^6*cos(f*x + e) + 645120*B*a^3*c^6*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx \\
&= -\frac{Ba^3c^6\sin(10\,fx+10\,e)}{5120\,f} + \frac{11}{256}\,(10\,Aa^3c^6 - 3\,Ba^3c^6)x \\
&\quad - \frac{(Aa^3c^6 - 3\,Ba^3c^6)\cos(9\,fx+9\,e)}{2304\,f} + \frac{(9\,Aa^3c^6 + 5\,Ba^3c^6)\cos(7\,fx+7\,e)}{1792\,f} \\
&\quad + \frac{(3\,Aa^3c^6 - Ba^3c^6)\cos(5\,fx+5\,e)}{64\,f} + \frac{(29\,Aa^3c^6 - 15\,Ba^3c^6)\cos(3\,fx+3\,e)}{192\,f} \\
&\quad + \frac{(33\,Aa^3c^6 - 19\,Ba^3c^6)\cos(fx+e)}{128\,f} - \frac{(6\,Aa^3c^6 - 5\,Ba^3c^6)\sin(8\,fx+8\,e)}{2048\,f} \\
&\quad - \frac{(32\,Aa^3c^6 - 51\,Ba^3c^6)\sin(6\,fx+6\,e)}{3072\,f} + \frac{(6\,Aa^3c^6 + 7\,Ba^3c^6)\sin(4\,fx+4\,e)}{256\,f} \\
&\quad + \frac{(144\,Aa^3c^6 - 25\,Ba^3c^6)\sin(2\,fx+2\,e)}{512\,f}
\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/5120*B*a^3*c^6*sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^6
)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*cos(9*f*x + 9*e)/f + 1/1792*(9*A*a^3
```

```
*c^6 + 5*B*a^3*c^6)*cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*cos
(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*cos(3*f*x + 3*e)/f +
1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*cos(f*x + e)/f - 1/2048*(6*A*a^3*c^6 -
5*B*a^3*c^6)*sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6)*sin(
6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*sin(4*f*x + 4*e)/f + 1/5
12*(144*A*a^3*c^6 - 25*B*a^3*c^6)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 16.01 (sec) , antiderivative size = 812, normalized size of antiderivative = 3.06

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^6\,dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^18*(6*A*a^3*c^6 - 2*B*a^3*c^6) + tan(e/2 + (f*x)/2)^16*
(22*A*a^3*c^6 - 18*B*a^3*c^6) + tan(e/2 + (f*x)/2)^8*(84*A*a^3*c^6 - 28*B*a
^3*c^6) + tan(e/2 + (f*x)/2)^14*((136*A*a^3*c^6)/3 - 8*B*a^3*c^6) + tan(e/2
 + (f*x)/2)^4*((136*A*a^3*c^6)/7 - (24*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^1
0*(116*A*a^3*c^6 - 60*B*a^3*c^6) - tan(e/2 + (f*x)/2)^19*((73*A*a^3*c^6)/64
 + (33*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^2*((202*A*a^3*c^6)/63 - (58*B*a
^3*c^6)/21) + tan(e/2 + (f*x)/2)^12*((328*A*a^3*c^6)/3 - 72*B*a^3*c^6) + ta
n(e/2 + (f*x)/2)^7*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) - tan(e/2 + (f
*x)/2)^13*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) + tan(e/2 + (f*x)/2)^6*
((456*A*a^3*c^6)/7 - (344*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^5*((449*A*a^3*
c^6)/48 - (577*B*a^3*c^6)/160) - tan(e/2 + (f*x)/2)^15*((449*A*a^3*c^6)/48
- (577*B*a^3*c^6)/160) + tan(e/2 + (f*x)/2)^3*((2117*A*a^3*c^6)/192 - (705*
B*a^3*c^6)/128) - tan(e/2 + (f*x)/2)^17*((2117*A*a^3*c^6)/192 - (705*B*a^3*
c^6)/128) + tan(e/2 + (f*x)/2)^9*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64)
 - tan(e/2 + (f*x)/2)^11*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64) + tan(e
/2 + (f*x)/2)*((73*A*a^3*c^6)/64 + (33*B*a^3*c^6)/128) + (58*A*a^3*c^6)/63
- (10*B*a^3*c^6)/21)/(f*(10*tan(e/2 + (f*x)/2)^2 + 45*tan(e/2 + (f*x)/2)^4
+ 120*tan(e/2 + (f*x)/2)^6 + 210*tan(e/2 + (f*x)/2)^8 + 252*tan(e/2 + (f*x)
/2)^10 + 210*tan(e/2 + (f*x)/2)^12 + 120*tan(e/2 + (f*x)/2)^14 + 45*tan(e/2
 + (f*x)/2)^16 + 10*tan(e/2 + (f*x)/2)^18 + tan(e/2 + (f*x)/2)^20 + 1)) + (
11*a^3*c^6*atan((11*a^3*c^6*tan(e/2 + (f*x)/2)*(10*A - 3*B))/(128*((55*A*a^
3*c^6)/64 - (33*B*a^3*c^6)/128)))*(10*A - 3*B))/(128*f)
```

3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal result

Integrand size = 36, antiderivative size = 222

$$\begin{aligned}&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx\\&=\frac{5}{128}a^3(9A-2B)c^5x+\frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f}\\&+\frac{5a^3(9A-2B)c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{5a^3(9A-2B)c^5\cos^3(e+fx)\sin(e+fx)}{192f}\\&+\frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\\&+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}\end{aligned}$$

```
[Out] 5/128*a^3*(9*A-2*B)*c^5*x+1/56*a^3*(9*A-2*B)*c^5*cos(f*x+e)^7/f+5/128*a^3*(
9*A-2*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(9*A-2*B)*c^5*cos(f*x+e)^3*s
in(f*x+e)/f+1/48*a^3*(9*A-2*B)*c^5*cos(f*x+e)^5*sin(f*x+e)/f-1/9*a^3*B*c^3*
cos(f*x+e)^7*(c-c*sin(f*x+e))^2/f+1/72*a^3*(9*A-2*B)*cos(f*x+e)^7*(c^5-c^5*
sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{a^3c^5(9A-2B)\cos^7(e+fx)}{56f}+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}$$
$$+\frac{a^3c^5(9A-2B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^5(9A-2B)\sin(e+fx)\cos^3(e+fx)}{192f}$$
$$+\frac{5a^3c^5(9A-2B)\sin(e+fx)\cos(e+fx)}{128f}$$
$$+\frac{5}{128}a^3c^5x(9A-2B)-\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (5*a^3*(9*A - 2*B)*c^5*x)/128 + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^7)/(56*f)
 + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(9*A
- 2*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(9*A - 2*B)*c^5*Cos[
e + f*x]^5*Sin[e + f*x])/(48*f) - (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e +
f*x])^2)/(9*f) + (a^3*(9*A - 2*B)*Cos[e + f*x]^7*(c^5 - c^5*Sin[e + f*x]))/
(72*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\
&= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\\
&\quad+\frac{1}{9}\left(a^3(9A-2B)c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^2\,dx\\
&= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\\
&\quad+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}\\
&\quad+\frac{1}{8}\left(a^3(9A-2B)c^4\right)\int \cos^6(e+fx)(c-c\sin(e+fx))\,dx
\end{aligned}$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} - \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f} + \frac{1}{8}\left(a^3(9A-2B)c^5\right)\int \cos^6(e+fx)\,dx$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f} + \frac{1}{48}\left(5a^3(9A-2B)c^5\right)\int \cos^4(e+fx)\,dx$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A-2B)c^5\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f} + \frac{1}{64}\left(5a^3(9A-2B)c^5\right)\int \cos^2(e+fx)\,dx$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A-2B)c^5\cos(e+fx)\sin(e+fx)}{128f} + \frac{5a^3(9A-2B)c^5\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f} + \frac{1}{128}\left(5a^3(9A-2B)c^5\right)\int 1\,dx$$

$$
\begin{aligned}
&= \frac{5}{128}a^3(9A-2B)c^5x + \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} \\
&+ \frac{5a^3(9A-2B)c^5\cos(e+fx)\sin(e+fx)}{128f} \\
&+ \frac{5a^3(9A-2B)c^5\cos^3(e+fx)\sin(e+fx)}{192f} \\
&+ \frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f} \\
&- \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} \\
&+ \frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.78 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$
\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx
$$
$$
= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5(2520(9A-2B)(e+fx)+504(20A-13B)\cos(e+fx)+336(18}{}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x)
+ 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 100
8*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[9*(
e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e + f*
x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6)
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{c^5\left(\left(3A-\frac{11B}{6}\right)\cos(3fx+3e)+\left(A-\frac{B}{2}\right)\cos(5fx+5e)+\frac{\left(A-\frac{B}{8}\right)\cos(7fx+7e)}{7}+(8A-B)\sin(2fx+2e)+\frac{\left(\frac{5A}{2}+B\right)\sin(4fx+4}{2}\right.}{32f}$
risch	$\frac{45a^3c^5xA}{128}-\frac{5a^3c^5xB}{64}+\frac{5c^5a^3\cos(fx+e)A}{32f}-\frac{13c^5a^3\cos(fx+e)B}{128f}+\frac{B\,a^3c^5\cos(9fx+9e)}{2304f}-\frac{\sin(8fx+8e)A\,a^3c}{1024f}$
parts	$\frac{(-2A\,a^3c^5-2B\,a^3c^5)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-2A\,a^3c^5+B\,a^3c^5)\cos(fx+e)}{f}+\frac{(-A\,a^3c^5+2B\,a^3c^5)\left(-\right.}{}$
derivativedivides	$\frac{A\,a^3c^5(fx+e)+2A\,a^3c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{6A\,a^3c^5\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}}{}$
default	$\frac{A\,a^3c^5(fx+e)+2A\,a^3c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{6A\,a^3c^5\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}}{}$
norman	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/32*c^5*((3*A-11/6*B)*cos(3*f*x+3*e)+(A-1/2*B)*cos(5*f*x+5*e)+1/7*(A-1/8*B
)*cos(7*f*x+7*e)+(8*A-B)*sin(2*f*x+2*e)+1/2*(5/2*A+B)*sin(4*f*x+4*e)+1/16*(
-1/2*A+B)*sin(8*f*x+8*e)+1/72*cos(9*f*x+9*e)*B+1/3*B*sin(6*f*x+6*e)+(5*A-13
/4*B)*cos(f*x+e)+45/4*f*x*A-5/2*f*x*B+64/7*A-352/63*B)*a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{896\,Ba^3c^5\cos\left(fx+e\right)^9+2304\,(A-B)a^3c^5\cos\left(fx+e\right)^7+315\,(9\,A-2\,B)a^3c^5fx-21\,\left(48\,(A-2\,B)a^3\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7
+ 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 - 8
*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^3
 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. 2(209) = 418.

Time = 1.14 (sec) , antiderivative size = 1753, normalized size of antiderivative = 7.90

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e +
 f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a**
3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f*x)
**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32
 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*sin(e
 + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*cos(e +
 f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a**3*c**5
*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)**6*cos(e
 + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*A*
a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(e + f*x)**
4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*
a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c**5*sin(e + f
*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f - 6*A*a**3*c**5*
sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**
7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) + A*a**3*c**5*
sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)**7/(35*f) + 16*A*
a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f*x)**3/f + 2*A*a**
3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8/64 + 35*B*a**3*c**
5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c**5*x*sin(e + f*x)**6/8
 + 105*B*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/32 - 45*B*a**3*c**5*x*
sin(e + f*x)**4*cos(e + f*x)**2/8 + 9*B*a**3*c**5*x*sin(e + f*x)**4/4 + 35*
B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/16 - 45*B*a**3*c**5*x*sin(e +
 f*x)**2*cos(e + f*x)**4/8 + 9*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**
2/2 - B*a**3*c**5*x*sin(e + f*x)**2 + 35*B*a**3*c**5*x*cos(e + f*x)**8/64 -
 15*B*a**3*c**5*x*cos(e + f*x)**6/8 + 9*B*a**3*c**5*x*cos(e + f*x)**4/4 - B
*a**3*c**5*x*cos(e + f*x)**2 + B*a**3*c**5*sin(e + f*x)**8*cos(e + f*x)/f -
 93*B*a**3*c**5*sin(e + f*x)**7*cos(e + f*x)/(64*f) + 8*B*a**3*c**5*sin(e +
 f*x)**6*cos(e + f*x)**3/(3*f) - 2*B*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)
/f - 511*B*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(192*f) + 33*B*a**3*c*
*5*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 16*B*a**3*c**5*sin(e + f*x)**4*cos(
e + f*x)**5/(5*f) - 4*B*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f - 385*B
*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(192*f) + 5*B*a**3*c**5*sin(e +
f*x)**3*cos(e + f*x)**3/f - 15*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)/(4*
f) + 64*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 16*B*a**3*c**5
```

```
*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**5*sin(e + f*x)**2*cos(
e + f*x)/f - 35*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**7/(64*f) + 15*B*a**3
*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) - 9*B*a**3*c**5*sin(e + f*x)*cos(e
 + f*x)**3/(4*f) + B*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f + 128*B*a**3*c**
5*cos(e + f*x)**9/(315*f) - 32*B*a**3*c**5*cos(e + f*x)**7/(35*f) + 4*B*a**
3*c**5*cos(e + f*x)**3/(3*f) - B*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(A
 + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(210) = 420$.

Time = 0.32 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.78

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^5 \, dx$$
$$= \frac{18432 \left(5 \cos\left(fx + e\right)^7 - 21 \cos\left(fx + e\right)^5 + 35 \cos\left(fx + e\right)^3 - 35 \cos\left(fx + e\right)\right) A a^3 c^5 + 129024 \left(3 \cos\left(f\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] 1/322560*(18432*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^3*c^5 + 129024*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^3*c^5 + 645120*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^3*c^5 - 105*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e)
 + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*sin(2*f
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a
^3*c^5 - 161280*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*(f*x +
e)*A*a^3*c^5 + 1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x +
 e)^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^5 + 18432*(5*cos(f*x
 + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^
5 - 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^5 + 210*(128*sin(2*f*x
 + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 7
68*sin(2*f*x + 2*e))*B*a^3*c^5 - 10080*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^5 + 60480*(12*f*x + 1
2*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^5 - 161280*(2*f*x + 2*
e - sin(2*f*x + 2*e))*B*a^3*c^5 + 645120*A*a^3*c^5*cos(f*x + e) - 322560*B*
a^3*c^5*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.32

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx &= \frac{Ba^3c^5\cos(9\,fx+9\,e)}{2304\,f}+\frac{Ba^3c^5\sin(6\,fx+6\,e)}{96\,f} \\ &+\frac{5}{128}\left(9\,Aa^3c^5-2\,Ba^3c^5\right)x+\frac{\left(8\,Aa^3c^5-Ba^3c^5\right)\cos(7\,fx+7\,e)}{1792\,f} \\ &+\frac{\left(2\,Aa^3c^5-Ba^3c^5\right)\cos(5\,fx+5\,e)}{64\,f}+\frac{\left(18\,Aa^3c^5-11\,Ba^3c^5\right)\cos(3\,fx+3\,e)}{192\,f} \\ &+\frac{\left(20\,Aa^3c^5-13\,Ba^3c^5\right)\cos(fx+e)}{128\,f}-\frac{\left(Aa^3c^5-2\,Ba^3c^5\right)\sin(8\,fx+8\,e)}{1024\,f} \\ &+\frac{\left(5\,Aa^3c^5+2\,Ba^3c^5\right)\sin(4\,fx+4\,e)}{128\,f}+\frac{\left(8\,Aa^3c^5-Ba^3c^5\right)\sin(2\,fx+2\,e)}{32\,f}\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] 1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f + 5
/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos(7
*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/192*(
18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 - 13*
B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x + 8*
e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8*A*a^3
*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.18

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$

$$=\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}\left(4\,A\,a^3\,c^5-2\,B\,a^3\,c^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}\left(8\,A\,a^3\,c^5-8\,B\,a^3\,c^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\left(\frac{8\,A\,a^3\,c^5}{7}-\frac{8\,B}{}\right.}{}$$

$$+\frac{5\,a^3\,c^5\,\mathrm{atan}\left(\frac{5\,a^3\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(9\,A-2\,B)}{64\left(\frac{45\,A\,a^3\,c^5}{64}-\frac{5\,B\,a^3\,c^5}{32}\right)}\right)(9\,A-2\,B)}{64\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^16*(4*A*a^3*c^5 - 2*B*a^3*c^5) + tan(e/2 + (f*x)/2)^14*
(8*A*a^3*c^5 - 8*B*a^3*c^5) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c^5)/7 - (8*B*
a^3*c^5)/7) + tan(e/2 + (f*x)/2)^8*(32*A*a^3*c^5 - 4*B*a^3*c^5) + tan(e/2 +
 (f*x)/2)^6*(24*A*a^3*c^5 - 24*B*a^3*c^5) + tan(e/2 + (f*x)/2)^12*(24*A*a^3
*c^5 - (16*B*a^3*c^5)/3) + tan(e/2 + (f*x)/2)^10*(40*A*a^3*c^5 - 40*B*a^3*c
^5) + tan(e/2 + (f*x)/2)^4*((88*A*a^3*c^5)/7 - (32*B*a^3*c^5)/7) - tan(e/2
+ (f*x)/2)^17*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + tan(e/2 + (f*x)/2)^5
*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^13*((149*A*a
^3*c^5)/32 + (83*B*a^3*c^5)/16) + tan(e/2 + (f*x)/2)^3*((189*A*a^3*c^5)/32
- (191*B*a^3*c^5)/48) - tan(e/2 + (f*x)/2)^15*((189*A*a^3*c^5)/32 - (191*B*
a^3*c^5)/48) + tan(e/2 + (f*x)/2)^7*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/1
6) - tan(e/2 + (f*x)/2)^11*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/16) + tan(
e/2 + (f*x)/2)*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + (4*A*a^3*c^5)/7 - (
22*B*a^3*c^5)/63)/(f*(9*tan(e/2 + (f*x)/2)^2 + 36*tan(e/2 + (f*x)/2)^4 + 84
*tan(e/2 + (f*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^8 + 126*tan(e/2 + (f*x)/2)^1
0 + 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^14 + 9*tan(e/2 + (f*x)
/2)^16 + tan(e/2 + (f*x)/2)^18 + 1)) + (5*a^3*c^5*atan((5*a^3*c^5*tan(e/2 +
 (f*x)/2)*(9*A - 2*B))/(64*((45*A*a^3*c^5)/64 - (5*B*a^3*c^5)/32)))*(9*A -
2*B))/(64*f)
```

3.40 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal result

Integrand size = 36, antiderivative size = 181

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\
&= \frac{5}{128}a^3(8A-B)c^4x+\frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} \\
&\quad+\frac{5a^3(8A-B)c^4\cos(e+fx)\sin(e+fx)}{128f}+\frac{5a^3(8A-B)c^4\cos^3(e+fx)\sin(e+fx)}{192f} \\
&\quad+\frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}
\end{aligned}$$

```
[Out] 5/128*a^3*(8*A-B)*c^4*x+1/56*a^3*(8*A-B)*c^4*cos(f*x+e)^7/f+5/128*a^3*(8*A-
B)*c^4*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(8*A-B)*c^4*cos(f*x+e)^3*sin(f*x+e
)/f+1/48*a^3*(8*A-B)*c^4*cos(f*x+e)^5*sin(f*x+e)/f-1/8*a^3*B*cos(f*x+e)^7*(
c^4-c^4*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used

$= \{3046, 2939, 2748, 2715, 8\}$

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{a^3c^4(8A-B)\cos^7(e+fx)}{56f} + \frac{a^3c^4(8A-B)\sin(e+fx)\cos^5(e+fx)}{48f} + \frac{5a^3c^4(8A-B)\sin(e+fx)\cos^3(e+fx)}{192f} + \frac{5a^3c^4(8A-B)\sin(e+fx)\cos(e+fx)}{128f} + \frac{5}{128}a^3c^4x(8A-B) - \frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*(8*A - B)*c^4*x)/128 + (a^3*(8*A - B)*c^4*Cos[e + f*x]^7)/(56*f) + (
5*a^3*(8*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(8*A - B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(8*A - B)*c^4*Cos[e + f*x]^5
*Sin[e + f*x])/(48*f) - (a^3*B*Cos[e + f*x]^7*(c^4 - c^4*Sin[e + f*x]))/(8*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx\\
&= -\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}\\
&\quad+\frac{1}{8}\left(a^3(8A-B)c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))\,dx\\
&= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}\\
&\quad+\frac{1}{8}\left(a^3(8A-B)c^4\right)\int \cos^6(e+fx)\,dx\\
&= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f}\\
&\quad-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}+\frac{1}{48}\left(5a^3(8A-B)c^4\right)\int \cos^4(e+fx)\,dx\\
&= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{5a^3(8A-B)c^4\cos^3(e+fx)\sin(e+fx)}{192f}\\
&\quad+\frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f}\\
&\quad-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}+\frac{1}{64}\left(5a^3(8A-B)c^4\right)\int \cos^2(e+fx)\,dx\\
&= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{5a^3(8A-B)c^4\cos(e+fx)\sin(e+fx)}{128f}\\
&\quad+\frac{5a^3(8A-B)c^4\cos^3(e+fx)\sin(e+fx)}{192f}\\
&\quad+\frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f}\\
&\quad-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}+\frac{1}{128}\left(5a^3(8A-B)c^4\right)\int 1\,dx
\end{aligned}$$

$$= \frac{5}{128}a^3(8A-B)c^4x + \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} + \frac{5a^3(8A-B)c^4\cos(e+fx)\sin(e+fx)}{128f} + \frac{5a^3(8A-B)c^4\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}$$

Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4(840(8A-B)(e+fx)+1680(A-B)\cos(e+fx)+1008(A-B)}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) + 1
680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Cos[
5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f*x)
] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*B*Si
n[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{c^4\left(3(A-B)\cos(3fx+3e)+(A-B)\cos(5fx+5e)+\frac{(A-B)\cos(7fx+7e)}{7}+(15A-B)\sin(2fx+2e)+\left(\frac{B}{2}+3A\right)\sin(4fx+4e)+\right.}{64f}$
risch	$\frac{5a^3c^4xA}{16}-\frac{5a^3c^4xB}{128}+\frac{5c^4a^3\cos(fx+e)A}{64f}-\frac{5c^4a^3\cos(fx+e)B}{64f}+\frac{B\,a^3c^4\sin(8fx+8e)}{1024f}+\frac{c^4a^3\cos(7fx+7e)A}{448f}-$
parts	$\frac{(-3A\,a^3c^4-B\,a^3c^4)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-3A\,a^3c^4+3B\,a^3c^4)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx}{5f}$
derivativedivides	$\frac{A\,a^3c^4(fx+e)+3A\,a^3c^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B\,a^3c^4\left(-\frac{\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\frac{35(}{}\right.}{}\right.}{}$
default	$\frac{A\,a^3c^4(fx+e)+3A\,a^3c^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B\,a^3c^4\left(-\frac{\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\right.}{}\right.}{}$
norman	$\frac{\frac{c^4a^3(488A-397B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192f}+\left(\frac{5}{16}A\,a^3c^4-\frac{5}{128}B\,a^3c^4\right)x+\frac{c^4a^3(88A+5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}-\frac{5c^4a^3(136A-353B)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{192f}}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/64*c^4*(3*(A-B)*cos(3*f*x+3*e)+(A-B)*cos(5*f*x+5*e)+1/7*(A-B)*cos(7*f*x+7
*e)+(15*A-B)*sin(2*f*x+2*e)+(1/2*B+3*A)*sin(4*f*x+4*e)+1/3*(A+B)*sin(6*f*x+
6*e)+1/16*B*sin(8*f*x+8*e)+5*(A-B)*cos(f*x+e)+20*f*x*A-5/2*f*x*B+64/7*A-64/
7*B)*a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.76

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{384\,(A-B)a^3c^4\cos{(fx+e)}^7+105\,(8\,A-B)a^3c^4fx+7\,\left(48\,Ba^3c^4\cos{(fx+e)}^7+8\,(8\,A-B)a^3c^4\cos{(f}\right.}{2688\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7*
(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8*A
 - B)*a^3*c^4*cos(f*x + e)^3 + 15*(8*A - B)*a^3*c^4*cos(f*x + e))*sin(f*x +
 e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(163) = 326$.

Time = 0.87 (sec) , antiderivative size = 1579, normalized size of antiderivative = 8.72

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*cos(e
 + f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f +
 11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*sin(e +
 f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c**4*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A*a**3*c**4*
sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**5
/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**3*c**4*
sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)**7/(35*f) + 8
*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f*x)**3/f + A*a*
*3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/128 + 35*B*a**3*c
**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*x*sin(e + f*x)**6
/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 45*B*a**3*c**4
*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*sin(e + f*x)**4/8 +
 35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 45*B*a**3*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 - B*a**3*c**4*x*sin(e + f*x)**2/2 + 35*B*a**3*c**4*x*cos(e + f*x)*
*8/128 - 15*B*a**3*c**4*x*cos(e + f*x)**6/16 + 9*B*a**3*c**4*x*cos(e + f*x)
**4/8 - B*a**3*c**4*x*cos(e + f*x)**2/2 - 93*B*a**3*c**4*sin(e + f*x)**7*co
s(e + f*x)/(128*f) + B*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f - 511*B*a**
3*c**4*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 33*B*a**3*c**4*sin(e + f*x
)**5*cos(e + f*x)/(16*f) + 2*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
- 3*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 385*B*a**3*c**4*sin(e + f*
x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)*
*3/(2*f) - 15*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*B*a**3*c**
```

```
4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**3/f + 3*B*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 35*B*a**3*c
**4*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 15*B*a**3*c**4*sin(e + f*x)*cos(
e + f*x)**5/(16*f) - 9*B*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + B*a
**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*B*a**3*c**4*cos(e + f*x)**7/(
35*f) - 8*B*a**3*c**4*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**4*cos(e + f*x)**3
/f - B*a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3*(-c*sin(e) + c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(170) = 340.

Time = 0.24 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.15

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{3072\left(5\cos\left(fx+e\right)^7-21\cos\left(fx+e\right)^5+35\cos\left(fx+e\right)^3-35\cos\left(fx+e\right)\right)Aa^3c^4+21504\left(3\cos\left(fx+\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3
*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c
^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^
5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f*x + e)^
5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*
c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{Ba^3c^4\sin(8fx+8e)}{1024f}+\frac{5}{128}(8Aa^3c^4-Ba^3c^4)x+\frac{(Aa^3c^4-Ba^3c^4)\cos(7fx+7e)}{448f}$$
$$+\frac{(Aa^3c^4-Ba^3c^4)\cos(5fx+5e)}{64f}+\frac{3(Aa^3c^4-Ba^3c^4)\cos(3fx+3e)}{64f}$$
$$+\frac{5(Aa^3c^4-Ba^3c^4)\cos(fx+e)}{64f}+\frac{(Aa^3c^4+Ba^3c^4)\sin(6fx+6e)}{192f}$$
$$+\frac{(6Aa^3c^4+Ba^3c^4)\sin(4fx+4e)}{128f}+\frac{(15Aa^3c^4-Ba^3c^4)\sin(2fx+2e)}{64f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x + 1
/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3*c
^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f +
5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*c^4)
*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)/f +
1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.65

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6Aa^3c^4-6Ba^3c^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(2Aa^3c^4-2Ba^3c^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(6Aa^3c^4-6}{}$$
$$+\frac{5a^3c^4\,\mathrm{atan}\left(\frac{5a^3c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8A-B)}{64\left(\frac{5Aa^3c^4}{8}-\frac{5Ba^3c^4}{64}\right)}\right)(8A-B)}{64f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^4*(6*A*a^3*c^4 - 6*B*a^3*c^4) + tan(e/2 + (f*x)/2)^12*(
2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)^6*(6*A*a^3*c^4 - 6*B*a^3*c^
4) + tan(e/2 + (f*x)/2)^14*(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)
```

```
^2*((2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7) + tan(e/2 + (f*x)/2)^8*(10*A*a^3*c^4
 - 10*B*a^3*c^4) + tan(e/2 + (f*x)/2)^10*(10*A*a^3*c^4 - 10*B*a^3*c^4) - ta
n(e/2 + (f*x)/2)^15*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + tan(e/2 + (f*x)
/2)^3*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^13*((6
1*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^5*((113*A*a^3*c
^4)/24 + (895*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^11*((113*A*a^3*c^4)/24 +
 (895*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^7*((85*A*a^3*c^4)/24 - (1765*B*a
^3*c^4)/192) - tan(e/2 + (f*x)/2)^9*((85*A*a^3*c^4)/24 - (1765*B*a^3*c^4)/1
92) + tan(e/2 + (f*x)/2)*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + (2*A*a^3*c
^4)/7 - (2*B*a^3*c^4)/7)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e/2 + (f*x)/2)
^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*tan(e/2 + (f*x)
/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*
x)/2)^16 + 1)) + (5*a^3*c^4*atan((5*a^3*c^4*tan(e/2 + (f*x)/2)*(8*A - B))/(
64*((5*A*a^3*c^4)/8 - (5*B*a^3*c^4)/64)))*(8*A - B))/(64*f)
```

3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= \frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos(e+fx)\sin(e+fx)}{16f}\\&+\frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f}\end{aligned}$$

```
[Out] 5/16*a^3*A*c^3*x-1/7*a^3*B*c^3*cos(f*x+e)^7/f+5/16*a^3*A*c^3*cos(f*x+e)*sin
(f*x+e)/f+5/24*a^3*A*c^3*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*A*c^3*cos(f*x+e)
^5*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2748, 2715, 8}

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= \frac{a^3Ac^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3Ac^3\sin(e+fx)\cos^3(e+fx)}{24f}\\&+\frac{5a^3Ac^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*A*c^3*x)/16 - (a^3*B*c^3*Cos[e + f*x]^7)/(7*f) + (5*a^3*A*c^3*Cos[e
+ f*x]*Sin[e + f*x])/(16*f) + (5*a^3*A*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(24
*f) + (a^3*A*c^3*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))\,dx \\
&= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\left(a^3Ac^3\right)\int \cos^6(e+fx)\,dx \\
&= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{6}\left(5a^3Ac^3\right)\int \cos^4(e+fx)\,dx \\
&= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad +\frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{8}\left(5a^3Ac^3\right)\int \cos^2(e+fx)\,dx
\end{aligned}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{16}(5a^3Ac^3)\int 1\,dx$$

$$= \frac{5}{16}a^3Ac^3x - \frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{a^3c^3(-192B\cos^7(e+fx)+7A(60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))))}{1344f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)]
+ 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{c^3 a^3 (420 f x A+7 \sin(6 f x+6 e) A+63 \sin(4 f x+4 e) A+315 A \sin(2 f x+2 e)-105 \cos(f x+e) B-3 \cos(7 f x+7 e) B-21 \cos(5 f x+\ldots}{1344 f}$
risch	$\frac{5 a^3 A c^3 x}{16}-\frac{5 B a^3 c^3 \cos(f x+e)}{64 f}-\frac{B a^3 c^3 \cos(7 f x+7 e)}{448 f}+\frac{A a^3 c^3 \sin(6 f x+6 e)}{192 f}-\frac{B a^3 c^3 \cos(5 f x+5 e)}{64 f}+\frac{3 A a^3 c^3 \ldots}{\ldots}$
derivativedivides	$\frac{A a^3 c^3 (f x+e)-A a^3 c^3 \left(-\frac{\left(\sin^5(f x+e)+\frac{5\left(\sin^3(f x+e)\right)}{4}+\frac{15 \sin(f x+e)}{8}\right)\cos(f x+e)}{6}+\frac{5 f x}{16}+\frac{5 e}{16}\right)+3 A a^3 c^3 \left(-\frac{\left(\sin^3(f x+e)+\frac{3\ldots}{}\right)}{}\right.}{\ldots}$
default	$\frac{A a^3 c^3 (f x+e)-A a^3 c^3 \left(-\frac{\left(\sin^5(f x+e)+\frac{5\left(\sin^3(f x+e)\right)}{4}+\frac{15 \sin(f x+e)}{8}\right)\cos(f x+e)}{6}+\frac{5 f x}{16}+\frac{5 e}{16}\right)+3 A a^3 c^3 \left(-\frac{\left(\sin^3(f x+e)+\frac{3\ldots}{}\right)}{}\right.}{\ldots}$
parts	$a^3 A c^3 x-\frac{B a^3 c^3 \cos(f x+e)}{f}-\frac{3 A a^3 c^3 \left(-\frac{\cos(f x+e)\sin(f x+e)}{2}+\frac{f x}{2}+\frac{e}{2}\right)}{f}+\frac{3 A a^3 c^3 \left(-\frac{\left(\sin^3(f x+e)+\frac{3 \sin(f x+e)}{2}\right) c\ldots}{4}\right.}{f}$
norman	$\frac{-\frac{2 B a^3 c^3}{7 f}-\frac{6 B a^3 c^3 \left(\tan^4\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{f}-\frac{10 B a^3 c^3 \left(\tan^8\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2 B a^3 c^3 \left(\tan^{12}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{f}+\frac{5 a^3 A c^3 x}{16}+\frac{11 A a^3 c^3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{8 f}+\ldots}{\ldots}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/1344*c^3*a^3*(420*f*x*A+7*sin(6*f*x+6*e)*A+63*sin(4*f*x+4*e)*A+315*A*sin(
2*f*x+2*e)-105*cos(f*x+e)*B-3*cos(7*f*x+7*e)*B-21*cos(5*f*x+5*e)*B-63*cos(3
*f*x+3*e)*B-192*B)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx =$$
$$-\frac{48\,Ba^3c^3\cos(fx+e)^7-105\,Aa^3c^3fx-7\left(8\,Aa^3c^3\cos(fx+e)^5+10\,Aa^3c^3\cos(fx+e)^3+15\,Aa^3c^3\cos\ldots\right.}{336\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*co
s(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(116) = 232$.

Time = 0.54 (sec) , antiderivative size = 682, normalized size of antiderivative = 5.83

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \begin{cases} -\frac{5Aa^3c^3x\sin^6(e+fx)}{16} - \frac{15Aa^3c^3x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{9Aa^3c^3x\sin^4(e+fx)}{8} - \frac{15Aa^3c^3x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{9Aa^3c^3x \\ x(A + B\sin(e))(a\sin(e) + a)^3(-c\sin(e) + c)^3 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*cos(e
 + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/
(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c*
*3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*cos(e +
f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**
3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)**6*cos(e
+ f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)
**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f + 3*B*a**3*c**3
*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)**7/(35*f) - 8
*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f*x)**3/f - B*a*
*3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*
sin(e) + c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(107) = 214$.

Time = 0.22 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.26

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx =$$
$$-\frac{35\left(4\sin(2fx + 2e)^3 + 60fx + 60e + 9\sin(4fx + 4e) - 48\sin(2fx + 2e)\right)Aa^3c^3 - 630(12fx + 1}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^3
 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 3
5*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{5}{16}Aa^3c^3x - \frac{Ba^3c^3\cos(7fx+7e)}{448f} - \frac{Ba^3c^3\cos(5fx+5e)}{64f} - \frac{3Ba^3c^3\cos(3fx+3e)}{64f} - \frac{5Ba^3c^3\cos(fx+e)}{64f} + \frac{Aa^3c^3\sin(6fx+6e)}{192f} + \frac{3Aa^3c^3\sin(4fx+4e)}{64f} + \frac{15Aa^3c^3\sin(2fx+2e)}{64f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos(
5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f*x
 + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x + 4
*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{5\,A\,a^3\,c^3\,x}{16} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\left(\frac{a^3\,c^3\,(672\,B-735\,A\,(e+f\,x))}{336}+\frac{35\,A\,a^3\,c^3\,(e+f\,x)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^3\,c^3\,(2016\,B-2205\,A\,(e+f\,x))}{336}+\frac{105\,A}{}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*A*a^3*c^3*x)/16 - (tan(e/2 + (f*x)/2)^12*((a^3*c^3*(672*B - 735*A*(e + f
*x)))/336 + (35*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^3*(
2016*B - 2205*A*(e + f*x)))/336 + (105*A*a^3*c^3*(e + f*x))/16) + tan(e/2 +
 (f*x)/2)^8*((a^3*c^3*(3360*B - 3675*A*(e + f*x)))/336 + (175*A*a^3*c^3*(e
+ f*x))/16) + (a^3*c^3*(96*B - 105*A*(e + f*x)))/336 + (5*A*a^3*c^3*(e + f*
x))/16 - (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^3)/6 - (85*A*a^3*c^3*tan(e/2 + (f*
x)/2)^5)/24 + (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (7*A*a^3*c^3*tan(e/2
 + (f*x)/2)^11)/6 + (11*A*a^3*c^3*tan(e/2 + (f*x)/2)^13)/8 - (11*A*a^3*c^3*
tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 36, antiderivative size = 138

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{1}{16}a^3(6A+B)c^2x-\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(6A+B)c^2\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^3(6A+B)c^2\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f}$$

```
[Out] 1/16*a^3*(6*A+B)*c^2*x-1/30*a^3*(6*A+B)*c^2*cos(f*x+e)^5/f+1/16*a^3*(6*A+B)
*c^2*cos(f*x+e)*sin(f*x+e)/f+1/24*a^3*(6*A+B)*c^2*cos(f*x+e)^3*sin(f*x+e)/f
-1/6*B*c^2*cos(f*x+e)^5*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = -\frac{a^3c^2(6A+B)\cos^5(e+fx)}{30f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^3c^2x(6A+B)-\frac{Bc^2\cos^5(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{6f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(6*A + B)*c^2*x)/16 - (a^3*(6*A + B)*c^2*Cos[e + f*x]^5)/(30*f) + (a^3
*(6*A + B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^3*(6*A + B)*c^2*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (B*c^2*Cos[e + f*x]^5*(a^3 + a^3*Sin[e + f
*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$

$$= -\frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f} + \frac{1}{6}\left(a^2(6A+B)c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))\,dx$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} - \frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f} + \frac{1}{6}\left(a^3(6A+B)c^2\right)\int \cos^4(e+fx)\,dx$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} + \frac{a^3(6A+B)c^2\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f} + \frac{1}{8}\left(a^3(6A+B)c^2\right)\int \cos^2(e+fx)\,dx$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} + \frac{a^3(6A+B)c^2\cos(e+fx)\sin(e+fx)}{16f} + \frac{a^3(6A+B)c^2\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f} + \frac{1}{16}\left(a^3(6A+B)c^2\right)\int 1\,dx$$

$$= \frac{1}{16}a^3(6A+B)c^2x - \frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} + \frac{a^3(6A+B)c^2\cos(e+fx)\sin(e+fx)}{16f} + \frac{a^3(6A+B)c^2\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f}$$

Mathematica [A] (verified)

Time = 6.89 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{a^3c^2(360Ae+60Be+360Afx+60Bfx-120(A+B)\cos(e+fx)-60(A+B)\cos(3(e+fx))-12A\text{co}}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*x
] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{\left(5(A+B)\cos(3fx+3e)+(A+B)\cos(5fx+5e)+5\left(-4A-\frac{B}{4}\right)\sin(2fx+2e)+\frac{5\left(-A+\frac{B}{2}\right)\sin(4fx+4e)}{2}+\frac{5B\sin(6fx+6e)}{12}+\right.}{80f}$
risch	$\frac{3a^3c^2xA}{8}+\frac{a^3c^2xB}{16}-\frac{c^2a^3\cos(fx+e)A}{8f}-\frac{c^2a^3\cos(fx+e)B}{8f}-\frac{B\,a^3c^2\sin(6fx+6e)}{192f}-\frac{c^2a^3\cos(5fx+5e)A}{80f}-$
parts	$-\frac{(-2A\,a^3c^2-2B\,a^3c^2)(2+\sin^2(fx+e))\cos(fx+e)}{3f}+\frac{(-2A\,a^3c^2+B\,a^3c^2)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{(A\,a^3}{}$
derivativedivides	$\frac{-\frac{A\,a^3c^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+A\,a^3c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2A\,a^3}{}}{}$
default	$\frac{-\frac{A\,a^3c^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+A\,a^3c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2A\,a^3}{}}{}$
norman	$\frac{(\frac{3}{8}A\,a^3c^2+\frac{1}{16}B\,a^3c^2)x+(\frac{3}{8}A\,a^3c^2+\frac{1}{16}B\,a^3c^2)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}A\,a^3c^2+\frac{3}{8}B\,a^3c^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}A\,a^3c^2}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] -1/80*(5*(A+B)*cos(3*f*x+3*e)+(A+B)*cos(5*f*x+5*e)+5*(-4*A-1/4*B)*sin(2*f*x
+2*e)+5/2*(-A+1/2*B)*sin(4*f*x+4*e)+5/12*B*sin(6*f*x+6*e)+10*(A+B)*cos(f*x+
e)-30*f*x*A-5*f*x*B+16*A+16*B)*c^2*a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx =$$
$$-\frac{48\,(A+B)a^3c^2\cos\left(fx+e\right)^5-15\,(6\,A+B)a^3c^2fx+5\,\left(8\,Ba^3c^2\cos\left(fx+e\right)^5-2\,(6\,A+B)a^3c^2\cos\left(\right.\right.}{240\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(8
*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A + B
)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(128) = 256$.

Time = 0.45 (sec) , antiderivative size = 910, normalized size of antiderivative = 6.59

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\begin{cases} \frac{3Aa^3c^2x\sin^4(e+fx)}{8}+\frac{3Aa^3c^2x\sin^2(e+fx)\cos^2(e+fx)}{4}-Aa^3c^2x\sin^2(e+fx)+\frac{3Aa^3c^2x\cos^4(e+fx)}{8}-Aa^3c^2x\cos^2(e- \\ x(A+B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c)^2 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*cos
(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*c**
2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e + f*
x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a**3*c
**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2*cos(e
+ f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**2*cos(e
+ f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15*B*a**3*c*
*2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c**2*x*cos(e
+ f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2*x*cos(e + f*
x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**3*c**2*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**3*c**
2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*sin(e + f*x)**2*cos
(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*B*a**3*
c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*B*a**3*c**2*cos(e + f
*x)**3/(3*f) - B*a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**3*(-c*sin(e) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(128) = 256$.

Time = 0.22 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.61

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx =$$
$$-\frac{64\left(3\cos\left(fx+e\right)^5 - 10\cos\left(fx+e\right)^3 + 15\cos\left(fx+e\right)\right)Aa^3c^2 + 640\left(\cos\left(fx+e\right)^3 - 3\cos\left(fx+e\right)\right)}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c
^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 - 30*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2 + 480*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a^3*c^2 - 960*(f*x + e)*A*a^3*c^2 + 64*(3*cos(f*x + e)^5 - 1
0*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2 + 640*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^2 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*
x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^2 + 60*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e)
)*B*a^3*c^2 + 960*A*a^3*c^2*cos(f*x + e) + 960*B*a^3*c^2*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.43

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx$$
$$= -\frac{Ba^3c^2\sin\left(6\,fx + 6\,e\right)}{192\,f} + \frac{1}{16}\left(6\,Aa^3c^2 + Ba^3c^2\right)x - \frac{\left(Aa^3c^2 + Ba^3c^2\right)\cos\left(5\,fx + 5\,e\right)}{80\,f}$$
$$- \frac{\left(Aa^3c^2 + Ba^3c^2\right)\cos\left(3\,fx + 3\,e\right)}{16\,f} - \frac{\left(Aa^3c^2 + Ba^3c^2\right)\cos\left(fx + e\right)}{8\,f}$$
$$+ \frac{\left(2\,Aa^3c^2 - Ba^3c^2\right)\sin\left(4\,fx + 4\,e\right)}{64\,f} + \frac{\left(16\,Aa^3c^2 + Ba^3c^2\right)\sin\left(2\,fx + 2\,e\right)}{64\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*c^2*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^3*c^2 + B*a^3*c^2)*x - 1/
80*(A*a^3*c^2 + B*a^3*c^2)*cos(5*f*x + 5*e)/f - 1/16*(A*a^3*c^2 + B*a^3*c^2
)*cos(3*f*x + 3*e)/f - 1/8*(A*a^3*c^2 + B*a^3*c^2)*cos(f*x + e)/f + 1/64*(2
*A*a^3*c^2 - B*a^3*c^2)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^3*c^2 + B*a^3*c^2
)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.88

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$

$$= \frac{a^3\,c^2\,\mathrm{atan}\left(\frac{a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(6\,A+B)}{8\,\left(\frac{3\,A\,a^3\,c^2}{4}+\frac{B\,a^3\,c^2}{8}\right)}\right)\,(6\,A+B)}{8\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(4\,A\,a^3\,c^2+4\,B\,a^3\,c^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,(2\,A\,a^3\,c^2+2\,B\,a^3\,c^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(4\,A\,a^3\,c^2+4}{}$$

$$- \frac{a^3\,c^2\,(6\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{8\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a^3*c^2*atan((a^3*c^2*tan(e/2 + (f*x)/2)*(6*A + B))/(8*((3*A*a^3*c^2)/4 +
(B*a^3*c^2)/8)))*(6*A + B))/(8*f) - (tan(e/2 + (f*x)/2)^4*(4*A*a^3*c^2 + 4*
B*a^3*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2 + (
f*x)/2)^6*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^10*(2*A*a^3*c^2
+ 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^2*((2*A*a^3*c^2)/5 + (2*B*a^3*c^2)/5) -
 tan(e/2 + (f*x)/2)^5*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/
2)^7*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^11*((5*A*a^3*c
^2)/4 - (B*a^3*c^2)/8) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c^2)/4 + (47*B*a^3*
c^2)/24) + tan(e/2 + (f*x)/2)^9*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/24) - tan
(e/2 + (f*x)/2)*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) + (2*A*a^3*c^2)/5 + (2*B*
a^3*c^2)/5)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e
/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e
/2 + (f*x)/2)^12 + 1)) - (a^3*c^2*(6*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

3.43 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 140

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\frac{1}{8}a^3(5A+2B)cx-\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f}$$
$$+\frac{a^3(5A+2B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f}$$
$$-\frac{(5A+2B)c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{20f}$$

```
[Out] 1/8*a^3*(5*A+2*B)*c*x-1/12*a^3*(5*A+2*B)*c*cos(f*x+e)^3/f+1/8*a^3*(5*A+2*B)
*c*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^2/f-1/20
*(5*A+2*B)*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used

$= \{3046, 2939, 2757, 2748, 2715, 8\}$

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= -\frac{a^3c(5A + 2B)\cos^3(e + fx)}{12f} - \frac{c(5A + 2B)\cos^3(e + fx)\,(a^3\sin(e + fx) + a^3)}{20f}$$
$$+ \frac{a^3c(5A + 2B)\sin(e + fx)\cos(e + fx)}{8f}$$
$$+ \frac{1}{8}a^3cx(5A + 2B) - \frac{aBc\cos^3(e + fx)(a\sin(e + fx) + a)^2}{5f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(5*A + 2*B)*c*x)/8 - (a^3*(5*A + 2*B)*c*Cos[e + f*x]^3)/(12*f) + (a^3*
(5*A + 2*B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(a +
 a*Sin[e + f*x])^2)/(5*f) - ((5*A + 2*B)*c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e
+ f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} + \frac{1}{5}(a(5A+2B)c)\int \cos^2(e+fx)(a+a\sin(e+fx))^2\,dx$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} - \frac{(5A+2B)c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{20f} + \frac{1}{4}\left(a^2(5A+2B)c\right)\int \cos^2(e+fx)(a+a\sin(e+fx))\,dx$$

$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} - \frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} - \frac{(5A+2B)c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{20f} + \frac{1}{4}\left(a^3(5A+2B)c\right)\int \cos^2(e+fx)\,dx$$

$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} + \frac{a^3(5A+2B)c\cos(e+fx)\sin(e+fx)}{8f} - \frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} - \frac{(5A+2B)c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{20f} + \frac{1}{8}\left(a^3(5A+2B)c\right)\int 1\,dx$$

$$=\frac{1}{8}a^3(5A+2B)cx-\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f}+\frac{a^3(5A+2B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f}-\frac{(5A+2B)c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{20f}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$

$$=\frac{a^3c\cos(e+fx)\left(-30(5A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(-8(10A+7B)+15(3A-2B)\sin}{120f\sqrt{\cos^2(e+fx}}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
[Out] (a^3*c*Cos[e + f*x]*(-30*(5*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]
 + Sqrt[Cos[e + f*x]^2]*(-8*(10*A + 7*B) + 15*(3*A - 2*B)*Sin[e + f*x] + 16
*(5*A + 2*B)*Sin[e + f*x]^2 + 30*(A + 2*B)*Sin[e + f*x]^3 + 24*B*Sin[e + f*
x]^4)))/(120*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{\left(\left(-\frac{2A}{3}-\frac{5B}{12}\right)\cos(3fx+3e)+\left(-\frac{A}{8}-\frac{B}{4}\right)\sin(4fx+4e)+\frac{\cos(5fx+5e)B}{20}+A\sin(2fx+2e)+\left(-2A-\frac{3B}{2}\right)\cos(fx+e)+\frac{5fxA}{2}\right.}{4f}$
risch	$\frac{5a^3cxA}{8}+\frac{a^3cxB}{4}-\frac{a^3c\cos(fx+e)A}{2f}-\frac{3a^3c\cos(fx+e)B}{8f}+\frac{B\,a^3c\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)A\,a^3c}{32f}-\frac{\sin(4}{}$
parts	$\frac{(-A\,a^3c-2B\,a^3c)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(2A\,a^3c+B\,a^3c)\cos(fx+e)}{f}+a^3cxA+$
derivativedivides	$\frac{-A\,a^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2A\,a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+\frac{B\,a^3c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(}{}\right.}{5}}{}$
default	$\frac{-A\,a^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2A\,a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+\frac{B\,a^3c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(}{}\right.}{5}}{}$
norman	$\frac{(\frac{5}{8}A\,a^3c+\frac{1}{4}B\,a^3c)x+(\frac{5}{8}A\,a^3c+\frac{1}{4}B\,a^3c)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,a^3c+\frac{5}{2}B\,a^3c)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,a^3c+\frac{5}{2}B\,a^3c}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*((-2/3*A-5/12*B)*cos(3*f*x+3*e)+(-1/8*A-1/4*B)*sin(4*f*x+4*e)+1/20*cos(
5*f*x+5*e)*B+A*sin(2*f*x+2*e)+(-2*A-3/2*B)*cos(f*x+e)+5/2*f*x*A+f*x*B-8/3*A
-28/15*B)*c*a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\frac{24\,Ba^3c\cos{(fx+e)}^5-80\,(A+B)a^3c\cos{(fx+e)}^3+15\,(5\,A+2\,B)a^3cfx-15\,(2\,(A+2\,B)a^3c\cos{(fx}}{120\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5*
A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a^3
*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(128) = 256.

Time = 0.30 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.47

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \begin{cases} -\frac{3Aa^3cx\sin^4(e+fx)}{8} - \frac{3Aa^3cx\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3Aa^3cx\cos^4(e+fx)}{8} + Aa^3cx + \frac{5Aa^3c\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{2Aa^3c}{} \\ x(A + B\sin(e))(a\sin(e) + a)^3(-c\sin(e) + c) \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3*c
*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e + f
*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**4/4 -
 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e + f*x)**
2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*a**3*c*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*c*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
*3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx =$$
$$-\frac{320\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^3c + 15\left(12fx + 12e + \sin(4fx+4e) - 8\sin(2fx+2e)\right)Aa^3c}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c - 3
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*(12
*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f*x +
 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^3*c*c
os(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{Ba^3c\cos(5\,fx+5\,e)}{80\,f}+\frac{Aa^3c\sin(2\,fx+2\,e)}{4\,f} + \frac{1}{8}\,(5\,Aa^3c+2\,Ba^3c)x-\frac{(8\,Aa^3c+5\,Ba^3c)\cos(3\,fx+3\,e)}{48\,f} - \frac{(4\,Aa^3c+3\,Ba^3c)\cos(fx+e)}{8\,f}-\frac{(Aa^3c+2\,Ba^3c)\sin(4\,fx+4\,e)}{32\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f - 1
/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*sin(
4*f*x + 4*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.79

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{a^3\,c\,\mathrm{atan}\left(\frac{a^3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(5\,A+2\,B)}{4\left(\frac{5\,A\,a^3\,c}{4}+\frac{B\,a^3\,c}{2}\right)}\right)(5\,A+2\,B)}{4\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8(4\,A\,a^3\,c+2\,B\,a^3\,c)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{3\,A\,a^3\,c}{4}-\frac{B\,a^3\,c}{2}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{7\,A\,a^3\,c}{2}+3\,B\,a^3\,c\right)}{f\,(} - \frac{a^3\,c\,(5\,A+2\,B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^3*c*atan((a^3*c*tan(e/2 + (f*x)/2)*(5*A + 2*B))/(4*((5*A*a^3*c)/4 + (B*a
^3*c)/2)))*(5*A + 2*B))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^3*c + 2*B*a^3*
c) - tan(e/2 + (f*x)/2)*((3*A*a^3*c)/4 - (B*a^3*c)/2) - tan(e/2 + (f*x)/2)^
3*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^7*((7*A*a^3*c)/2 + 3*B*a
```

```
^3*c) + tan(e/2 + (f*x)/2)^9*((3*A*a^3*c)/4 - (B*a^3*c)/2) + tan(e/2 + (f*x
)/2)^6*(8*A*a^3*c + 8*B*a^3*c) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c)/3 + (8*B
*a^3*c)/3) + tan(e/2 + (f*x)/2)^4*((16*A*a^3*c)/3 + (4*B*a^3*c)/3) + (4*A*a
^3*c)/3 + (14*B*a^3*c)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/
2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2
)^10 + 1)) - (a^3*c*(5*A + 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{5a^3(3A+4B)x}{2c}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3(3A+4B)c^3\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}$$

```
[Out] -5/2*a^3*(3*A+4*B)*x/c+5/3*a^3*(3*A+4*B)*cos(f*x+e)^3/c/f-5/2*a^3*(3*A+4*B)
*cos(f*x+e)*sin(f*x+e)/c/f+a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^4+
2*a^3*(3*A+4*B)*c^3*cos(f*x+e)^5/f/(c^2-c^2*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\sin(e+fx)\cos(e+fx)}{2cf}-\frac{5a^3x(3A+4B)}{2c}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-5*a^3*(3*A + 4*B)*x)/(2*c) + (5*a^3*(3*A + 4*B)*Cos[e + f*x]^3)/(3*c*f) -
 (5*a^3*(3*A + 4*B)*Cos[e + f*x]*Sin[e + f*x])/(2*c*f) + (a^3*(A + B)*c^3*C
os[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) + (2*a^3*(3*A + 4*B)*c^3*Cos[e +
f*x]^5)/(f*(c^2 - c^2*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} - \left(a^3(3A+4B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2} - \left(5a^3(3A+4B)\right)\int \frac{\cos^4(e+fx)}{c-c\sin(e+fx)}\,dx$$

$$= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2} - \frac{\left(5a^3(3A+4B)\right)\int \cos^2(e+fx)\,dx}{c}$$

$$= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2} - \frac{\left(5a^3(3A+4B)\right)\int 1\,dx}{2c}$$

$$= -\frac{5a^3(3A+4B)x}{2c} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2}$$

Mathematica [A] (verified)

Time = 11.64 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A+4B)(e+fx)-3(16A\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3*(
e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 + 5*e
 + 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{65a^3\left(\frac{4\left(4A+\frac{23B}{3}\right)\cos(2fx+2e)}{65}+\frac{(A+4B)\sin(3fx+3e)}{65}-\frac{B\cos(4fx+4e)}{195}+\frac{4\left(-3fxA-4fxB+\frac{24}{5}A+\frac{94}{15}B\right)\cos(fx+e)}{13}+\left(A+\frac{68B}{65}\right)\right.}{8cf\cos(fx+e)}$
derivativedivides	$\frac{2a^3\left(-\frac{\left(2B+\frac{A}{2}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-2B-\frac{A}{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{fc}$
default	$\frac{2a^3\left(-\frac{\left(2B+\frac{A}{2}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-2B-\frac{A}{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{fc}$
risch	$-\frac{15a^3xA}{2c}-\frac{10a^3xB}{c}+\frac{2a^3e^{i(fx+e)}A}{cf}+\frac{31a^3e^{i(fx+e)}B}{8cf}+\frac{2a^3e^{-i(fx+e)}A}{cf}+\frac{31a^3e^{-i(fx+e)}B}{8cf}+\frac{16a^3A}{fc(e^{i(fx+e)}-}$
norman	$\frac{-\frac{17Aa^3+20Ba^3}{cf}+\frac{5a^3(3A+4B)x}{2c}-\frac{(5Aa^3+2Ba^3)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{(17Aa^3+18Ba^3)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{(21Aa^3+34Ba^3)(\tan^9}{3cf}}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 65/8*a^3*(4/65*(4*A+23/3*B)*cos(2*f*x+2*e)+1/65*(A+4*B)*sin(3*f*x+3*e)-1/19
5*B*cos(4*f*x+4*e)+4/13*(-3*f*x*A-4*f*x*B+24/5*A+94/15*B)*cos(f*x+e)+(A+68/
65*B)*sin(f*x+e)+16/13*A+19/13*B)/c/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{2\,Ba^3\cos\left(fx+e\right)^4-(3\,A+10\,B)a^3\cos\left(fx+e\right)^3+15\,(3\,A+4\,B)a^3fx-24\,(A+2\,B)a^3\cos\left(fx+e\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*a^3*cos(f*x + e)^4 - (3*A + 10*B)*a^3*cos(f*x + e)^3 + 15*(3*A +
4*B)*a^3*f*x - 24*(A + 2*B)*a^3*cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3*A
 + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*cos(f*x + e) - (2*B*a^3*cos(f*x + e)^3
 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*cos(f*x + e)^2 - 3*(7*A + 12*B)
*a^3*cos(f*x + e) + 48*(A + B)*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*s
in(f*x + e) + c*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. 2(144) = 288.

Time = 3.79 (sec) , antiderivative size = 4255, normalized size of antiderivative = 27.28

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f
*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*
tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 + f*x/2)
**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x
*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6
 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2
 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f)
 - 135*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*ta
```

n(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4
 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
+ f*x/2) - 6*c*f) + 135*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x
/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*ta
n(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*tan(e/2 + f*x/2)/(6*c*f*
tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**
5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/
2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x/(6*c*f*tan(
e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 102*A*a**3*tan(e/2 + f*x/2)**
6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 +
 f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c
*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 54*A*a**3*tan(e/
2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c
*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/
2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 336*
A*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x
/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*t
an(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
6*c*f) + 96*A*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 378*A*a**3*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x/2)
7 - 6*c*f*tan(e/2 + f*x/2)6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 42*A*a**3*tan(e/2 + f*x/2)/(6*c*f*tan(e/2
 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*
c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x
/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 144*A*a**3/(6*c*f*tan(e/2 + f*x/
2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan
(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
+ 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*
f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)
5 - 18*c*f*tan(e/2 + f*x/2)4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(
e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 60*B*a**3*f*x*tan(e/2 +
 f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*
tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)*
*3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 180*B*a
3*f*x*tan(e/2 + f*x/2)5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*
x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*
tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6
*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*

```
x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*t
an(e/2 + f*x/2) - 6*c*f) - 180*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)
**2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*
tan(e/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 1
8*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f
*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 6
0*B*a**3*f*x/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*
f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 120*B
*a**3*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/
2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*ta
n(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6
*c*f) + 108*B*a**3*tan(e/2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 372*B*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 192*B*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan
(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
 f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 456*B*a**3*tan(e/2 + f*x/2)*
*2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
+ f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*
c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 68*B*a**3*tan(e
/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f
*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)
**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 188*B*
a**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/
2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 1
8*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f), Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(152) = 304$.

Time = 0.35 (sec) , antiderivative size = 1139, normalized size of antiderivative = 7.30

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*a^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a
^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((sin(f*x + e)/(c
os(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*s
in(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/c) + 9*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
 + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 6*B*a^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*a^3/(c - c*sin(f*x + e)/(cos(f*x +
 e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$

$$-\frac{\frac{15\,(3\,Aa^3+4\,Ba^3)(fx+e)}{c}+\frac{96\,(Aa^3+Ba^3)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+12\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,Ba\right.}{}}{6\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(tan(1/2*f
*x + 1/2*e) - 1)) + 2*(3*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*tan(1/2*f*
x + 1/2*e)^5 - 24*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*tan(1/2*f*x + 1/2
*e)^4 - 48*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*a^3*tan(1/2*f*x + 1/2*e) - 12*B*a^3*tan(1/2*f*x + 1/2*e) - 24*A*a^3 -
46*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f
```

Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$=\frac{24\,A\,a^3-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\,A\,a^3+\frac{34\,B\,a^3}{3}\right)+\frac{94\,B\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(9\,A\,a^3+18\,B\,a^3)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(17}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\right.}$$

$$-\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(3\,A+4\,B)}{15\,A\,a^3+20\,B\,a^3}\right)\,(3\,A+4\,B)}{c\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x)),x)
```

```
[Out] (24*A*a^3 - tan(e/2 + (f*x)/2)*(7*A*a^3 + (34*B*a^3)/3) + (94*B*a^3)/3 - ta
n(e/2 + (f*x)/2)^5*(9*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^6*(17*A*a^3 +
20*B*a^3) - tan(e/2 + (f*x)/2)^3*(16*A*a^3 + 32*B*a^3) + tan(e/2 + (f*x)/2)
^4*(56*A*a^3 + 62*B*a^3) + tan(e/2 + (f*x)/2)^2*(63*A*a^3 + 76*B*a^3))/(f*(
c - c*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 - 3*c*tan(e/2 + (f*x)/2
)^3 + 3*c*tan(e/2 + (f*x)/2)^4 - 3*c*tan(e/2 + (f*x)/2)^5 + c*tan(e/2 + (f*
x)/2)^6 - c*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*
(3*A + 4*B))/(15*A*a^3 + 20*B*a^3))*(3*A + 4*B))/(c*f)
```

3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 163

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx &= \frac{5a^3(2A+5B)x}{2c^2}-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}\\&\quad-\frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}\end{aligned}$$

```
[Out] 5/2*a^3*(2*A+5*B)*x/c^2-5/2*a^3*(2*A+5*B)*cos(f*x+e)/c^2/f+1/3*a^3*(A+B)*c^
3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^5-2/3*a^3*(2*A+5*B)*c*cos(f*x+e)^5/f/(c-c
*sin(f*x+e))^3-5/6*a^3*(2*A+5*B)*cos(f*x+e)^3/f/(c^2-c^2*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2938, 2759, 2758, 2761, 8}

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx &= \frac{a^3c^3(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}\\&\quad-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{5a^3x(2A+5B)}{2c^2}-\frac{2a^3c(2A+5B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}\end{aligned}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (5*a^3*(2*A + 5*B)*x)/(2*c^2) - (5*a^3*(2*A + 5*B)*Cos[e + f*x])/(2*c^2*f)
+ (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - (2*a^3*(2
*A + 5*B)*c*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^3) - (5*a^3*(2*A + 5*
B)*Cos[e + f*x]^3)/(6*f*(c^2 - c^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{1}{3}\left(a^3(2A+5B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}\\
&\quad+\frac{1}{3}\left(5a^3(2A+5B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}\\
&\quad-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{\left(5a^3(2A+5B)\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx}{2c}\\
&= -\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}\\
&\quad-\frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}\\
&\quad-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{\left(5a^3(2A+5B)\right)\int 1\,dx}{2c^2}\\
&= \frac{5a^3(2A+5B)x}{2c^2}-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}\\
&\quad-\frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 11.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.72

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$=\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(32(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{2a^3\left(\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A+5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{fc^2}$
default	$\frac{2a^3\left(\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A+5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{fc^2}$
parallelrisch	$-\frac{\left(\left(30fxA+75fxB-56A-\frac{607}{4}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-10fxA-25fxB-\frac{19}{3}A-\frac{115}{12}B\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+(-30fxA-75fxB\right.}{2fc^2\left(-3\right.}$
risch	$\frac{5a^3xA}{c^2}+\frac{25a^3xB}{2c^2}+\frac{iBa^3e^{2i(fx+e)}}{8c^2f}-\frac{a^3e^{i(fx+e)}A}{2c^2f}-\frac{5a^3e^{i(fx+e)}B}{2c^2f}-\frac{a^3e^{-i(fx+e)}A}{2c^2f}-\frac{5a^3e^{-i(fx+e)}B}{2c^2f}-$
norman	$\frac{\frac{\left(8Aa^3+25Ba^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{46Aa^3+118Ba^3}{3cf}-\frac{5a^3(2A+5B)x}{2c}-\frac{\left(34Aa^3+77Ba^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(38Aa^3+93Ba^3\right)\tan}{cf}}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^2*((1/2*B*tan(1/2*f*x+1/2*e)^3+(-A-5*B)*tan(1/2*f*x+1/2*e)^2-1/2*
B*tan(1/2*f*x+1/2*e)-A-5*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A+5*B)*arctan
(tan(1/2*f*x+1/2*e))-(-4*A-12*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(16*A+16*B)/(ta
n(1/2*f*x+1/2*e)-1)^3-1/2*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{3\,Ba^3\cos{(fx+e)}^4 - 6\,(A+4\,B)a^3\cos{(fx+e)}^3 - 30\,(2\,A+5\,B)a^3fx - 16\,(A+B)a^3 + (15\,(2\,A+5\,B)}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5*
B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^3)
*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x +
e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a^3*c
os(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 79*B)*
a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. 2(151) = 302.

Time = 7.40 (sec) , antiderivative size = 4665, normalized size of antiderivative = 28.62

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((30*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7
- 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f
*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 +
f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 90*A*a**3*f*x*tan(e/2
+ f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 +
 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*
tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f
```

```
*x/2) - 6*c**2*f) + 150*A*a**3*f*x*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 +
f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 -
 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*
tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 210*A*a**3*f
*x*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 +
f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 +
 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*
tan(e/2 + f*x/2) - 6*c**2*f) + 210*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f
*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 +
f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 1
50*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f
*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 +
f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 +
 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 90*A*a**3*f*x*tan(e/2 + f*x/2)/(6
*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan
(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/
2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2
*f) - 30*A*a**3*f*x/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x
/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42
*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan
(e/2 + f*x/2) - 6*c**2*f) + 48*A*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2
 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**
5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2
*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 204*A*a**
3*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f
*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 +
42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*t
an(e/2 + f*x/2) - 6*c**2*f) + 212*A*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(
e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2
)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c
**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 432*A*
a**3*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
+ f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4
 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*
f*tan(e/2 + f*x/2) - 6*c**2*f) + 256*A*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*t
an(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 228
*A*a**3*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
+ f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4
 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*
f*tan(e/2 + f*x/2) - 6*c**2*f) + 92*A*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 -
18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*t
an(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*
```

x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 75*B*a**3*f*x*tan(e/2 +
f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 3
0*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*ta
n(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x
/2) - 6*c**2*f) - 225*B*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*
x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 4
2*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*ta
n(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 375*B*a**3*f*x
*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*
x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 4
2*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*ta
n(e/2 + f*x/2) - 6*c**2*f) - 525*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*t
an(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 525
*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*t
an(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*
x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 1
8*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 375*B*a**3*f*x*tan(e/2 + f*x/2)**2/
(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*t
an(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*
x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c*
*2*f) + 225*B*a**3*f*x*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*
c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(
e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2
)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 75*B*a**3*f*x/(6*c**2*f*tan
(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/
2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*
c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 150*B
*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**
4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2
*f*tan(e/2 + f*x/2) - 6*c**2*f) - 462*B*a**3*tan(e/2 + f*x/2)**5/(6*c**2*f*
tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f
*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 65
6*B*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(
e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2
)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c
2*f*tan(e/2 + f*x/2) - 6*c2*f) - 996*B*a**3*tan(e/2 + f*x/2)**3/(6*c**2
*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2
+ f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3
 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) +
 718*B*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*t
an(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*

```
x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 1
8*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 558*B*a**3*tan(e/2 + f*x/2)/(6*c**2
*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2
+ f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3
 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) +
 236*B*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 +
 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*
tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f
*x/2) - 6*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e
) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(156) = 312$.

Time = 0.34 (sec) , antiderivative size = 1386, normalized size of antiderivative = 8.50

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*a^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) - 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 7*c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*c^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/c^2) + 4*A*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/c^2) + 12*B*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^
2) + 6*A*a^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
```

```
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 6*B*a^3*((9*sin(f*x + e
)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*
c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c^2) - 2*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3) + 6*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 -
 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*a^3*(3*sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{15\,(2\,Aa^3+5\,Ba^3)(fx+e)}{c^2} + \frac{6\left(Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-10\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Aa^3-10\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c^2}}{6\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a^3
*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*e)^2
 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A*a^3
+ 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$=\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A+5\,B)}{10\,A\,a^3+25\,B\,a^3}\right)\,(2\,A+5\,B)}{c^2\,f}$$
$$-\frac{\frac{46\,A\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(38\,A\,a^3+93\,B\,a^3)+\frac{118\,B\,a^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(8\,A\,a^3+25\,B\,a^3)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{f\,\left(-c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-5\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(2*A + 5*B))/(10*A*a^3 + 25*B*a^3))*(
2*A + 5*B))/(c^2*f) - ((46*A*a^3)/3 - tan(e/2 + (f*x)/2)*(38*A*a^3 + 93*B*a
^3) + (118*B*a^3)/3 + tan(e/2 + (f*x)/2)^6*(8*A*a^3 + 25*B*a^3) - tan(e/2 +
 (f*x)/2)^5*(34*A*a^3 + 77*B*a^3) - tan(e/2 + (f*x)/2)^3*(72*A*a^3 + 166*B*
a^3) + tan(e/2 + (f*x)/2)^4*((106*A*a^3)/3 + (328*B*a^3)/3) + tan(e/2 + (f*
x)/2)^2*((128*A*a^3)/3 + (359*B*a^3)/3))/(f*(5*c^2*tan(e/2 + (f*x)/2)^2 - 7
*c^2*tan(e/2 + (f*x)/2)^3 + 7*c^2*tan(e/2 + (f*x)/2)^4 - 5*c^2*tan(e/2 + (f
*x)/2)^5 + 3*c^2*tan(e/2 + (f*x)/2)^6 - c^2*tan(e/2 + (f*x)/2)^7 + c^2 - 3*
c^2*tan(e/2 + (f*x)/2)))
```

3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 153

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx &= -\frac{a^3(A+6B)x}{c^3}+\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}\\ &\quad-\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}+\frac{2a^3(A+6B)c^3\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}\end{aligned}$$

```
[Out] -a^3*(A+6*B)*x/c^3+a^3*(A+6*B)*cos(f*x+e)/c^3/f+1/5*a^3*(A+B)*c^3*cos(f*x+e
)^7/f/(c-c*sin(f*x+e))^6-2/15*a^3*(A+6*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))
^4+2/3*a^3*(A+6*B)*c^3*cos(f*x+e)^3/f/(c^3-c^3*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2759, 2761, 8}

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx &= \frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}\\ &\quad+\frac{2a^3c^3(A+6B)\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{a^3x(A+6B)}{c^3}-\frac{2a^3c(A+6B)\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}\end{aligned}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(A + 6*B)*x)/c^3) + (a^3*(A + 6*B)*Cos[e + f*x])/(c^3*f) + (a^3*(A +
 B)*c^3*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - (2*a^3*(A + 6*B)*c*C
os[e + f*x]^5)/(15*f*(c - c*Sin[e + f*x])^4) + (2*a^3*(A + 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(c^3 - c^3*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{1}{5}\left(a^3(A+6B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}\\
&\quad+\frac{1}{3}\left(a^3(A+6B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}\\
&\quad+\frac{2a^3(A+6B)\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\left(a^3(A+6B)\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx}{c^2}\\
&= \frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}\\
&\quad-\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}+\frac{2a^3(A+6B)\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\left(a^3(A+6B)\right)\int 1\,dx}{c^3}\\
&= -\frac{a^3(A+6B)x}{c^3}+\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}\\
&\quad-\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}+\frac{2a^3(A+6B)\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(153) = 306.

Time = 11.41 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.07

$$
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx
$$

$$
= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(11A+21B)\left(\right.\right.}{}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B*Co
```

```
s[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e + f*
x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*
x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*
(c - c*Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2a^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{}{3(\mathrm{ta}}\right.}{f\,c^3}$
default	$\frac{2a^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{}{3(\mathrm{ta}}\right.}{f\,c^3}$
risch	$-\frac{a^3xA}{c^3}-\frac{6a^3xB}{c^3}+\frac{B\,a^3e^{i(fx+e)}}{2c^3f}+\frac{B\,a^3e^{-i(fx+e)}}{2c^3f}+\frac{-\frac{112A\,a^3e^{2i(fx+e)}}{3}-24iA\,a^3e^{3i(fx+e)}+\frac{56iA\,a^3e^{i(fx+e)}}{3}+}{}$
parallelrisch	$-\frac{\left(\left(\left(\frac{233}{2}-60fx\right)B-10fxA+24A\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(\left(-\frac{33}{2}+30fx\right)B+5fxA-\frac{16A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(fxA+6fxB-\frac{243}{10}B\right.\right.}{f\,c^3\left(-10\,\mathrm{c}\right.}$
norman	$\frac{-\frac{5a^3(A+6B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{30a^3(A+6B)x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{5a^3(A+6B)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{14a^3(A+6B)x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^3*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A+6*B)*arctan(tan(1/2*f*x+1/2*e))-
(2*A+6*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(8*A-8*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5
*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(40*A+24*B)/(tan(1/2*f*x+1/2*e)-1
)^3-1/4*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(150) = 300.

Time = 0.26 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.20

$$\int\frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{15\,Ba^3\cos\left(fx+e\right)^4+60\,(A+6\,B)a^3fx-24\,(A+B)a^3-(15\,(A+6\,B)a^3fx-(46\,A+231\,B)a^3)\cos(}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*a^3*cos(f*x + e)^4 + 60*(A + 6*B)*a^3*f*x - 24*(A + B)*a^3 - (15
*(A + 6*B)*a^3*f*x - (46*A + 231*B)*a^3)*cos(f*x + e)^3 - (45*(A + 6*B)*a^3
*f*x + 2*(A + 66*B)*a^3)*cos(f*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(6*A +
 31*B)*a^3)*cos(f*x + e) - (15*B*a^3*cos(f*x + e)^3 + 60*(A + 6*B)*a^3*f*x
+ 24*(A + B)*a^3 - (15*(A + 6*B)*a^3*f*x + 2*(23*A + 108*B)*a^3)*cos(f*x +
e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(4*A + 29*B)*a^3)*cos(f*x + e))*sin(f*x +
 e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e)
- 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x
 + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.

Time = 13.60 (sec) , antiderivative size = 4665, normalized size of antiderivative = 30.49

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*A*a**3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**
7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c*
*3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(
e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*A*a**3*f*x*t
an(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x
/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 +
225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) - 165*A*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**
3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/
2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2
)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**
3*f) + 225*A*a**3*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 -
75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f
*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2
+ f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 225*A*a**3*f*x*tan(
e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)
**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225
*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*ta
n(e/2 + f*x/2) - 15*c**3*f) + 165*A*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f
*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 +
 f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**
3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f
```

) - 75*A*a**3*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3
*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/
2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2
)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*A*a**3*f*x/(15*c**3*f*t
an(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f
*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3
- 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f)
- 60*A*a**3*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*
tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 +
f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**
2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 120*A*a**3*tan(e/2 + f*x/2)**
5/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3
*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/
2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2)
- 15*c**3*f) - 460*A*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c
3*f*tan(e/2 + f*x/2)4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 320*A*a**3*tan
(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2
)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 22
5*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f) - 452*A*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*ta
n(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*
x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 -
165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) +
200*A*a**3*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan
(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*
x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 +
75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 52*A*a**3/(15*c**3*f*tan(e/2 + f
*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 -
225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 90*B*a**
3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/
2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2
)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75
*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 450*B*a**3*f*x*tan(e/2 + f*x/2)**6/
(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f
*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2
+ f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) - 990*B*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2
)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225
*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1350*B*a**3*
f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2
+ f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)*

```
*4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 1350*B*a**3*f*x*tan(e/2 + f*x/2)**3/(
15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*
tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 +
 f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) + 990*B*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)
**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*
c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*ta
n(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 450*B*a**3*f*
x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x
/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 +
225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) + 90*B*a**3*f*x/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c
**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 180*B*a**3*tan
(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2
)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 22
5*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f) + 870*B*a**3*tan(e/2 + f*x/2)**5/(15*c**3*f*ta
n(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*
x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 -
 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
 2010*B*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f
*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2
+ f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)*
*2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 2220*B*a**3*tan(e/2 + f*x/2)
**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c*
*3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(
e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/
2) - 15*c**3*f) - 2232*B*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/
2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 22
5*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*
tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1230*B*a**3
*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/
2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 2
25*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) - 282*B*a**3/(15*c**3*f*tan(e/2 + f*x/2)**7 -
 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*
f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2
 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A +
B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. 2(150) = 300.

Time = 0.35 (sec) , antiderivative size = 1685, normalized size of antiderivative = 11.01

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*a^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*c^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1
1*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c^3) + A*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/c^3) + A*a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 9*A*a^3*(5*sin(f*x + e)/(cos(
f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 3*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
```

```
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) +
6*A*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5) + 6*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{\frac{30\,Ba^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^3} - \frac{15\,(Aa^3+6\,Ba^3)(fx+e)}{c^3} - \frac{4\left(15\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Ba^3}{}}{15\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^3) - 15*(A*a^3 + 6*B*a^3)*(f
*x + e)/c^3 - 4*(15*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*a^3*tan(1/2*f*x + 1
/2*e)^4 - 30*A*a^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*a^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 420*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 5
0*A*a^3*tan(1/2*f*x + 1/2*e) - 270*B*a^3*tan(1/2*f*x + 1/2*e) + 13*A*a^3 +
63*B*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.20

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{\frac{52\,A\,a^3}{15}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{40\,A\,a^3}{3}+82\,B\,a^3\right)+\frac{94\,B\,a^3}{5}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(4\,A\,a^3+12\,B\,a^3\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(8\,A\right.}{f\,\left(-c^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+5\,c^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-11\,c^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+15\right.} - \frac{2\,a^3\,\mathrm{atan}\left(\frac{2\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A+6\,B\right)}{2\,A\,a^3+12\,B\,a^3}\right)\,\left(A+6\,B\right)}{c^3\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] ((52*A*a^3)/15 - tan(e/2 + (f*x)/2)*((40*A*a^3)/3 + 82*B*a^3) + (94*B*a^3)/
5 + tan(e/2 + (f*x)/2)^6*(4*A*a^3 + 12*B*a^3) - tan(e/2 + (f*x)/2)^5*(8*A*a
^3 + 58*B*a^3) - tan(e/2 + (f*x)/2)^3*((64*A*a^3)/3 + 148*B*a^3) + tan(e/2
+ (f*x)/2)^4*((92*A*a^3)/3 + 134*B*a^3) + tan(e/2 + (f*x)/2)^2*((452*A*a^3)
/15 + (744*B*a^3)/5))/(f*(11*c^3*tan(e/2 + (f*x)/2)^2 - 15*c^3*tan(e/2 + (f
*x)/2)^3 + 15*c^3*tan(e/2 + (f*x)/2)^4 - 11*c^3*tan(e/2 + (f*x)/2)^5 + 5*c^
3*tan(e/2 + (f*x)/2)^6 - c^3*tan(e/2 + (f*x)/2)^7 + c^3 - 5*c^3*tan(e/2 + (
f*x)/2))) - (2*a^3*atan((2*a^3*tan(e/2 + (f*x)/2)*(A + 6*B))/(2*A*a^3 + 12*
B*a^3))*(A + 6*B))/(c^3*f)
```

3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 151

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a^3Bx}{c^4}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} +\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] a^3*B*x/c^4+1/7*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7-2/5*a^3*B*c
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5+2/3*a^3*B*c^2*cos(f*x+e)^3/f/(c^2-c^2*si
n(f*x+e))^3-2*a^3*B*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2759, 8}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a^3c^3(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}+\frac{a^3Bx}{c^4} +\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*B*x)/c^4 + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^
7) - (2*a^3*B*c*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) + (2*a^3*B*c^2
*Cos[e + f*x]^3)/(3*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a^3*B*Cos[e + f*x])/
(f*(c^4 - c^4*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\left(a^3Bc^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\left(a^3B\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} \\
&\quad + \frac{2a^3B\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3} - \frac{(a^3B)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{c^2} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} \\
&\quad + \frac{2a^3B\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3} - \frac{2a^3B\cos(e+fx)}{f\,(c^4-c^4\sin(e+fx))} + \frac{(a^3B)\int 1\,dx}{c^4} \\
&= \frac{a^3Bx}{c^4} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} \\
&\quad + \frac{2a^3B\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3} - \frac{2a^3B\cos(e+fx)}{f\,(c^4-c^4\sin(e+fx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 356 vs. $2(151) = 302$.

Time = 11.47 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.36

$$
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx
$$

$$
= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(120(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12(15A+29B\right.}{}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e +
f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x)/2]
 - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2
] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/
2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(
c - c*Sin[e + f*x])^4)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{2a^3\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160}{4\left(\tan\left(\right.\right.}\right.}{f\,c^4}$
default	$\frac{2a^3\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160}{4\left(\tan\left(\right.\right.}\right.}{f\,c^4}$
parallelrisch	$-\frac{2\left(-\frac{B\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(\frac{7}{2}fxB+A-B\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B\left(-\frac{21fx}{2}+8\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{35}{2}fxB+5A-\frac{55}{3}B\right)\left(t\right.\right.}{}$
risch	$\frac{a^3Bx}{c^4}-\frac{2\left(-337B\,a^3-15A\,a^3-2520iB\,a^3e^{5i(fx+e)}+6160iB\,a^3e^{3i(fx+e)}-1624iB\,a^3e^{i(fx+e)}+105A\,a^3e^{6i(fx+e)}+735\right.}{105\left(e^{i(fx+e)}-i\right)^7}$
norman	$\frac{-\frac{2336B\,a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{6544B\,a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{a^3xB}{c}-\frac{304B\,a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cf}+\frac{7a^3xB\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{\left(870A\,a^3-3142B\right.}{}}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^4*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)-1)-1/2*
(12*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(60*A+20*B)/(tan(1/2*f*x+1/2*e)-1)^
3-1/7*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/4*(160*A+96*B)/(tan(1/2*f*x+1/
2*e)-1)^4-1/6*(192*A+192*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/5*(240*A+208*B)/(tan
(1/2*f*x+1/2*e)-1)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(149) = 298.

Time = 0.27 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.40

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{840\,Ba^3fx+(105\,Ba^3fx+(15\,A+337\,B)a^3)\cos{(fx+e)}^4+120\,(A+B)a^3-(315\,Ba^3fx+(45\,A-6}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4
+ 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 - 2
4*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (A -
```

```
13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^3*f*
x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*B)*a^
3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*sin(f*x
 + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e
)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(
f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. $2(141) = 282$.

Time = 23.73 (sec) , antiderivative size = 2951, normalized size of antiderivative = 19.54

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*a**3*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7
- 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c
**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*t
an(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 1050*A*a**
3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2
+ f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2
)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 +
735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 630*A*a**3*tan(e/2 + f*x/2)**2/
(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**
4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan
(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f
*x/2) - 105*c**4*f) - 30*A*a**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*
f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 105*B*a**3*f*x*tan(e/
2 + f*x/2)**7/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)
**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3
675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4
*f*tan(e/2 + f*x/2) - 105*c**4*f) - 735*B*a**3*f*x*tan(e/2 + f*x/2)**6/(105
*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*
tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2
 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2
) - 105*c**4*f) + 2205*B*a**3*f*x*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 +
 f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)*
*5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 22
05*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) -
 3675*B*a**3*f*x*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*
c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*
```

```
tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2
 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 3675*B*a**3*f*x*
tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 +
f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)*
*4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 73
5*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 2205*B*a**3*f*x*tan(e/2 + f*x/2)*
*2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*
c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*
tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2
+ f*x/2) - 105*c**4*f) + 735*B*a**3*f*x*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/
2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/
2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 -
 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f
) - 105*B*a**3*f*x/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f
*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**
4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735
*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 210*B*a**3*tan(e/2 + f*x/2)**6/(10
5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f) - 1680*B*a**3*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*
x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5
 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*
c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 38
50*B*a**3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*
tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2
 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/
2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 7840*B*a**3*tan(e/2 + f
*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 +
 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c
**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*ta
n(e/2 + f*x/2) - 105*c**4*f) + 5334*B*a**3*tan(e/2 + f*x/2)**2/(105*c**4*f*
tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2
+ f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2
)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*
c**4*f) - 2128*B*a**3*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x/2)**7 - 73
5*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*
f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e
/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 334*B*a**3/(10
5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e)
 + c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2118 vs. 2(149) = 298.

Time = 0.39 (sec) , antiderivative size = 2118, normalized size of antiderivative = 14.03

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(5*B*a^3*((203*sin(f*x + e)/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f
*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^4) + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1)
 - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*
c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x + e)/(cos(f*x + e)
 + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^3*(49*sin(f*x + e)/(cos(f
*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7) - 12*A*a^3*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x
+ e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e
```

)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*
c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 12*B*a^3*(14*sin(f*x
+ e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2
)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7) + 6*A*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + 18*B*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*
c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7))/f

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{\frac{105\,(fx+e)Ba^3}{c^4} - \frac{2\left(105\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-105\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+840\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+525\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1925\,Ba^3\tan\left(\dots\right.}{c^4 \dots}}{105 \dots}$$

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")

[Out] 1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105*
B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a^3
*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3*ta
n(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*tan(1/
2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*a^3)/
(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f

Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{B\,a^3\,x}{c^4}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(\frac{a^3\,(1680\,B-2205\,B\,(e+f\,x))}{105}+21\,B\,a^3\,(e+f\,x)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{a^3\,(7840\,B-3675\,B\,(e+f\,x))}{105}+35\,B\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (B*a^3*x)/c^4 - (tan(e/2 + (f*x)/2)^5*((a^3*(1680*B - 2205*B*(e + f*x)))/10
5 + 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^3*((a^3*(7840*B - 3675*B*(e +
f*x)))/105 + 35*B*a^3*(e + f*x)) + (a^3*(30*A - 334*B + 105*B*(e + f*x)))/1
05 + tan(e/2 + (f*x)/2)^6*((a^3*(210*A - 210*B + 735*B*(e + f*x)))/105 - 7*
B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^2*((a^3*(630*A - 5334*B + 2205*B*(e +
 f*x)))/105 - 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^4*((a^3*(1050*A - 38
50*B + 3675*B*(e + f*x)))/105 - 35*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)*((
a^3*(2128*B - 735*B*(e + f*x)))/105 + 7*B*a^3*(e + f*x)) - B*a^3*(e + f*x))
/(c^4*f*(tan(e/2 + (f*x)/2) - 1)^7)
```

3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 77

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

```
[Out] 1/9*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*(A-8*B)*c^2*co
s(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 2750}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{a^3c^3(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2(A-8B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*(A - 8
*B)*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
```

```
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{1}{9}\left(a^3(A-8B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(77) = 154.

Time = 12.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.68

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$
$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(315(A-B)\cos\left(\frac{1}{2}(e+fx)\right)-189(A-B)\,\text{co}\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

[Out] -1/504*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(315 *(A - B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(5*(e + f*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9*B*Cos[(7*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2] + 105*A*Sin[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e + f*x))/2] - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*Sin[(9*(e + f*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^5)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.

Time = 1.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.25

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-A+B)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(23A+5B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+5(-A+B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(11A+3B\right.}{f c^5\left(\tan\left(\frac{fx}{2}+\right.\right.}$
derivativedivides	$\frac{2a^3\left(-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{14A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{992A+}{6\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f c^5}$
default	$\frac{2a^3\left(-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{14A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{992A+}{6\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f c^5}$
risch	$\frac{16B\,a^3}{63}-\frac{2A\,a^3}{63}-10iA\,a^3 e^{5i(fx+e)}+\frac{2iB\,a^3 e^{i(fx+e)}}{7}-2iB\,a^3 e^{7i(fx+e)}+2iA\,a^3 e^{7i(fx+e)}-\frac{2iA\,a^3 e^{i(fx+e)}}{7}+10iB\,a^3 e^{5i(fx-}$
norman	$\frac{-\frac{16A\,a^3-2B\,a^3}{63cf}-\frac{2A\,a^3\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(A\,a^3-B\,a^3\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(2A\,a^3-2B\,a^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7cf}+\frac{6\left(3A\,a^3-3B\,a^3\right)\left(t\right.}{cf}}{}$

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)

[Out] -2*(A*tan(1/2*f*x+1/2*e)^8+(-A+B)*tan(1/2*f*x+1/2*e)^7+1/3*(23*A+5*B)*tan(1/2*f*x+1/2*e)^6+5*(-A+B)*tan(1/2*f*x+1/2*e)^5+(11*A+3*B)*tan(1/2*f*x+1/2*e)^4+3*(-A+B)*tan(1/2*f*x+1/2*e)^3+1/7*(25*A+3*B)*tan(1/2*f*x+1/2*e)^2+1/7*(-A+B)*tan(1/2*f*x+1/2*e)+8/63*A-1/63*B)*a^3/f/c^5/(tan(1/2*f*x+1/2*e)-1)^9

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(75) = 150.

Time = 0.26 (sec) , antiderivative size = 331, normalized size of antiderivative = 4.30

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{(A-8\,B)a^3\cos\left(fx+e\right)^5-(4\,A+31\,B)a^3\cos\left(fx+e\right)^4+(19\,A+37\,B)a^3\cos\left(fx+e\right)^3+4\,(13\,A+2\ldots}{63\left(c^5f\cos\left(fx+e\right)^5+5\,c^5f\cos\left(fx+e\right)^4-8\,c^5f\cos(\ldots\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (19
*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(A
+ B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 + (5
*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*(A +
 B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5
+ 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2
 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*
x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f
*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. 2(68) = 136.

Time = 40.13 (sec) , antiderivative size = 3262, normalized size of antiderivative = 42.36

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*A*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9 -
 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c*
*5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*ta
n(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 +
 f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*A*a**3*tan(e/2
+ f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8
 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938
*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f
*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2
 + f*x/2) - 63*c**5*f) - 966*A*a**3*tan(e/2 + f*x/2)**6/(63*c**5*f*tan(e/2
+ f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)
```

7 - 5292*c5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7
938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**
5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 630*A*
a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/
2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x
/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4
+ 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c
5*f*tan(e/2 + f*x/2) - 63*c5*f) - 1386*A*a**3*tan(e/2 + f*x/2)**4/(63*c
5*f*tan(e/2 + f*x/2)9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*ta
n(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 +
f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)
3 - 2268*c5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c*
*5*f) + 378*A*a**3*tan(e/2 + f*x/2)**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f
*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/
2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x
/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 450*A*a**3*tan(e/2 + f*
x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**
5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan
(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f
*x/2) - 63*c**5*f) + 18*A*a**3*tan(e/2 + f*x/2)/(63*c**5*f*tan(e/2 + f*x/2)
9 - 567*c5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 52
92*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5
*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(
e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 16*A*a**3/(63*
c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*t
an(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2
+ f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2
)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c
5*f) - 126*B*a3*tan(e/2 + f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 56
7*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*
f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e
/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*
x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 210*B*a**3*tan(e/2 + f
*x/2)**6/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 +
2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c*
*5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*ta
n(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 +
f*x/2) - 63*c**5*f) - 630*B*a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f
*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7
 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938
*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f
*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**
3*tan(e/2 + f*x/2)**4/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +

```
f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)
**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5
292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5
*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**3*tan(e/2 + f*x/2)**3/(63*c**5*
f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/
2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x
/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3
- 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f
) - 54*B*a**3*tan(e/2 + f*x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(
e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f
*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**
2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 18*B*a**3*tan(e/2 + f*x/2)/(
63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*
f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e
/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*
x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 6
3*c**5*f) + 2*B*a**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +
f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)*
*6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 52
92*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*
f*tan(e/2 + f*x/2) - 63*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3/(-c*sin(e) + c)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2701 vs. $2(75) = 150$.

Time = 0.30 (sec) , antiderivative size = 2701, normalized size of antiderivative = 35.08

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
```

e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
 10*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) - 30*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c
^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 8*B*a^3*(9*sin(f*x + e)/(cos(f*x + e
) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 - 126*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)/(c^5 - 9*c^5*s
in(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*
x + e) + 1)^9) + 42*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*si

```
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
- 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(co
s(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)
+ 42*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(75) = 150$.

Time = 0.50 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.70

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\, dx =$$
$$-\frac{2\left(63\, Aa^3 \tan\left(\frac{1}{2}\, fx + \frac{1}{2}\, e\right)^8 - 63\, Aa^3 \tan\left(\frac{1}{2}\, fx + \frac{1}{2}\, e\right)^7 + 63\, Ba^3 \tan\left(\frac{1}{2}\, fx + \frac{1}{2}\, e\right)^7 + 483\, Aa^3 \tan\left(\frac{1}{2}\, fx -\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*B*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^3*t
an(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*tan(1/
2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/2*f*x
 + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x + 1/2
*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e) + 8*A*a
^3 - B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.49

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1013\,A\,a^3}{16}+\frac{149\,B\,a^3}{16}-\frac{113\,A\,a^3\,\cos(2\,e+2\,f\,x)}{4}+\frac{37\,A\,a^3\,\cos(3\,e+3\,f\,x)}{8}+\frac{7\,A\,a^3\,\cos(4\,e+4\,f\,x)}{16}-\frac{41\,B\,a^3\,\cos(2}{4}\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((1013*A*a^3)/16 + (149*B*a^3)/16 - (113*A*a^3*cos(2*
e + 2*f*x))/4 + (37*A*a^3*cos(3*e + 3*f*x))/8 + (7*A*a^3*cos(4*e + 4*f*x))/
16 - (41*B*a^3*cos(2*e + 2*f*x))/4 + (19*B*a^3*cos(3*e + 3*f*x))/8 + (7*B*a
^3*cos(4*e + 4*f*x))/16 + (63*A*a^3*sin(2*e + 2*f*x))/8 + (9*A*a^3*sin(3*e
+ 3*f*x))/2 - (9*A*a^3*sin(4*e + 4*f*x))/16 - (63*B*a^3*sin(2*e + 2*f*x))/8
 - (9*B*a^3*sin(3*e + 3*f*x))/2 + (9*B*a^3*sin(4*e + 4*f*x))/16 - (257*A*a^
3*cos(e + f*x))/8 - (23*B*a^3*cos(e + f*x))/8 - (63*A*a^3*sin(e + f*x))/2 +
 (63*B*a^3*sin(e + f*x))/2))/(63*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/
2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*
e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/1
6 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6} dx$

Optimal result

Integrand size = 36, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6} dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{a^3(2A-9B)c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+1/99*a^3*(2*A-9*B)*c^2
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/693*a^3*(2*A-9*B)*c*cos(f*x+e)^7/f/(c-
c*sin(f*x+e))^7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6} dx = \frac{a^3c^3(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3c^2(2A-9B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{a^3c(2A-9B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (a^3*(2*A
- 9*B)*c^2*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (a^3*(2*A - 9*B)
*c*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{1}{11}\left(a^3(2A-9B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{1}{99}\left(a^3(2A-9B)c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$

$$=\frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3(2A-9B)c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(118) = 236.

Time = 12.46 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.65

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$=\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(462(11A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-594(5A+2B\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A
+ 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Cos[
(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/2]
+ 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*(e +
 f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A*Sin[
(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*x))/2]
 - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*Sin[(9*(
e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(B-2A)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{35A}{3}+B\right)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2\left(-\frac{23A}{3}+3B\right)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2\left(4\right.\right.}{}$
risch	$-\frac{2ia^3\left(2iA+2970iA\,e^{4i(fx+e)}+693B\,e^{9i(fx+e)}+693iB\,e^{8i(fx+e)}-2310A\,e^{7i(fx+e)}-110iA\,e^{2i(fx+e)}-4158B\,e^{7i(fx+e)}-\right.}{}$
derivativedivides	$\frac{2a^3\left(-\frac{3008A+2880B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2960A+1968B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{1460A+78}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{f\,c^6}$
default	$\frac{2a^3\left(-\frac{3008A+2880B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2960A+1968B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{1460A+78}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{f\,c^6}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*(A*tan(1/2*f*x+1/2*e)^10+(B-2*A)*tan(1/2*f*x+1/2*e)^9+(35/3*A+B)*tan(1/2
*f*x+1/2*e)^8+2*(-23/3*A+3*B)*tan(1/2*f*x+1/2*e)^7+2*(46/3*A+B)*tan(1/2*f*x
+1/2*e)^6+2*(-11*A+4*B)*tan(1/2*f*x+1/2*e)^5+12/7*(13*A+B)*tan(1/2*f*x+1/2*
e)^4+2/7*(-25*A+11*B)*tan(1/2*f*x+1/2*e)^3+1/7*(269/9*A+2*B)*tan(1/2*f*x+1/
2*e)^2+1/7*(-16/9*A+B)*tan(1/2*f*x+1/2*e)+79/693*A-1/77*B)*a^3/f/c^6/(tan(1
/2*f*x+1/2*e)-1)^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{(2A-9B)a^3\cos(fx+e)^6+6(2A-9B)a^3\cos(fx+e)^5-(25A+234B)a^3\cos(fx+e)^4+7(23A+}{693\left(c^6f\cos(fx+e)^6-5c^6f\cos(fx+e\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 -
(25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 + 28
*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(A +
 B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x + e)^
4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x + e)^2
+ 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e))/(c^6*f*cos(
f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*co
s(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f +
 (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 -
 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(105) = 210$.

Time = 64.76 (sec) , antiderivative size = 4816, normalized size of antiderivative = 40.81

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-1386*A*a**3*tan(e/2 + f*x/2)**10/(693*c**6*f*tan(e/2 + f*x/2)**
11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320
166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690
*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**
6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 2772
*A*a**3*tan(e/2 + f*x/2)**9/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*
tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan
(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/
2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 +
 f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x
/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 16170*A*a**3*tan(e/2
+ f*x/2)**8/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)
**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8
 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 +
320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114
345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c*
*6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 21252*A*a**3*tan(e/2 + f*x/2)**7/(693
*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**
6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f
*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*ta
n(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e
/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f
*x/2) - 693*c**6*f) - 42504*A*a**3*tan(e/2 + f*x/2)**6/(693*c**6*f*tan(e/2
+ f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*
x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2
)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**
5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 -
 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6
*f) + 30492*A*a**3*tan(e/2 + f*x/2)**5/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7
623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345
*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c*
*6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*
f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*ta
n(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 30888*A*a*
*3*tan(e/2 + f*x/2)**4/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e
/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2
+ f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f
*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/
2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**
2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 9900*A*a**3*tan(e/2 + f*x/
2)**3/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 +
 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228
690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166
*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c*
```

*6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*t
an(e/2 + f*x/2) - 693*c**6*f) - 5918*A*a**3*tan(e/2 + f*x/2)**2/(693*c**6*f
*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan
(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/
2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 +
 f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*
x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) -
 693*c**6*f) + 352*A*a**3*tan(e/2 + f*x/2)/(693*c**6*f*tan(e/2 + f*x/2)**11
 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 11
4345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32016
6*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c
6*f*tan(e/2 + f*x/2)4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*
f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 158*A*
a**3/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 +
38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 2286
90*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*
c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**
6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*ta
n(e/2 + f*x/2) - 693*c**6*f) - 1386*B*a**3*tan(e/2 + f*x/2)**9/(693*c**6*f*
tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(
e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2
 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 +
f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x
/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) -
693*c**6*f) - 1386*B*a**3*tan(e/2 + f*x/2)**8/(693*c**6*f*tan(e/2 + f*x/2)*
*11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32
0166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 22869
0*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c*
*6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 831
6*B*a**3*tan(e/2 + f*x/2)**7/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f
*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*ta
n(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e
/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2
+ f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*
x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 2772*B*a**3*tan(e/2
+ f*x/2)**6/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)
10 + 38115*c6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8
 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 +
320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114
345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c*
*6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 11088*B*a**3*tan(e/2 + f*x/2)**5/(693
*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**
6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f
*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*ta

```
n(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e
/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f
*x/2) - 693*c**6*f) - 2376*B*a**3*tan(e/2 + f*x/2)**4/(693*c**6*f*tan(e/2 +
 f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x
/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)
**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5
 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 -
38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*
f) - 4356*B*a**3*tan(e/2 + f*x/2)**3/(693*c**6*f*tan(e/2 + f*x/2)**11 - 762
3*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c
**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6
*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*
tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(
e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 396*B*a**3*t
an(e/2 + f*x/2)**2/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 +
 f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*
x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2
)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**
4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 +
7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 198*B*a**3*tan(e/2 + f*x/2)/(6
93*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c
**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6
*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*
tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan
(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 +
 f*x/2) - 693*c**6*f) + 18*B*a**3/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c
**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6
*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*
tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan
(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2
 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f), Ne(f, 0)), (x*(A
 + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c)**6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3390 vs. $2(115) = 230$.

Time = 0.31 (sec) , antiderivative size = 3390, normalized size of antiderivative = 28.73

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*A*a^3*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^3*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*A*a^
3*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
```

^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*B*a^3*(341*sin
(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 -
11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4
62*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*A*a^3*(253*sin(f*x + e)
/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11) + 12*B*a^3*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*
sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 48*B*a^3*(11*sin(f*x + e)/(cos(f*
x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 165*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 231*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)
/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^

```
6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(115) = 230$.

Time = 0.76 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.99

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = -\frac{2\left(693\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-1386\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+693\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+8085\,Aa^3\tan\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/693*(693*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*A*a^3*tan(1/2*f*x + 1/2*e)
^9 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*tan(1/2*f*x + 1/2*e)^8 +
 693*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 41
58*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 1386
*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 5544*B
*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 1188*B*a
^3*tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 2178*B*a^3*
tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 198*B*a^3*tan(
1/2*f*x + 1/2*e)^2 - 176*A*a^3*tan(1/2*f*x + 1/2*e) + 99*B*a^3*tan(1/2*f*x
+ 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(tan(1/2*f*x + 1/2*e) - 1)^11)
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.46

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = -\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(565\,A\,a^3\,\cos\left(2\,e+2\,f\,x\right)-\frac{837\,B\,a^3}{16}-922\,A\,a^3-\frac{3527\,A\,a^3\,\cos(3\,e+3\,f\,x)}{32}-29\,A\,a^3\,\cos\left(4\,e\right.\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*(565*A*a^3*cos(2*e + 2*f*x) - (837*B*a^3)/16 - 922*A
*a^3 - (3527*A*a^3*cos(3*e + 3*f*x))/32 - 29*A*a^3*cos(4*e + 4*f*x) + (81*A
*a^3*cos(5*e + 5*f*x))/32 + (225*B*a^3*cos(2*e + 2*f*x))/4 - (207*B*a^3*cos
```

```
(3*e + 3*f*x))/16 + (9*B*a^3*cos(4*e + 4*f*x))/16 - (9*B*a^3*cos(5*e + 5*f*
x))/16 - (1617*A*a^3*sin(2*e + 2*f*x))/8 - (5049*A*a^3*sin(3*e + 3*f*x))/32
 + (407*A*a^3*sin(4*e + 4*f*x))/16 + (77*A*a^3*sin(5*e + 5*f*x))/32 + (693*
B*a^3*sin(2*e + 2*f*x))/8 + (99*B*a^3*sin(3*e + 3*f*x))/2 - (99*B*a^3*sin(4
*e + 4*f*x))/16 + (6635*A*a^3*cos(e + f*x))/16 + 18*B*a^3*cos(e + f*x) + (1
3629*A*a^3*sin(e + f*x))/16 - (693*B*a^3*sin(e + f*x))/2))/(693*c^6*f*((231
*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 +
(3*f*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(
1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 +
 (9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3(3A-10B)c\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}$$

```
[Out] 1/13*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/143*a^3*(3*A-10*B)*
c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/1287*a^3*(3*A-10*B)*c*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^8+2/9009*a^3*(3*A-10*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
7
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 2750}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \frac{a^3c^3(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3c^2(3A-10B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}+\frac{2a^3c(3A-10B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (a^3*(3*A
 - 10*B)*c^2*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(3*A -
 10*B)*c*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(3*A - 10
*B)*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}\,dx\\ &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{1}{13}\left(a^3(3A-10B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} \\
&\quad + \frac{1}{143}\left(2a^3(3A-10B)c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} \\
&\quad + \frac{2a^3(3A-10B)c\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8} + \frac{(2a^3(3A-10B))\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx}{1287} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} \\
&\quad + \frac{2a^3(3A-10B)c\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8} + \frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 339 vs. $2(156) = 312$.

Time = 13.83 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.17

$$
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = -\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6006(9A+5B)\cos\left(\frac{1}{2}(e+fx)\right)-7722(4A\right.}{}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -1/144144*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(
6006*(9*A + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2] -
 9009*A*Cos[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[(7*
(e + f*x))/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2] +
130*B*Cos[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(e + f
*x)/2] + 27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2] - 6864
*A*Sin[(5*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[(7*(e +
 f*x))/2] - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2] + 3*A*S
in[(13*(e + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.60

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3A+B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(17A+\frac{B}{3}\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-33A+\frac{23B}{3}\right)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\right.}{}$
derivativedivides	$\frac{2a^3\left(-\frac{6888A+3928B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18816A+14464B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3072A+3072B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{8832A+8576B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\right.}{}$
default	$\frac{2a^3\left(-\frac{6888A+3928B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18816A+14464B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3072A+3072B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{8832A+8576B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\right.}{}$
risch	$-\frac{4\left(-10B\,a^3+3A\,a^3-30030iB\,a^3e^{7i(fx+e)}+39iA\,a^3e^{i(fx+e)}+12012iB\,a^3e^{9i(fx+e)}-130iB\,a^3e^{i(fx+e)}-234A\,a^3e^{2i(fx+\right.}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*(A*tan(1/2*f*x+1/2*e)^12+(-3*A+B)*tan(1/2*f*x+1/2*e)^11+(17*A+1/3*B)*tan
(1/2*f*x+1/2*e)^10+(-33*A+23/3*B)*tan(1/2*f*x+1/2*e)^9+(72*A-B)*tan(1/2*f*x
+1/2*e)^8+(-82*A+50/3*B)*tan(1/2*f*x+1/2*e)^7+(666/7*A-38/21*B)*tan(1/2*f*x
+1/2*e)^6+(-426/7*A+90/7*B)*tan(1/2*f*x+1/2*e)^5+(857/21*A-2/63*B)*tan(1/2*
f*x+1/2*e)^4+(-263/21*A+215/63*B)*tan(1/2*f*x+1/2*e)^3+(37/231*B+389/77*A)*
tan(1/2*f*x+1/2*e)^2+(-79/231*A+97/693*B)*tan(1/2*f*x+1/2*e)+310/3003*A-97/
9009*B)*a^3/f/c^7/(tan(1/2*f*x+1/2*e)-1)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(152) = 304$.

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.04

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = -\frac{2\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^7-12\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^6-49\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^5+7\,(\cdots}{9009\,\left(c^7f\cos\left(fx+\cdots\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] -1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x +
e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x +
e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f*x
+ e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 10*B)
*a^3*cos(f*x + e)^6 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 35*(3*A - 10*B)*
```

```
a^3*cos(f*x + e)^4 - 63*(5*A + 31*B)*a^3*cos(f*x + e)^3 - 252*(8*A + 21*B)*
a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3)*sin(
f*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x
 + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos
(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*
c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 +
80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. 2(143) = 286.

Time = 102.64 (sec) , antiderivative size = 6669, normalized size of antiderivative = 42.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Piecewise((-18018*A*a**3*tan(e/2 + f*x/2)**12/(9009*c**7*f*tan(e/2 + f*x/2)
**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)*
*11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)
**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2
)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/
2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2
)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) -
 9009*c**7*f) + 54054*A*a**3*tan(e/2 + f*x/2)**11/(9009*c**7*f*tan(e/2 + f*
x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x
/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*
x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f
*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 +
f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f
*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/
2) - 9009*c**7*f) - 306306*A*a**3*tan(e/2 + f*x/2)**10/(9009*c**7*f*tan(e/2
 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2
+ f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2
 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/
2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e
/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/
2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 +
 f*x/2) - 9009*c**7*f) + 594594*A*a**3*tan(e/2 + f*x/2)**9/(9009*c**7*f*tan
(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(
e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan
(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*ta
n(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*t
an(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*ta
```

n(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e
/2 + f*x/2) - 9009*c**7*f) - 1297296*A*a**3*tan(e/2 + f*x/2)**8/(9009*c**7*
f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f
*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*
f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7
*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**
7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7
*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*
tan(e/2 + f*x/2) - 9009*c**7*f) + 1477476*A*a**3*tan(e/2 + f*x/2)**7/(9009*
c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c
7*f*tan(e/2 + f*x/2)11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*
c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444
*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 1159458
3*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574
*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c*
*7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 1714284*A*a**3*tan(e/2 + f*x/2)**6/(
9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702
702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 644
1435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 154
59444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11
594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 25
76574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 1171
17*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 1096524*A*a**3*tan(e/2 + f*x/2)
5/(9009*c7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12
+ 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10
+ 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8
+ 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6
 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4
 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 +
 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 735306*A*a**3*tan(e/2 + f*
x/2)**4/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)*
*12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)*
*10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)
8 + 15459444*c7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2
)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2
)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)*
*2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 225654*A*a**3*tan(e/2
+ f*x/2)**3/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x
/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x
/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*
x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f
*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f
*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x
/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 91026*A*a**3*tan(e
/2 + f*x/2)**2/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 +

f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 +
f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 +
f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2
+ f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2
+ f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 +
f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 6162*A*a**3*tan
(e/2 + f*x/2)/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f
*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f
*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 +
f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 +
f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 +
f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f
*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 1860*A*a**3/(900
9*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702
*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 644143
5*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 154594
44*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594
583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 25765
74*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*
c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 18018*B*a**3*tan(e/2 + f*x/2)**11/
(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 70
2702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 64
41435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15
459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 1
1594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2
576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117
117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 6006*B*a**3*tan(e/2 + f*x/2)**
10/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 +
702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 +
6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 +
15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6
+ 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4
+ 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 +
117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 138138*B*a**3*tan(e/2 + f*x
/2)**9/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**
12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**
10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)*
*8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)
6 + 11594583*c7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)
4 + 2576574*c7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**
2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 18018*B*a**3*tan(e/2 +
f*x/2)**8/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2
)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2
)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/
2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x

```
/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x
/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2
)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 300300*B*a**3*tan(e/
2 + f*x/2)**7/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f
*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f
*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 +
f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 +
 f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 +
 f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f
*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 32604*B*a**3*tan
(e/2 + f*x/2)**6/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2
+ f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2
+ f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2
 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/
2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/
2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2
+ f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 231660*B*a**3
*tan(e/2 + f*x/2)**5/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(
e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(
e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan
(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*ta
n(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*ta
n(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(
e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 572*B*a**
3*tan(e/2 + f*x/2)**4/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan
(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan
(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*ta
n(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*t
an(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*t
an(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan
(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 61490*B*
a**3*tan(e/2 + f*x/2)**3/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*
tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*
tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f
*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*
f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*
f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*
tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 2886*
B*a**3*tan(e/2 + f*x/2)**2/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*
f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*
f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7
*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**
7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**
7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*
f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 252
```

```
2*B*a**3*tan(e/2 + f*x/2)/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f
*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f
*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*
f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7
*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7
*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f
*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 194*
B*a**3/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**
12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**
10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)*
*8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)
**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)
**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**
2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f), Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)**3/(-c*sin(e) + c)**7, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4078 vs. $2(152) = 304$.

Time = 0.39 (sec) , antiderivative size = 4078, normalized size of antiderivative = 26.14

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

```
[Out] -2/45045*(6*A*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 6*B*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
```

sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^3*(3796*sin(f*x + e)/(cos(f*x + e) + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 105*A*a^3*(611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^3*(611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f

*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^
7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13) + 8*B*a^3*(559*sin(f*x + e)/(cos(f*x + e) + 1) - 3354*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 12298*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 30745
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 37323*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 - 49764*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24024*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 - 18018*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 43)/(c^7 -
13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x +
e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) -
462*A*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 858*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
51*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(f*x + e) +
1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x
+ e) + 1)^13) - 1386*B*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5
20*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1)
+ 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +

```
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 28
6*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*
x + e)^13/(cos(f*x + e) + 1)^13))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(152) = 304$.

Time = 0.43 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.70

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = -\frac{2\left(9009\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{12} - 27027\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{11} + 9009\,Ba^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{11} + 153153\,A\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/9009*(9009*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 27027*A*a^3*tan(1/2*f*x + 1/2
*e)^11 + 9009*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 153153*A*a^3*tan(1/2*f*x + 1/
2*e)^10 + 3003*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 297297*A*a^3*tan(1/2*f*x + 1
/2*e)^9 + 69069*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*tan(1/2*f*x + 1
/2*e)^8 - 9009*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*tan(1/2*f*x + 1/
2*e)^7 + 150150*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*tan(1/2*f*x + 1
/2*e)^6 - 16302*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*tan(1/2*f*x + 1
/2*e)^5 + 115830*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*tan(1/2*f*x +
1/2*e)^4 - 286*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*tan(1/2*f*x + 1/
2*e)^3 + 30745*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*tan(1/2*f*x + 1/2
*e)^2 + 1443*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*tan(1/2*f*x + 1/2*e)
 + 1261*B*a^3*tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7*f*(tan(1/2*
f*x + 1/2*e) - 1)^13)
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.21

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \frac{2\cos\left(\frac{e}{2} + \frac{fx}{2}\right)\left(\frac{2363\,B\,a^3}{32} - \frac{279183\,A\,a^3}{16} + \frac{220269\,A\,a^3\,\cos(2\,e+2\,f\,x)}{16} - \frac{46095\,A\,a^3\,\cos(3\,e+3\,f\,x)}{16} - \frac{20829\,A\,a^3\,\cos(4\,e+4\,f\,x)}{16} + \right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^7,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((2363*B*a^3)/32 - (279183*A*a^3)/16 + (220269*A*a^3*
cos(2*e + 2*f*x))/16 - (46095*A*a^3*cos(3*e + 3*f*x))/16 - (20829*A*a^3*cos
(4*e + 4*f*x))/16 + (2811*A*a^3*cos(5*e + 5*f*x))/16 + (231*A*a^3*cos(6*e +
 6*f*x))/16 - (8995*B*a^3*cos(2*e + 2*f*x))/64 + (497*B*a^3*cos(3*e + 3*f*x
))/16 + (3725*B*a^3*cos(4*e + 4*f*x))/32 - (361*B*a^3*cos(5*e + 5*f*x))/16
- (77*B*a^3*cos(6*e + 6*f*x))/64 - (19305*A*a^3*sin(2*e + 2*f*x))/4 - (8108
1*A*a^3*sin(3*e + 3*f*x))/16 + (15015*A*a^3*sin(4*e + 4*f*x))/16 + (3237*A*
a^3*sin(5*e + 5*f*x))/16 - (117*A*a^3*sin(6*e + 6*f*x))/8 + (77649*B*a^3*si
n(2*e + 2*f*x))/64 + (27027*B*a^3*sin(3*e + 3*f*x))/32 - (1001*B*a^3*sin(4*
e + 4*f*x))/8 - (559*B*a^3*sin(5*e + 5*f*x))/32 + (117*B*a^3*sin(6*e + 6*f*
x))/64 + (26979*A*a^3*cos(e + f*x))/4 + 40*B*a^3*cos(e + f*x) + (173745*A*a
^3*sin(e + f*x))/8 - (80223*B*a^3*sin(e + f*x))/16))/(9009*c^7*f*((1287*2^(
1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/2 + pi/4 + (f
*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64 - (143*2^(1/2
)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e)/2 + pi/4 + (9
*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/64 + (2^(1/2)
*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 197

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}$$

```
[Out] 1/15*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+1/195*a^3*(4*A-11*B)*
c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/715*a^3*(4*A-11*B)*c*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^9+2/6435*a^3*(4*A-11*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
8+2/45045*a^3*(4*A-11*B)*cos(f*x+e)^7/c/f/(c-c*sin(f*x+e))^7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 2750}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \frac{a^3c^3(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{a^3c^2(4A-11B)\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+\frac{a^3c(4A-11B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (a^3*(4*A
 - 11*B)*c^2*Cos[e + f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (a^3*(4*A -
11*B)*c*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(4*A - 11*B
)*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(4*A - 11*B)*Cos
[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^3c^3)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}\,dx$$

$$
\begin{aligned}
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{1}{15}\left(a^3(4A-11B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} \\
&\quad + \frac{1}{65}\left(a^3(4A-11B)c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} \\
&\quad + \frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{9}} + \frac{1}{715}\left(2a^3(4A-11B)\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{8}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} \\
&\quad + \frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{9}} + \frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{8}} \\
&\quad + \frac{\left(2a^3(4A-11B)\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx}{6435c} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} \\
&\quad + \frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{9}} + \frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{8}} \\
&\quad + \frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^{7}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 14.87 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.85

$$
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx
= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6435(72A+47B)\cos\left(\frac{1}{2}(e+fx)\right)-10010(26A\right.}{}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^8,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(6435*(72*A
 + 47*B)*Cos[(e + f*x)/2] - 10010*(26*A + 23*B)*Cos[(3*(e + f*x))/2] - 7207
2*A*Cos[(5*(e + f*x))/2] - 117117*B*Cos[(5*(e + f*x))/2] + 5460*A*Cos[(7*(e
 + f*x))/2] + 30030*B*Cos[(7*(e + f*x))/2] - 420*A*Cos[(11*(e + f*x))/2] +
1155*B*Cos[(11*(e + f*x))/2] + 4*A*Cos[(15*(e + f*x))/2] - 11*B*Cos[(15*(e
+ f*x))/2] + 437580*A*Sin[(e + f*x)/2] + 373230*B*Sin[(e + f*x)/2] + 240240
```

```
*A*Sin[(3*(e + f*x))/2] + 285285*B*Sin[(3*(e + f*x))/2] - 60060*A*Sin[(5*(e
 + f*x))/2] - 150150*B*Sin[(5*(e + f*x))/2] - 45045*B*Sin[(7*(e + f*x))/2]
- 1820*A*Sin[(9*(e + f*x))/2] + 5005*B*Sin[(9*(e + f*x))/2] + 60*A*Sin[(13*
(e + f*x))/2] - 165*B*Sin[(13*(e + f*x))/2]))/(1441440*c^8*f*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^8)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.51

method	result
risch	$\frac{4ia^3\left(-4iA-5460iA\,e^{4i(fx+e)}+45045B\,e^{11i(fx+e)}-302445iB\,e^{8i(fx+e)}-240240A\,e^{9i(fx+e)}+420iA\,e^{2i(fx+e)}-285285\right.}{}$
parallelrisch	$-\frac{2\left(A\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A+B)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(71A-B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+10(-6A+B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(74\right.}{}}{}$
derivativedivides	$\frac{2a^3\left(-\frac{188A+38B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{58816A+40000B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{52736A+49664B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{24320}{13\left(\tan\left(\right.\right.}\right.}{}$
default	$\frac{2a^3\left(-\frac{188A+38B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{58816A+40000B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{52736A+49664B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{24320}{13\left(\tan\left(\right.\right.}\right.}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x,method=_RETURN
VERBOSE)
```

```
[Out] 4/45045*I*a^3*(-4*I*A-5460*I*A*exp(4*I*(f*x+e))+45045*B*exp(11*I*(f*x+e))-3
02445*I*B*exp(8*I*(f*x+e))-240240*A*exp(9*I*(f*x+e))+420*I*A*exp(2*I*(f*x+e
))-285285*B*exp(9*I*(f*x+e))+11*I*B+437580*A*exp(7*I*(f*x+e))+72072*I*A*exp
(10*I*(f*x+e))+373230*B*exp(7*I*(f*x+e))+117117*I*B*exp(10*I*(f*x+e))-60060
*A*exp(5*I*(f*x+e))+230230*I*B*exp(6*I*(f*x+e))-150150*B*exp(5*I*(f*x+e))-1
155*I*B*exp(2*I*(f*x+e))-1820*A*exp(3*I*(f*x+e))-463320*I*A*exp(8*I*(f*x+e)
)+5005*B*exp(3*I*(f*x+e))-30030*I*B*exp(4*I*(f*x+e))+60*A*exp(I*(f*x+e))+26
0260*I*A*exp(6*I*(f*x+e))-165*B*exp(I*(f*x+e)))/f/c^8/(exp(I*(f*x+e))-I)^15
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(192) = 384.

Time = 0.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{2\,(4\,A-11\,B)a^3\cos\left(fx+e\right)^8+16\,(4\,A-11\,B)a^3\cos\left(fx+e\right)^7-49\,(4\,A-11\,B)a^3\cos\left(fx+e\right)^6-168\,(\cdots}{45045\,(c^8\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="fricas")
```

```
[Out] 1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x +
e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x +
e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f*x
+ e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f*x +
 e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A - 11
*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*A - 11
*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3*A + 8*
B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A + B)*a^3)
*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32*c^8*f*co
s(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4 - 112*c^8
*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x + e) + 128*
c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8*f*cos(f*x +
 e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 192*c^8*f*cos(f
*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8821 vs. 2(178) = 356.

Time = 162.97 (sec) , antiderivative size = 8821, normalized size of antiderivative = 44.78

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)
```

```
[Out] Piecewise((-90090*A*a**3*tan(e/2 + f*x/2)**14/(45045*c**8*f*tan(e/2 + f*x/2
)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2
)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*
x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2
 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(
e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*t
```

an(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*
tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*ta
n(e/2 + f*x/2) - 45045*c**8*f) + 360360*A*a**3*tan(e/2 + f*x/2)**13/(45045*
c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*
c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 614864
25*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 22
5450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 +
289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**
6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)
4 + 20495475*c8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)
2 + 675675*c8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 2132130*A*a**3*tan(e
/2 + f*x/2)**12/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2
+ f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/
2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*ta
n(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*
f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c*
*8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*
c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*
c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f)
+ 5405400*A*a**3*tan(e/2 + f*x/2)**11/(45045*c**8*f*tan(e/2 + f*x/2)**15 -
675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 -
20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**1
1 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/
2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f
*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2
+ f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2
+ f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 +
f*x/2) - 45045*c**8*f) - 13351338*A*a**3*tan(e/2 + f*x/2)**10/(45045*c**8*
f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*
f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c*
*8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 2254502
25*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 2898
64575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 1
35270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 +
20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 +
675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 20420400*A*a**3*tan(e/2 +
f*x/2)**9/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x
/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f
*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2
+ f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan
(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*
tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*
f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*
f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 28
249650*A*a**3*tan(e/2 + f*x/2)**8/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 6756

75*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 2049
5475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 1
35270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9
 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)
7 - 225450225*c8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x
/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*
x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/
2) - 45045*c**8*f) + 26357760*A*a**3*tan(e/2 + f*x/2)**7/(45045*c**8*f*tan(
e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(
e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*t
an(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**
8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*
c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 1352701
35*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495
475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 67567
5*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 22052030*A*a**3*tan(e/2 + f*x/2
)**6/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**1
4 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)*
*12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x
/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 +
 f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/
2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(
e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(
e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 12132120
*A*a**3*tan(e/2 + f*x/2)**5/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**
8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c
8*f*tan(e/2 + f*x/2)12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 1352701
35*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289
864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 -
225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5
 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**
3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 4
5045*c**8*f) - 6177990*A*a**3*tan(e/2 + f*x/2)**4/(45045*c**8*f*tan(e/2 + f
*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f
*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2
+ f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan
(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*
tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8
*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**
8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*
f*tan(e/2 + f*x/2) - 45045*c**8*f) + 1729000*A*a**3*tan(e/2 + f*x/2)**3/(45
045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729
725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61
486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10
+ 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)*

*8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/
2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*
x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*
x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 530670*A*a**3*ta
n(e/2 + f*x/2)**2/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/
2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(
e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*
tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**
8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*
c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 6148642
5*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 472972
5*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*
f) + 37200*A*a**3*tan(e/2 + f*x/2)/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675
675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 204
95475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 -
135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**
9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2
)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*
x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f
*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x
/2) - 45045*c**8*f) - 8486*A*a**3/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 6756
75*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 2049
5475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 1
35270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9
 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)
7 - 225450225*c8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x
/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*
x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/
2) - 45045*c**8*f) - 90090*B*a**3*tan(e/2 + f*x/2)**13/(45045*c**8*f*tan(e/
2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/
2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan
(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*
f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c*
*8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135
*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 2049547
5*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*
c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 30030*B*a**3*tan(e/2 + f*x/2)**12
/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 +
4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12
+ 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)*
*10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x
/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 +
f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2
+ f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2
+ f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 900900*B*a**

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3*tan(e/2 + f*x/2)**11/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*t
an(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f
*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c*
*8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 28986457
5*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 22545
0225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61
486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4
729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*
c**8*f) + 612612*B*a**3*tan(e/2 + f*x/2)**10/(45045*c**8*f*tan(e/2 + f*x/2)
**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)
**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x
/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2
+ f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e
/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*ta
n(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*t
an(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan
(e/2 + f*x/2) - 45045*c**8*f) - 2912910*B*a**3*tan(e/2 + f*x/2)**9/(45045*c
**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c
**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 6148642
5*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225
450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 +
289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6
 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)*
*4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)*
*2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 1583010*B*a**3*tan(e/
2 + f*x/2)**8/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 +
f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2
+ f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(
e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*
tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8
*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c*
*8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c*
*8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) -
 3655080*B*a**3*tan(e/2 + f*x/2)**7/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 67
5675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20
495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 -
 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)*
*9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/
2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f
*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 +
f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*
x/2) - 45045*c**8*f) + 1161160*B*a**3*tan(e/2 + f*x/2)**6/(45045*c**8*f*tan
(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan
(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*
tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c*
```

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*8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575
*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270
135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 2049
5475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 6756
75*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 1831830*B*a**3*tan(e/2 + f*x/2
)**5/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**1
4 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)*
*12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x
/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 +
 f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/
2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(
e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(
e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 210210*B
*a**3*tan(e/2 + f*x/2)**4/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*
f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**
8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135
*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 28986
4575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 22
5450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 -
 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3
- 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 450
45*c**8*f) - 340340*B*a**3*tan(e/2 + f*x/2)**3/(45045*c**8*f*tan(e/2 + f*x/
2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/
2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f
*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/
2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan
(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*
tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f
*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*t
an(e/2 + f*x/2) - 45045*c**8*f) - 4620*B*a**3*tan(e/2 + f*x/2)**2/(45045*c*
*8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c*
*8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425
*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 2254
50225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 2
89864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6
+ 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**
4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**
2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 12210*B*a**3*tan(e/2 +
 f*x/2)/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)
**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/
2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 +
f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/
2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan
(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*t
an(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*t
```

```
an(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 814*B
*a**3/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**
14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)
**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*
x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2
+ f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e
/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan
(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan
(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f), Ne(f, 0)
), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c)**8, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4765 vs. $2(192) = 384$.

Time = 0.41 (sec) , antiderivative size = 4765, normalized size of antiderivative = 24.19

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="maxima")
```

```
[Out] 2/45045*(3*A*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891
345*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 2867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*s
in(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 643
5*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x
+ e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^1
4 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + B*a^3*(17715*sin(f*x + e)/
(cos(f*x + e) + 1) - 78960*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 342160*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891345*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3043040*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 3912480*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3687255
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2867865*sin(f*x + e)^9/(cos(f*x + e)
```

+ 1)^9 - 1585584*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 7*A*a^3*(7845*sin(f*x + e)/(cos(f*x + e) + 1) - 54915*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 668850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1444443*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3063060*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2357355*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1414413*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 630630*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 210210*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 952)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 12*A*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x +

e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*
x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15
) - 12*B*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12
/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1)
+ 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
+ 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11
/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 10
5*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f
*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + 6*A*a^3*(675
*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 20475*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 46410*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
122265*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 95095*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*sin(f*x
+ e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e
) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30
03*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) +
1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^1
2 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/
(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + 18*B*a
^3*(675*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 20475*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 46410*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 122265*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 95095*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*s
in(f*x + e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(
f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6

/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435
*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*si
n(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e)
 + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x +
 e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) -
 48*B*a^3*(60*sin(f*x + e)/(cos(f*x + e) + 1) - 420*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 1820*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5460*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 9009*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15015*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12870*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 12870*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 5005*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 - 3003*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 4)/(c^8 - 1
5*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x +
e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f
*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) +
1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15))/f

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(192) = 384$.

Time = 0.45 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.48

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = -\frac{2\left(45045\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}-180180\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+45045\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+106\right.}{}$$

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="giac")

[Out] -2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x +
1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f*x
 + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1/2*
f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*tan(
1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*A*a^3
*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 14124825*A
*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 1317888

```
0*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 110
26015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/2*e)^6 -
6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x + 1/2*e)^5
+ 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*x + 1/2*e)^
4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*f*x + 1/2*e)
^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*f*x + 1/2*e)^
2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x + 1/2*e) + 42
43*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.93

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{544369\,A\,a^3}{4}-\frac{21791\,B\,a^3}{4}-\frac{257861\,A\,a^3\,\cos(2\,e+2\,f\,x)}{2}+\frac{3497111\,A\,a^3\,\cos(3\,e+3\,f\,x)}{128}+\frac{72047\,A\,a^3\,\cos(4\,e+4\,f\,x)}{4}\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^8,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((544369*A*a^3)/4 - (21791*B*a^3)/4 - (257861*A*a^3*c
os(2*e + 2*f*x))/2 + (3497111*A*a^3*cos(3*e + 3*f*x))/128 + (72047*A*a^3*co
s(4*e + 4*f*x))/4 - (378579*A*a^3*cos(5*e + 5*f*x))/128 - (1059*A*a^3*cos(6
*e + 6*f*x))/2 + (4251*A*a^3*cos(7*e + 7*f*x))/128 + (219769*B*a^3*cos(2*e
+ 2*f*x))/32 - (191389*B*a^3*cos(3*e + 3*f*x))/128 - 1672*B*a^3*cos(4*e + 4
*f*x) + (38841*B*a^3*cos(5*e + 5*f*x))/128 + (1551*B*a^3*cos(6*e + 6*f*x))/
32 - (429*B*a^3*cos(7*e + 7*f*x))/128 + (2633345*A*a^3*sin(2*e + 2*f*x))/64
 + (7210775*A*a^3*sin(3*e + 3*f*x))/128 - (89375*A*a^3*sin(4*e + 4*f*x))/8
- (504205*A*a^3*sin(5*e + 5*f*x))/128 + (29765*A*a^3*sin(6*e + 6*f*x))/64 +
 (4235*A*a^3*sin(7*e + 7*f*x))/128 - (451165*B*a^3*sin(2*e + 2*f*x))/64 - (
854425*B*a^3*sin(3*e + 3*f*x))/128 + (9295*B*a^3*sin(4*e + 4*f*x))/8 + (464
75*B*a^3*sin(5*e + 5*f*x))/128 - (3025*B*a^3*sin(6*e + 6*f*x))/64 - (385*B*
a^3*sin(7*e + 7*f*x))/128 - (5734111*A*a^3*cos(e + f*x))/128 + (126929*B*a^
3*cos(e + f*x))/128 - (25501905*A*a^3*sin(e + f*x))/128 + (3970395*B*a^3*si
n(e + f*x))/128))/(45045*c^8*f*((6435*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/12
8 - (5005*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/128 - (3003*2^(1/2)*cos(
(5*e)/2 + pi/4 + (5*f*x)/2))/128 + (1365*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*
x)/2))/128 + (455*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/128 - (105*2^(1/
2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/128 - (15*2^(1/2)*cos((13*e)/2 + pi/4
 + (13*f*x)/2))/128 + (2^(1/2)*cos((15*e)/2 - pi/4 + (15*f*x)/2))/128))
```

3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 190

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx \\
&= -\frac{35(4A-5B)c^4x}{8a} - \frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} \\
&\quad - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} \\
&\quad - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))}
\end{aligned}$$

```
[Out] -35/8*(4*A-5*B)*c^4*x/a-35/12*(4*A-5*B)*c^4*cos(f*x+e)^3/a/f-35/8*(4*A-5*B)
*c^4*cos(f*x+e)*sin(f*x+e)/a/f-a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e)
)^5-2*a^2*(4*A-5*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^3-7/4*(4*A-5*B)*c^4
*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used

$= \{3046, 2938, 2759, 2758, 2761, 2715, 8\}$

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$$
$$= -\frac{a^4c^4(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5} - \frac{2a^2c^4(4A-5B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^3} - \frac{35c^4(4A-5B)\cos^3(e+fx)}{12af} - \frac{7c^4(4A-5B)\cos^5(e+fx)}{4f(a\sin(e+fx)+a)} - \frac{35c^4(4A-5B)\sin(e+fx)\cos(e+fx)}{8af} - \frac{35c^4x(4A-5B)}{8a}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-35*(4*A - 5*B)*c^4*x)/(8*a) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]^3)/(12*a*f
) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(8*a*f) - (a^4*(A - B)*c
^4*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5) - (2*a^2*(4*A - 5*B)*c^4*Cos[
e + f*x]^7)/(f*(a + a*Sin[e + f*x])^3) - (7*(4*A - 5*B)*c^4*Cos[e + f*x]^5)
/(4*f*(a + a*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx \\
&= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \left(a^3(4A-5B)c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^4}\,dx \\
&= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} \\
&\quad - \left(7a(4A-5B)c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^2}\,dx \\
&= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} \\
&\quad - \frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))} - \frac{1}{4}\left(35(4A-5B)c^4\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx)}\,dx
\end{aligned}$$

$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))} - \frac{(35(4A-5B)c^4)\int\cos^2(e+fx)\,dx}{4a}$$

$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))} - \frac{(35(4A-5B)c^4)\int 1\,dx}{8a}$$

$$= -\frac{35(4A-5B)c^4x}{8a} - \frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 12.48 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.44

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(3072(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-420(4A-5B)(e\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(3072*(A - B)
*Sin[(e + f*x)/2] - 420*(4*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 24*(47*A - 75*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]) + 8*(A - 5*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) +
24*(5*A - 12*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)] + 3*
B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[4*(e + f*x)]))/(96*a*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{2c^4\left(-\frac{16A-16B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\left(\frac{5A}{2}-\frac{47B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(11A-15B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5A}{2}-\frac{55B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(35A-55B)\left(\mathrm{t}\right.}{(}\right.}{}$
default	$\frac{2c^4\left(-\frac{16A-16B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\left(\frac{5A}{2}-\frac{47B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(11A-15B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5A}{2}-\frac{55B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(35A-55B)\left(\mathrm{t}\right.}{(}\right.}{}$
parallelrisch	$-\frac{7c^4\left(\left(30fxA-\frac{75}{2}fxB+\frac{1189}{14}A-\frac{1433}{14}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(30fxA-\frac{75}{2}fxB+\frac{139}{14}A-\frac{215}{14}B\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+9\left(A-\frac{3B}{2}\right)\cos\right.}{}$
risch	$-\frac{35c^4xA}{2a}+\frac{175c^4xB}{8a}-\frac{47c^4\mathrm{e}^{i(fx+e)}A}{8af}+\frac{75c^4\mathrm{e}^{i(fx+e)}B}{8af}-\frac{47c^4\mathrm{e}^{-i(fx+e)}A}{8af}+\frac{75c^4\mathrm{e}^{-i(fx+e)}B}{8af}-\frac{32c^4}{fa\left(\mathrm{e}^{i(fx-}\right.}$
norman	$\frac{-\frac{166A\,c^4-206B\,c^4}{3af}-\frac{35(4A-5B)c^4x}{8a}-\frac{\left(108A\,c^4-167B\,c^4\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4af}-\frac{\left(148A\,c^4-175B\,c^4\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4af}-\frac{\left(204A\,c^4-3\right.}{}}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*c^4/a*(-(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1)-((5/2*A-47/8*B)*tan(1/2*f*x+
1/2*e)^7+(11*A-15*B)*tan(1/2*f*x+1/2*e)^6+(5/2*A-55/8*B)*tan(1/2*f*x+1/2*e)
^5+(35*A-55*B)*tan(1/2*f*x+1/2*e)^4+(-5/2*A+55/8*B)*tan(1/2*f*x+1/2*e)^3+(1
07/3*A-175/3*B)*tan(1/2*f*x+1/2*e)^2+(-5/2*A+47/8*B)*tan(1/2*f*x+1/2*e)+35/
3*A-55/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^4-35/8*(4*A-5*B)*arctan(tan(1/2*f*x+1/
2*e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{6\,Bc^4\cos\left(fx+e\right)^5-8\,(A-5\,B)c^4\cos\left(fx+e\right)^4+(52\,A-113\,B)c^4\cos\left(fx+e\right)^3+105\,(4\,A-5\,B)c^4}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(6*B*c^4*cos(f*x + e)^5 - 8*(A - 5*B)*c^4*cos(f*x + e)^4 + (52*A - 11
3*B)*c^4*cos(f*x + e)^3 + 105*(4*A - 5*B)*c^4*f*x + 96*(3*A - 5*B)*c^4*cos(
f*x + e)^2 + 384*(A - B)*c^4 + 3*(35*(4*A - 5*B)*c^4*f*x + (204*A - 239*B)*
```

```
c^4)*cos(f*x + e) - (6*B*c^4*cos(f*x + e)^4 + 2*(4*A - 17*B)*c^4*cos(f*x +
e)^3 - 105*(4*A - 5*B)*c^4*f*x + 3*(20*A - 49*B)*c^4*cos(f*x + e)^2 - 3*(76
*A - 111*B)*c^4*cos(f*x + e) + 384*(A - B)*c^4)*sin(f*x + e))/(a*f*cos(f*x
+ e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6690 vs. $2(173) = 346$.

Time = 6.94 (sec) , antiderivative size = 6690, normalized size of antiderivative = 35.21

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-420*A*c**4*f*x*tan(e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9
+ 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2
+ f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 9
6*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f
*x/2) + 24*a*f) - 420*A*c**4*f*x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/
2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*ta
n(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)*
*4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e
/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2
 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96
*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 +
f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*
f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**6/(24*a*f*
tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)*
*7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan
(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
+ 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**5/(
24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 +
f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*
a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x
/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/
2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan
(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5
 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/
2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2
 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*
a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*
x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f
*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*
tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**
```

```
8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e
/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 +
 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*c**
4*f*x*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2
)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*ta
n(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**
3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*
c**4*f*x/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*
tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)
**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan
(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 888*A*c**4*tan(e/2 +
 f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*
f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/
2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*t
an(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 648*A*c**4*tan(e/2
 + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*
a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*
x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f
*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 3720*A*c**4*tan(
e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 +
96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 +
 f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*
a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1800*A*c**4*t
an(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8
 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/
2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 +
96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 6168*A*c**
4*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)
**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan
(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3
 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1592*A*
c**4*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x
/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*
tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)
**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 4664
*A*c**4*tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 +
f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a
*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x
/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 4
40*A*c**4*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f
*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*
f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/
2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 13
28*A*c**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f
*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2
```

```
)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*ta
n(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x*tan(
e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 +
96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 +
 f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*
a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*
x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)
**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan
(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3
 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 2100*B*
c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 +
 f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*
a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*
x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) +
2100*B*c**4*f*x*tan(e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*ta
n(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6
 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e
/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*
a*f) + 3150*B*c**4*f*x*tan(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24
*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*
x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*
f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2
) + 24*a*f) + 3150*B*c**4*f*x*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)*
*9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e
/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4
+ 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2
+ f*x/2) + 24*a*f) + 2100*B*c**4*f*x*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 +
f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*
f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x
/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*t
an(e/2 + f*x/2) + 24*a*f) + 2100*B*c**4*f*x*tan(e/2 + f*x/2)**2/(24*a*f*tan
(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7
+ 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/
2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 2
4*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x*tan(e/2 + f*x/2)/(24*a*f*
tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)*
*7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan
(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
+ 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x/(24*a*f*tan(e/2 + f*x/
2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*ta
n(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)*
*4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e
/2 + f*x/2) + 24*a*f) + 1050*B*c**4*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f
*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f
*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/
```

```
2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta
n(e/2 + f*x/2) + 24*a*f) + 1002*B*c**4*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2
+ f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*
a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f
*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f
*tan(e/2 + f*x/2) + 24*a*f) + 4122*B*c**4*tan(e/2 + f*x/2)**6/(24*a*f*tan(e
/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 +
96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2
+ f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*
a*f*tan(e/2 + f*x/2) + 24*a*f) + 2970*B*c**4*tan(e/2 + f*x/2)**5/(24*a*f*ta
n(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7
 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e
/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 +
24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 6918*B*c**4*tan(e/2 + f*x/2)**4/(24*a*f
*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)
**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*ta
n(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 2470*B*c**4*tan(e/2 + f*x/2)**3/(24*
a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x
/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f
*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)
**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 5590*B*c**4*tan(e/2 + f*x/2)**2/(
24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 +
f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*
a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x
/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 598*B*c**4*tan(e/2 + f*x/2)/(2
4*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f
*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a
*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/
2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 1648*B*c**4/(24*a*f*tan(e/2 + f
*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f
*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/
2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta
n(e/2 + f*x/2) + 24*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**4/(
a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1796 vs. $2(182) = 364$.

Time = 0.33 (sec) , antiderivative size = 1796, normalized size of antiderivative = 9.45

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
```

="maxima")

```
[Out] 1/12*(B*c^4*((19*sin(f*x + e)/(cos(f*x + e) + 1) + 211*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 91*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 219*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 165*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 165*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 64)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 6*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 4*a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a*sin(f*x + e)^8/(cos(f*x + e) + 1)^
8 + a*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 45*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a) - 4*A*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 9*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) + 16*B*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)
 + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4
8*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 72*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f
*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a) - 144*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 96*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)
/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e
```

```
)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 96
*A*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1))) + 24*B*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 24*A*c^4/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.72

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{105\,(4\,Ac^4-5\,Bc^4)(fx+e)}{a}+\frac{768\,(Ac^4-Bc^4)}{a(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}+\frac{2\left(60\,Ac^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7-141\,Bc^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+264\,Ac^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^6-36}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/24*(105*(4*A*c^4 - 5*B*c^4)*(f*x + e)/a + 768*(A*c^4 - B*c^4)/(a*(tan(1/
2*f*x + 1/2*e) + 1)) + 2*(60*A*c^4*tan(1/2*f*x + 1/2*e)^7 - 141*B*c^4*tan(1
/2*f*x + 1/2*e)^7 + 264*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 360*B*c^4*tan(1/2*f*
x + 1/2*e)^6 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 165*B*c^4*tan(1/2*f*x + 1/
2*e)^5 + 840*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1320*B*c^4*tan(1/2*f*x + 1/2*e)
^4 - 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*B*c^4*tan(1/2*f*x + 1/2*e)^3 + 8
56*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1400*B*c^4*tan(1/2*f*x + 1/2*e)^2 - 60*A*
c^4*tan(1/2*f*x + 1/2*e) + 141*B*c^4*tan(1/2*f*x + 1/2*e) + 280*A*c^4 - 440
*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a))/f
```

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{55\,A\,c^4}{3}-\frac{299\,B\,c^4}{12}\right)+\frac{166\,A\,c^4}{3}-\frac{206\,B\,c^4}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(27\,A\,c^4-\frac{167\,B\,c^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\right.}$$

$$-\frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,A-5\,B)}{140\,A\,c^4-175\,B\,c^4}\right)\,(4\,A-5\,B)}{4\,a\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((55*A*c^4)/3 - (299*B*c^4)/12) + (166*A*c^4)/3 - (20
6*B*c^4)/3 + tan(e/2 + (f*x)/2)^7*(27*A*c^4 - (167*B*c^4)/4) + tan(e/2 + (f
*x)/2)^8*(37*A*c^4 - (175*B*c^4)/4) + tan(e/2 + (f*x)/2)^5*(75*A*c^4 - (495
*B*c^4)/4) + tan(e/2 + (f*x)/2)^6*(155*A*c^4 - (687*B*c^4)/4) + tan(e/2 + (
f*x)/2)^4*(257*A*c^4 - (1153*B*c^4)/4) + tan(e/2 + (f*x)/2)^3*((199*A*c^4)/
3 - (1235*B*c^4)/12) + tan(e/2 + (f*x)/2)^2*((583*A*c^4)/3 - (2795*B*c^4)/1
2))/(f*(a + a*tan(e/2 + (f*x)/2) + 4*a*tan(e/2 + (f*x)/2)^2 + 4*a*tan(e/2 +
 (f*x)/2)^3 + 6*a*tan(e/2 + (f*x)/2)^4 + 6*a*tan(e/2 + (f*x)/2)^5 + 4*a*tan
(e/2 + (f*x)/2)^6 + 4*a*tan(e/2 + (f*x)/2)^7 + a*tan(e/2 + (f*x)/2)^8 + a*t
an(e/2 + (f*x)/2)^9)) - (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(4*A - 5*B)
)/(140*A*c^4 - 175*B*c^4))*(4*A - 5*B))/(4*a*f)
```

3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 157

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{5(3A-4B)c^3x}{2a}-\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af}-\frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\frac{2a^3(3A-4B)c^3\cos^5(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)^2}$$

```
[Out] -5/2*(3*A-4*B)*c^3*x/a-5/3*(3*A-4*B)*c^3*cos(f*x+e)^3/a/f-5/2*(3*A-4*B)*c^3
*cos(f*x+e)*sin(f*x+e)/a/f-a^3*(A-B)*c^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4-
2*a^3*(3*A-4*B)*c^3*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{a^3c^3(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{2a^3c^3(3A-4B)\cos^5(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{5c^3(3A-4B)\cos^3(e+fx)}{3af}-\frac{5c^3(3A-4B)\sin(e+fx)\cos(e+fx)}{2af}-\frac{5c^3x(3A-4B)}{2a}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-5*(3*A - 4*B)*c^3*x)/(2*a) - (5*(3*A - 4*B)*c^3*Cos[e + f*x]^3)/(3*a*f) -
 (5*(3*A - 4*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - (a^3*(A - B)*c^3*C
os[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) - (2*a^3*(3*A - 4*B)*c^3*Cos[e +
f*x]^5)/(f*(a^2 + a^2*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \left(a^2(3A-4B)c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \left(5(3A-4B)c^3\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx)}\,dx$$

$$= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \frac{\left(5(3A-4B)c^3\right)\int \cos^2(e+fx)\,dx}{a}$$

$$= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \frac{\left(5(3A-4B)c^3\right)\int 1\,dx}{2a}$$

$$= -\frac{5(3A-4B)c^3x}{2a} - \frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}$$

Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A-4B)(e+fx)+(48A\cdots\right.}{\cdots}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*(e
 + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 + 5*
e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{65\left(\frac{4\left(-4A+\frac{23B}{3}\right)\cos(2fx+2e)}{65}+\frac{(A-4B)\sin(3fx+3e)}{65}-\frac{B\cos(4fx+4e)}{195}+\frac{4\left(-3fxA+4fxB-\frac{24}{5}A+\frac{94}{15}B\right)\cos(fx+e)}{13}+\left(A-\frac{68B}{65}\right)\mathrm{s}\cdots\right.}{8af\cos(fx+e)}$
derivativedivides	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{5(3\cdots}{\cdots}\right.}{fa}$
default	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{5(3\cdots}{\cdots}\right.}{fa}$
risch	$-\frac{15c^3xA}{2a}+\frac{10c^3xB}{a}-\frac{2c^3\mathrm{e}^{i(fx+e)}A}{af}+\frac{31c^3\mathrm{e}^{i(fx+e)}B}{8af}-\frac{2c^3\mathrm{e}^{-i(fx+e)}A}{af}+\frac{31c^3\mathrm{e}^{-i(fx+e)}B}{8af}-\frac{16c^3A}{fa\left(\mathrm{e}^{i(fx+e)}+i\cdots\right.}$
norman	$\frac{-\frac{72Ac^3-94Bc^3}{3af}-\frac{5(3A-4B)c^3x}{2a}-\frac{\left(9Ac^3-18Bc^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(17Ac^3-20Bc^3\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(21Ac^3-34Bc^3\right)\tan\left(\frac{f}{\cdots}\right.}{3af}\cdots}{\cdots}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 65/8*(4/65*(-4*A+23/3*B)*cos(2*f*x+2*e)+1/65*(A-4*B)*sin(3*f*x+3*e)-1/195*B
*cos(4*f*x+4*e)+4/13*(-3*f*x*A+4*f*x*B-24/5*A+94/15*B)*cos(f*x+e)+(A-68/65*
B)*sin(f*x+e)-16/13*A+19/13*B)*c^3/a/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{2\,Bc^3\cos{(fx+e)}^4+(3\,A-10\,B)c^3\cos{(fx+e)}^3+15\,(3\,A-4\,B)c^3fx+24\,(A-2\,B)c^3\cos{(fx+e)}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*c^3*cos(f*x + e)^4 + (3*A - 10*B)*c^3*cos(f*x + e)^3 + 15*(3*A -
4*B)*c^3*f*x + 24*(A - 2*B)*c^3*cos(f*x + e)^2 + 48*(A - B)*c^3 + 3*(5*(3*A
 - 4*B)*c^3*f*x + (23*A - 28*B)*c^3)*cos(f*x + e) + (2*B*c^3*cos(f*x + e)^3
 + 15*(3*A - 4*B)*c^3*f*x - 3*(A - 4*B)*c^3*cos(f*x + e)^2 + 3*(7*A - 12*B)
*c^3*cos(f*x + e) - 48*(A - B)*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*s
in(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. 2(139) = 278.

Time = 3.83 (sec) , antiderivative size = 4255, normalized size of antiderivative = 27.10

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)
**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x
*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 135*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
```

```
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 102*A*c**3*tan(e/2 + f*x/2)**
6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c**3*tan(e/
2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a
*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/
2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 336*
A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) - 96*A*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) - 378*A*c**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 42*A*c**3*tan(e/2 + f*x/2)/(6*a*f*tan(e/2
 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*
a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x
/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 144*A*c**3/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 +
 f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*
tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)*
*3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c
**3*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*
x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*
tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
```

```
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)
**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*
tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6
0*B*c**3*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*
f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 120*B
*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) + 108*B*c**3*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 372*B*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 192*B*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan
(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
 f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 456*B*c**3*tan(e/2 + f*x/2)*
*2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 68*B*c**3*tan(e
/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 188*B*
c**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f), Ne(f, 0)), (x*
(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(151) = 302$.

Time = 0.33 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.13

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/3*(B*c^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*c^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 9*B*c^3*((sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) - 18*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 18*B*c^3*((sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x
 + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 18*A*c^3*(a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1))) + 6*B*c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*c^3/(a + a*sin(f*x + e)/(cos(f*x +
e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{\frac{15\,(3\,Ac^3-4\,Bc^3)(fx+e)}{a}+\frac{96\,(Ac^3-Bc^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-12\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+24\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,Bc^3}{}}{6\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*c^3 - 4*B*c^3)*(f*x + e)/a + 96*(A*c^3 - B*c^3)/(a*(tan(1/2*f
*x + 1/2*e) + 1)) + 2*(3*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 12*B*c^3*tan(1/2*f*
x + 1/2*e)^5 + 24*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*c^3*tan(1/2*f*x + 1/2
*e)^4 + 48*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*c^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*c^3*tan(1/2*f*x + 1/2*e) + 12*B*c^3*tan(1/2*f*x + 1/2*e) + 24*A*c^3 -
46*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.03

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(7\,A\,c^3-\frac{34\,B\,c^3}{3}\right)+24\,A\,c^3-\frac{94\,B\,c^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(9\,A\,c^3-18\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6(1}{f\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\right.} - \frac{5\,c^3\,\mathrm{atan}\left(\frac{5\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(3\,A-4\,B)}{15\,A\,c^3-20\,B\,c^3}\right)\,(3\,A-4\,B)}{a\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(7*A*c^3 - (34*B*c^3)/3) + 24*A*c^3 - (94*B*c^3)/3 +
tan(e/2 + (f*x)/2)^5*(9*A*c^3 - 18*B*c^3) + tan(e/2 + (f*x)/2)^6*(17*A*c^3
- 20*B*c^3) + tan(e/2 + (f*x)/2)^3*(16*A*c^3 - 32*B*c^3) + tan(e/2 + (f*x)/
2)^4*(56*A*c^3 - 62*B*c^3) + tan(e/2 + (f*x)/2)^2*(63*A*c^3 - 76*B*c^3))/(f
*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)
/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e/2 + (
f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7)) - (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2
)*(3*A - 4*B))/(15*A*c^3 - 20*B*c^3))*(3*A - 4*B))/(a*f)
```

3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{3(2A-3B)c^2x}{2a}-\frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}$$

```
[Out] -3/2*(2*A-3*B)*c^2*x/a-3/2*(2*A-3*B)*c^2*cos(f*x+e)/a/f-a^2*(A-B)*c^2*cos(f
*x+e)^5/f/(a+a*sin(f*x+e))^3-1/2*(2*A-3*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+
e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2758, 2761, 8}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{a^2c^2(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{3c^2(2A-3B)\cos(e+fx)}{2af} - \frac{c^2(2A-3B)\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{3c^2x(2A-3B)}{2a}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*(2*A - 3*B)*c^2*x)/(2*a) - (3*(2*A - 3*B)*c^2*Cos[e + f*x])/(2*a*f) - (
a^2*(A - B)*c^2*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3) - ((2*A - 3*B)*c
^2*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx\\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\left(a(2A-3B)c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx\\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}\\
&\quad -\frac{1}{2}\left(3(2A-3B)c^2\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx\\
&= -\frac{3(2A-3B)c^2\cos(e+fx)}{2af}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}\\
&\quad -\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}-\frac{\left(3(2A-3B)c^2\right)\int 1\,dx}{2a}\\
&= -\frac{3(2A-3B)c^2x}{2a}-\frac{3(2A-3B)c^2\cos(e+fx)}{2af}\\
&\quad -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.59

$$
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A-3B)(e+fx)+4(\cdots}{4af\left(\cdots\right.}
$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] -1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[
(e + f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*
(e + f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e + 9
*f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(a*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{4c^2\left(\frac{(-A+3B)\cos(2fx+2e)}{8}-\frac{B\sin(3fx+3e)}{32}+\frac{\left(-3fxA+\frac{9}{2}fxB-5A+7B\right)\cos(fx+e)}{4}+\left(A-\frac{33B}{32}\right)\sin(fx+e)-\frac{9A}{8}+\frac{11B}{8}\right)}{af\cos(fx+e)}$
derivativedivides	$\frac{2c^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{fa}$
default	$\frac{2c^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{fa}$
risch	$-\frac{3c^2xA}{a}+\frac{9c^2xB}{2a}-\frac{c^2e^{i(fx+e)}A}{2af}+\frac{3c^2e^{i(fx+e)}B}{2af}-\frac{c^2e^{-i(fx+e)}A}{2af}+\frac{3c^2e^{-i(fx+e)}B}{2af}-\frac{8c^2A}{fa\left(e^{i(fx+e)}+i\right)}+$
norman	$\frac{\frac{\left(6Ac^2-4Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(8Ac^2-9Bc^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(20Ac^2-15Bc^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(22Ac^2-20Bc^2\right)\left(\tan^5\left(\frac{fx}{2}\right.\right.}{af}}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4*c^2*(1/8*(-A+3*B)*cos(2*f*x+2*e)-1/32*B*sin(3*f*x+3*e)+1/4*(-3*f*x*A+9/2*
f*x*B-5*A+7*B)*cos(f*x+e)+(A-33/32*B)*sin(f*x+e)-9/8*A+11/8*B)/a/f/cos(f*x+
e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.52

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$=\frac{Bc^2\cos{(fx+e)}^3-3\,(2\,A-3\,B)c^2fx-2\,(A-3\,B)c^2\cos{(fx+e)}^2-8\,(A-B)c^2-(3\,(2\,A-3\,B)c^2fx}{2\,(af\,\mathrm{co}}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*x
 + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos(f
*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^2*c
os(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(99) = 198$.

Time = 2.03 (sec) , antiderivative size = 2365, normalized size of antiderivative = 20.04

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2
*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/
2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan(e/2 + f*x/2)**4/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*
A*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) - 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 6*A*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) - 16*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*tan(e/2 + f*x/2)**
3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 3
6*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f
*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*ta
n(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 20*A*c**2/(2*a*f*tan(e/2 +
 f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*
tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/
2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*f*x*tan(e/2 + f*x/2)**3/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*
B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
```

```
tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 +
f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*t
an(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*
tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 14*B*c**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 42*B*c**2*tan(e/2 + f*x/2)**2/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 10*B
*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) + 28*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*t
an(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**2/(
a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(112) = 224$.

Time = 0.31 (sec) , antiderivative size = 608, normalized size of antiderivative = 5.15

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \frac{Bc^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-2Ac^2\left(\frac{}{a+\frac{a\sin}{\cos(}}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 4*B*
c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
```

```
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) - 4*A*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*c^2*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{\frac{3\,(2\,Ac^2-3\,Bc^2)(fx+e)}{a}+\frac{16\,(Ac^2-Bc^2)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}-\frac{2\left(Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^2\tan\left(\frac{1}{2}\,fx\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a}}{2\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f*
x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x +
1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e)^2 - B*c^2*tan(1/2*f*x + 1/2*e) - 2*
A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.04

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,c^2-5\,B\,c^2\right)+10\,A\,c^2-14\,B\,c^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(2\,A\,c^2-7\,B\,c^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(8\,A}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right.} - \frac{3\,c^2\,\mathrm{atan}\left(\frac{3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A-3\,B)}{6\,A\,c^2-9\,B\,c^2}\right)\,(2\,A-3\,B)}{a\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(2*A*c^2 - 5*B*c^2) + 10*A*c^2 - 14*B*c^2 + tan(e/2 +
 (f*x)/2)^3*(2*A*c^2 - 7*B*c^2) + tan(e/2 + (f*x)/2)^4*(8*A*c^2 - 9*B*c^2)
+ tan(e/2 + (f*x)/2)^2*(18*A*c^2 - 21*B*c^2))/(f*(a + a*tan(e/2 + (f*x)/2)
+ 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2
)^4 + a*tan(e/2 + (f*x)/2)^5)) - (3*c^2*atan((3*c^2*tan(e/2 + (f*x)/2)*(2*A
 - 3*B))/(6*A*c^2 - 9*B*c^2))*(2*A - 3*B))/(a*f)
```

3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(A-2B)cx}{a}+\frac{Bc\cos(e+fx)}{af}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] -(A-2*B)*c*x/a+B*c*cos(f*x+e)/a/f-2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3046, 2936, 2718}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{2c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx(A-2B)}{a}+\frac{Bc\cos(e+fx)}{af}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 2*B)*c*x)/a) + (B*c*Cos[e + f*x])/(a*f) - (2*(A - B)*c*Cos[e + f*x]
)/(f*(a + a*Sin[e + f*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx \\
&= -\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))} - \frac{c\int (aA-2aB+aB\sin(e+fx))\,dx}{a^2} \\
&= -\frac{(A-2B)cx}{a} - \frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))} - \frac{(Bc)\int \sin(e+fx)\,dx}{a} \\
&= -\frac{(A-2B)cx}{a} + \frac{Bc\cos(e+fx)}{af} - \frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. $2(57) = 114$.

Time = 5.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\left(-((A-2B)x) + \frac{B\cos(e)\cos(fx)}{f} - \frac{B\sin(e)\sin(fx)}{f} + \frac{4(A-B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(c - c\sin(e+fx}{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]
```

[Out] ((-((A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A - B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{2c\left(-\frac{-2B+2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa}$
default	$\frac{2c\left(-\frac{-2B+2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa}$
parallelrisch	$\frac{2c\left(\frac{B\cos(2fx+2e)}{4}+\left(-\frac{1}{2}fxA+fxB-A+\frac{3}{2}B\right)\cos(fx+e)+(A-B)\sin(fx+e)-A+\frac{5B}{4}\right)}{af\cos(fx+e)}$
risch	$-\frac{cxA}{a}+\frac{2cxB}{a}+\frac{Bc\,e^{i(fx+e)}}{2af}+\frac{Bc\,e^{-i(fx+e)}}{2af}-\frac{4cA}{fa\left(e^{i(fx+e)}+i\right)}+\frac{4cB}{fa\left(e^{i(fx+e)}+i\right)}$
norman	$\frac{\frac{2Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{4Ac-6Bc}{af}-\frac{(A-2B)cx}{a}+\frac{2Bc\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2(4Ac-5Bc)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(4Ac-4Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 2/f*c/a*(-(-2*B+2*A)/(tan(1/2*f*x+1/2*e)+1)+B/(1+tan(1/2*f*x+1/2*e)^2)-(A-2*B)*arctan(tan(1/2*f*x+1/2*e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.05

$$\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(A-2B)cfx-Bc\cos(fx+e)^2+2(A-B)c+((A-2B)cfx+(2A-3B)c)\cos(fx+e)+((A-}{af\cos(fx+e)+af\sin(fx+e)+af}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] -((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x + (2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A - B)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(49) = 98$.

Time = 1.06 (sec) , antiderivative size = 828, normalized size of antiderivative = 14.53

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(
e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2)**3
+ a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e/2 + f
*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*
f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2
+ a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f
*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*
x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
 + a*f) + 4*B*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2
+ f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)/(a*f*tan
(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
6*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(57) = 114$.

Time = 0.40 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.49

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{2\left(Bc\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Ac\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)\right)}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{\frac{(Ac-2\,Bc)(fx+e)}{a}+\frac{2\left(2\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac-3\,Bc\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(1
/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f*x
 + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(4\,A\,c-4\,B\,c)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,c-6\,B\,c}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}-\frac{A\,c\,f\,x-2\,B\,c\,f\,x}{a\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] - (4*A*c - 6*B*c + tan(e/2 + (f*x)/2)^2*(4*A*c - 4*B*c) - 2*B*c*tan(e/2 + (
f*x)/2))/(f*(a + a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2
+ (f*x)/2)^3)) - (A*c*f*x - 2*B*c*f*x)/(a*f)
```

3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 35

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

```
[Out] B*sec(f*x+e)/a/c/f+A*tan(f*x+e)/a/c/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2748, 3852, 8}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\,dx}{ac}\\
&= \frac{B\sec(e+fx)}{acf}+\frac{A\int \sec^2(e+fx)\,dx}{ac}\\
&= \frac{B\sec(e+fx)}{acf}-\frac{A\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{acf}\\
&= \frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),
x]
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	size
parallelrisch	$\frac{-2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2B}{fac\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	42
risch	$\frac{2iA+2B\,\mathrm{e}^{i(fx+e)}}{\left(\mathrm{e}^{i(fx+e)}-i\right)\left(\mathrm{e}^{i(fx+e)}+i\right)acf}$	56
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{acf}$	57
default	$\frac{-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{acf}$	57
norman	$\frac{-\frac{2B}{acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}-\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	123

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-2*A*tan(1/2*f*x+1/2*e)-2*B)/f/a/c/(tan(1/2*f*x+1/2*e)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{A\sin{(fx+e)}+B}{acf\cos{(fx+e)}}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] (A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(26) = 52.

Time = 0.72 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \begin{cases} -\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} - \frac{2B}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f\neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B/
(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e
) + a)*(-c*sin(e) + c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \frac{\frac{A\tan(fx+e)}{ac}+\frac{B}{ac\cos(fx+e)}}{f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] (A*tan(f*x + e)/(a*c) + B/(a*c*cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = -\frac{2\left(A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+B\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)acf}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)
```

Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=-\frac{2\left(B+A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,c\,f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
[Out] -(2*(B + A*tan(e/2 + (f*x)/2)))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} + \frac{(2A-B)\tan(e+fx)}{3ac^2f}$$

```
[Out] 1/3*(A+B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+1/3*(2*A-B)*tan(f*x+e)/a/c^2/
f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 3852, 8}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{(2A-B)\tan(e+fx)}{3ac^2f} + \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + ((2*A - B)*Tan[e
+ f*x])/(3*a*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{ac}\\
&= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{(2A-B)\int \sec^2(e+fx)\,dx}{3ac^2}\\
&= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}-\frac{(2A-B)\text{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{3ac^2f}\\
&= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{(2A-B)\tan(e+fx)}{3ac^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{\cos(e+fx)(6B-2(A+B)\cos(e+fx)+(4A-2B)\cos(2(e+fx))+8A\sin(e+fx)-4B\sin(e+fx)}{12ac^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2
),x]
[Out] (Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)]
+ 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e + f
*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{2i\left(4iA\,\mathrm{e}^{i(fx+e)}-2iB\,\mathrm{e}^{i(fx+e)}+3B\,\mathrm{e}^{2i(fx+e)}+2A-B\right)}{3\left(\mathrm{e}^{i(fx+e)}-i\right)^3\left(\mathrm{e}^{i(fx+e)}+i\right)a\,c^2f}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a\,c^2f}$
default	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a\,c^2f}$
parallelrisch	$\frac{-6A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6A-6B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-2A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2A-2B}{3fa\,c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
norman	$\frac{\frac{2A-4B}{6acf}-\frac{4(2A-B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{(2A-4B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{(8A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}+\frac{(14A-16B)\left(\tan^2\right.}{6acf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
[Out] -2/3*I*(4*I*A*exp(I*(f*x+e))-2*I*B*exp(I*(f*x+e))+3*B*exp(2*I*(f*x+e))+2*A-
B)/(exp(I*(f*x+e))-I)^3/(exp(I*(f*x+e))+I)/a/c^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$=-\frac{(2\,A-B)\cos\left(fx+e\right)^2+(2\,A-B)\sin\left(fx+e\right)-A+2\,B}{3\,(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right))}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*f
*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(51) = 102.

Time = 2.23 (sec) , antiderivative size = 578, normalized size of antiderivative = 9.17

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$=\begin{cases}-\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}+\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)^2}\end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c
**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*A
*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 +
 f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 + f*x/
2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c
**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/2)**4
- 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*ta
n(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*B*tan(e/2
 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3
+ 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*tan(e/2 + f*x
/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**2),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(60) = 120.

Time = 0.23 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.22

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = -\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin(f*x
+ e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = -\frac{\frac{3(A-B)}{ac^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{9A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-12A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7A+B}{ac^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}}{6f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)/(
a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{2\left(\frac{3B}{2}+A\,\cos\left(e+f\,x\right)+B\,\cos\left(e+f\,x\right)+2\,A\,\sin\left(e+f\,x\right)-B\,\sin\left(e+f\,x\right)+A\,\cos\left(2\,e+2\,f\,x\right)-\frac{B\,c}{}\right.}{3\,a\,c^2\,f\,\left(2\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

```
[Out] (2*((3*B)/2 + A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) - B*sin(e
+ f*x) + A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))
/2 - (B*sin(2*e + 2*f*x))/2))/(3*a*c^2*f*(2*cos(e + f*x) - sin(2*e + 2*f*x)
))
```

3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))} + \frac{2(3A-2B)\tan(e+fx)}{15ac^3f}$$

```
[Out] 1/5*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/15*(3*A-2*B)*sec(f*x+e)/a/f
/(c^3-c^3*sin(f*x+e))+2/15*(3*A-2*B)*tan(f*x+e)/a/c^3/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2751, 3852, 8}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{2(3A-2B)\tan(e+fx)}{15ac^3f} + \frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))} + \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(5*a*c*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*Sec[
e + f*x])/(15*a*f*(c^3 - c^3*Sin[e + f*x])) + (2*(3*A - 2*B)*Tan[e + f*x])/
(15*a*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))} + \frac{(2(3A-2B))\int \sec^2(e+fx)\,dx}{15ac^3}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))} - \frac{(2(3A-2B))\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{15ac^3f}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))} + \frac{2(3A-2B)\tan(e+fx)}{15ac^3f}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)(80B+5(-9A+B)\cos(e+fx)+32(3A-2B)\cos(2(e+fx))+9A\cos(3(e+fx))-B\,\text{c}}{240}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3
),x]
```

```
[Out] -1/240*(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Cos
[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e + f
*x] - 80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] - 24
*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(a*c^3*f*(-1 + Sin[e + f*x])^
3*(1 + Sin[e + f*x]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4\left(-3iA+12A\,\mathrm{e}^{i(fx+e)}+10B\,\mathrm{e}^{3i(fx+e)}+2iB-8B\,\mathrm{e}^{i(fx+e)}-10iB\,\mathrm{e}^{2i(fx+e)}+15iA\,\mathrm{e}^{2i(fx+e)}\right)}{15\left(\mathrm{e}^{i(fx+e)}-i\right)^{5}\left(\mathrm{e}^{i(fx+e)}+i\right)f\,c^{3}a}$
parallelrisch	$\frac{-30A\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(60A-30B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A+40B)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-40B\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(18A+8B)}{15f\,c^{3}a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}$
derivativedivides	$\frac{-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a\,c^{3}f}$
default	$\frac{-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a\,c^{3}f}$
norman	$\frac{-\frac{12A+2B}{15afc}+\frac{2(6A-7B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3afc}-\frac{2A\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{2(-8B+7A)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5afc}+\frac{2(2A-B)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{2(9A-4}{}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*(-3*I*A+12*A*exp(I*(f*x+e))+10*B*exp(3*I*(f*x+e))+2*I*B-8*B*exp(I*(f*
x+e))-10*I*B*exp(2*I*(f*x+e))+15*I*A*exp(2*I*(f*x+e)))/(exp(I*(f*x+e))-I)^5
/(exp(I*(f*x+e))+I)/f/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{4\,(3\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(2\,(3\,A-2\,B)\cos\left(fx+e\right)^{2}-9\,A+6\,B\right)\sin\left(fx+e\right)-6\,A+9\,B}{15\,\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A +
 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(85) = 170$.

Time = 4.55 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.12

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*
a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*
f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 60*A*
tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2
+ f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2
)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f*x/2)*
*3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*
a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*
f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 18*A*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(
e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f
*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) -
 15*a*c**3*f) - 12*A/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2
 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/
2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75
*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3
*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)**3/(15*a*c**3*f*
tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/
2) - 15*a*c**3*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a*c**3*f*tan(e/2 + f*x/2)*
*6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75
*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f)
 + 8*B*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(
e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f
*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 2*B/(15*a*c**3*f*t
an(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
+ f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2
) - 15*a*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) +
 c)**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(98) = 196$.

Time = 0.23 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.15

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^3}\,dx = -\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 1\right)}{ac^3 - \frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} + \frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \cdots\right.}{ac^3 - \frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{5ac^3}{(\cos\cdots}\right.}{15f}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm ="maxima")

[Out] -2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos (f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5* a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10* sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^ 4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^ 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.64

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^3}\,dx = -\frac{\frac{15(A-B)}{ac^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} + \frac{105A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 + 15B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 270A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 30B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 360A\tan\left(\frac{1}{2}fx+\frac{1}{2}\cdots\right.}{ac^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}}{60f}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm ="giac")

[Out] -1/60*(15*(A - B)/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (105*A*tan(1/2*f*x + 1/2*e)^4 + 15*B*tan(1/2*f*x + 1/2*e)^4 - 270*A*tan(1/2*f*x + 1/2*e)^3 + 30 *B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 - 40*B*tan(1/2*f*x + 1/2*e)^2 - 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A - 7*B)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{2\left(\frac{5\,B\,\sin(e+f\,x)}{2}-\frac{15\,A\,\cos(e+f\,x)}{4}-\frac{5\,B\,\cos(e+f\,x)}{8}-\frac{15\,A\,\sin(e+f\,x)}{4}-\frac{5\,B}{2}-3\,A\,\cos\left(2\,e+2\,f\,x\right)+\frac{3\,A\,\cos(3\,e+3\,f\,x)}{4}\right.}{15\,a\,c^3\,f\,\left(\frac{\cos(3\,e+3\,f\,x)}{4}-\right.}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

```
[Out] (2*((5*B*sin(e + f*x))/2 - (15*A*cos(e + f*x))/4 - (5*B*cos(e + f*x))/8 - (
15*A*sin(e + f*x))/4 - (5*B)/2 - 3*A*cos(2*e + 2*f*x) + (3*A*cos(3*e + 3*f*
x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*x
) + (3*A*sin(3*e + 3*f*x))/4 + (B*sin(2*e + 2*f*x))/2 - (B*sin(3*e + 3*f*x)
)/2))/(15*a*c^3*f*(cos(3*e + 3*f*x)/4 - (5*cos(e + f*x))/4 + sin(2*e + 2*f*
x)))
```

3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)} + \frac{2(4A-3B)\tan(e+fx)}{35ac^4f}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+1/35*(4*A-3*B)*sec(f*x+e)/a/f
/(c^2-c^2*sin(f*x+e))^2+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e))+
2/35*(4*A-3*B)*tan(f*x+e)/a/c^4/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2751, 3852, 8}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{2(4A-3B)\tan(e+fx)}{35ac^4f} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(7*a*c*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*Sec[
e + f*x])/(35*a*f*(c^2 - c^2*Sin[e + f*x])^2) + ((4*A - 3*B)*Sec[e + f*x])/
(35*a*f*(c^4 - c^4*Sin[e + f*x])) + (2*(4*A - 3*B)*Tan[e + f*x])/(35*a*c^4*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{ac}$$
$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2}$$
$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(3(4A-3B))\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{35ac^3}$$
$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)} + \frac{(2(4A-3B))\int \sec^2(e+fx)\,dx}{35ac^4}$$
$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)} - \frac{(2(4A-3B))\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{35ac^4f}$$
$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)} + \frac{2(4A-3B)\tan(e+fx)}{35ac^4f}$$

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.69

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(560B+(-406A+182B)\cos(e}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4
),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x)]
 + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] +
 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*A*Si
n[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288*B*Si
n[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(2240*a*c^
4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

method	result
risch	$\frac{4i\left(56iA\,\mathrm{e}^{3i(fx+e)}-42iB\,\mathrm{e}^{3i(fx+e)}+35B\,\mathrm{e}^{4i(fx+e)}-24iA\,\mathrm{e}^{i(fx+e)}+56A\,\mathrm{e}^{2i(fx+e)}+18iB\,\mathrm{e}^{i(fx+e)}-42B\,\mathrm{e}^{2i(fx+e)}-4A+}{35\left(\mathrm{e}^{i(fx+e)}-i\right)^{7}\left(\mathrm{e}^{i(fx+e)}+i\right)a\,c^{4}f}$
parallelrisch	$\frac{-70A\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A-70B)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-350A+140B)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A-210B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{35f\,c^{4}a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\mathrm{ta}\right.}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-}{a\,c^{4}f}$
default	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-}{a\,c^{4}f}$
norman	$\frac{\frac{(6A-2B)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{26A-2B}{35afc}-\frac{12(4A-3B)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5afc}-\frac{2A\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{2(4A-18B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5afc}-\frac{4(3A-}{}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}\right.\right.}$

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNVE RBOSE)

[Out] 4/35*I*(56*I*A*exp(3*I*(f*x+e))-42*I*B*exp(3*I*(f*x+e))+35*B*exp(4*I*(f*x+e))-24*I*A*exp(I*(f*x+e))+56*A*exp(2*I*(f*x+e))+18*I*B*exp(I*(f*x+e))-42*B*e xp(2*I*(f*x+e))-4*A+3*B)/(exp(I*(f*x+e))-I)^7/(exp(I*(f*x+e))+I)/a/c^4/f

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$$
$$=\frac{2\,(4\,A-3\,B)\cos{(fx+e)}^{4}-9\,(4\,A-3\,B)\cos{(fx+e)}^{2}+\left(6\,(4\,A-3\,B)\cos{(fx+e)}^{2}-20\,A+15\,B\right)\mathrm{si}}{35\,\left(3\,ac^{4}f\cos{(fx+e)}^{3}-4\,ac^{4}f\cos{(fx+e)}-\left(ac^{4}f\cos{(fx+e)}^{3}-4\,ac^{4}f\cos{(fx+e)}\right)\mathrm{s}\right.}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm ="fricas")

[Out] 1/35*(2*(4*A - 3*B)*cos(f*x + e)^4 - 9*(4*A - 3*B)*cos(f*x + e)^2 + (6*(4*A - 3*B)*cos(f*x + e)^2 - 20*A + 15*B)*sin(f*x + e) + 15*A - 20*B)/(3*a*c^4* f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f*cos(f*x + e)^3 - 4*a*c ^4*f*cos(f*x + e))*sin(f*x + e))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2468 vs. $2(122) = 244$.

Time = 9.50 (sec) , antiderivative size = 2468, normalized size of antiderivative = 17.38

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*A*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210
*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c*
*4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*
tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*
tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2
 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*
x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)*
*2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*A*tan(e/2 + f*x/2)*
*5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 49
0*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c
**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f
*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*tan(e/2 + f*x/2)**4/(35*a*c**4*f*t
an(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/
2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f
*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2)
 - 35*a*c**4*f) + 14*A*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/2)**8
 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c
**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) -
154*A*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*t
an(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f
*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 86*A*tan(e/2 + f*
x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 +
490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a
*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4
*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 26*A/(35*a*c**4*f*tan(e/2 + f*x/2)**8
- 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490
*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c*
*4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 7
0*B*tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan
(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2
+ f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x
/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 140*B*tan(e/2 + f*x
/2)**5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7
```

```
+ 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490
*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c*
*4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 210*B*tan(e/2 + f*x/2)**4/(35*a*c**4
*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*ta
n(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2
 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*
x/2) - 35*a*c**4*f) + 112*B*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/
2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6
 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*
f) - 42*B*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4
*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*ta
n(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2
 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 12*B*tan(e/2
+ f*x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**
7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 4
90*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*
c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 2*B/(35*a*c**4*f*tan(e/2 + f*x/2)*
*8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 -
490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f),
 Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(137) = 274$.

Time = 0.24 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.36

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^4}\,dx =$$
$$-\frac{2\left(\frac{A\left(\frac{43\sin(fx+e)}{\cos(fx+e)+1}-\frac{77\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{175\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{35\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-13\right)}{ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14ac^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{14ac^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{14ac^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{6ac^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{ac^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}} - \frac{\cdots}{ac^{\cdots}}\right)}{35f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -2/35*(A*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/(
a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x +
```

```
e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 56*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 70*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(a*c^4 - 6*a*c^4*sin(f*
x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a*c^4*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = -\frac{\frac{35\,(A-B)}{ac^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{525\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+35\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1960\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+4025\,A\tan\left(\frac{1}{2}\,fx\right.}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/280*(35*(A - B)/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*A*tan(1/2*f*x
+ 1/2*e)^6 + 35*B*tan(1/2*f*x + 1/2*e)^6 - 1960*A*tan(1/2*f*x + 1/2*e)^5 +
280*B*tan(1/2*f*x + 1/2*e)^5 + 4025*A*tan(1/2*f*x + 1/2*e)^4 - 665*B*tan(1/
2*f*x + 1/2*e)^4 - 4480*A*tan(1/2*f*x + 1/2*e)^3 + 1120*B*tan(1/2*f*x + 1/2
*e)^3 + 3143*A*tan(1/2*f*x + 1/2*e)^2 - 791*B*tan(1/2*f*x + 1/2*e)^2 - 1176
*A*tan(1/2*f*x + 1/2*e) + 392*B*tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/(a*c^4
*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.68

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{2\left(\frac{35\,B}{4}+\frac{91\,A\cos(e+f\,x)}{4}-\frac{7\,B\cos(e+f\,x)}{4}+14\,A\sin\left(e+f\,x\right)-\frac{21\,B\sin(e+f\,x)}{2}+14\,A\cos\left(2\,e+2\,f\,x\right)-\frac{39\,A\cos(\right.}{}}{}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

```
[Out] (2*((35*B)/4 + (91*A*cos(e + f*x))/4 - (7*B*cos(e + f*x))/4 + 14*A*sin(e +
f*x) - (21*B*sin(e + f*x))/2 + 14*A*cos(2*e + 2*f*x) - (39*A*cos(3*e + 3*f*
x))/4 - A*cos(4*e + 4*f*x) - (21*B*cos(2*e + 2*f*x))/2 + (3*B*cos(3*e + 3*f
*x))/4 + (3*B*cos(4*e + 4*f*x))/4 - (91*A*sin(2*e + 2*f*x))/4 - 6*A*sin(3*e
 + 3*f*x) + (13*A*sin(4*e + 4*f*x))/8 + (7*B*sin(2*e + 2*f*x))/4 + (9*B*sin
(3*e + 3*f*x))/2 - (B*sin(4*e + 4*f*x))/8))/(35*a*c^4*f*((7*cos(e + f*x))/2
 - (3*cos(3*e + 3*f*x))/2 - (7*sin(2*e + 2*f*x))/2 + sin(4*e + 4*f*x)/4))
```

3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2} dx$

Optimal result

Integrand size = 36, antiderivative size = 240

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2} dx \\
&= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} \\
&\quad + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f} \\
&\quad - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} \\
&\quad + \frac{6a^4(4A-7B)c^5\cos^7(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

```
[Out] 105/8*(4*A-7*B)*c^5*x/a^2+35/4*(4*A-7*B)*c^5*cos(f*x+e)^3/a^2/f+105/8*(4*A-
7*B)*c^5*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a
*sin(f*x+e))^7+2/3*a^3*(4*A-7*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^5+6*a^
4*(4*A-7*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^3+21/4*(4*A-7*B)*c^5*co
s(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3046, 2938, 2759, 2758, 2761, 2715, 8}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx\\&=-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}+\frac{2a^3c^5(4A-7B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^5}\\&\quad+\frac{35c^5(4A-7B)\cos^3(e+fx)}{4a^2f}+\frac{21c^5(4A-7B)\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\\&\quad+\frac{105c^5(4A-7B)\sin(e+fx)\cos(e+fx)}{8a^2f}\\&\quad+\frac{105c^5x(4A-7B)}{8a^2}+\frac{6a^4c^5(4A-7B)\cos^7(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^3}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (105*(4*A - 7*B)*c^5*x)/(8*a^2) + (35*(4*A - 7*B)*c^5*Cos[e + f*x]^3)/(4*a^
2*f) + (105*(4*A - 7*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f) - (a^5*(A
- B)*c^5*Cos[e + f*x]^11)/(3*f*(a + a*Sin[e + f*x])^7) + (2*a^3*(4*A - 7*B)
*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^5) + (6*a^4*(4*A - 7*B)*c^5*
Cos[e + f*x]^7)/(f*(a^2 + a^2*Sin[e + f*x])^3) + (21*(4*A - 7*B)*c^5*Cos[e
+ f*x]^5)/(4*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
```

```
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx\\ &= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7}-\frac{1}{3}\left(a^4(4A-7B)c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^6}\,dx\end{aligned}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \left(3a^2(4A-7B)c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7B)c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} + \left(21(4A-7B)c^5\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7B)c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)} + \frac{\left(105(4A-7B)c^5\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx)}\,dx}{4a}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7B)c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)} + \frac{\left(105(4A-7B)c^5\right)\int \cos^2(e+fx)\,dx}{4a^2}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7B)c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)} + \frac{\left(105(4A-7B)c^5\right)\int 1\,dx}{8a^2}$$

$$= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7B)c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 11.80 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.48

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(2048(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-1024(A-B)\left(\mathrm{co}\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B)
*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 10
24*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(9
5*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*(A -
7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7*A - 2
4*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)]))/(96*a^2*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{2c^5\left(\frac{\left(\frac{7A}{2}-\frac{95B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(23A-49B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{7A}{2}-\frac{103B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(71A-161B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}\right.}{}$
default	$\frac{2c^5\left(\frac{\left(\frac{7A}{2}-\frac{95B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(23A-49B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{7A}{2}-\frac{103B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(71A-161B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}\right.}{}$
parallelrisch	$-\frac{75c^5\left(\frac{\left(-84fxA+147fxB-\frac{2543}{15}A+\frac{4514}{15}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{5}+\frac{\left(-49fxB+28fxA+\frac{712}{45}B-\frac{481}{45}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{5}+\frac{\left(-84fxA+147fx\right.}{}\right.}{}$
risch	$\frac{105c^5xA}{2a^2}-\frac{735c^5xB}{8a^2}+\frac{7ic^5e^{2i(fx+e)}A}{8a^2f}-\frac{3ic^5e^{2i(fx+e)}B}{a^2f}+\frac{95c^5e^{i(fx+e)}A}{8a^2f}-\frac{217c^5e^{i(fx+e)}B}{8a^2f}+\frac{95c^5e^{-i(fx+e}}{8a^2f}$
norman	Expression too large to display

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^5/a^2*((((7/2*A-95/8*B)*tan(1/2*f*x+1/2*e)^7+(23*A-49*B)*tan(1/2*f*x+1
/2*e)^6+(7/2*A-103/8*B)*tan(1/2*f*x+1/2*e)^5+(71*A-161*B)*tan(1/2*f*x+1/2*e
)^4+(-7/2*A+103/8*B)*tan(1/2*f*x+1/2*e)^3+(215/3*A-497/3*B)*tan(1/2*f*x+1/2
*e)^2+(-7/2*A+95/8*B)*tan(1/2*f*x+1/2*e)+71/3*A-161/3*B)/(1+tan(1/2*f*x+1/2
*e)^2)^4+105/8*(4*A-7*B)*arctan(tan(1/2*f*x+1/2*e))-1/2*(-64*A+64*B)/(tan(1
/2*f*x+1/2*e)+1)^2-(-48*A+80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(64*A-64*B)/(tan
(1/2*f*x+1/2*e)+1)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.54

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = -\frac{6\,Bc^5\cos\left(fx+e\right)^6+4\,(2\,A-11\,B)c^5\cos\left(fx+e\right)^5+(76\,A-241\,B)c^5\cos\left(fx+e\right)^4-2\,(212\,A-431}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A -
 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(4*
A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A - 3
485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 2141*B)
*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(f*x +
 e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x + 3*(16
4*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)*c^5*f*
x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. 2(224) = 448.

Time = 21.52 (sec) , antiderivative size = 10608, normalized size of antiderivative = 44.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((1260*A*c**5*f*x*tan(e/2 + f*x/2)**11/(24*a**2*f*tan(e/2 + f*x/2)
**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 31
```

2*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*
tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 +
f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**
2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*tan(e/2 + f*x
/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*
f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2
+ f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)
3 + 168*a2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2
*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f*x/2)**11 +
72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2
*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/
2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2
)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72
*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c**5*f*x*tan(e/2 + f*x/2)**
8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a*
*2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(
e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x
/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 +
168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) +
22680*A*c**5*f*x*tan(e/2 + f*x/2)**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a
2*f*tan(e/2 + f*x/2)10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*ta
n(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f
*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4
+ 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2
*f*tan(e/2 + f*x/2) + 24*a**2*f) + 27720*A*c**5*f*x*tan(e/2 + f*x/2)**6/(24
*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*
tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**
5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a
2*f*tan(e/2 + f*x/2)2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 27720
*A*c**5*f*x*tan(e/2 + f*x/2)**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f
*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
+ f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)
6 + 528*a2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312
*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*ta
n(e/2 + f*x/2) + 24*a**2*f) + 22680*A*c**5*f*x*tan(e/2 + f*x/2)**4/(24*a**2
*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e
/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/
2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 4
32*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f
*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c*
*5*f*x*tan(e/2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(
e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*
x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +

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 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2
*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
 + f*x/2) + 24*a**2*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**2/(24*a**2*f*tan
(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f
*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
+ 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**
2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e
/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*
tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/
2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 +
432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2
 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2)
+ 24*a**2*f) + 1260*A*c**5*f*x/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*
tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
+ f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)*
*6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*
a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan
(e/2 + f*x/2) + 24*a**2*f) + 2472*A*c**5*tan(e/2 + f*x/2)**10/(24*a**2*f*ta
n(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 +
f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a*
*2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(
e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 7752*A*c**5*tan
(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/
2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 +
432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2
 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2)
+ 24*a**2*f) + 16016*A*c**5*tan(e/2 + f*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)
**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 31
2*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*
tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 +
 f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**
2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 31968*A*c**5*tan(e/2 + f*x/2)
**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*
a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*ta
n(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f
*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3
+ 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f)
+ 36752*A*c**5*tan(e/2 + f*x/2)**6/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**
2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(
e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x
/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 +
312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f
```

*tan(e/2 + f*x/2) + 24*a**2*f) + 50192*A*c**5*tan(e/2 + f*x/2)**5/(24*a**2*
f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/
2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2
)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 43
2*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*
tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 39168*A*c**
5*tan(e/2 + f*x/2)**4/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 +
 f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)*
*8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*
a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*ta
n(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*
x/2) + 24*a**2*f) + 35360*A*c**5*tan(e/2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f
*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9
 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a*
*2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(
e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x
/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 19912*A*c**5*tan(e/2 + f
*x/2)**2/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2
*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/
2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2
)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**
2*f) + 9384*A*c**5*tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a*
*2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan
(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*
x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 +
 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*
f*tan(e/2 + f*x/2) + 24*a**2*f) + 3952*A*c**5/(24*a**2*f*tan(e/2 + f*x/2)**
11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*
a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*ta
n(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f
*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2
 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 2205*B*c**5*f*x*tan(e/2 + f*x/2
)**11/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 16
8*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*
tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 +
 f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**
3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f
) - 6615*B*c**5*f*x*tan(e/2 + f*x/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 +
72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*
f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2
 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)
4 + 312*a2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*
a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 15435*B*c**5*f*x*tan(e/2 + f*x/2)**9
/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**

2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e
/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/
2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 1
68*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 2
8665*B*c**5*f*x*tan(e/2 + f*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a*
*2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan
(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*
x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 +
312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*
f*tan(e/2 + f*x/2) + 24*a**2*f) - 39690*B*c**5*f*x*tan(e/2 + f*x/2)**7/(24*
a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*t
an(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5
+ 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a*
*2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 48510*
B*c**5*f*x*tan(e/2 + f*x/2)**6/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*
tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
+ f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)*
*6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*
a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan
(e/2 + f*x/2) + 24*a**2*f) - 48510*B*c**5*f*x*tan(e/2 + f*x/2)**5/(24*a**2*
f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/
2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2
)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 43
2*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*
tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 39690*B*c**
5*f*x*tan(e/2 + f*x/2)**4/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e
/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x
/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +
528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*
f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
+ f*x/2) + 24*a**2*f) - 28665*B*c**5*f*x*tan(e/2 + f*x/2)**3/(24*a**2*f*tan
(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f
*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
+ 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**
2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e
/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 15435*B*c**5*f*x
*tan(e/2 + f*x/2)**2/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 +
f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**
8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a
2*f*tan(e/2 + f*x/2)5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan
(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x
/2) + 24*a**2*f) - 6615*B*c**5*f*x*tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*
x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9
+ 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**

2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e
/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/
2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 2205*B*c**5*f*x/(24*a**2*
f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/
2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2
)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 43
2*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*
tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 4410*B*c**5
*tan(e/2 + f*x/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 +
 f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)*
*8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*
a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*ta
n(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*
x/2) + 24*a**2*f) - 13278*B*c**5*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f
*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9
 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a*
*2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(
e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x
/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 29096*B*c**5*tan(e/2 + f
*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2
*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/
2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2
)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**
2*f) - 54072*B*c**5*tan(e/2 + f*x/2)**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 7
2*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f
*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2
+ f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)*
*4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a
2*f*tan(e/2 + f*x/2) + 24*a2*f) - 67220*B*c**5*tan(e/2 + f*x/2)**6/(24*
a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*t
an(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5
 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a*
*2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 85196*
B*c**5*tan(e/2 + f*x/2)**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(
e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*
x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +
 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2
*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
 + f*x/2) + 24*a**2*f) - 70632*B*c**5*tan(e/2 + f*x/2)**4/(24*a**2*f*tan(e/
2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/
2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 5
28*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f
*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2

```
+ f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 60968*B*c**5*tan(e/
2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)*
*10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432
*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*t
an(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 +
f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 2
4*a**2*f) - 35218*B*c**5*tan(e/2 + f*x/2)**2/(24*a**2*f*tan(e/2 + f*x/2)**1
1 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a
**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan
(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*
x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 +
 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 16374*B*c**5*tan(e/2 + f*x/2)/(2
4*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f
*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2
+ f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)*
*5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*
a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 6928
*B*c**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*
f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2
 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)
**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2
*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**5/(a*sin(e) + a)**2, Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2982 vs. $2(228) = 456$.

Time = 0.36 (sec) , antiderivative size = 2982, normalized size of antiderivative = 12.42

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/12*(B*c^5*((603*sin(f*x + e)/(cos(f*x + e) + 1) + 1297*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2228*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2628*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3014*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2618*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1980*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 1100*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 495*sin(f*x + e)^9/(c
os(f*x + e) + 1)^9 + 165*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 256)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^2*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 18*a^2*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 22*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a
```

^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 18*a^2*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 13*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7*a^2*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 3*a^2*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + a^2*
sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 165*arctan(sin(f*x + e)/(cos(f*x +
 e) + 1))/a^2) - 20*A*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^2) + 40*B*c^5*((75*sin(f*x + e)/(cos(f*x + e)
 + 1) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)
 + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*A*c^5*((57*sin(f*x +
 e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 + a^2*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) + 40*B*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2
*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 160*A*c^5*
((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +

```
 e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*
a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 160*B*c^5*((12*s
in(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 80*A*c^5*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^2) + 40*B*c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 8
*A*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 40*A
*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos
(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3) + 8*B*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = \frac{\frac{315\,(4\,Ac^5-7\,Bc^5)(fx+e)}{a^2}+\frac{256\left(9\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+11\,Ac^5-1\right.}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1/
2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e) -
36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x + 1/
2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f*x +
 1/2*e)^7 + 552*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*tan(1/2*f*x + 1/2
```

```
*e)^6 + 84*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*tan(1/2*f*x + 1/2*e)^5
+ 1704*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*tan(1/2*f*x + 1/2*e)^4 - 8
4*A*c^5*tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 1720*A*
c^5*tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 84*A*c^5*t
an(1/2*f*x + 1/2*e) + 285*B*c^5*tan(1/2*f*x + 1/2*e) + 568*A*c^5 - 1288*B*c
^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(391\,A\,c^5-\frac{2729\,B\,c^5}{4}\right)+\frac{494\,A\,c^5}{3}-\frac{866\,B\,c^5}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(103\,A\,c^5-\frac{735\,B\,c^5}{4}\right)+\tan\left(\frac{e}{2}+\frac{f}{2}\right.}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}+3\,a^2\,\mathrm{t}\right.}$$
$$+\frac{105\,c^5\,\mathrm{atan}\left(\frac{105\,c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4\,A-7\,B)}{420\,A\,c^5-735\,B\,c^5}\right)(4\,A-7\,B)}{4\,a^2\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(391*A*c^5 - (2729*B*c^5)/4) + (494*A*c^5)/3 - (866*B*c
^5)/3 + tan(e/2 + (f*x)/2)^10*(103*A*c^5 - (735*B*c^5)/4) + tan(e/2 + (f*x)
/2)^9*(323*A*c^5 - (2213*B*c^5)/4) + tan(e/2 + (f*x)/2)^7*(1332*A*c^5 - 225
3*B*c^5) + tan(e/2 + (f*x)/2)^4*(1632*A*c^5 - 2943*B*c^5) + tan(e/2 + (f*x)
/2)^8*((2002*A*c^5)/3 - (3637*B*c^5)/3) + tan(e/2 + (f*x)/2)^3*((4420*A*c^5
)/3 - (7621*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((2489*A*c^5)/3 - (17609*B*c^5
)/12) + tan(e/2 + (f*x)/2)^6*((4594*A*c^5)/3 - (16805*B*c^5)/6) + tan(e/2 +
 (f*x)/2)^5*((6274*A*c^5)/3 - (21299*B*c^5)/6))/(f*(7*a^2*tan(e/2 + (f*x)/2
)^2 + 13*a^2*tan(e/2 + (f*x)/2)^3 + 18*a^2*tan(e/2 + (f*x)/2)^4 + 22*a^2*ta
n(e/2 + (f*x)/2)^5 + 22*a^2*tan(e/2 + (f*x)/2)^6 + 18*a^2*tan(e/2 + (f*x)/2
)^7 + 13*a^2*tan(e/2 + (f*x)/2)^8 + 7*a^2*tan(e/2 + (f*x)/2)^9 + 3*a^2*tan(
e/2 + (f*x)/2)^10 + a^2*tan(e/2 + (f*x)/2)^11 + a^2 + 3*a^2*tan(e/2 + (f*x)
/2))) + (105*c^5*atan((105*c^5*tan(e/2 + (f*x)/2)*(4*A - 7*B))/(420*A*c^5 -
 735*B*c^5))*(4*A - 7*B))/(4*a^2*f)
```

3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 180

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx &= \frac{35(A-2B)c^4x}{2a^2}+\frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f}+\frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f}\\&\quad-\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}+\frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}\end{aligned}$$

```
[Out] 35/2*(A-2*B)*c^4*x/a^2+35/3*(A-2*B)*c^4*cos(f*x+e)^3/a^2/f+35/2*(A-2*B)*c^4
*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+
e))^6+2*a^2*(A-2*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4+14*(A-2*B)*c^4*co
s(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx &= -\frac{a^4c^4(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{35c^4(A-2B)\cos^3(e+fx)}{3a^2f}+\frac{2a^2c^4(A-2B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\\&\quad+\frac{35c^4(A-2B)\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{35c^4x(A-2B)}{2a^2}+\frac{14c^4(A-2B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (35*(A - 2*B)*c^4*x)/(2*a^2) + (35*(A - 2*B)*c^4*Cos[e + f*x]^3)/(3*a^2*f)
+ (35*(A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(2*a^2*f) - (a^4*(A - B)*c^4
*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^6) + (2*a^2*(A - 2*B)*c^4*Cos[e
+ f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (14*(A - 2*B)*c^4*Cos[e + f*x]^5)/(f
*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx\\
&= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}-\left(a^3(A-2B)c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}\\
&\quad+\left(7a(A-2B)c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\
&= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}\\
&\quad+\frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}+\frac{\left(35(A-2B)c^4\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx)}\,dx}{a}\\
&= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f}-\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}\\
&\quad+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}+\frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}\\
&\quad+\frac{\left(35(A-2B)c^4\right)\int \cos^2(e+fx)\,dx}{a^2}\\
&= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f}+\frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f}\\
&\quad-\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}\\
&\quad+\frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}+\frac{\left(35(A-2B)c^4\right)\int 1\,dx}{2a^2}\\
&= \frac{35(A-2B)c^4x}{2a^2}+\frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f}+\frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f}\\
&\quad-\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}+\frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.47 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(128(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-64(A-B)\left(\cos\left(\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)*
Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(
5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(
A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A - 71*B
)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e + f*x)]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{2c^4\left(\frac{\left(\frac{A}{2}-3B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6A-17B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(12A-36B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6A-\frac{53B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{35(}{}\right.}{f\,a^2}$
default	$\frac{2c^4\left(\frac{\left(\frac{A}{2}-3B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6A-17B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(12A-36B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6A-\frac{53B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{35(}{}\right.}{f\,a^2}$
parallelrisch	$-\frac{21c^4\left(\left(-20fxA+40fxB-\frac{277}{7}A+\frac{1712}{21}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(-40fxB+20fxA+\frac{290}{21}B-\frac{191}{21}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+(-20fxA+40f\right.}{}$
risch	$\frac{35c^4xA}{2a^2}-\frac{35c^4xB}{a^2}+\frac{ic^4e^{2i(fx+e)}A}{8a^2f}-\frac{3ic^4e^{2i(fx+e)}B}{4a^2f}+\frac{3c^4e^{i(fx+e)}A}{a^2f}-\frac{71c^4e^{i(fx+e)}B}{8a^2f}+\frac{3c^4e^{-i(fx+e)}A}{a^2f}-$
norman	$\frac{\frac{\left(111Ac^4-212Bc^4\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(33Ac^4-70Bc^4\right)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(979Ac^4-2004Bc^4\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{\left(1893Ac^4-37\right.}{}}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^4/a^2*(((1/2*A-3*B)*tan(1/2*f*x+1/2*e)^5+(6*A-17*B)*tan(1/2*f*x+1/2*e
)^4+(12*A-36*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+3*B)*tan(1/2*f*x+1/2*e)+6*A-53
/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+35/2*(A-2*B)*arctan(tan(1/2*f*x+1/2*e))-1/
2*(-32*A+32*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-16*A+32*B)/(tan(1/2*f*x+1/2*e)+1)
-1/3*(32*A-32*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \frac{2\,Bc^4\cos\left(fx+e\right)^5-(3\,A-16\,B)c^4\cos\left(fx+e\right)^4+2\,(15\,A-38\,B)c^4\cos\left(fx+e\right)^3-210\,(A-2\,B)c^4 fx}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(2*B*c^4*cos(f*x + e)^5 - (3*A - 16*B)*c^4*cos(f*x + e)^4 + 2*(15*A - 3
8*B)*c^4*cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(A
- 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*cos(f*x + e)^2 - (105*(A - 2*B)*c^4*f
*x + 2*(97*A - 202*B)*c^4)*cos(f*x + e) - (2*B*c^4*cos(f*x + e)^4 + (3*A -
14*B)*c^4*cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4*cos(
f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218*B)*c^
4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) -
 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. 2(175) = 350.

Time = 12.86 (sec) , antiderivative size = 7337, normalized size of antiderivative = 40.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((105*A*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**9
 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 +
 f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2
+ 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/2)*
```

*8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*
f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
630*A*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*
f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 +
f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4
+ 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan(e/2 + f*x/2)**6/(6*a**2
*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2
+ f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5
+ 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1260*A*c**
4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2
+ f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**
6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2
*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
+ f*x/2) + 6*a**2*f) + 1260*A*c**4*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/
2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)*
*7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**
2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2
+ f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan
(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)
8 + 36*a2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a*
*2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/
2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) + 630*A*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)
9 + 18*a2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a*
*2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/
2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)*
*2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/
2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*
f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
105*A*c**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
+ f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) + 198*A*c**4*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 +
18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*t
an(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*
x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 666*A*c**4*tan(e/2 + f*x/2)**7/(6*a
2*f*tan(e/2 + f*x/2)9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e

/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)
5 + 72*a2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a*
*2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1066*A*
c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2
+ f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6
 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*
f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
 f*x/2) + 6*a**2*f) + 2094*A*c**4*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f
*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 +
60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*t
an(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1842*A*c**4*tan(e/2 + f*
x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*
a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(
e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2
)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*
f) + 2214*A*c**4*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**
2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2
 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**
4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1302*A*c**4*tan(e/2 + f*x/2)**2/(6*a**2*f
*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 +
f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 +
 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*
tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 786*A*c**4*t
an(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)*
*8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**
2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
 6*a**2*f) + 328*A*c**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 +
 f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6
+ 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f
*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
f*x/2) + 6*a**2*f) - 210*B*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 +
 f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7
+ 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f
*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 630*B*c**4*f*x*tan(e/2
 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8
+ 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f
*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 +
f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*
a**2*f) - 1260*B*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9
 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 +

f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2
+ 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2100*B*c**4*f*x*tan(e/2 + f*x/2)
6/(6*a2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
2520*B*c**4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**
2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2
+ f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**
4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2520*B*c**4*f*x*tan(e/2 + f*x/2)**4/(6*a*
*2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/
2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)*
*5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**
2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2100*B*c
4*f*x*tan(e/2 + f*x/2)3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e
/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)
6 + 72*a2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a*
*2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) - 1260*B*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(
e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2
)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a
2*f*tan(e/2 + f*x/2)4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e
/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 630*B*c**4*f*x*ta
n(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
+ f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) - 210*B*c**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/
2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)*
*6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**
2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
+ f*x/2) + 6*a**2*f) - 420*B*c**4*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 +
f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 +
60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*
tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 1272*B*c**4*tan(e/2 + f
*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36
*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan
(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/
2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) - 2320*B*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a*
*2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/
2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)*
*4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**

```
2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 3960*B*c**4*tan(e/2 + f*x/2)**5/(6*a**2*
f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 +
 f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5
+ 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f
*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 3960*B*c**4
*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*
x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 7
2*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*ta
n(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 4280*B*c**4*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2
)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a
**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e
/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2688*B*c**4*tan(e/2 + f*x/2)
**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 1560*B*c**4*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan
(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/
2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*
a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) - 660*B*c**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a*
*2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/
2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)*
*4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**
2*f*tan(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) +
 c)**4/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(172) = 344$.

Time = 0.35 (sec) , antiderivative size = 2094, normalized size of antiderivative = 11.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(A*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
```

s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 4*B*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*B*c^4*((57*sin(f*x + e)/(cos(f*x + e
) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2
4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^
2) + 16*A*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 24
*B*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 12*A*c^4*
((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*B*c^4*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^2) - 2*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) +
1)^3) + 8*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*

```
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*B*c^4*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(172) = 344$.

Time = 0.33 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{105\,(Ac^4-2\,Bc^4)(fx+e)}{a^2} + \frac{2\left(99\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-210\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+333\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-636\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+533\,Ac\right.}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*tan(1/2*f*x + 1/2*e)
^8 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*tan(1/2*f*x + 1/2*e)^7 -
636*B*c^4*tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 1160*
B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 1980*B*c
^4*tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1980*B*c^4*t
an(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^4*tan(
1/2*f*x + 1/2*e)^3 + 651*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*tan(1/2*
f*x + 1/2*e)^2 + 393*A*c^4*tan(1/2*f*x + 1/2*e) - 780*B*c^4*tan(1/2*f*x + 1
/2*e) + 164*A*c^4 - 330*B*c^4)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2
*e)^2 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.30

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(131\,A\,c^4-260\,B\,c^4\right)+\frac{164\,A\,c^4}{3}-110\,B\,c^4+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(33\,A\,c^4-70\,B\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+6\right.}$$

$$+ \frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A-2\,B)}{35\,A\,c^4-70\,B\,c^4}\right)\,(A-2\,B)}{a^2\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(131*A*c^4 - 260*B*c^4) + (164*A*c^4)/3 - 110*B*c^4 + t
an(e/2 + (f*x)/2)^8*(33*A*c^4 - 70*B*c^4) + tan(e/2 + (f*x)/2)^7*(111*A*c^4
 - 212*B*c^4) + tan(e/2 + (f*x)/2)^2*(217*A*c^4 - 448*B*c^4) + tan(e/2 + (f
*x)/2)^4*(307*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^5*(349*A*c^4 - 660*B*
c^4) + tan(e/2 + (f*x)/2)^6*((533*A*c^4)/3 - (1160*B*c^4)/3) + tan(e/2 + (f
*x)/2)^3*(369*A*c^4 - (2140*B*c^4)/3))/(f*(6*a^2*tan(e/2 + (f*x)/2)^2 + 10*
a^2*tan(e/2 + (f*x)/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(e/2 + (
f*x)/2)^5 + 10*a^2*tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^7 + 3*a^
2*tan(e/2 + (f*x)/2)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*tan(e/2 + (
f*x)/2))) + (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(A - 2*B))/(35*A*c^4 -
70*B*c^4))*(A - 2*B))/(a^2*f)
```

3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 162

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx\\&=\frac{5(2A-5B)c^3x}{2a^2}+\frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}\\&\quad+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{5(2A-5B)c^3\cos^3(e+fx)}{6f(a^2+a^2\sin(e+fx))}\end{aligned}$$

```
[Out] 5/2*(2*A-5*B)*c^3*x/a^2+5/2*(2*A-5*B)*c^3*cos(f*x+e)/a^2/f-1/3*a^3*(A-B)*c^
3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5+2/3*a*(2*A-5*B)*c^3*cos(f*x+e)^5/f/(a+a
*sin(f*x+e))^3+5/6*(2*A-5*B)*c^3*cos(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2938, 2759, 2758, 2761, 8}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx\\&=-\frac{a^3c^3(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{5c^3(2A-5B)\cos(e+fx)}{2a^2f}\\&\quad+\frac{5c^3(2A-5B)\cos^3(e+fx)}{6f(a^2\sin(e+fx)+a^2)}+\frac{5c^3x(2A-5B)}{2a^2}+\frac{2ac^3(2A-5B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^3}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*(2*A - 5*B)*c^3*x)/(2*a^2) + (5*(2*A - 5*B)*c^3*Cos[e + f*x])/(2*a^2*f)
- (a^3*(A - B)*c^3*Cos[e + f*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (2*a*(2*A
 - 5*B)*c^3*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^3) + (5*(2*A - 5*B)*c
^3*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}-\frac{1}{3}\left(a^2(2A-5B)c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\
&= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}\\
&\quad+\frac{1}{3}\left(5(2A-5B)c^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx\\
&= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}\\
&\quad+\frac{5(2A-5B)c^3\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)}+\frac{\left(5(2A-5B)c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{2a}\\
&= \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}\\
&\quad+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}\\
&\quad+\frac{5(2A-5B)c^3\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)}+\frac{\left(5(2A-5B)c^3\right)\int 1\,dx}{2a^2}\\
&= \frac{5(2A-5B)c^3x}{2a^2}+\frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}\\
&\quad+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{5(2A-5B)c^3\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^3\left(64(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-32(A-B)\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*S
in[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(7*
A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*(2*
A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*B)*C
os[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2c^3\left(\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A-5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}\right.}{f\,a^2}$
default	$\frac{2c^3\left(\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A-5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}\right.}{f\,a^2}$
parallelrisch	$-\frac{c^3\left(\left(-30fxA+75fxB-56A+\frac{607}{4}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(10fxA-25fxB-\frac{19}{3}A+\frac{115}{12}B\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-30fxA+75fxB\right.\right.}{2f\,a^2\left(3\,\mathrm{c}\right.}$
risch	$\frac{5c^3xA}{a^2}-\frac{25c^3xB}{2a^2}-\frac{iB\,c^3\mathrm{e}^{2i(fx+e)}}{8a^2f}+\frac{c^3\mathrm{e}^{i(fx+e)}A}{2a^2f}-\frac{5c^3\mathrm{e}^{i(fx+e)}B}{2a^2f}+\frac{c^3\mathrm{e}^{-i(fx+e)}A}{2a^2f}-\frac{5c^3\mathrm{e}^{-i(fx+e)}B}{2a^2f}+\frac{iB\,c}{}$
norman	$\frac{\frac{\left(8A\,c^3-25B\,c^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(34A\,c^3-77B\,c^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(38A\,c^3-93B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(148A\,c^3-352B\,c^3\right)\left(\tan^3\right.}{af}}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^3/a^2*((-1/2*B*tan(1/2*f*x+1/2*e)^3+(A-5*B)*tan(1/2*f*x+1/2*e)^2+1/2*
B*tan(1/2*f*x+1/2*e)+A-5*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A-5*B)*arctan
(tan(1/2*f*x+1/2*e))-1/2*(-16*A+16*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-4*A+12*B)/
(tan(1/2*f*x+1/2*e)+1)-1/3*(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.80

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \frac{3\,Bc^3\cos\left(fx+e\right)^4+6\,(A-4\,B)c^3\cos\left(fx+e\right)^3-30\,(2\,A-5\,B)c^3fx+16\,(A-B)c^3+(15\,(2\,A-5\,B)$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*c^3*cos(f*x + e)^4 + 6*(A - 4*B)*c^3*cos(f*x + e)^3 - 30*(2*A - 5*
B)*c^3*f*x + 16*(A - B)*c^3 + (15*(2*A - 5*B)*c^3*f*x - (62*A - 131*B)*c^3)
*cos(f*x + e)^2 - (15*(2*A - 5*B)*c^3*f*x + 2*(26*A - 71*B)*c^3)*cos(f*x +
e) + (3*B*c^3*cos(f*x + e)^3 - 30*(2*A - 5*B)*c^3*f*x - 3*(2*A - 9*B)*c^3*c
os(f*x + e)^2 - 16*(A - B)*c^3 - (15*(2*A - 5*B)*c^3*f*x + 2*(34*A - 79*B)*
c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
 - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. 2(148) = 296.

Time = 7.42 (sec) , antiderivative size = 4665, normalized size of antiderivative = 28.80

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((30*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2
+ f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 +
 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*
tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
```

```
*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 +
f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*
tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f
*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1
50*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f
*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 +
f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 +
 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2 + f*x/2)/(6
*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan
(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/
2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) + 30*A*c**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 48*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2
 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**
5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2
*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 204*A*c**
3*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f
*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*t
an(e/2 + f*x/2) + 6*a**2*f) + 212*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 432*A*
c**3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 256*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*t
an(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 228
*A*c**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 92*A*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
```

x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x*tan(e/2 +
f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*
x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 4
2*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*ta
n(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x
*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 525*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*t
an(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 525
*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*t
an(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x*tan(e/2 + f*x/2)**2/
(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*t
an(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*
x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a*
*2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 150*B
*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 462*B*c**3*tan(e/2 + f*x/2)**5/(6*a**2*f*
tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f
*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 65
6*B*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(
e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2
)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a
2*f*tan(e/2 + f*x/2) + 6*a2*f) - 996*B*c**3*tan(e/2 + f*x/2)**3/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 718*B*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*t
an(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*

```
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 558*B*c**3*tan(e/2 + f*x/2)/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 236*B*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 +
 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*
tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e
) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(152) = 304$.

Time = 0.33 (sec) , antiderivative size = 1378, normalized size of antiderivative = 8.51

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/3*(B*c^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) - 4*A*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*
x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a^2) + 12*B*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
^2) - 6*A*c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
```

```
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*B*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3
*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a^2) + 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3) - 6*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*c^3*(3*sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{15\,(2\,Ac^3-5\,Bc^3)(fx+e)}{a^2} - \frac{6\left(Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^3+10\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}}{6\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*c^3 - 5*B*c^3)*(f*x + e)/a^2 - 6*(B*c^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 10*B*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c^3
*tan(1/2*f*x + 1/2*e) - 2*A*c^3 + 10*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*a^2) + 16*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 12*A*c^3*tan(1/2*f*x + 1/2*e) - 24*B*c^3*tan(1/2*f*x + 1/2*e) + 5*A*c^3
- 11*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.07

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(38\,A\,c^3-93\,B\,c^3)+\frac{46\,A\,c^3}{3}-\frac{118\,B\,c^3}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(8\,A\,c^3-25\,B\,c^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(34\,A}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+3\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+5\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+7\,a^2\right.}$$
$$+\frac{5\,c^3\,\mathrm{atan}\left(\frac{5\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,A-5\,B)}{10\,A\,c^3-25\,B\,c^3}\right)\,(2\,A-5\,B)}{a^2\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(38*A*c^3 - 93*B*c^3) + (46*A*c^3)/3 - (118*B*c^3)/3 +
tan(e/2 + (f*x)/2)^6*(8*A*c^3 - 25*B*c^3) + tan(e/2 + (f*x)/2)^5*(34*A*c^3
- 77*B*c^3) + tan(e/2 + (f*x)/2)^3*(72*A*c^3 - 166*B*c^3) + tan(e/2 + (f*x)
/2)^4*((106*A*c^3)/3 - (328*B*c^3)/3) + tan(e/2 + (f*x)/2)^2*((128*A*c^3)/3
 - (359*B*c^3)/3))/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2
)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2*tan(e
/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)
)) + (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2)*(2*A - 5*B))/(10*A*c^3 - 25*B*c^
3))*(2*A - 5*B))/(a^2*f)
```

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 108

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx\\&=\frac{(A-4B)c^2x}{a^2}+\frac{(A-4B)c^2\cos(e+fx)}{a^2f}\\&\quad-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

```
[Out] (A-4*B)*c^2*x/a^2+(A-4*B)*c^2*cos(f*x+e)/a^2/f-1/3*a^2*(A-B)*c^2*cos(f*x+e)
^5/f/(a+a*sin(f*x+e))^4+2/3*(A-4*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2759, 2761, 8}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx\\&=\frac{c^2(A-4B)\cos(e+fx)}{a^2f}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\\&\quad+\frac{c^2x(A-4B)}{a^2}+\frac{2c^2(A-4B)\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((A - 4*B)*c^2*x)/a^2 + ((A - 4*B)*c^2*Cos[e + f*x])/(a^2*f) - (a^2*(A - B)
*c^2*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^4) + (2*(A - 4*B)*c^2*Cos[e
+ f*x]^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} - \frac{1}{3}\left(a(A-4B)c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{((A-4B)c^2)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{a}$$

$$= \frac{(A-4B)c^2\cos(e+fx)}{a^2f} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{((A-4B)c^2)\int 1\,dx}{a^2}$$

$$= \frac{(A-4B)c^2x}{a^2} + \frac{(A-4B)c^2\cos(e+fx)}{a^2f} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. $2(108) = 216$.

Time = 11.22 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.17

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-4(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A -
 B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)*(c - c*Sin[e + f*x])^2)/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c^2\left(-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
default	$\frac{2c^2\left(-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
risch	$\frac{c^2xA}{a^2}-\frac{4c^2xB}{a^2}-\frac{B\,c^2 e^{i(fx+e)}}{2a^2f}-\frac{B\,c^2 e^{-i(fx+e)}}{2a^2f}+\frac{8iA\,c^2 e^{i(fx+e)}+8A\,c^2 e^{2i(fx+e)}-24iB\,c^2 e^{i(fx+e)}-16B\,c^2 e^{2i(f}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$
parallelrisch	$\frac{3c^2\left(\left(fxA-4fxB+\frac{4}{3}A-8B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(\left(-\frac{11}{6}+4fx\right)B-fxA+\frac{4A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+\left(fxA-4fxB+\frac{4}{3}A-\frac{14}{3}B\right)\sin\left(\frac{fx}{2}+\right.\right.}{f\,a^2\left(3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\sin\left(\frac{3fx}{2}+\frac{3e}{2}\right)+3\right.}$
norman	$\frac{\frac{c^2(A-4B)x}{a}+\frac{\left(8A\,c^2-30B\,c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c^2(A-4B)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{8A\,c^2-38B\,c^2}{3af}-\frac{8B\,c^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{2\left(4A\,c^2-61B\right.}{}}{}$

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 2/f*c^2/a^2*(-1/2*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A-8*B)/(tan(1/2*f*x+1/2*e)+1)^3-4*B/(tan(1/2*f*x+1/2*e)+1)-B/(1+tan(1/2*f*x+1/2*e)^2)+(A-4*B)*arctan(tan(1/2*f*x+1/2*e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(104) = 208.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{3\,Bc^2\cos(fx+e)^3+6\,(A-4\,B)c^2fx-4\,(A-B)c^2-(3\,(A-4\,B)c^2fx-(8\,A-23\,B)c^2)\cos(fx+e}{3\,(a^2f\,c}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A - 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x + 2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*x + e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. 2(102) = 204.

Time = 3.98 (sec) , antiderivative size = 2474, normalized size of antiderivative = 22.91

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x*t
an(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a*
*2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5
 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)
**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*A*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)*
*5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*A*c**2*
tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 24*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 - 12*B*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(e/2
 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*t
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x
/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
```

```
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(
e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 12*B*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c**2*tan(
e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*
f*tan(e/2 + f*x/2) + 3*a**2*f) - 78*B*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) - 74*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 90*B*c**2*tan(e/2 + f*x
/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*
f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) - 38*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(104) = 208$.

Time = 0.32 (sec) , antiderivative size = 833, normalized size of antiderivative = 7.71

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*
c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 2*B*c^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
```

```
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \frac{\frac{3\,(Ac^2-4\,Bc^2)(fx+e)}{a^2}-\frac{6\,Bc^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{8\,\left(3\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Ac^2+4\,Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*e)
 + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,A\,c^2-30\,B\,c^2\right)+\frac{8\,A\,c^2}{3}-\frac{38\,B\,c^2}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(8\,A\,c^2-26\,B\,c^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{8\,A\,c^2}{3}\right.}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\,\mathrm{t}\right.} + \frac{2\,c^2\,\mathrm{atan}\left(\frac{2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A-4\,B)}{2\,A\,c^2-8\,B\,c^2}\right)\,(A-4\,B)}{a^2\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(8*A*c^2 - 30*B*c^2) + (8*A*c^2)/3 - (38*B*c^2)/3 + tan
(e/2 + (f*x)/2)^3*(8*A*c^2 - 26*B*c^2) + tan(e/2 + (f*x)/2)^2*((8*A*c^2)/3
- (74*B*c^2)/3) - 8*B*c^2*tan(e/2 + (f*x)/2)^4)/(f*(4*a^2*tan(e/2 + (f*x)/2
)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2
 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2))) + (2*c^2*atan((2*c^2*tan(e
/2 + (f*x)/2)*(A - 4*B))/(2*A*c^2 - 8*B*c^2))*(A - 4*B))/(a^2*f)
```

3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{Bcx}{a^2}+\frac{(A-7B)c\cos(e+fx)}{3a^2f(1+\sin(e+fx))} - \frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

```
[Out] -B*c*x/a^2+1/3*(A-7*B)*c*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-2/3*(A-B)*c*cos(f*
x+e)/f/(a+a*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3046, 2936, 2814, 2727}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{c(A-7B)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{Bcx}{a^2} - \frac{2c(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*c*x)/a^2) + ((A - 7*B)*c*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) -
(2*(A - B)*c*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx\\
&= -\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{c\int \frac{aA-4aB+3aB\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}\\
&= -\frac{Bcx}{a^2}-\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{((A-7B)c)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\
&= -\frac{Bcx}{a^2}-\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(A-7B)c\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. $2(72) = 144$.

Time = 6.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.17

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A-3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)-14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e+\frac{3}{}\right.\right.}{6a^2f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e\right.\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*f
*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin[
(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^2*f
*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{Bcx}{a^2}+\frac{2Ac\,e^{2i(fx+e)}-8iBc\,e^{i(fx+e)}-6Bc\,e^{2i(fx+e)}-\frac{2Ac}{3}+\frac{14Bc}{3}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$
derivativedivides	$\frac{2c\left(-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
default	$\frac{2c\left(-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
parallelrisch	$-\frac{2\left(\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(\frac{3}{2}fxB+A+B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B\left(\frac{3fx}{2}+4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{fxB}{2}+\frac{A}{3}+\frac{5B}{3}\right)c}{f\,a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$\frac{-\frac{2Ac+10Bc}{3af}-\frac{Bcx}{a}-\frac{16Bc\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8Bc\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{(14Ac+22Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(10Ac+2}{}}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -B*c*x/a^2+2/3*(3*A*c*exp(2*I*(f*x+e))-12*I*B*c*exp(I*(f*x+e))-9*B*c*exp(2*
I*(f*x+e))-A*c+7*B*c)/f/a^2/(exp(I*(f*x+e))+I)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(68) = 136.

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.31

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{6\,Bcfx-(3\,Bcfx+(A-7\,B)c)\cos{(fx+e)}^2+2\,(A-B)c+(3\,Bcfx+(A+5\,B)c)\cos{(fx+e)}+(6\,B}{3\,\left(a^2f\cos{(fx+e)}^2-a^2f\cos{(fx+e)}-2\,a^2f-(a^2f\cos{(fx}\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c + (
3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c*f
*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(70) = 140.

Time = 2.09 (sec) , antiderivative size = 702, normalized size of antiderivative = 9.75

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \begin{cases} -\frac{6Ac\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{2Ac}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} \\ \frac{x(A+B\sin{(e)})(-c\sin{(e)}+c)}{(a\sin{(e)}+a)^2} \end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(3*
a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*B*c*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)
**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*a**2*f*tan(e/2 +
 f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) - 6*B*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f
```

```
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c*tan(e
/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 +
9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*B*c/(3*a**2*f*tan(e/2 + f*x/2)**
3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), N
e(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(68) = 136$.

Time = 0.30 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.28

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(Bc\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\cdots\right.}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin\cdots}{(\cos\cdots}}\cdots\right.}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{\frac{3(fx+e)Bc}{a^2}+\frac{2\left(3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+12Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac+5Bc\right)}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/2
*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/2*
f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{B\,c\,x}{a^2} - \frac{\left(\frac{c\,(6\,A+6\,B+9\,B\,(e+f\,x))}{3}-3\,B\,c\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\left(\frac{c\,(24\,B+9\,B\,(e+f\,x))}{3}-3\,B\,c\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] - (B*c*x)/a^2 - (tan(e/2 + (f*x)/2)^2*((c*(6*A + 6*B + 9*B*(e + f*x)))/3 -
3*B*c*(e + f*x)) + tan(e/2 + (f*x)/2)*((c*(24*B + 9*B*(e + f*x)))/3 - 3*B*c
*(e + f*x)) + (c*(2*A + 10*B + 3*B*(e + f*x)))/3 - B*c*(e + f*x))/(a^2*f*(t
an(e/2 + (f*x)/2) + 1)^3)
```

3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = -\frac{(A-B)\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))} + \frac{(2A+B)\tan(e+fx)}{3a^2cf}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+1/3*(2*A+B)*tan(f*x+e)/a^2/c
/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 3852, 8}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{(2A+B)\tan(e+fx)}{3a^2cf} - \frac{(A-B)\sec(e+fx)}{3cf\,(a^2\sin(e+fx)+a^2)}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/3*((A - B)*Sec[e + f*x])/(c*f*(a^2 + a^2*Sin[e + f*x])) + ((2*A + B)*Tan
[e + f*x])/(3*a^2*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{ac}\\
&= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}+\frac{(2A+B)\int \sec^2(e+fx)\,dx}{3a^2c}\\
&= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}-\frac{(2A+B)\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3a^2cf}\\
&= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}+\frac{(2A+B)\tan(e+fx)}{3a^2cf}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \frac{\cos(e+fx)(-6B-2(A-B)\cos(e+fx)+2(2A+B)\cos(2(e+fx))-8A\sin(e+fx)-4B\sin(e+f\ldots}{12a^2cf(-1+\sin(e+fx))(1+\sin(e+fx))^2}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)]
 - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e +
f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

method	result
risch	$\frac{2i\left(4iA\,e^{i(fx+e)}+2iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2A-B\right)}{3\left(e^{i(fx+e)}+i\right)^3\left(e^{i(fx+e)}-i\right)a^2cf}$
parallelrisch	$\frac{-6A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-6A-6B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-2A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2A-2B}{3f\,a^2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
derivativedivides	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2cf}$
default	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2cf}$
norman	$\frac{-\frac{2A+4B}{6acf}-\frac{4(2A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}+\frac{A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{(2A+4B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{(8A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}-\frac{(14A+16B)(\tan\ldots}{6acf}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3*I*(4*I*A*exp(I*(f*x+e))+2*I*B*exp(I*(f*x+e))+3*B*exp(2*I*(f*x+e))-2*A-B
)/(exp(I*(f*x+e))+I)^3/(exp(I*(f*x+e))-I)/a^2/c/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = -\frac{(2\,A+B)\cos\left(fx+e\right)^2-(2\,A+B)\sin\left(fx+e\right)-A-2\,B}{3\,\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*f
*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(51) = 102.

Time = 2.22 (sec) , antiderivative size = 578, normalized size of antiderivative = 9.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)} \end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*A
*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 +
 f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 + f*x/
2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**
2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/2)**4
+ 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*ta
n(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*B*tan(e/2
 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3
- 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*tan(e/2 + f*x
/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(58) = 116$.

Time = 0.22 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.27

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))}\,dx = \frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(f*x +
 e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*
c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))}\,dx = -\frac{\frac{3(A+B)}{a^2c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{9A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+12A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7A-B}{a^2c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{6f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)/(
a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 12.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \frac{2\left(\frac{3\,B}{2}-A\,\cos\left(e+f\,x\right)+B\,\cos\left(e+f\,x\right)+2\,A\,\sin\left(e+f\,x\right)+B\,\sin\left(e+f\,x\right)-A\,\cos\left(2\,e+2\,f\,x\right)-B\,c}{3\,a^2\,c\,f\,\left(2\,\cos\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

```
[Out] (2*((3*B)/2 - A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) + B*sin(e
+ f*x) - A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))
/2 + (B*sin(2*e + 2*f*x))/2))/(3*a^2*c*f*(2*cos(e + f*x) + sin(2*e + 2*f*x)
))
```

3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{A\tan^3(e+fx)}{3a^2c^2f}$$

```
[Out] 1/3*B*sec(f*x+e)^3/a^2/c^2/f+A*tan(f*x+e)/a^2/c^2/f+1/3*A*tan(f*x+e)^3/a^2/
c^2/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2748, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{A\tan^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*Tan[e + f*x])/(a^2*c^2*f) + (A*Tan[e
+ f*x]^3)/(3*a^2*c^2*f)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\,dx}{a^2c^2} \\
&= \frac{B\sec^3(e+fx)}{3a^2c^2f} + \frac{A\int \sec^4(e+fx)\,dx}{a^2c^2} \\
&= \frac{B\sec^3(e+fx)}{3a^2c^2f} - \frac{A\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{a^2c^2f} \\
&= \frac{B\sec^3(e+fx)}{3a^2c^2f} + \frac{A\tan(e+fx)}{a^2c^2f} + \frac{A\tan^3(e+fx)}{3a^2c^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)}{3a^2c^2f} + \frac{A\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{a^2c^2f}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(a
^2*c^2*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

method	result
risch	$\frac{4iA\,\mathrm{e}^{2i(fx+e)}+\frac{8B\,\mathrm{e}^{3i(fx+e)}}{3}+\frac{4iA}{3}}{\left(\mathrm{e}^{i(fx+e)}-i\right)^{3}\left(\mathrm{e}^{i(fx+e)}+i\right)^{3}a^{2}c^{2}f}$
parallelrisch	$\frac{6A\sin(fx+e)+2A\sin(3fx+3e)+3\cos(fx+e)B+\cos(3fx+3e)B+4B}{3a^{2}c^{2}f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^{2}c^{2}f}$
default	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^{2}c^{2}f}$
norman	$\frac{-\frac{2B}{3acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}-\frac{2A\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{2B\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^{2}\left(\frac{fx}{2}+\right.\right.}{3acf}}{a\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 4/3*(3*I*A*exp(2*I*(f*x+e))+2*B*exp(3*I*(f*x+e))+I*A)/(exp(I*(f*x+e))-I)^3/
(exp(I*(f*x+e))+I)^3/a^2/c^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx=\frac{\left(2\,A\cos\left(fx+e\right)^{2}+A\right)\sin\left(fx+e\right)+B}{3\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(56) = 112$.

Time = 2.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 7.56

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \begin{cases} -\frac{6A\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f} + \frac{4A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)^2} \end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) - 6*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) - 2*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f
*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne(f, 0)), (
x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{\frac{\left(\tan(fx+e)^3+3\tan(fx+e)\right)A}{a^2c^2} + \frac{B}{a^2c^2\cos(fx+e)^3}}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x + e
)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = -\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+B\right)}{3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^2c^2f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/2
*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 -
1)^3*a^2*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = -\frac{2\left(3\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,B\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+B\right)}{3\,a^2\,c^2\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(B + 3*A*tan(e/2 + (f*x)/2) - 2*A*tan(e/2 + (f*x)/2)^3 + 3*A*tan(e/2 +
(f*x)/2)^5 + 3*B*tan(e/2 + (f*x)/2)^4))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^2
- 1)^3)
```

3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 93

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}$$

```
[Out] 1/5*(A+B)*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+1/5*(4*A-B)*tan(f*x+e)/a^
2/c^3/f+1/15*(4*A-B)*tan(f*x+e)^3/a^2/c^3/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + ((4*A - B)*Ta
n[e + f*x])/(5*a^2*c^3*f) + ((4*A - B)*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^2c^2} \\
&= \frac{(A+B)\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))}+\frac{(4A-B)\int\sec^4(e+fx)\,dx}{5a^2c^3} \\
&= \frac{(A+B)\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))}-\frac{(4A-B)\text{Subst}\left(\int (1+x^2)\,dx,x,-\tan(e+fx)\right)}{5a^2c^3f} \\
&= \frac{(A+B)\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(93) = 186.

Time = 2.00 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.55

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-240B+54(A+B)\cos(e+f}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*A*
Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*Cos
[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e + f*
x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)
] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1 + Sin[
e + f*x])^3*(1 + Sin[e + f*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

method	result
risch	$-\frac{4 i \left(24 i A\,{\mathrm e}^{3 i \left(f x +e \right)}-6 i B\,{\mathrm e}^{3 i \left(f x +e \right)}+15 B\,{\mathrm e}^{4 i \left(f x +e \right)}+8 i A\,{\mathrm e}^{i \left(f x +e \right)}+8 A\,{\mathrm e}^{2 i \left(f x +e \right)}-2 i B\,{\mathrm e}^{i \left(f x +e \right)}-2 B\,{\mathrm e}^{2 i \left(f x +e \right)}+4 A -B \right)}{15 \left({\mathrm e}^{i \left(f x +e \right)}+i \right)^{3} \left({\mathrm e}^{i \left(f x +e \right)}-i \right)^{5} f \,c^{3} a^{2}}$
parallelrisch	$\frac{-30 A \left(\tan^{7}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+\left(30 A -30 B \right) \left(\tan^{6}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+\left(10 A +20 B \right) \left(\tan^{5}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+\left(-50 A -10 B \right) \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+(-}{15 f \,c^{3} a^{2} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3} \left(\tan \left(\frac{f}{2}\right.\right.}$
derivativedivides	$\frac{-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2 \left(\frac{A}{4}-\frac{B}{4}\right)}{3 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2 \left(\frac{5 A}{16}-\frac{3 B}{16}\right)}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2 \left(A +B \right)}{5 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2 A +2 B}{2 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{3 A}{2}+B}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{2}}-}{a^{2} c^{3} f}$
default	$\frac{-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2 \left(\frac{A}{4}-\frac{B}{4}\right)}{3 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2 \left(\frac{5 A}{16}-\frac{3 B}{16}\right)}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2 \left(A +B \right)}{5 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2 A +2 B}{2 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{3 A}{2}+B}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{2}}-}{a^{2} c^{3} f}$
norman	$\frac{\frac{6 A -4 B}{10 a f c}-\frac{4 \left(4 A -B \right) \left(\tan^{5}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 a f c}-\frac{A \left(\tan^{10}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a f c}+\frac{\left(14 A -16 B \right) \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{10 a f c}+\frac{\left(6 A -4 B \right) \left(\tan^{8}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2 a f c}+\frac{\left(2 A -8 B \right) \left(\tan\right.}{3 a}}{a \left(1+\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right) \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right.\right.}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] -4/15*I*(24*I*A*exp(3*I*(f*x+e))-6*I*B*exp(3*I*(f*x+e))+15*B*exp(4*I*(f*x+e
))+8*I*A*exp(I*(f*x+e))+8*A*exp(2*I*(f*x+e))-2*I*B*exp(I*(f*x+e))-2*B*exp(2
*I*(f*x+e))+4*A-B)/(exp(I*(f*x+e))+I)^3/(exp(I*(f*x+e))-I)^5/f/c^3/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^3}\, dx = -\frac{2\,(4\,A - B)\cos\left(fx + e\right)^4 - (4\,A - B)\cos\left(fx + e\right)^2 + \left(2\,(4\,A - B)\cos\left(fx + e\right)^2 + 4\,A - B\right)\sin\left(fx + \right.}{15\,\left(a^2c^3f\cos\left(fx + e\right)^3\sin\left(fx + e\right) - a^2c^3f\cos\left(fx + e\right)^3\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B)
*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e)^
3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(82) = 164.

Time = 9.03 (sec) , antiderivative size = 2674, normalized size of antiderivative = 28.75

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^3}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 30*A*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 50*A*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
```

```
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 42*A*tan(e/2 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 18*A*tan(e/2 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*
a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a
**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a*
*2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c
**3*f) - 6*A/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 30*B*tan(e/2
 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 20*B*tan(e/2
 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 10*B*tan(e/2
 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 16*B*tan(e/2
 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 18*B*tan(e/2
 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 12*B*tan(e/2
 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*
x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x
/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/
2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 6*B/(15*a**2*c*
*3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**
3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3
*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*
f*tan(e/2 + f*x/2) - 15*a**2*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**2*(-c*sin(e) + c)**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 651, normalized size of antiderivative = 7.00

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^3}\,dx$$
$$= \frac{2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7} + 3\right)}{a^2c^3 - \frac{2a^2c^3\sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^2c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{6a^2c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{6a^2c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{2a^2c^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{2a^2c^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{a^2c^3\sin(fx+e)^8}{(\cos(fx+e)+1)^8}} - \frac{}{a^2 \cdots}\right.}{15f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/(a^2*c^
3 - 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*
c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) - 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)/(a^2*c^3 - 2*a^2*c^3*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.34 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.38

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^3}\,dx =$$
$$-\frac{\frac{5\left(15A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 9B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 24A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 12B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 13A - 7B\right)}{a^2c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3} + \frac{165A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 + 45B\tan \cdots}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 - 9*B*tan(1/2*f*x + 1/2*e)^2 + 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A - 7*B)/(a^2*c^3*(ta
n(1/2*f*x + 1/2*e) + 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 + 45*B*tan(1/2*f
*x + 1/2*e)^4 - 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 + 70*B*tan(1/2*f*x + 1/2*e)^2 - 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A + 13*B)/(a^2*c^3*(tan(1/
2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.97

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{\left(\frac{8\,A}{15}-\frac{2\,B}{15}-\frac{16\,A\,\sin(e+f\,x)}{15}+\frac{4\,B\,\sin(e+f\,x)}{15}\right)\,\cos\left(e+f\,x\right)^2+\frac{2\,A}{15}-\frac{8\,B}{15}-\frac{8\,A\,\sin(e+f\,x)}{15}+\frac{2\,B\,\sin(e+f\,x)}{15}}{a^2\,c^3\,f\,\left(2\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^3\right)}$$
$$-\frac{\frac{2\,A}{5}+\frac{2\,B}{5}-\frac{2\,A\,\sin(e+f\,x)}{5}-\frac{2\,B\,\sin(e+f\,x)}{5}}{a^2\,c^3\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{16\,A}{15}-\frac{4\,B}{15}\right)}{a^2\,c^3\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x)
```

```
[Out] ((2*A)/15 - (8*B)/15 - (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 - (2*B)/15 - (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x
))/15))/(a^2*c^3*f*(2*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^3)) - ((
2*A)/5 + (2*B)/5 - (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^2*c^3*f*
(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((16*A)/15 - (4*B)/15))/(a^2*c^3*f*(2
*sin(e + f*x) - 2))
```

3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 135

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/35*(5*A-2*B)*sec(f*x+
e)^3/a^2/f/(c^4-c^4*sin(f*x+e))+4/35*(5*A-2*B)*tan(f*x+e)/a^2/c^4/f+4/105*(
5*A-2*B)*tan(f*x+e)^3/a^2/c^4/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + ((5*A - 2*B
)*Sec[e + f*x]^3)/(35*a^2*f*(c^4 - c^4*Sin[e + f*x])) + (4*(5*A - 2*B)*Tan[
e + f*x])/(35*a^2*c^4*f) + (4*(5*A - 2*B)*Tan[e + f*x]^3)/(105*a^2*c^4*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\,(c^2-c^2\sin(e+fx))^2} + \frac{(5A-2B)\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(4(5A-2B))\int\sec^4(e+fx)\,dx}{35a^2c^4}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} - \frac{(4(5A-2B))\text{Subst}\left(\int\left(1+x^2\right)\,dx, x, -\tan(e+fx)\right)}{35a^2c^4f}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f} + \frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.

Time = 2.40 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2688B+42(25A+4B)\cos}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -1/13440*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[2*
(e + f*x)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos[4*
(e + f*x)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e
 + f*x)] - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e + f*x
)] - 96*B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e + f*x)
] - 300*A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e + f*x)]
 - 128*B*Sin[5*(e + f*x)]))/(a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f
*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{16\left(-6iB\,e^{2i(fx+e)}+2iB+70iA\,e^{4i(fx+e)}-28iB\,e^{4i(fx+e)}+20A\,e^{i(fx+e)}+40A\,e^{3i(fx+e)}+42B\,e^{5i(fx+e)}+15iA\,e^{2i(fx+}\right.}{105\left(e^{i(fx+e)}-i\right)^{7}\left(e^{i(fx+e)}+i\right)^{3}a^{2}c^{4}f}$
parallelrisch	$\frac{-210A\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(420A-210B)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-280A+280B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-560A-280B)\left(\tan^{6}\left(\frac{fx}{2}+\right.\right.}{}$
derivativedivides	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}}{a^{2}c^{4}f}$
default	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}}{a^{2}c^{4}f}$
norman	$\frac{-\frac{20A+6B}{35afc}-\frac{4(10A+11B)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15afc}-\frac{2A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{2(30A-47B)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35afc}-\frac{2(7A-4B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3afc}+\frac{2(2}{}}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] -16/105*(-6*I*B*exp(2*I*(f*x+e))+2*I*B+70*I*A*exp(4*I*(f*x+e))-28*I*B*exp(4
*I*(f*x+e))+20*A*exp(I*(f*x+e))+40*A*exp(3*I*(f*x+e))+42*B*exp(5*I*(f*x+e))
+15*I*A*exp(2*I*(f*x+e))-8*B*exp(I*(f*x+e))-16*B*exp(3*I*(f*x+e))-5*I*A)/(e
xp(I*(f*x+e))-I)^7/(exp(I*(f*x+e))+I)^3/a^2/c^4/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{16\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-8\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(8\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-12\,(5\,A-2\,B\right.}{105\,\left(a^{2}c^{4}f\cos\left(fx+e\right)^{5}+2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\,c\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*(
5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*si
n(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*x +
 e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4228 vs. 2(122) = 244.

Time = 17.58 (sec) , antiderivative size = 4228, normalized size of antiderivative = 31.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**9/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10
 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**
8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)
**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/
2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x
/2) - 105*a**2*c**4*f) + 420*A*tan(e/2 + f*x/2)**8/(105*a**2*c**4*f*tan(e/2
 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e
/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan
(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*t
an(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*
tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*A*tan(e/2 + f*x/2)**7/(105*a**2*c
**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2
*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a*
*2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*
a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420
*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 560*A*tan(e/2 + f*x/2)**
6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)*
*9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)
**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x
/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*
x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 420*A*tan(e
/2 + f*x/2)**5/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(
e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan
(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*
tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f
*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f)
+ 280*A*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**
2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a*
*2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470
*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 31
5*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*
a**2*c**4*f) - 760*A*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f*tan(e/2 + f*x/2)*
*10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2
)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x
/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f
```

*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 240*A*tan(e/2 + f*x/2)**2/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*ta n(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f* tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4* f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4 *f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 30*A*tan(e/2 + f*x/2)/(105*a**2*c* *4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2* c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a** 2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a **2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420* a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 60*A/(105*a**2*c**4*f*tan (e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*t an(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f *tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4 *f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c** 4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 210*B*tan(e/2 + f*x/2)**8/(105*a* *2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315* a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 147 0*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 280*B*tan(e/2 + f*x/ 2)**7/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x /2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f* x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*B*t an(e/2 + f*x/2)**6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f* tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f *tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c** 4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c* *4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4 *f) - 168*B*tan(e/2 + f*x/2)**5/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420 *a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 84 0*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 28*B*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/ 2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f* x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 136*B*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f*t an(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f *tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4

```
*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c*
*4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c
**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 264*B*tan(e/2 + f*x/2)**2/(105*
a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 31
5*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1
470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4
- 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2
 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 72*B*tan(e/2 + f*x
/2)/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2
)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/
2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f
*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 +
f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 18*B/(105
*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 3
15*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 -
1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4
 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**
2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f), Ne(f, 0)), (x*(A +
 B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(129) = 258$.

Time = 0.25 (sec) , antiderivative size = 835, normalized size of antiderivative = 6.19

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(36*sin(f*x + e)/(cos(f*x + e) + 1) - 132*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 68*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 84*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 140*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 140*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 105
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2
*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10) + 5*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
24*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 28*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 56*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(c
```

```
os(f*x + e) + 1)^7 + 42*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9 - 6)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(129) = 258$.

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.05

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^4}\,dx = -\frac{\frac{35\left(9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,A-5\,B\right)}{a^2c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3} + \frac{1365\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+210\,B\tan(\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/840*(35*(9*A*tan(1/2*f*x + 1/2*e)^2 - 6*B*tan(1/2*f*x + 1/2*e)^2 + 15*A*
tan(1/2*f*x + 1/2*e) - 9*B*tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(tan(
1/2*f*x + 1/2*e) + 1)^3) + (1365*A*tan(1/2*f*x + 1/2*e)^6 + 210*B*tan(1/2*f
*x + 1/2*e)^6 - 5775*A*tan(1/2*f*x + 1/2*e)^5 - 105*B*tan(1/2*f*x + 1/2*e)^
5 + 12250*A*tan(1/2*f*x + 1/2*e)^4 - 175*B*tan(1/2*f*x + 1/2*e)^4 - 14350*A
*tan(1/2*f*x + 1/2*e)^3 + 910*B*tan(1/2*f*x + 1/2*e)^3 + 10185*A*tan(1/2*f*
x + 1/2*e)^2 - 756*B*tan(1/2*f*x + 1/2*e)^2 - 3955*A*tan(1/2*f*x + 1/2*e) +
 427*B*tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(tan(1/2*f*x + 1/2*e)
- 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.46

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^4}\,dx = -\frac{\left(\frac{32\,A}{21}-\frac{64\,B}{105}-\frac{16\,A\,\sin(e+f\,x)}{21}+\frac{32\,B\,\sin(e+f\,x)}{105}\right)\cos\left(e+f\,x\right)^4+\left(\frac{8\,A}{7}+\frac{12\,B}{35}-\frac{8\,A\,\sin(e+f\,x)}{7}-\frac{12\,B\,\sin(e+f\,x)}{35}+\cdots\right.}{a^2\,c^4\,f\,\left(4\cos\left(e+\cdots\right)\right.}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x)
```

```
[Out] -((10*B)/21 - (4*A)/21 + (10*A*sin(e + f*x))/21 - (4*B*sin(e + f*x))/21 + c
os(e + f*x)^3*((8*A)/7 + (12*B)/35 - (8*A*sin(e + f*x))/7 - (12*B*sin(e + f
*x))/35 + ((4*sin(e + f*x) - 4)*((4*A)/7 + (6*B)/35))/2) - cos(e + f*x)^2*(
(16*A)/21 - (32*B)/105 - (8*A*sin(e + f*x))/7 + (16*B*sin(e + f*x))/35) + c
os(e + f*x)^4*((32*A)/21 - (64*B)/105 - (16*A*sin(e + f*x))/21 + (32*B*sin(
e + f*x))/105))/(a^2*c^4*f*(4*cos(e + f*x)^3*sin(e + f*x) - 4*cos(e + f*x)^
3 + 2*cos(e + f*x)^5))
```

3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 175

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx &= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}\\ &\quad+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}\end{aligned}$$

```
[Out] 1/9*(A+B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+1/21*(2*A-B)*sec(f*x+e)
^3/a^2/c^3/f/(c-c*sin(f*x+e))^2+1/21*(2*A-B)*sec(f*x+e)^3/a^2/f/(c^5-c^5*si
n(f*x+e))+4/21*(2*A-B)*tan(f*x+e)/a^2/c^5/f+4/63*(2*A-B)*tan(f*x+e)^3/a^2/c
^5/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 3852}

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx &= \frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}\\ &\quad+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}\end{aligned}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*
Sec[e + f*x]^3)/(21*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + ((2*A - B)*Sec[e +
f*x]^3)/(21*a^2*f*(c^5 - c^5*Sin[e + f*x])) + (4*(2*A - B)*Tan[e + f*x])/(2
1*a^2*c^5*f) + (4*(2*A - B)*Tan[e + f*x]^3)/(63*a^2*c^5*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(5(2A-B))\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{21a^2c^4}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\sec^3(e+fx)}{21a^2f(c^5-c^5\sin(e+fx))} + \frac{(4(2A-B))\int \sec^4(e+fx)\,dx}{21a^2c^5}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\sec^3(e+fx)}{21a^2f(c^5-c^5\sin(e+fx))} - \frac{(4(2A-B))\mathrm{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{21a^2c^5f}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\sec^3(e+fx)}{21a^2f(c^5-c^5\sin(e+fx))} + \frac{4(2A-B)\tan(e+fx)}{21a^2c^5f} + \frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}$$

Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.88

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-10752B+180(31A-5B)\cos(}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f*x
)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e + f*x
)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*
x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e + f*x
] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e + f*x)]
- 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e + f*x)
] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5*(e + f*
x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*c^5*f*(-1
 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

method	result
risch	$\frac{16i\left(72iA\,\mathrm{e}^{5i(fx+e)}-36iB\,\mathrm{e}^{5i(fx+e)}+42B\,\mathrm{e}^{6i(fx+e)}+4iA\,\mathrm{e}^{3i(fx+e)}+54A\,\mathrm{e}^{4i(fx+e)}-2iB\,\mathrm{e}^{3i(fx+e)}-27B\,\mathrm{e}^{4i(fx+e)}-12i}{63\left(\mathrm{e}^{i(fx+e)}-i\right)^{9}\left(\mathrm{e}^{i(fx+e)}+i\right)^{3}f\,c^{5}a^{2}}$
parallelrisch	$\frac{-126A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(378A-126B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-546A+252B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-126A-378B)\left(\tan^{8}\left(\right.\right.}{}$
derivativedivides	$\frac{-\frac{-\frac{A}{16}+\frac{B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{7A}{64}-\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a}$
default	$\frac{-\frac{-\frac{A}{16}+\frac{B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{7A}{64}-\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a}$
norman	$\frac{(4A-8B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{(6A-2B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{38A+2B}{63afc}+\frac{8(2A-B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7acf}-\frac{2A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{4(92A-6}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 16/63*I*(72*I*A*exp(5*I*(f*x+e))-36*I*B*exp(5*I*(f*x+e))+42*B*exp(6*I*(f*x+
e))+4*I*A*exp(3*I*(f*x+e))+54*A*exp(4*I*(f*x+e))-2*I*B*exp(3*I*(f*x+e))-27*
B*exp(4*I*(f*x+e))-12*I*A*exp(I*(f*x+e))+24*A*exp(2*I*(f*x+e))+6*I*B*exp(I*
(f*x+e))-12*B*exp(2*I*(f*x+e))-2*A+B)/(exp(I*(f*x+e))-I)^9/(exp(I*(f*x+e))+
I)^3/f/c^5/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$=\frac{8\,(2\,A-B)\cos\left(fx+e\right)^{6}-36\,(2\,A-B)\cos\left(fx+e\right)^{4}+15\,(2\,A-B)\cos\left(fx+e\right)^{2}+\left(24\,(2\,A-B)\cos\left(\right.\right.}{63\left(3\,a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}-\left(a^{2}c^{5}f\cos\left(fx+\right.\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A -
B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x + e
```

```
)^2 - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a^2*c^5*f*cos(f*x + e)^5 -
4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^5*f*cos(f*
x + e)^3)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5868 vs. 2(160) = 320.

Time = 33.91 (sec) , antiderivative size = 5868, normalized size of antiderivative = 33.53

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*A*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12
 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)*
*10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/
2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f
*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 +
 f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2
+ f*x/2) - 63*a**2*c**5*f) + 378*A*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*tan
(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*
tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c
**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2
*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**
2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*A*tan(e/2 + f*x/2)**9/(63
*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 +
756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9
- 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/
2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x
/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 126*A*tan(e/2
 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2
 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(
e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*
f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5
*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f)
 + 756*A*tan(e/2 + f*x/2)**7/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**
2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*
a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 22
68*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 +
 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3
 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) -
```

63*a**2*c**5*f) - 588*A*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/
2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f
*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e
/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan
(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) - 612*A*tan(e/2 + f*x/2)**5/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
2*c5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 900*A*tan(e/2 + f*x/2)**
4/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**
11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2
)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*
x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 +
f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2
+ f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 470*A*ta
n(e/2 + f*x/2)**3/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*ta
n(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f
*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**
5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*
c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2
*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c*
*5*f) - 78*A*tan(e/2 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378
*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 -
126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8
+ 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2
)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/
2) - 63*a**2*c**5*f) + 102*A*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x
/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 +
f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(
e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*ta
n(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*t
an(e/2 + f*x/2) - 63*a**2*c**5*f) - 38*A/(63*a**2*c**5*f*tan(e/2 + f*x/2)**
12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2
)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*
x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 +
f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2
+ f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/
2 + f*x/2) - 63*a**2*c**5*f) - 126*B*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*t
an(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*

f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c*
*5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2
*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a*
*2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a
2*c5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 252*B*tan(e/2 + f*x/2)**9/(
63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11
+ 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**
9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2
)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*
x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f
*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 378*B*tan(e
/2 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e
/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*ta
n(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f
*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**
5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*
f) + 84*B*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a*
*2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126
*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5
+ 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**
3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2)
- 63*a**2*c**5*f) - 72*B*tan(e/2 + f*x/2)**5/(63*a**2*c**5*f*tan(e/2 + f*x/
2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f
*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e
/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan
(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) - 324*B*tan(e/2 + f*x/2)**4/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
2*c5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 256*B*tan(e/2 + f*x/2)**
3/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**
11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2
)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*
x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 +
 f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2
+ f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 150*B*ta
n(e/2 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*ta
n(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f
*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**

```
5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*
c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2
*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c*
*5*f) + 12*B*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a*
*2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126
*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5
+ 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**
3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2)
- 63*a**2*c**5*f) - 2*B/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**
5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*
c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a*
*2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 75
6*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a
**2*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c
)**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(168) = 336$.

Time = 0.30 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1
)^9 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 - 19)/(a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
+ 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36
*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2
*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^
5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) + B*(6*sin(f*x + e)/(cos(f*x + e)
+ 1) - 75*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 128*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 - 162*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 36*sin(f*x + e)^5/
```

```
(cos(f*x + e) + 1)^5 + 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 189*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8 + 126*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 63
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1)/(a^2*c^5 - 6*a^2*c^5*sin(f*x +
e)/(cos(f*x + e) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*
a^2*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 36*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2
*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 2*a^2*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 - a^2*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12))/f
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = -\frac{\frac{21\left(21\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-24\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,A-13\,B\right)}{a^2c^5\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}+\frac{3591\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+315\,B}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 36
*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c^5
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*tan
(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*x +
1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e)^6 -
 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 107730*A
*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*tan(1/2*
f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f*x + 1/2
*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*e) + 2196
*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x + 1/2*e) -
1)^9))/f
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.93

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2\left(7A-14B-14A\sin\left(e+fx\right)+7B\sin\left(e+fx\right)+30A\cos\left(e+fx\right)^2-76A\cos\left(e+fx\right)^3-72A\,c\right.}{}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x)
```

```
[Out] (2*(7*A - 14*B - 14*A*sin(e + f*x) + 7*B*sin(e + f*x) + 30*A*cos(e + f*x)^2
 - 76*A*cos(e + f*x)^3 - 72*A*cos(e + f*x)^4 + 57*A*cos(e + f*x)^5 + 16*A*c
os(e + f*x)^6 - 15*B*cos(e + f*x)^2 - 4*B*cos(e + f*x)^3 + 36*B*cos(e + f*x
)^4 + 3*B*cos(e + f*x)^5 - 8*B*cos(e + f*x)^6 - 40*A*cos(e + f*x)^2*sin(e +
 f*x) + 76*A*cos(e + f*x)^3*sin(e + f*x) + 48*A*cos(e + f*x)^4*sin(e + f*x)
 - 19*A*cos(e + f*x)^5*sin(e + f*x) + 20*B*cos(e + f*x)^2*sin(e + f*x) + 4*
B*cos(e + f*x)^3*sin(e + f*x) - 24*B*cos(e + f*x)^4*sin(e + f*x) - B*cos(e
+ f*x)^5*sin(e + f*x)))/(63*a^2*c^5*f*(8*cos(e + f*x)^3*sin(e + f*x) - 2*co
s(e + f*x)^5*sin(e + f*x) - 8*cos(e + f*x)^3 + 6*cos(e + f*x)^5))
```

3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 243

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{21(3A-8B)c^5x}{2a^3}-\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} \\
&\quad -\frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f} \\
&\quad -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8}+\frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} \\
&\quad -\frac{6a^5(3A-8B)c^5\cos^7(e+fx)}{5f\left(a^2+a^2\sin(e+fx)\right)^4}-\frac{42a^5(3A-8B)c^5\cos^5(e+fx)}{5f\left(a^4+a^4\sin(e+fx)\right)^2}
\end{aligned}$$

```
[Out] -21/2*(3*A-8*B)*c^5*x/a^3-7*(3*A-8*B)*c^5*cos(f*x+e)^3/a^3/f-21/2*(3*A-8*B)
*c^5*cos(f*x+e)*sin(f*x+e)/a^3/f-1/5*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a*sin
(f*x+e))^8+2/15*a^3*(3*A-8*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6-6/5*a^5
*(3*A-8*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^4-42/5*a^5*(3*A-8*B)*c^5
*cos(f*x+e)^5/f/(a^4+a^4*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = -\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f}+\frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6}-\frac{21c^5(3A-8B)\sin(e+fx)\cos(e+fx)}{2a^3f}-\frac{21c^5x(3A-8B)}{2a^3}-\frac{42a^5c^5(3A-8B)\cos^5(e+fx)}{5f\left(a^4\sin(e+fx)+a^4\right)^2}-\frac{6a^5c^5(3A-8B)\cos^7(e+fx)}{5f\left(a^2\sin(e+fx)+a^2\right)^4}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-21*(3*A - 8*B)*c^5*x)/(2*a^3) - (7*(3*A - 8*B)*c^5*Cos[e + f*x]^3)/(a^3*f
) - (21*(3*A - 8*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(2*a^3*f) - (a^5*(A - B)
*c^5*Cos[e + f*x]^11)/(5*f*(a + a*Sin[e + f*x])^8) + (2*a^3*(3*A - 8*B)*c^5
*Cos[e + f*x]^9)/(15*f*(a + a*Sin[e + f*x])^6) - (6*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^7)/(5*f*(a^2 + a^2*Sin[e + f*x])^4) - (42*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^5)/(5*f*(a^4 + a^4*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^8}\,dx\\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8}-\frac{1}{5}\left(a^4(3A-8B)c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^7}\,dx\\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8}+\frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6}\\
&\quad+\frac{1}{5}\left(3a^2(3A-8B)c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8}+\frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6}\\
&\quad-\frac{6a(3A-8B)c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^4}-\frac{1}{5}\left(21(3A-8B)c^5\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx
\end{aligned}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} - \frac{6a(3A-8B)c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^4} - \frac{42(3A-8B)c^5\cos^5(e+fx)}{5af(a+a\sin(e+fx))^2} - \frac{(21(3A-8B)c^5)\int\frac{\cos^4(e+fx)}{a+a\sin(e+fx)}dx}{a^2}$$

$$= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} - \frac{6a(3A-8B)c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^4} - \frac{42(3A-8B)c^5\cos^5(e+fx)}{5af(a+a\sin(e+fx))^2} - \frac{(21(3A-8B)c^5)\int\cos^2(e+fx)\,dx}{a^3}$$

$$= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} - \frac{6a(3A-8B)c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^4} - \frac{42(3A-8B)c^5\cos^5(e+fx)}{5af(a+a\sin(e+fx))^2} - \frac{(21(3A-8B)c^5)\int 1\,dx}{2a^3}$$

$$= -\frac{21(3A-8B)c^5x}{2a^3} - \frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} - \frac{6a(3A-8B)c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^4} - \frac{42(3A-8B)c^5\cos^5(e+fx)}{5af(a+a\sin(e+fx))^2}$$

Mathematica [A] (verified)

Time = 12.08 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(768(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-384(A-B)\left(\cos\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)*
Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64
*(21*A - 31*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 128*(54*A - 119*B)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 630*(3*A - 8*B)
*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*(32*A - 127*B)*Cos[
e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 5*B*Cos[3*(e + f*x)]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(A - 8*B)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^10*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c^5\left(-\frac{\left(\frac{A}{2}-4B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-31B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(16A-64B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+4B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8A-\frac{95B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\right.}{}$
default	$\frac{2c^5\left(-\frac{\left(\frac{A}{2}-4B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-31B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(16A-64B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+4B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8A-\frac{95B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\right.}{}$
parallelrisch	$-\frac{63c^5\left(\left(\frac{19297}{378}B-\frac{1223}{63}A-10fxA+\frac{80}{3}fxB\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{40}{3}fxB+5fxA-\frac{2543}{378}B+\frac{341}{126}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{39439}{3780}B\right.\right.}{}$
risch	$-\frac{63c^5xA}{2a^3}+\frac{84c^5xB}{a^3}-\frac{B\,c^5\mathrm{e}^{3i(fx+e)}}{24a^3f}-\frac{ic^5\mathrm{e}^{2i(fx+e)}A}{8a^3f}+\frac{ic^5\mathrm{e}^{2i(fx+e)}B}{a^3f}-\frac{4c^5\mathrm{e}^{i(fx+e)}A}{a^3f}+\frac{127c^5\mathrm{e}^{i(fx+e)}B}{8a^3f}-$
norman	Expression too large to display

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^5/a^3*(-((1/2*A-4*B)*tan(1/2*f*x+1/2*e)^5+(8*A-31*B)*tan(1/2*f*x+1/2*
e)^4+(16*A-64*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+4*B)*tan(1/2*f*x+1/2*e)+8*A-9
5/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3-21/2*(3*A-8*B)*arctan(tan(1/2*f*x+1/2*e))
-1/4*(-256*A+256*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(32*A-96*B)/(tan(1/2*f*x+1
/2*e)+1)^2-(32*A-80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(96*A-32*B)/(tan(1/2*f*x+
1/2*e)+1)^3-1/5*(128*A-128*B)/(tan(1/2*f*x+1/2*e)+1)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{10\,Bc^5\cos(fx+e)^6+15\,(A-6\,B)c^5\cos(fx+e)^5+10\,(21\,A-74\,B)c^5\cos(fx+e)^4-1260\,(3\,A-8}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/30*(10*B*c^5*cos(f*x + e)^6 + 15*(A - 6*B)*c^5*cos(f*x + e)^5 + 10*(21*A
 - 74*B)*c^5*cos(f*x + e)^4 - 1260*(3*A - 8*B)*c^5*f*x - 192*(A - B)*c^5 +
(315*(3*A - 8*B)*c^5*f*x + (2373*A - 6128*B)*c^5)*cos(f*x + e)^3 + (945*(3*
A - 8*B)*c^5*f*x - 2*(753*A - 2248*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8
*B)*c^5*f*x + 2*(323*A - 848*B)*c^5)*cos(f*x + e) + (10*B*c^5*cos(f*x + e)^
5 - 5*(3*A - 20*B)*c^5*cos(f*x + e)^4 + 5*(39*A - 128*B)*c^5*cos(f*x + e)^3
 - 1260*(3*A - 8*B)*c^5*f*x + 192*(A - B)*c^5 + (315*(3*A - 8*B)*c^5*f*x -
2*(1089*A - 2744*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8*B)*c^5*f*x + 2*(3
07*A - 832*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2
 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. 2(228) = 456.

Time = 37.88 (sec) , antiderivative size = 10608, normalized size of antiderivative = 43.65

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-945*A*c**5*f*x*tan(e/2 + f*x/2)**11/(30*a**3*f*tan(e/2 + f*x/2)
**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 7
50*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3
*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(
e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x
/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*tan(e/2
 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
```

*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 +
 f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)
9 + 750*a3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 13
80*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3
*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/
2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 23625*A*c**5*f*x
*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)*
*8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 13
80*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*
f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
 + f*x/2) + 30*a**3*f) - 35910*A*c**5*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*ta
n(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 +
 f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)*
*7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 11
40*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f
*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 43470*A*c
5*f*x*tan(e/2 + f*x/2)6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*ta
n(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 +
f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)*
*6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 75
0*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*
tan(e/2 + f*x/2) + 30*a**3*f) - 43470*A*c**5*f*x*tan(e/2 + f*x/2)**5/(30*a*
*3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*ta
n(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 +
f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)*
*5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390
*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 35
910*A*c**5*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a*
*3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan
(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 +
f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)*
*4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*
a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 23625*A*c**5*f*x*tan(e/2 + f*x/2)**3
/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a*
*3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan
(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 +
f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**
3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*
f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 +
 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**
3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan
(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 +

```
f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2
 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*tan(e/2 + f*x
/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390
*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*
tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2
 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2
)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 945*A*c**5*f*x/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2
 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2
)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 +
1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**
3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e
/2 + f*x/2) + 30*a**3*f) - 1950*A*c**5*tan(e/2 + f*x/2)**10/(30*a**3*f*tan(
e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f
*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7
 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140
*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*t
an(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 9270*A*c**5
*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)*
*8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 13
80*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*
f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
 + f*x/2) + 30*a**3*f) - 24780*A*c**5*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/
2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x
/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 +
 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a
**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan
(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 42540*A*c**5*
tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**
8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 138
0*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f
*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
+ f*x/2) + 30*a**3*f) - 66936*A*c**5*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2
 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/
2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 +
1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a*
*3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(
e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 69960*A*c**5*t
an(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f
*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8
 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380
*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*
tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 +
```

f*x/2) + 30*a**3*f) - 70548*A*c**5*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 49620*A*c**5*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 29418*A*c**5*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 12930*A*c**5*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 2976*A*c**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2520*B*c**5*f*x*tan(e/2 + f*x/2)**11/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 12600*B*c**5*f*x*tan(e/2 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 32760*B*c**5*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 63000*B*c**5*f*x*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**1

```
1 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*
a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*
tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2
 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)
**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 95760*B*c**5*f*x*tan(e/2 +
 f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**1
0 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*
a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*
tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2
+ f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
+ 30*a**3*f) + 115920*B*c**5*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f
*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**
9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380
*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f
*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2
+ f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 115920*B*c**5*f*x*
tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**
8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 138
0*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f
*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
+ f*x/2) + 30*a**3*f) + 95760*B*c**5*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan
(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 +
f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**
7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 114
0*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*
tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 63000*B*c*
*5*f*x*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan
(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f
*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**
6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750
*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) + 32760*B*c**5*f*x*tan(e/2 + f*x/2)**2/(30*a**
3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan
(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f
*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**
5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*
a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 126
00*B*c**5*f*x*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f
*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2
 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/
2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 +
 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3
*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2520*B*c**5*f*x/(30*a**3*f*tan(e/2 + f*x
/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
```

+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
3*f*tan(e/2 + f*x/2)6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*t
an(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 5040*B*c**5*tan(e/2
+ f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)*
*10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 114
0*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*
f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/
2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2
) + 30*a**3*f) + 25140*B*c**5*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/
2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a*
*3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*ta
n(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 64180*B*c**5*tan(e/2
+ f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**
10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140
*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f
*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2
+ f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
+ 30*a**3*f) + 116360*B*c**5*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/
2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a*
*3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*ta
n(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 173416*B*c**5*tan(e/2
+ f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)*
*10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 114
0*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*
f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/
2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2
) + 30*a**3*f) + 190720*B*c**5*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x
/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
3*f*tan(e/2 + f*x/2)6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*t
an(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 184448*B*c**5*tan(e/
2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
10 + 390*a3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) + 133720*B*c**5*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*
x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*

```
a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*
tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
 f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 77768*B*c**5*tan(e/
2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) + 34540*B*c**5*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)
**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 7
50*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3
*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(
e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x
/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 7916*B*c**5/(30*a**3*f*
tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2
 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2
)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 +
1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3
*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f), Ne(f, 0)
), (x*(A + B*sin(e))*(-c*sin(e) + c)**5/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3282 vs. $2(231) = 462$.

Time = 0.38 (sec) , antiderivative size = 3282, normalized size of antiderivative = 13.51

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^5*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(
a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
+ a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos
```

(f*x + e) + 1))/a^3) - A*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 5*B*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*A*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 60*B*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 20*A*c^5*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 20*B*c^5*((95*sin(f*x + e)/(cos(f*x +

```
e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^5*(20*sin(f*x + e)/(cos(f*x + e)
 + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 40*A*c^5*(5*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 20*
B*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5
*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5) + 30*A*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a
^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^5*(5*
sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{315\,(3\,Ac^5-8\,Bc^5)(fx+e)}{a^3}+\frac{10\left(3\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+48\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-186\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+96\,A\cdots\right.}{\left(\tan\left(\frac{1}{2}\,fx\cdots\right.\right.}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

[Out] -1/30*(315*(3*A*c^5 - 8*B*c^5)*(f*x + e)/a^3 + 10*(3*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^5 + 48*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 186*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 96*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 384*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^5*tan(1/2*f*x + 1/2*e) + 24*B*c^5*tan(1/2*f*x + 1/2*e) + 48*A*c^5 - 190*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a^3) + 64*(30*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 75*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 135*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 345*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 255*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 595*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 165*A*c^5*tan(1/2*f*x + 1/2*e) - 395*B*c^5*tan(1/2*f*x + 1/2*e) + 39*A*c^5 - 94*B*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f

Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.06

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(431\,A\,c^5-\frac{3454\,B\,c^5}{3}\right)+\frac{496\,A\,c^5}{5}-\frac{3958\,B\,c^5}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(65\,A\,c^5-168\,B\,c^5\right)+\tan\left(\frac{e}{2}+\right.}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}+5\right.} - \frac{21\,c^5\,\mathrm{atan}\left(\frac{21\,c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(3\,A-8\,B)}{63\,A\,c^5-168\,B\,c^5}\right)(3\,A-8\,B)}{a^3\,f}$$

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^3,x)

[Out] - (tan(e/2 + (f*x)/2)*(431*A*c^5 - (3454*B*c^5)/3) + (496*A*c^5)/5 - (3958*B*c^5)/15 + tan(e/2 + (f*x)/2)^10*(65*A*c^5 - 168*B*c^5) + tan(e/2 + (f*x)/2)^9*(309*A*c^5 - 838*B*c^5) + tan(e/2 + (f*x)/2)^8*(826*A*c^5 - (6418*B*c^5)/3) + tan(e/2 + (f*x)/2)^7*(1418*A*c^5 - (11636*B*c^5)/3) + tan(e/2 + (f*x)/2)^3*(1654*A*c^5 - (13372*B*c^5)/3) + tan(e/2 + (f*x)/2)^5*(2332*A*c^5 - (19072*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((4903*A*c^5)/5 - (38884*B*c^5)/15) + tan(e/2 + (f*x)/2)^6*((11156*A*c^5)/5 - (86708*B*c^5)/15) + tan(e/2 + (f*x)/2)^4*((11758*A*c^5)/5 - (92224*B*c^5)/15))/(f*(13*a^3*tan(e/2 + (f*x)/2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*a^3*tan(e/2 + (f*x)/2)^4 + 46*a^3*tan(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (f*x)/2)^6 + 38*a^3*tan(e/2 + (f*x)/2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*a^3*tan(e/2 + (f*x)/2)^9 + 5*a^3*tan(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x)/2)^11 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (21*c^5*atan((21*c^5*tan(e/2 + (f*x)/2)*(3*A - 8*B))/(63*A*c^5 - 168*B*c^5))*(3*A - 8*B))/(a^3*f)

3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 201

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx &= -\frac{7(2A-7B)c^4x}{2a^3}-\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f}\\ &\quad-\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5}\\ &\quad-\frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3}-\frac{7(2A-7B)c^4\cos^3(e+fx)}{6f\left(a^3+a^3\sin(e+fx)\right)}\end{aligned}$$

```
[Out] -7/2*(2*A-7*B)*c^4*x/a^3-7/2*(2*A-7*B)*c^4*cos(f*x+e)/a^3/f-1/5*a^4*(A-B)*c
^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^7+2/15*a^2*(2*A-7*B)*c^4*cos(f*x+e)^7/f/
(a+a*sin(f*x+e))^5-14/15*(2*A-7*B)*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^3-7/
6*(2*A-7*B)*c^4*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2938, 2759, 2758, 2761, 8}

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx &= -\frac{a^4c^4(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}-\frac{7c^4(2A-7B)\cos(e+fx)}{2a^3f}-\frac{7c^4(2A-7B)\cos^3(e+fx)}{6f\left(a^3\sin(e+fx)+a^3\right)}\\ &\quad-\frac{7c^4x(2A-7B)}{2a^3}+\frac{2a^2c^4(2A-7B)\cos^7(e+fx)}{15f(a\sin(e+fx)+a)^5}-\frac{14c^4(2A-7B)\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^3}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*(2*A - 7*B)*c^4*x)/(2*a^3) - (7*(2*A - 7*B)*c^4*Cos[e + f*x])/(2*a^3*f)
 - (a^4*(A - B)*c^4*Cos[e + f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (2*a^2*(
2*A - 7*B)*c^4*Cos[e + f*x]^7)/(15*f*(a + a*Sin[e + f*x])^5) - (14*(2*A - 7
*B)*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^3) - (7*(2*A - 7*B)*c^4*
Cos[e + f*x]^3)/(6*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} - \frac{1}{5}\left(a^3(2A-7B)c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} + \frac{1}{15}\left(7a(2A-7B)c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3} - \frac{\left(7(2A-7B)c^4\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{3a}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3} - \frac{7(2A-7B)c^4\cos^3(e+fx)}{6f\left(a^3+a^3\sin(e+fx)\right)} - \frac{\left(7(2A-7B)c^4\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{2a^2}$$

$$= -\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3} - \frac{7(2A-7B)c^4\cos^3(e+fx)}{6f\left(a^3+a^3\sin(e+fx)\right)} - \frac{\left(7(2A-7B)c^4\right)\int 1\,dx}{2a^3}$$

$$= -\frac{7(2A-7B)c^4x}{2a^3} - \frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3} - \frac{7(2A-7B)c^4\cos^3(e+fx)}{6f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 11.63 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(384(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-192(A-B)\left(\cos\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)*
Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64*
(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)*Sin
[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 210*(2*A - 7*B)*(e
+ f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 60*(A - 7*B)*Cos[e + f*x]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^8*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2c^4\left(-\frac{-128A+128B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8A-24B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{64A-64B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{64A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}\right.\right.}{2}}{}\right.}{f\,a^3}$
default	$\frac{2c^4\left(-\frac{-128A+128B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8A-24B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{64A-64B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{64A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}\right.\right.}{2}}{}\right.}{f\,a^3}$
parallelrisch	$-\frac{7c^4\left(\left(\frac{471}{7}B-\frac{281}{14}A-10fxA+35fxB\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{35}{2}fxB+5fxA-\frac{761}{84}B+\frac{131}{42}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{1937}{140}B+\frac{291}{70}\right.\right.}{}$
risch	$-\frac{7c^4xA}{a^3}+\frac{49c^4xB}{2a^3}+\frac{iB\,c^4e^{2i(fx+e)}}{8a^3f}-\frac{c^4e^{i(fx+e)}A}{2a^3f}+\frac{7c^4e^{i(fx+e)}B}{2a^3f}-\frac{c^4e^{-i(fx+e)}A}{2a^3f}+\frac{7c^4e^{-i(fx+e)}B}{2a^3f}-$
norman	Expression too large to display

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^4/a^3*(-1/4*(-128*A+128*B)/(tan(1/2*f*x+1/2*e)+1)^4-(8*A-24*B)/(tan(1
/2*f*x+1/2*e)+1)-1/5*(64*A-64*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(64*A-32*B)/(
tan(1/2*f*x+1/2*e)+1)^3+16*B/(tan(1/2*f*x+1/2*e)+1)^2-(-1/2*B*tan(1/2*f*x+1
/2*e)^3+(A-7*B)*tan(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)+A-7*B)/(1+tan
(1/2*f*x+1/2*e)^2)^2-7/2*(2*A-7*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(189) = 378$.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.95

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \frac{15\,Bc^4\cos{(fx+e)}^5 - 30\,(A-6\,B)c^4\cos{(fx+e)}^4 + 420\,(2\,A-7\,B)c^4fx + 96\,(A-B)c^4 - (105\,(2\,A-}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/30*(15*B*c^4*cos(f*x + e)^5 - 30*(A - 6*B)*c^4*cos(f*x + e)^4 + 420*(2*A
- 7*B)*c^4*f*x + 96*(A - B)*c^4 - (105*(2*A - 7*B)*c^4*f*x + (554*A - 1819*
B)*c^4)*cos(f*x + e)^3 - (315*(2*A - 7*B)*c^4*f*x - 2*(134*A - 619*B)*c^4)*
cos(f*x + e)^2 + 6*(35*(2*A - 7*B)*c^4*f*x + 2*(74*A - 249*B)*c^4)*cos(f*x
+ e) - (15*B*c^4*cos(f*x + e)^4 + 15*(2*A - 11*B)*c^4*cos(f*x + e)^3 - 420*
(2*A - 7*B)*c^4*f*x + 96*(A - B)*c^4 + (105*(2*A - 7*B)*c^4*f*x - 2*(262*A
- 827*B)*c^4)*cos(f*x + e)^2 - 6*(35*(2*A - 7*B)*c^4*f*x + 2*(66*A - 241*B)
*c^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
 e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. $2(185) = 370$.

Time = 23.49 (sec) , antiderivative size = 7337, normalized size of antiderivative = 36.50

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-210*A*c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)*
*9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*
a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*ta
```

n(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(e
/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)
8 + 360*a3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780
*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*t
an(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 +
f*x/2) + 30*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/
2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2
)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 78
0*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*
tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4200*A*c**
4*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e
/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/
2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 6
00*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f
*tan(e/2 + f*x/2) + 30*a**3*f) - 5460*A*c**4*f*x*tan(e/2 + f*x/2)**5/(30*a*
*3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x
/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 +
360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) -
5460*A*c**4*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a
3*f*tan(e/2 + f*x/2)8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan
(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*
x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4200*A*c**4*f*x*tan(e/2 + f*x/2
)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*
a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*ta
n(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f
*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 3
0*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)
9 + 150*a3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600
*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*t
an(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(
e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**
8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a
3*f*tan(e/2 + f*x/2)5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan
(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*
x/2) + 30*a**3*f) - 210*A*c**4*f*x/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**
3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e
/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/
2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 1
50*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 480*A*c**4*tan(e/2 + f*x/2)**8/(3
0*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*
tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 +

```
 f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**
3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*
f) - 1980*A*c**4*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a
**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan
(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*
x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5420*A*c**4*tan(e/2 + f*x/2)**6
/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3
*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/
2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2
)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 7060*A*c**4*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 15
0*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*
tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 +
 f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**
2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 10308*A*c**4*tan(e/2 + f*x/2
)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*
a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*ta
n(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f
*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 3
0*a**3*f) - 7940*A*c**4*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9
+ 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**
3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e
/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 6036*A*c**4*tan(e/2 + f*
x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 3
60*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f
*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2
+ f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
+ 30*a**3*f) - 2860*A*c**4*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9
+ 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**
3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e
/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 668*A*c**4/(30*a**3*f*ta
n(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f
*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5
+ 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**
3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 735*B*
c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*ta
n(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f
*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4
+ 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**
3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 3675*B*c**4*f*x*tan(e/2 + f*x/2)**8/(30
*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*t
an(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 +
```

f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3
 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f
) + 8820*B*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 15
0*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*
tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 +
 f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**
2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 14700*B*c**4*f*x*tan(e/2 + f
*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 +
360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*
f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2
 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
 + 30*a**3*f) + 19110*B*c**4*f*x*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f
*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7
 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**
3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e
/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 19110*B*c**4*f*
x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
 f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**
6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a
3*f*tan(e/2 + f*x/2)3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan
(e/2 + f*x/2) + 30*a**3*f) + 14700*B*c**4*f*x*tan(e/2 + f*x/2)**3/(30*a**3*
f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2
 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)
5 + 780*a3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360
*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 88
20*B*c**4*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3
*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/
2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2
)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15
0*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 3675*B*c**4*f*x*tan(e/2 + f*x/2)/(
30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f
*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2
+ f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)*
*3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3
*f) + 735*B*c**4*f*x/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6
 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a*
*3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(
e/2 + f*x/2) + 30*a**3*f) + 1470*B*c**4*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(
e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x
/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 +
780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*
f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 7290*B*c
4*tan(e/2 + f*x/2)7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2
 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)

```
**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600
*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) + 17410*B*c**4*tan(e/2 + f*x/2)**6/(30*a**3*f*
tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 +
 f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**
5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a
**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2621
0*B*c**4*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*ta
n(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f
*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4
+ 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**
3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 33798*B*c**4*tan(e/2 + f*x/2)**4/(30*a*
*3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x
/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 +
360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) +
 28750*B*c**4*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3
*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/
2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2
)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15
0*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 20406*B*c**4*tan(e/2 + f*x/2)**2/(
30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f
*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2
+ f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)*
*3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3
*f) + 10070*B*c**4*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a*
*3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(
e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x
/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2308*B*c**4/(30*a**3*f*tan(e/2 +
 f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**
7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a
**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan
(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f), Ne(f, 0)), (x*
(A + B*sin(e))*(-c*sin(e) + c)**4/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2394 vs. $2(189) = 378$.

Time = 0.35 (sec) , antiderivative size = 2394, normalized size of antiderivative = 11.91

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^4}{(a + a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
```

hm="maxima")

```
[Out] 1/15*(B*c^4*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5
*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*A*c^4*((105
*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^3) + 24*B*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) - 8*A*c^4*((95*sin(f*x + e)/(cos(f*x + e)
 + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) + 12*B*c^4*((95*sin(f*x + e)/(cos(f*x + e)
 + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^4*(20*sin(f*x + e)/(cos(f*x + e) +
 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
```

```
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 24*A*c^4*(5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 16*B*
c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) + 24*A*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^4*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.44

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{105\,(2\,Ac^4-7\,Bc^4)(fx+e)}{a^3}-\frac{30\left(Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^4+14\,B\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(105*(2*A*c^4 - 7*B*c^4)*(f*x + e)/a^3 - 30*(B*c^4*tan(1/2*f*x + 1/2*
e)^3 - 2*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 14*B*c^4*tan(1/2*f*x + 1/2*e)^2 - B
*c^4*tan(1/2*f*x + 1/2*e) - 2*A*c^4 + 14*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 +
1)^2*a^3) + 32*(15*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^4*tan(1/2*f*x + 1/
2*e)^4 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^3
 + 130*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 380*B*c^4*tan(1/2*f*x + 1/2*e)^2 + 80
*A*c^4*tan(1/2*f*x + 1/2*e) - 250*B*c^4*tan(1/2*f*x + 1/2*e) + 19*A*c^4 - 5
9*B*c^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 15.72 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{286\,A\,c^4}{3}-\frac{1007\,B\,c^4}{3}\right)+\frac{334\,A\,c^4}{15}-\frac{1154\,B\,c^4}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(16\,A\,c^4-49\,B\,c^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+5\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+12\,a^3\right.} - \frac{7\,c^4\,\mathrm{atan}\left(\frac{7\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-7\,B)}{14\,A\,c^4-49\,B\,c^4}\right)(2\,A-7\,B)}{a^3\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((286*A*c^4)/3 - (1007*B*c^4)/3) + (334*A*c^4)/15 - (
1154*B*c^4)/15 + tan(e/2 + (f*x)/2)^8*(16*A*c^4 - 49*B*c^4) + tan(e/2 + (f*
x)/2)^7*(66*A*c^4 - 243*B*c^4) + tan(e/2 + (f*x)/2)^6*((542*A*c^4)/3 - (174
1*B*c^4)/3) + tan(e/2 + (f*x)/2)^5*((706*A*c^4)/3 - (2621*B*c^4)/3) + tan(e
/2 + (f*x)/2)^3*((794*A*c^4)/3 - (2875*B*c^4)/3) + tan(e/2 + (f*x)/2)^2*((1
006*A*c^4)/5 - (3401*B*c^4)/5) + tan(e/2 + (f*x)/2)^4*((1718*A*c^4)/5 - (56
33*B*c^4)/5))/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*x)/2)^3
 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a^3*tan(e
/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f*x)/2)^8
+ a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (7*c^4*atan
((7*c^4*tan(e/2 + (f*x)/2)*(2*A - 7*B))/(14*A*c^4 - 49*B*c^4))*(2*A - 7*B))
/(a^3*f)
```

3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 153

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-6B)c^3x}{a^3}-\frac{(A-6B)c^3\cos(e+fx)}{a^3f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}-\frac{2a^3(A-6B)c^3\cos^3(e+fx)}{3f\left(a^3+a^3\sin(e+fx)\right)^2}$$

```
[Out] -(A-6*B)*c^3*x/a^3-(A-6*B)*c^3*cos(f*x+e)/a^3/f-1/5*a^3*(A-B)*c^3*cos(f*x+e
)^7/f/(a+a*sin(f*x+e))^6+2/15*a*(A-6*B)*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
^4-2/3*a^3*(A-6*B)*c^3*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2759, 2761, 8}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{c^3(A-6B)\cos(e+fx)}{a^3f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{2a^3c^3(A-6B)\cos^3(e+fx)}{3f\left(a^3\sin(e+fx)+a^3\right)^2}-\frac{c^3x(A-6B)}{a^3}+\frac{2ac^3(A-6B)\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(((A - 6*B)*c^3*x)/a^3) - ((A - 6*B)*c^3*Cos[e + f*x])/(a^3*f) - (a^3*(A -
 B)*c^3*Cos[e + f*x]^7)/(5*f*(a + a*Sin[e + f*x])^6) + (2*a*(A - 6*B)*c^3*C
os[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^4) - (2*a^3*(A - 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(a^3 + a^3*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx\\
&= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}-\frac{1}{5}\left(a^2(A-6B)c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}\\
&\quad+\frac{1}{3}\left((A-6B)c^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\
&= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}\\
&\quad-\frac{2(A-6B)c^3\cos^3(e+fx)}{3af(a+a\sin(e+fx))^2}-\frac{\left((A-6B)c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{a^2}\\
&= -\frac{(A-6B)c^3\cos(e+fx)}{a^3f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}\\
&\quad+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}-\frac{2(A-6B)c^3\cos^3(e+fx)}{3af(a+a\sin(e+fx))^2}-\frac{\left((A-6B)c^3\right)\int 1\,dx}{a^3}\\
&= -\frac{(A-6B)c^3x}{a^3}-\frac{(A-6B)c^3\cos(e+fx)}{a^3f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}\\
&\quad+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}-\frac{2(A-6B)c^3\cos^3(e+fx)}{3af(a+a\sin(e+fx))^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(153) = 306.

Time = 11.42 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.01

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-24(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*x)
/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e + f*x
```

```
)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5)*(c - c*Sin[e + f*x])^3)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2c^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{32}{5\left(\tan\left(\right.\right.}}{f\,a^3}$
default	$\frac{2c^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{32}{5\left(\tan\left(\right.\right.}}{f\,a^3}$
risch	$-\frac{c^3xA}{a^3}+\frac{6c^3xB}{a^3}+\frac{B\,c^3e^{i(fx+e)}}{2a^3f}+\frac{B\,c^3e^{-i(fx+e)}}{2a^3f}+\frac{\frac{112A\,c^3e^{2i(fx+e)}}{3}-24iA\,c^3e^{3i(fx+e)}+\frac{56iA\,c^3e^{i(fx+e)}}{3}-12A\,c}{}$
parallelrisch	$-\frac{c^3\left(\left(\left(\frac{233}{2}+60fx\right)B-10fxA-24A\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(\left(-30fx-\frac{33}{2}\right)B+5fxA+\frac{16A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{243}{10}B+\frac{24}{5}A+fx\right.\right.}{f\,a^3\left(-10\cos\left(\right.\right.}$
norman	$-\frac{5c^3(A-6B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}-\frac{14c^3(A-6B)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{30c^3(A-6B)x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{5c^3(A-6B)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{c^3(}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^3/a^3*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A-6*B)*arctan(tan(1/2*f*x+1/2*e))-
1/4*(-64*A+64*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-8*A-8*B)/(tan(1/2*f*x+1/2*e
)+1)^2-(2*A-6*B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(32*A-32*B)/(tan(1/2*f*x+1/2*e)
+1)^5-1/3*(40*A-24*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(147) = 294$.

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$=\frac{15\,Bc^3\cos\left(fx+e\right)^4+60\,(A-6\,B)c^3fx+24\,(A-B)c^3-\left(15\,(A-6\,B)c^3fx+(46\,A-231\,B)c^3\right)\cos\left(fx\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*c^3*cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (15
*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*cos(f*x + e)^3 - (45*(A - 6*B)*c^3
*f*x - 2*(A - 66*B)*c^3)*cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*A -
 31*B)*c^3)*cos(f*x + e) + (15*B*c^3*cos(f*x + e)^3 + 60*(A - 6*B)*c^3*f*x
- 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*cos(f*x +
e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*cos(f*x + e))*sin(f*x +
 e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e)
- 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x
 + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. 2(143) = 286.

Time = 13.77 (sec) , antiderivative size = 4665, normalized size of antiderivative = 30.49

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*A*c**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f*x*t
an(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x
/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**
3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/
2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2
)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f
*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*A*c**3*f*x*tan(
e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225
*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f
*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**
3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
```

) - 75*A*c**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3
*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/
2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 15*A*c**3*f*x/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 60*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*A*c**3*tan(e/2 + f*x/2)**
5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/
2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 460*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
3*f*tan(e/2 + f*x/2)4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 320*A*c**3*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 452*A*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
200*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan
(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*
x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 52*A*c**3/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**
3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*x*tan(e/2 + f*x/2)**6/
(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f
*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2
+ f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*
f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)*

```
*4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*f*x*tan(e/2 + f*x/2)**3/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*
x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x
/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**3*f*x/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 180*B*c**3*tan
(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 870*B*c**3*tan(e/2 + f*x/2)**5/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
 2010*B*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 2220*B*c**3*tan(e/2 + f*x/2)
**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) + 2232*B*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/
2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 22
5*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1230*B*c**3
*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2
25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 282*B*c**3/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A +
B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. 2(147) = 294.

Time = 0.35 (sec) , antiderivative size = 1679, normalized size of antiderivative = 10.97

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(3*B*c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
 + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11
*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^3) - A*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a^3) + 3*B*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) - A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*A*c^3*(5*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^3*(5*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^
3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*
```

```
c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^3*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \frac{\frac{30\,Bc^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3} - \frac{15\,(Ac^3-6\,Bc^3)(fx+e)}{a^3} - \frac{4\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Bc^3}{}}{15\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(f
*x + e)/a^3 - 4*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*tan(1/2*f*x + 1
/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 5
0*A*c^3*tan(1/2*f*x + 1/2*e) - 270*B*c^3*tan(1/2*f*x + 1/2*e) + 13*A*c^3 -
63*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.18

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{40\,A\,c^3}{3}-82\,B\,c^3\right)+\frac{52\,A\,c^3}{15}-\frac{94\,B\,c^3}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(4\,A\,c^3-12\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(8}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+11\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+1}$$

$$-\frac{2\,c^3\,\mathrm{atan}\left(\frac{2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A-6\,B)}{2\,A\,c^3-12\,B\,c^3}\right)\,(A-6\,B)}{a^3\,f}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((40*A*c^3)/3 - 82*B*c^3) + (52*A*c^3)/15 - (94*B*c^3
)/5 + tan(e/2 + (f*x)/2)^6*(4*A*c^3 - 12*B*c^3) + tan(e/2 + (f*x)/2)^5*(8*A
*c^3 - 58*B*c^3) + tan(e/2 + (f*x)/2)^3*((64*A*c^3)/3 - 148*B*c^3) + tan(e/
2 + (f*x)/2)^4*((92*A*c^3)/3 - 134*B*c^3) + tan(e/2 + (f*x)/2)^2*((452*A*c^
3)/15 - (744*B*c^3)/5))/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2 +
(f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 + 5*
a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2 +
 (f*x)/2))) - (2*c^3*atan((2*c^3*tan(e/2 + (f*x)/2)*(A - 6*B))/(2*A*c^3 - 1
2*B*c^3))*(A - 6*B))/(a^3*f)
```

3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 110

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{Bc^2x}{a^3}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2Bc^2\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] B*c^2*x/a^3-1/5*a^2*(A-B)*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5-2/3*B*c^2*c
os(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*B*c^2*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2759, 8}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{2Bc^2\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bc^2x}{a^3}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*c^2*x)/a^3 - (a^2*(A - B)*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^
5) - (2*B*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*B*c^2*Cos[e
 + f*x])/(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}+\left(aBc^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}-\frac{\left(Bc^2\right)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{a}\\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}\\
&\quad+\frac{2Bc^2\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}+\frac{\left(Bc^2\right)\int 1\,dx}{a^3}\\
&= \frac{Bc^2x}{a^3}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2Bc^2\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(110) = 220.

Time = 11.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.47

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-12(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^5)*(c - c*Sin[e + f*x])^2)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,a^3}$
default	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,a^3}$
parallelrisch	$-\frac{2c^2\left(-\frac{B\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(-\frac{5}{2}fxB+A-B\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-5fx-4)B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-5fxB+2A-\frac{34}{3}B\right)\right.}{f\,a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
risch	$\frac{B\,c^2x}{a^3}-\frac{2\left(-30A\,c^2e^{2i(fx+e)}+15A\,c^2e^{4i(fx+e)}+250B\,c^2e^{2i(fx+e)}-180iB\,c^2e^{3i(fx+e)}+140iB\,c^2e^{i(fx+e)}-75B\,c^2e^{4i(\right.}{15f\,a^3\left(e^{i(fx+e)}+i\right)^5}$
norman	$\frac{\frac{B\,c^2x}{a}+\frac{8B\,c^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}+\frac{48B\,c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}+\frac{B\,c^2x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{6A\,c^2-46B\,c^2}{15fa}+\frac{40B\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3fa}+\frac{64B\,c^2}{}}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^2/a^3*(B*arctan(tan(1/2*f*x+1/2*e))-1/4*(-32*A+32*B)/(tan(1/2*f*x+1/2
*e)+1)^4-(A-B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1
)^5-1/3*(24*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3+4*A/(tan(1/2*f*x+1/2*e)+1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(106) = 212$.

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.54

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{60\,Bc^2fx - (15\,Bc^2fx - (3\,A-43\,B)c^2)\cos{(fx+e)}^3 - 12\,(A-B)c^2 - (45\,Bc^2fx - (9\,A+11\,B)c^2)\,c}{15\,\left(a^3f\cos{(fx+e)}^3 + 3\,a\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 12
*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^
2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 - (15
*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9*B)*
c^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos
(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.

Time = 7.72 (sec) , antiderivative size = 1647, normalized size of antiderivative = 14.97

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*
tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*A*c**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
```

```
15*B*c**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*
f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*c**2*f*x*tan(
e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**3/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) + 75*B*c**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*c**2*f*x/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) + 30*B*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1
20*B*c**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 340*B*c**2*tan(e/2 + f
*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 200*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(
e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/
2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 46*B*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*si
n(e) + c)**2/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 1134, normalized size of antiderivative = 10.31

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*c^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
```

```
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 2*A*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$=\frac{\frac{15\,(fx+e)Bc^2}{a^3}-\frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-170\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*c^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*c^
2*tan(1/2*f*x + 1/2*e)^4 - 60*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c^2*tan(1
```

```
/2*f*x + 1/2*e)^2 - 170*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*c^2*tan(1/2*f*
x + 1/2*e) + 3*A*c^2 - 23*B*c^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 16.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{c^2\,(120\,B+150\,B\,(e+f\,x))}{15}-10\,B\,c^2\,(e+f\,x)\right)+\frac{c^2\,(46\,B-6\,A+15\,B\,(e+f\,x))}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{c^2\,(30\,B-\dots}{\dots}\right.}{\dots}$$
$$+\frac{B\,c^2\,x}{a^3}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^3*((c^2*(120*B + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f
*x)) + (c^2*(46*B - 6*A + 15*B*(e + f*x)))/15 + tan(e/2 + (f*x)/2)^4*((c^2*
(30*B - 30*A + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) + tan(e/2 + (f*x)/2
)^2*((c^2*(340*B - 60*A + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f*x)) + tan(
e/2 + (f*x)/2)*((c^2*(200*B + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) - B*
c^2*(e + f*x))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5) + (B*c^2*x)/a^3
```

3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{a(A-11B)c\cos(e+fx)}{15f\left(a^2+a^2\sin(e+fx)\right)^2} + \frac{(A+4B)c\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -2/5*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^3+1/15*a*(A-11*B)*c*cos(f*x+e)/f
/(a^2+a^2*sin(f*x+e))^2+1/15*(A+4*B)*c*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3046, 2936, 2829, 2727}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{c(A+4B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)} + \frac{ac(A-11B)\cos(e+fx)}{15f\left(a^2\sin(e+fx)+a^2\right)^2} - \frac{2c(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*(A - B)*c*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) + (a*(A - 11*B)*c*
Cos[e + f*x])/(15*f*(a^2 + a^2*Sin[e + f*x])^2) + ((A + 4*B)*c*Cos[e + f*x]
)/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{c\int \frac{aA-6aB+5aB\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{((A+4B)c)\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(A+4B)c\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 6.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{c\left(-15(A+B)\cos\left(e+\frac{fx}{2}\right)+5(A+B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)-25B\sin\left(\frac{fx}{2}\right)-15B\sin\left(2e+\frac{3fx}{2}\right)+\right.}{30a^3f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(
f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*
x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
parallelrisch	$-\frac{2c\left(A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(5A-B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{(A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}+\frac{4A}{15}+\frac{B}{15}\right)}{f\,a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
derivativedivides	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^3}$
default	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^3}$
risch	$\frac{-\frac{10Bc\,e^{2i(fx+e)}}{3}+2iBc\,e^{3i(fx+e)}-\frac{2iBc\,e^{i(fx+e)}}{3}+2Bc\,e^{4i(fx+e)}-\frac{2iAc\,e^{i(fx+e)}}{3}+\frac{2Ac\,e^{2i(fx+e)}}{3}+\frac{2Ac}{15}+\frac{8Bc}{15}+2iAc\,e^{3i(fx-}}{f\,a^3\left(e^{i(fx+e)}+i\right)^5}$
norman	$\frac{-\frac{8Ac+2Bc}{15fa}-\frac{2Ac\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(23Ac-3Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-\frac{2(7Ac+7Bc)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{2(11Ac-Bc)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2a^2\left(\tan\right.}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*c*(A*tan(1/2*f*x+1/2*e)^4+(A+B)*tan(1/2*f*x+1/2*e)^3+1/3*(5*A-B)*tan(1/2
*f*x+1/2*e)^2+1/3*(A+B)*tan(1/2*f*x+1/2*e)+4/15*A+1/15*B)/f/a^3/(tan(1/2*f*
x+1/2*e)+1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{(A+4B)c\cos(fx+e)^3 - (2A-7B)c\cos(fx+e)^2 + 3(A-B)c\cos(fx+e) + 6(A-B)c - ((A+4}{15\left(a^3f\cos(fx+e)^3 + 3a^3f\cos(fx+e)^2 - 2a^3f\cos(fx+e) - 4a^3f + \left(a^3f\cos(\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B)
*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c*c
os(f*x + e) + 6*(A - B)*c)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a
^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(97) = 194.

Time = 4.42 (sec) , antiderivative size = 1035, normalized size of antiderivative = 10.05

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
```

```
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 10*B*c*tan(e
/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f*x/2)/(15*a**3*f*ta
n(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*
x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*
a**3*f) - 2*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(
e) + c)/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(97) = 194$.

Time = 0.21 (sec) , antiderivative size = 733, normalized size of antiderivative = 7.12

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 2*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{15\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 15*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 25*A*c*tan(1/2*f*x + 1/2*e)^2 - 5*B*c*tan(1/2*f
*x + 1/2*e)^2 + 5*A*c*tan(1/2*f*x + 1/2*e) + 5*B*c*tan(1/2*f*x + 1/2*e) + 4
*A*c + B*c)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{41\,A\,c}{4}-\frac{B\,c}{4}-\frac{11\,A\,c\,\cos(e+f\,x)}{2}+\frac{B\,c\,\cos(e+f\,x)}{2}+5\,A\,c\,\sin\left(e+f\,x\right)+5\,B\,c\,\sin\left(e+f\,x\right)-\frac{3\,A}{}\right.}{15\,a^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,c}{}\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((41*A*c)/4 - (B*c)/4 - (11*A*c*cos(e + f*x))/2 + (B*
c*cos(e + f*x))/2 + 5*A*c*sin(e + f*x) + 5*B*c*sin(e + f*x) - (3*A*c*cos(2*
e + 2*f*x))/4 + (3*B*c*cos(2*e + 2*f*x))/4 - (5*A*c*sin(2*e + 2*f*x))/4 - (
5*B*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f
*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2
- pi/4 + (5*f*x)/2))/4))
```

3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))} + \frac{2(3A+2B)\tan(e+fx)}{15a^3cf}$$

```
[Out] -1/5*(A-B)*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/15*(3*A+2*B)*sec(f*x+e)/c/
f/(a^3+a^3*sin(f*x+e))+2/15*(3*A+2*B)*tan(f*x+e)/a^3/c/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2938, 2751, 3852, 8}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{2(3A+2B)\tan(e+fx)}{15a^3cf} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3\sin(e+fx)+a^3)} - \frac{(A-B)\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/5*((A - B)*Sec[e + f*x])/(a*c*f*(a + a*Sin[e + f*x])^2) - ((3*A + 2*B)*S
ec[e + f*x])/(15*c*f*(a^3 + a^3*Sin[e + f*x])) + (2*(3*A + 2*B)*Tan[e + f*x
])/(15*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} + \frac{(3A+2B)\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{5a^2c}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))} + \frac{(2(3A+2B))\int\sec^2(e+fx)\,dx}{15a^3c}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))} - \frac{(2(3A+2B))\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{15a^3cf}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))} + \frac{2(3A+2B)\tan(e+fx)}{15a^3cf}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$$
$$= \frac{\cos(e+fx)(-80B-5(9A+B)\cos(e+fx)+32(3A+2B)\cos(2(e+fx))+9A\cos(3(e+fx))+B\cos(}{240a^3c}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e +
 f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] - 8
0*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*Sin[
3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a^3*c*f*(-1 + Sin[e + f*x])*(1
+ Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4\left(-3iA-12A\,\mathrm{e}^{i(fx+e)}+10B\,\mathrm{e}^{3i(fx+e)}-2iB-8B\,\mathrm{e}^{i(fx+e)}+10iB\,\mathrm{e}^{2i(fx+e)}+15iA\,\mathrm{e}^{2i(fx+e)}\right)}{15\left(\mathrm{e}^{i(fx+e)}+i\right)^{5}\left(\mathrm{e}^{i(fx+e)}-i\right)a^{3}cf}$
parallelrisch	$\frac{-30A\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A-30B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A-40B)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-40B\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(18A-}{15fca^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(-2B+2A)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{3}cf}$
default	$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(-2B+2A)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{3}cf}$
norman	$\frac{\frac{12A-2B}{15cfa}-\frac{2(6A+7B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cfa}-\frac{2A\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cfa}+\frac{2(9A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cfa}+\frac{2(2A-7B)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5cfa}-\frac{2(9A+4B)(\mathrm{t}}{3}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*(-3*I*A-12*A*exp(I*(f*x+e))+10*B*exp(3*I*(f*x+e))-2*I*B-8*B*exp(I*(f*
x+e))+10*I*B*exp(2*I*(f*x+e))+15*I*A*exp(2*I*(f*x+e)))/(exp(I*(f*x+e))+I)^5
/(exp(I*(f*x+e))-I)/a^3/c/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$$
$$=\frac{4\,(3\,A+2\,B)\cos{(fx+e)}^{2}+\left(2\,(3\,A+2\,B)\cos{(fx+e)}^{2}-9\,A-6\,B\right)\sin{(fx+e)}-6\,A-9\,B}{15\,\left(a^{3}cf\cos{(fx+e)}^{3}-2\,a^{3}cf\cos{(fx+e)}\sin{(fx+e)}-2\,a^{3}cf\cos{(fx+e)}\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A -
6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x
+ e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(85) = 170$.

Time = 4.54 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.12

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*
a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*
f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*
tan(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
+ f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f*x/2)*
*3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*
a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*
f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 18*A*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(
e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f
*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) -
 15*a**3*c*f) + 12*A/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/
2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75
*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c
*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**3/(15*a**3*c*f*
tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/
2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a**3*c*f*tan(e/2 + f*x/2)*
*6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75
*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f)
 - 8*B*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(
e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f
*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 2*B/(15*a**3*c*f*t
an(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
+ f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2
) - 15*a**3*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e
) + c)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(96) = 192$.

Time = 0.21 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.15

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))}\,dx$$
$$= \frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1\right)}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} - \frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}- \cdots\right.}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin \cdots}{(\cos(f \cdots}}\right.}{15f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a
^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6) - 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - a^3*c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))}\,dx =$$
$$-\frac{\frac{15(A+B)}{a^3c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)} + \frac{105A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-15B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+270A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+30B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+360A\tan\left(\frac{1}{2}fx+ \cdots\right.}{a^3c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}}{60f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A + B)/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 - 15*B*tan(1/2*f*x + 1/2*e)^4 + 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 + 40*B*tan(1/2*f*x
 + 1/2*e)^2 + 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 + 7*B)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx =$$

$$-\frac{2\left(\frac{15\,A\,\cos(e+f\,x)}{4}-\frac{5\,B}{2}-\frac{5\,B\,\cos(e+f\,x)}{8}-\frac{15\,A\,\sin(e+f\,x)}{4}-\frac{5\,B\,\sin(e+f\,x)}{2}+3\,A\,\cos\left(2\,e+2\,f\,x\right)-\frac{3\,A\,\cos(3\,e+3\,f\,x)}{4}\right.}{15\,a^3\,c\,f\,\left(\frac{5\,\cos(e+f\,x)}{4}-\right.}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*((15*A*cos(e + f*x))/4 - (5*B)/2 - (5*B*cos(e + f*x))/8 - (15*A*sin(e +
 f*x))/4 - (5*B*sin(e + f*x))/2 + 3*A*cos(2*e + 2*f*x) - (3*A*cos(3*e + 3*f
*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*
x) + (3*A*sin(3*e + 3*f*x))/4 - (B*sin(2*e + 2*f*x))/2 + (B*sin(3*e + 3*f*x
))/2))/(15*a^3*c*f*((5*cos(e + f*x))/4 - cos(3*e + 3*f*x)/4 + sin(2*e + 2*f
*x)))
```

3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = -\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}+\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}$$

```
[Out] -1/5*(A-B)*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+1/5*(4*A+B)*tan(f*x+e)/a
^3/c^2/f+1/15*(4*A+B)*tan(f*x+e)^3/a^3/c^2/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}-\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3\sin(e+fx)+a^3)}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] -1/5*((A - B)*Sec[e + f*x]^3)/(c^2*f*(a^3 + a^3*Sin[e + f*x])) + ((4*A + B)
*Tan[e + f*x])/(5*a^3*c^2*f) + ((4*A + B)*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{a^2c^2}\\
&= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))}+\frac{(4A+B)\int \sec^4(e+fx)\,dx}{5a^3c^2}\\
&= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))}-\frac{(4A+B)\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{5a^3c^2f}\\
&= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}+\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(90) = 180.

Time = 1.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.63

$$
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx
$$

$$
= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(240B+54(A-B)\cos(e+fx)
$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A*C
os[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*Cos[
4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e + f*x
)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)]
 + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 + Sin[e
 + f*x])^2*(1 + Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

method	result
risch	$\frac{4i\left(24iA\,e^{3i(fx+e)}+6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}-8A\,e^{2i(fx+e)}+2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}-4A-B\right)}{15\left(e^{i(fx+e)}+i\right)^{5}\left(e^{i(fx+e)}-i\right)^{3}f\,c^{2}a^{3}}$
parallelrisch	$\frac{-30A\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-30A-30B)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10A-20B)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(50A-10B)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\cdots}{15f\,c^{2}a^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}\left(\tan\cdots\right)}$
derivativedivides	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{B}{4}+\frac{A}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{5A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2B-2A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}\cdots}{a^{3}c^{2}f}$
default	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{B}{4}+\frac{A}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{5A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2B-2A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}\cdots}{a^{3}c^{2}f}$
norman	$\frac{-\frac{6A+4B}{10cfa}-\frac{4(4A+B)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cfa}+\frac{A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{(14A+16B)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10cfa}-\frac{(6A+4B)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2cfa}-\frac{(2A+8B)\cdots}{\cdots}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^{2}\left(\tan\left(\frac{fx}{2}\cdots\right)\right)}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 4/15*I*(24*I*A*exp(3*I*(f*x+e))+6*I*B*exp(3*I*(f*x+e))+15*B*exp(4*I*(f*x+e)
)+8*I*A*exp(I*(f*x+e))-8*A*exp(2*I*(f*x+e))+2*I*B*exp(I*(f*x+e))-2*B*exp(2*
I*(f*x+e))-4*A-B)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^3/f/c^2/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = -\frac{2\,(4\,A+B)\cos\left(fx+e\right)^4-(4\,A+B)\cos\left(fx+e\right)^2-\left(2\,(4\,A+B)\cos\left(fx+e\right)^2+4\,A+B\right)\sin\left(fx+e\right)}{15\,\left(a^3c^2f\cos\left(fx+e\right)^3\sin\left(fx+e\right)+a^3c^2f\cos\left(fx+e\right)^3\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B)
*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e)^
3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(82) = 164.

Time = 9.00 (sec) , antiderivative size = 2674, normalized size of antiderivative = 29.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 30*A*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 50*A*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
```

30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 42*A*tan(e/2 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 18*A*tan(e/2 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*
a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a
3*c2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a*
*3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c
2*f) + 6*A/(15*a3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 30*B*tan(e/2
 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 20*B*tan(e/2
 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 10*B*tan(e/2
 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) + 16*B*tan(e/2
 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 18*B*tan(e/2
 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 12*B*tan(e/2
 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*
x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x
/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/
2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 6*B/(15*a**3*c*
*2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**
2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2
*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*
f*tan(e/2 + f*x/2) - 15*a**3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**3*(-c*sin(e) + c)**2), True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(84) = 168$.

Time = 0.22 (sec) , antiderivative size = 650, normalized size of antiderivative = 7.22

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^2}\,dx = \frac{2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-3\right)}{a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}+\frac{\cdots}{a^3c\cdots}\right.}{15f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/(a^3*c^
2 + 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*
c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) + B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3)/(a^3*c^2 + 2*a^3*c^2*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 0.47 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.46

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^2}\,dx = -\frac{\frac{5\left(15A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+9B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-24A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-12B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13A+7B\right)}{a^3c^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}+\frac{165A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45B\tan\left(\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 + 9*B*tan(1/2*f*x + 1/2*e)^2 - 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 - 45*B*tan(1/2*f
*x + 1/2*e)^4 + 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 - 70*B*tan(1/2*f*x + 1/2*e)^2 + 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(tan(1/
2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\left(\frac{8A}{15}+\frac{2B}{15}+\frac{16A\sin(e+fx)}{15}+\frac{4B\sin(e+fx)}{15}\right)\cos(e+fx)^2+\frac{2A}{15}+\frac{8B}{15}+\frac{8A\sin(e+fx)}{15}+\frac{2B\sin(e+fx)}{15}}{a^3c^2f\left(2\cos(e+fx)^3\sin(e+fx)+2\cos(e+fx)^3\right)} - \frac{\frac{2A}{5}-\frac{2B}{5}+\frac{2A\sin(e+fx)}{5}-\frac{2B\sin(e+fx)}{5}}{a^3c^2f\,(2\sin(e+fx)+2)} - \frac{\cos(e+fx)\left(\frac{16A}{15}+\frac{4B}{15}\right)}{a^3c^2f\,(2\sin(e+fx)+2)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x)
```

```
[Out] ((2*A)/15 + (8*B)/15 + (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 + (2*B)/15 + (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x
))/15))/(a^3*c^2*f*(2*cos(e + f*x)^3*sin(e + f*x) + 2*cos(e + f*x)^3)) - ((
2*A)/5 - (2*B)/5 + (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^3*c^2*f*
(2*sin(e + f*x) + 2)) - (cos(e + f*x)*((16*A)/15 + (4*B)/15))/(a^3*c^2*f*(2
*sin(e + f*x) + 2))
```

3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 84

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan^5(e+fx)}{5a^3c^3f}$$

```
[Out] 1/5*B*sec(f*x+e)^5/a^3/c^3/f+A*tan(f*x+e)/a^3/c^3/f+2/3*A*tan(f*x+e)^3/a^3/
c^3/f+1/5*A*tan(f*x+e)^5/a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2748, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{A\tan^5(e+fx)}{5a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*Tan[e + f*x])/(a^3*c^3*f) + (2*A*Tan[
e + f*x]^3)/(3*a^3*c^3*f) + (A*Tan[e + f*x]^5)/(5*a^3*c^3*f)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\,dx}{a^3c^3}\\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\int \sec^6(e+fx)\,dx}{a^3c^3}\\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}-\frac{A\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{a^3c^3f}\\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan^5(e+fx)}{5a^3c^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx\\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{a^3c^3f}
\end{aligned}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3
+ Tan[e + f*x]^5/5))/(a^3*c^3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\frac{16iA\,\mathrm{e}^{2i(fx+e)}}{3}+\frac{32B\,\mathrm{e}^{5i(fx+e)}}{5}+\frac{32iA\,\mathrm{e}^{4i(fx+e)}}{3}+\frac{16iA}{15}}{\left(\mathrm{e}^{i(fx+e)}+i\right)^{5}\left(\mathrm{e}^{i(fx+e)}-i\right)^{5}f\,c^{3}a^{3}}$
parallelrisch	$\frac{-2A\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-2B\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{8A\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{116A\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15}-4B\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{8A\left(\tan^{3}\left(\frac{fx}{2}\right.\right.}{3}}{f\,c^{3}a^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{11A}{8}+\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}}{a^{3}c^{3}f}$
default	$\frac{-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{11A}{8}+\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}}{a^{3}c^{3}f}$
norman	$\frac{-\frac{2B}{5acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{2A\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{76A\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}+\frac{2A\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{76A\left(\tan^{7}\left(\frac{fx}{2}\right.\right.}{15cfa}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 16/15*(5*I*A*exp(2*I*(f*x+e))+6*B*exp(5*I*(f*x+e))+10*I*A*exp(4*I*(f*x+e))+
I*A)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^5/f/c^3/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\left(8\,A\cos\left(fx+e\right)^{4}+4\,A\cos\left(fx+e\right)^{2}+3\,A\right)\sin\left(fx+e\right)+3\,B}{15\,a^{3}c^{3}f\cos\left(fx+e\right)^{5}}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/(
a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(78) = 156$.

Time = 6.10 (sec) , antiderivative size = 1098, normalized size of antiderivative = 13.07

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^3}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 -
 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 -
 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 -
 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f*x/2
)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/
2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/
2)**2 - 15*a**3*c**3*f) - 116*A*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*tan(e/2
 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/
2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/
2 + f*x/2)**2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**3/(15*a**3*c**3*f*
tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f
*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f
*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*A*tan(e/2 + f*x/2)/(15*a**3*c**
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*B*tan(e/2 + f*x/2)**8/(15*a
**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*
a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*
a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 60*B*tan(e/2 + f*x/2)**
4/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8
 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**
4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 6*B/(15*a**3*c**
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((
a*sin(e) + a)**3*(-c*sin(e) + c)**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\frac{\left(3\,\tan(fx+e)^5+10\,\tan(fx+e)^3+15\,\tan(fx+e)\right)A}{a^3c^3}+\frac{3\,B}{a^3c^3\cos(fx+e)^5}}{15\,f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*A/(a^3*c^3)
+ 3*B/(a^3*c^3*cos(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+58\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\right.}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5 a}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*tan
(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/2*
e)^4 - 20*A*tan(1/2*f*x + 1/2*e)^3 + 15*A*tan(1/2*f*x + 1/2*e) + 3*B)/((tan
(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+15\,B\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-20\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+58\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+30\,B\tan\left(\frac{e}{2}+\right.\right.}{15\,a^3\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^5}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(3*B + 15*A*tan(e/2 + (f*x)/2) - 20*A*tan(e/2 + (f*x)/2)^3 + 58*A*tan(e
/2 + (f*x)/2)^5 - 20*A*tan(e/2 + (f*x)/2)^7 + 15*A*tan(e/2 + (f*x)/2)^9 + 3
0*B*tan(e/2 + (f*x)/2)^4 + 15*B*tan(e/2 + (f*x)/2)^8))/(15*a^3*c^3*f*(tan(e
/2 + (f*x)/2)^2 - 1)^5)
```

3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 121

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx &= \frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}\\ &\quad+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}\end{aligned}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+1/7*(6*A-B)*tan(f*x+e)/a^
3/c^4/f+2/21*(6*A-B)*tan(f*x+e)^3/a^3/c^4/f+1/35*(6*A-B)*tan(f*x+e)^5/a^3/c
^4/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 3852}

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx &= \frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}\\ &\quad+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))}\end{aligned}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + ((6*A - B)*Ta
n[e + f*x])/(7*a^3*c^4*f) + (2*(6*A - B)*Tan[e + f*x]^3)/(21*a^3*c^4*f) + (
(6*A - B)*Tan[e + f*x]^5)/(35*a^3*c^4*f)
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^3c^3}\\
&= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(6A-B)\int \sec^6(e+fx)\,dx}{7a^3c^4}\\
&= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}-\frac{(6A-B)\text{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{7a^3c^4f}\\
&= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}\\
&\quad+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 325 vs. $2(121) = 242$.

Time = 2.89 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.69

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-8960B+1500(A+B)\cos(e$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -1/53760*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e +
f*x)] + 750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(e +
 f*x)] + 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e +
 f*x)] - 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin[e +
f*x] + 2560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e + f*x)
] - 7680*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*(e + f*
x)] - 300*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[5*(e + f
*x)] - 75*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)]))/(a^3*c^4*f*(-1 + Sin
[e + f*x])^4*(1 + Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{16i(120iA\,e^{5i(fx+e)}-20iB\,e^{5i(fx+e)}+70B\,e^{6i(fx+e)}+60iA\,e^{3i(fx+e)}+30A\,e^{4i(fx+e)}-10iB\,e^{3i(fx+e)}-5B\,e^{4i(fx+e)}+}{105(e^{i(fx+e)}+i)^5(e^{i(fx+e)}-i)^7 f c^4 a^3}$
parallelrisch	$\frac{-210A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A-210B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A+140B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-630A-70B)\left(\tan^{8}\left(\frac{fx}{2}+\right.\right.}{}$
derivativedivides	$\frac{-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}}{a^3c}$
default	$\frac{-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}}{a^3c}$
norman	$\frac{\frac{(-2B+2A)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cfa}-\frac{2A+2B}{7cfa}-\frac{152(6A-B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}-\frac{2A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{4B\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cfa}+\frac{(312A-752B)(}{105}}{}$

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)

[Out] -16/105*I*(120*I*A*exp(5*I*(f*x+e))-20*I*B*exp(5*I*(f*x+e))+70*B*exp(6*I*(f
*x+e))+60*I*A*exp(3*I*(f*x+e))+30*A*exp(4*I*(f*x+e))-10*I*B*exp(3*I*(f*x+e)
)-5*B*exp(4*I*(f*x+e))+12*I*A*exp(I*(f*x+e))+24*A*exp(2*I*(f*x+e))-2*I*B*ex
p(I*(f*x+e))-4*B*exp(2*I*(f*x+e))+6*A-B)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e
))-I)^7/f/c^4/a^3

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = -\frac{8\,(6\,A-B)\cos\left(fx+e\right)^{6}-4\,(6\,A-B)\cos\left(fx+e\right)^{4}-(6\,A-B)\cos\left(fx+e\right)^{2}+\left(8\,(6\,A-B)\cos\left(fx\right.\right.}{105\,\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}c^{4}\right.}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")

[Out] -1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B)
*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^2
+ 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f*x
+ e) - a^3*c^4*f*cos(f*x + e)^5)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6135 vs. 2(109) = 218.

Time = 33.49 (sec) , antiderivative size = 6135, normalized size of antiderivative = 50.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)

[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**11/(105*a**3*c**4*f*tan(e/2 + f*x/2)**1
2 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)
10 + 1050*a3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x
/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 +
f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2
+ f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2
 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**10/(105*a**3*c**4*f*

tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4
*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c
4*f*tan(e/2 + f*x/2)8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**
3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a
3*c4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*
a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**9
/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**
11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/
2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*
x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 +
f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2
+ f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 630*A*t
an(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*
tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**
4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c
4*f*tan(e/2 + f*x/2)7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3
*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a*
*3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3
*c**4*f) - 756*A*tan(e/2 + f*x/2)**7/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12
- 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**
10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2
)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*
x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 +
f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 +
f*x/2) - 105*a**3*c**4*f) + 1092*A*tan(e/2 + f*x/2)**6/(105*a**3*c**4*f*ta
n(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f
*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**
4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*
c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**
3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*
*3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 156*A*tan(e/2 + f*x/2)**5/(
105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11
- 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)
9 + 525*a3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/
2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*
x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 +
f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 780*A*tan
(e/2 + f*x/2)**4/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*ta
n(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*
f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**
4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c
4*f*tan(e/2 + f*x/2)4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c
4*f) - 90*A*tan(e/2 + f*x/2)3/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 2
10*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10

+ 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**
8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2
)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x
/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*
x/2) - 105*a**3*c**4*f) + 330*A*tan(e/2 + f*x/2)**2/(105*a**3*c**4*f*tan(e/
2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan
(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*
tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4
*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c*
*4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c
4*f*tan(e/2 + f*x/2) - 105*a3*c**4*f) - 150*A*tan(e/2 + f*x/2)/(105*a**
3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*
a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 +
 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4
 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)*
*2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 30*A/(105*a**3*c
4*f*tan(e/2 + f*x/2)12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**
3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*
a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 21
00*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 -
1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2
+ 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 210*B*tan(e/2 + f*x
/2)**10/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f
*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2
 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e
/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan
(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*t
an(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) +
140*B*tan(e/2 + f*x/2)**9/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*
c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a
3*c4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100
*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 +
 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 1
05*a**3*c**4*f) - 70*B*tan(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2
)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*
x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 +
 f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/
2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(
e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan
(e/2 + f*x/2) - 105*a**3*c**4*f) - 224*B*tan(e/2 + f*x/2)**7/(105*a**3*c**4
*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c
4*f*tan(e/2 + f*x/2)10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**
3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*

```
a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 105
0*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 2
10*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 532*B*tan(e/2 + f*x/2)
**6/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2
)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f
*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 +
 f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2
 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e
/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 376*
B*tan(e/2 + f*x/2)**5/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4
*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*
c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**
3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a
**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420
*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a
**3*c**4*f) - 220*B*tan(e/2 + f*x/2)**4/(105*a**3*c**4*f*tan(e/2 + f*x/2)**
12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2
)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*
x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 +
 f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2
 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/
2 + f*x/2) - 105*a**3*c**4*f) - 160*B*tan(e/2 + f*x/2)**3/(105*a**3*c**4*f*
tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4
*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c
**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**
3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a
**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*
a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 90*B*tan(e/2 + f*x/2)**2/
(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**1
1 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2
)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x
/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f
*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 +
 f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 60*B*tan
(e/2 + f*x/2)/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e
/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*t
an(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f
*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4
*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c*
*4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4
*f) - 30*B/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2
+ f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(
e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*ta
n(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*
tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*
```

```
f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f)
, Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e) + c)**4), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(114) = 228$.

Time = 0.24 (sec) , antiderivative size = 1019, normalized size of antiderivative = 8.42

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 45*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 80*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 110*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 188*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 266*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 112*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 35
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 70*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 105*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3*c^4*s
in(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*
a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(
cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) - 3*A
*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 35*s
in(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(114) = 228.

Time = 0.38 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = -\frac{\frac{7\left(165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-75\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+540\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+750\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{a^3c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/1680*(7*(165*A*tan(1/2*f*x + 1/2*e)^4 - 75*B*tan(1/2*f*x + 1/2*e)^4 + 54
0*A*tan(1/2*f*x + 1/2*e)^3 - 210*B*tan(1/2*f*x + 1/2*e)^3 + 750*A*tan(1/2*f
*x + 1/2*e)^2 - 280*B*tan(1/2*f*x + 1/2*e)^2 + 480*A*tan(1/2*f*x + 1/2*e) -
 170*B*tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e)
+ 1)^5) + (2205*A*tan(1/2*f*x + 1/2*e)^6 + 525*B*tan(1/2*f*x + 1/2*e)^6 - 1
0080*A*tan(1/2*f*x + 1/2*e)^5 - 1470*B*tan(1/2*f*x + 1/2*e)^5 + 21945*A*tan
(1/2*f*x + 1/2*e)^4 + 2555*B*tan(1/2*f*x + 1/2*e)^4 - 26460*A*tan(1/2*f*x +
 1/2*e)^3 - 2240*B*tan(1/2*f*x + 1/2*e)^3 + 18963*A*tan(1/2*f*x + 1/2*e)^2
+ 1407*B*tan(1/2*f*x + 1/2*e)^2 - 7476*A*tan(1/2*f*x + 1/2*e) - 434*B*tan(1
/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\left(\frac{16\,A}{35}-\frac{8\,B}{105}-\frac{32\,A\,\sin(e+f\,x)}{35}+\frac{16\,B\,\sin(e+f\,x)}{105}\right)\,\cos\left(e+f\,x\right)^4+\left(\frac{4\,A}{35}-\frac{2\,B}{105}-\frac{16\,A\,\sin(e+f\,x)}{35}+\frac{8\,B\,\sin(e+f\,x)}{105}\right)\,\cos\left(\right.}{a^3\,c^4\,f\,\left(2\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^5\right)} - \frac{\frac{2\,A}{7}+\frac{2\,B}{7}-\frac{2\,A\,\sin(e+f\,x)}{7}-\frac{2\,B\,\sin(e+f\,x)}{7}}{a^3\,c^4\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)} - \frac{\cos\left(e+f\,x\right)\,\left(\frac{32\,A}{35}-\frac{16\,B}{105}\right)}{a^3\,c^4\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x)
```

```
[Out] ((2*A)/35 - (12*B)/35 - (12*A*sin(e + f*x))/35 + (2*B*sin(e + f*x))/35 + co
s(e + f*x)^2*((4*A)/35 - (2*B)/105 - (16*A*sin(e + f*x))/35 + (8*B*sin(e +
f*x))/105) + cos(e + f*x)^4*((16*A)/35 - (8*B)/105 - (32*A*sin(e + f*x))/35
 + (16*B*sin(e + f*x))/105))/(a^3*c^4*f*(2*cos(e + f*x)^5*sin(e + f*x) - 2*
cos(e + f*x)^5)) - ((2*A)/7 + (2*B)/7 - (2*A*sin(e + f*x))/7 - (2*B*sin(e +
 f*x))/7)/(a^3*c^4*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((32*A)/35 - (16
*B)/105))/(a^3*c^4*f*(2*sin(e + f*x) - 2))
```

3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 162

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\,(c^5-c^5\sin(e+fx))}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}$$

```
[Out] 1/9*(A+B)*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/63*(7*A-2*B)*sec(f*x+
e)^5/a^3/f/(c^5-c^5*sin(f*x+e))+2/21*(7*A-2*B)*tan(f*x+e)/a^3/c^5/f+4/63*(7
*A-2*B)*tan(f*x+e)^3/a^3/c^5/f+2/105*(7*A-2*B)*tan(f*x+e)^5/a^3/c^5/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\,(c^5-c^5\sin(e+fx))}+\frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B
)*Sec[e + f*x]^5)/(63*a^3*f*(c^5 - c^5*Sin[e + f*x])) + (2*(7*A - 2*B)*Tan[
e + f*x])/(21*a^3*c^5*f) + (4*(7*A - 2*B)*Tan[e + f*x]^3)/(63*a^3*c^5*f) +
(2*(7*A - 2*B)*Tan[e + f*x]^5)/(105*a^3*c^5*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{(2(7A-2B))\int\sec^6(e+fx)\,dx}{21a^3c^5}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} - \frac{(2(7A-2B))\text{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{21a^3c^5f}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f} + \frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f} + \frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 373 vs. $2(162) = 324$.

Time = 3.75 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.30

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-184320B+1125(49A+13B\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*(e
 + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A*Co
s[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 325*B
*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)] - 12
25*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x] + 92
160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*x)] - 1
36192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4*(e + f*
x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*Sin[5*(e +
 f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168*A*Sin[7*(
e + f*x)] - 2048*B*Sin[7*(e + f*x)]))/(1290240*a^3*c^5*f*(-1 + Sin[e + f*x]
)^5*(1 + Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{32(-2iB\,e^{2i(fx+e)}+2iB+315iA\,e^{6i(fx+e)}+133iA\,e^{4i(fx+e)}-38iB\,e^{4i(fx+e)}-90iB\,e^{6i(fx+e)}+28A\,e^{i(fx+e)}+180B\,e^{7i}}{315(e^{i(fx+e)}+i)^5(e^{i(fx}}$
parallelrisch	$\frac{-630A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(1260A-630B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-420A+840B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3360A-840B)\left(\tan^{1}\right.}{}$
derivativedivides	$\frac{-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{\frac{51A}{16}+\frac{21B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{}$
default	$\frac{-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{\frac{51A}{16}+\frac{21B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{}$
norman	$\frac{\frac{14A-40B}{252cfa}+\frac{2(602A-697B)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}-\frac{A\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2cfa}-\frac{(1036A-296B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}+\frac{(476A-136B)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{315cfa}}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] -32/315*(-2*I*B*exp(2*I*(f*x+e))+2*I*B+315*I*A*exp(6*I*(f*x+e))+133*I*A*exp
(4*I*(f*x+e))-38*I*B*exp(4*I*(f*x+e))-90*I*B*exp(6*I*(f*x+e))+28*A*exp(I*(f
*x+e))+180*B*exp(7*I*(f*x+e))-8*B*exp(I*(f*x+e))+7*I*A*exp(2*I*(f*x+e))+112
*A*exp(3*I*(f*x+e))+140*A*exp(5*I*(f*x+e))-40*B*exp(5*I*(f*x+e))-32*B*exp(3
*I*(f*x+e))-7*I*A)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^9/f/c^5/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.14

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = -\frac{32\,(7\,A-2\,B)\cos\left(fx+e\right)^6-16\,(7\,A-2\,B)\cos\left(fx+e\right)^4-4\,(7\,A-2\,B)\cos\left(fx+e\right)^2-\left(16\,(7\,A-2\right.}{315\,\left(a^3c^5f\cos\left(fx+e\right)^7+2\,a^3c^5\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*(
7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^6 - 24*(7*A - 2*B)
```

```
*cos(f*x + e)^4 - 10*(7*A - 2*B)*cos(f*x + e)^2 - 49*A + 14*B)*sin(f*x + e)
 - 14*A + 49*B)/(a^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(
f*x + e) - 2*a^3*c^5*f*cos(f*x + e)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8396 vs. $2(150) = 300$.

Time = 59.13 (sec) , antiderivative size = 8396, normalized size of antiderivative = 51.83

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^5}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*tan(e/2 + f*x/2)**13/(315*a**3*c**5*f*tan(e/2 + f*x/2)**1
4 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e
/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*t
an(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**
5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1260*A*tan(e/2 + f*x/2)**12/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 42
0*A*tan(e/2 + f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*
c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a
**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8
 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 +
f*x/2) - 315*a**3*c**5*f) - 3360*A*tan(e/2 + f*x/2)**10/(315*a**3*c**5*f*ta
n(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*
f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*
c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*
a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4
- 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**
2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 966*A*tan(e/2 +
f*x/2)**9/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2
```

```
+ f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(
e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f
*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c
**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**
3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a
**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a*
*3*c**5*f) + 4956*A*tan(e/2 + f*x/2)**8/(315*a**3*c**5*f*tan(e/2 + f*x/2)**
14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/
2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
 f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(
e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*
tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5
*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c*
*5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 7224*A*tan(e/2 + f*x/2)**7/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 13
44*A*tan(e/2 + f*x/2)**6/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*
c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a
**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8
 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 +
f*x/2) - 315*a**3*c**5*f) + 3766*A*tan(e/2 + f*x/2)**5/(315*a**3*c**5*f*tan
(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f
*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c
**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a
**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 63
00*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 -
 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2
 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 700*A*tan(e/2 + f
*x/2)**4/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 +
 f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e
/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*
tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c*
*5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3
*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a*
*3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**
3*c**5*f) - 2660*A*tan(e/2 + f*x/2)**3/(315*a**3*c**5*f*tan(e/2 + f*x/2)**1
4 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2
```

)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e
/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*t
an(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**
5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1120*A*tan(e/2 + f*x/2)**2/(315*a
3*c5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 3
15*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11
 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2
 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e
/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 70*
A*tan(e/2 + f*x/2)/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f
*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c*
*5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a*
*3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 141
75*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 +
 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**
3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2)
 - 315*a**3*c**5*f) - 140*A/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a*
*3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 504
0*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10
- 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)
8 - 14175*a3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*
x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 +
 f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2
 + f*x/2) - 315*a**3*c**5*f) - 630*B*tan(e/2 + f*x/2)**12/(315*a**3*c**5*f*
tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**
5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**
3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 1417
5*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 +
 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**
4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)
2 + 1260*a3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 840*B*tan(e/2
+ f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e
/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*t
an(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**
5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**
3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*
a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 31
5*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315
*a**3*c**5*f) - 840*B*tan(e/2 + f*x/2)**10/(315*a**3*c**5*f*tan(e/2 + f*x/2
)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f
*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/

```
2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5
*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c
**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3
*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 1176*B*tan(e/2 + f*x/2)**9/(3
15*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13
 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)
**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f
*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/
2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan
(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*t
an(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) -
 966*B*tan(e/2 + f*x/2)**8/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**
3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040
*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 -
 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)*
*8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x
/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 +
f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2
+ f*x/2) - 315*a**3*c**5*f) + 2064*B*tan(e/2 + f*x/2)**7/(315*a**3*c**5*f*t
an(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5
*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3
*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175
*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 +
6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4
 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)*
*2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 3216*B*tan(e/2
+ f*x/2)**6/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/
2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*ta
n(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5
*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3
*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a
**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315
*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*
a**3*c**5*f) - 176*B*tan(e/2 + f*x/2)**5/(315*a**3*c**5*f*tan(e/2 + f*x/2)*
*14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x
/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2
+ f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan
(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f
*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**
5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c
**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 110*B*tan(e/2 + f*x/2)**4/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
```

```
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 40
*B*tan(e/2 + f*x/2)**3/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c*
*5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**
3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 630
0*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 -
 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)*
*5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/
2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*
x/2) - 315*a**3*c**5*f) - 680*B*tan(e/2 + f*x/2)**2/(315*a**3*c**5*f*tan(e/
2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*ta
n(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5
*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3
*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*
a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 50
40*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 +
1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 200*B*tan(e/2 + f*x/
2)/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2
)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f
*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/
2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*
f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**
5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5
*f) - 50*B/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2
 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan
(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*
f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*
c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a*
*3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*
a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a
**3*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e) + c
)**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(154) = 308$.

Time = 0.26 (sec) , antiderivative size = 1201, normalized size of antiderivative = 7.41

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
```

```
hm="maxima")
[Out] -2/315*(B*(100*sin(f*x + e)/(cos(f*x + e) + 1) - 340*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 55*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 88*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1608*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1032*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
483*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 588*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 420*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 420*sin(f*x + e)^11/(c
os(f*x + e) + 1)^11 - 315*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 25)/(a^3*
c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^
3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c
^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^
5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*si
n(f*x + e)^14/(cos(f*x + e) + 1)^14) - 7*A*(5*sin(f*x + e)/(cos(f*x + e) +
1) - 80*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 190*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 269*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 96*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 516*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 69*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 240*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 + 30*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 90*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 + 45*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 10)/(a^3*c^5 -
4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(
f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x + e) +
 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*sin(f*x
+ e)^14/(cos(f*x + e) + 1)^14))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(154) = 308$.

Time = 0.37 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.40

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^5}\,dx =$$

$$-\frac{\frac{21\left(435\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-225\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+1470\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-690\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2060\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-940\,B\tan\left(\frac{1}{2}\,f\cdots\right.}{a^3c^5\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/20160*(21*(435*A*tan(1/2*f*x + 1/2*e)^4 - 225*B*tan(1/2*f*x + 1/2*e)^4 +
 1470*A*tan(1/2*f*x + 1/2*e)^3 - 690*B*tan(1/2*f*x + 1/2*e)^3 + 2060*A*tan(
1/2*f*x + 1/2*e)^2 - 940*B*tan(1/2*f*x + 1/2*e)^2 + 1330*A*tan(1/2*f*x + 1/
2*e) - 590*B*tan(1/2*f*x + 1/2*e) + 353*A - 163*B)/(a^3*c^5*(tan(1/2*f*x +
1/2*e) + 1)^5) + (31185*A*tan(1/2*f*x + 1/2*e)^8 + 4725*B*tan(1/2*f*x + 1/2
*e)^8 - 185220*A*tan(1/2*f*x + 1/2*e)^7 - 11340*B*tan(1/2*f*x + 1/2*e)^7 +
546840*A*tan(1/2*f*x + 1/2*e)^6 + 15120*B*tan(1/2*f*x + 1/2*e)^6 - 961380*A
*tan(1/2*f*x + 1/2*e)^5 + 3780*B*tan(1/2*f*x + 1/2*e)^5 + 1101618*A*tan(1/2
*f*x + 1/2*e)^4 - 24318*B*tan(1/2*f*x + 1/2*e)^4 - 828492*A*tan(1/2*f*x + 1
/2*e)^3 + 33852*B*tan(1/2*f*x + 1/2*e)^3 + 404208*A*tan(1/2*f*x + 1/2*e)^2
- 19368*B*tan(1/2*f*x + 1/2*e)^2 - 116172*A*tan(1/2*f*x + 1/2*e) + 6732*B*t
an(1/2*f*x + 1/2*e) + 16373*A - 223*B)/(a^3*c^5*(tan(1/2*f*x + 1/2*e) - 1)^
9))/f
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\left(\frac{128\,B}{315}-\frac{64\,A}{45}+\frac{32\,A\,\sin(e+f\,x)}{45}-\frac{64\,B\,\sin(e+f\,x)}{315}\right)\cos\left(e+f\,x\right)^6+\left(\frac{8\,A\,\sin(e+f\,x)}{9}-\frac{20\,B}{63}-\frac{8\,A}{9}+\frac{20\,B\,\sin(e+f\,x)}{63}-\right.}{}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x)
```

```
[Out] ((4*A)/45 - (14*B)/45 - (14*A*sin(e + f*x))/45 + (4*B*sin(e + f*x))/45 - co
s(e + f*x)^5*((8*A)/9 + (20*B)/63 - (8*A*sin(e + f*x))/9 - (20*B*sin(e + f*
x))/63 + ((4*sin(e + f*x) - 4)*((4*A)/9 + (10*B)/63))/2) + cos(e + f*x)^2*(
(8*A)/45 - (16*B)/315 - (4*A*sin(e + f*x))/9 + (8*B*sin(e + f*x))/63) + cos
(e + f*x)^4*((32*A)/45 - (64*B)/315 - (16*A*sin(e + f*x))/15 + (32*B*sin(e
+ f*x))/105) - cos(e + f*x)^6*((64*A)/45 - (128*B)/315 - (32*A*sin(e + f*x)
)/45 + (64*B*sin(e + f*x))/315))/(a^3*c^5*f*(4*cos(e + f*x)^5*sin(e + f*x)
- 4*cos(e + f*x)^5 + 2*cos(e + f*x)^7))
```

3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 205

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}$$

```
[Out] 1/11*(A+B)*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+1/99*(8*A-3*B)*sec(f*x
+e)^5/a^3/f/(c^3-c^3*sin(f*x+e))^2+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^6-c
^6*sin(f*x+e))+2/33*(8*A-3*B)*tan(f*x+e)/a^3/c^6/f+4/99*(8*A-3*B)*tan(f*x+e
)^3/a^3/c^6/f+2/165*(8*A-3*B)*tan(f*x+e)^5/a^3/c^6/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2751, 3852}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+\frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + ((8*A - 3*
B)*Sec[e + f*x]^5)/(99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + ((8*A - 3*B)*Sec
[e + f*x]^5)/(99*a^3*f*(c^6 - c^6*Sin[e + f*x])) + (2*(8*A - 3*B)*Tan[e + f
*x])/(33*a^3*c^6*f) + (4*(8*A - 3*B)*Tan[e + f*x]^3)/(99*a^3*c^6*f) + (2*(8
*A - 3*B)*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f(c^2-c^2\sin(e+fx))^3} + \frac{(8A-3B)\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{11a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f(c^2-c^2\sin(e+fx))^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^3-c^3\sin(e+fx))^2} + \frac{(7(8A-3B))\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{99a^3c^5}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f(c^2-c^2\sin(e+fx))^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^3-c^3\sin(e+fx))^2} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^6-c^6\sin(e+fx))} + \frac{(2(8A-3B))\int \sec^6(e+fx)\,dx}{33a^3c^6}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f(c^2-c^2\sin(e+fx))^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^3-c^3\sin(e+fx))^2} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^6-c^6\sin(e+fx))} - \frac{(2(8A-3B))\text{Subst}\left(\int (1+2x^2+x^4)\,dx, x, -\tan(e+fx)\right)}{33a^3c^6f}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f(c^2-c^2\sin(e+fx))^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^3-c^3\sin(e+fx))^2} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f(c^6-c^6\sin(e+fx))} + \frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f} + \frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f} + \frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}$$

Mathematica [A] (verified)

Time = 5.41 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.96

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{1013760B - 3850(107A-3B)\cos(e+fx) + 135168(8A-3B)\cos(2(e+fx)) - 127330A\cos(3(e+fx))}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^6),x]
```

```
[Out] (1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e
+ f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*Co
s[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] - 105
0*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f*x)]
 + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(e + f
*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Sin[e +
f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*A*Sin[3
*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)] - 5250*
B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e + f*x)] +
```

```
37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin[7*(e + f*
x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B*Sin[8*(e +
 f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

method	result
risch	$\frac{32i\left(-48iA\,\mathrm{e}^{i(fx+e)}+30iB\,\mathrm{e}^{3i(fx+e)}+495B\,\mathrm{e}^{8i(fx+e)}-102iB\,\mathrm{e}^{5i(fx+e)}+528A\,\mathrm{e}^{6i(fx+e)}+18iB\,\mathrm{e}^{i(fx+e)}-198B\,\mathrm{e}^{6i(fx+}\right.}{495(\mathrm{e}}$
parallelrisch	$\frac{-990A\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2970A-990B)\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3630A+1980B)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4950A-2970B)}{}$
derivativedivides	$-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{20A+20B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2(53A+51B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{92A+84B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{169A}{4}+\frac{99B}{4}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{217A}{2}}{3(\tan(\frac{f}{2}}$
default	$-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{20A+20B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2(53A+51B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{92A+84B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{169A}{4}+\frac{99B}{4}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{217A}{2}}{3(\tan(\frac{f}{2}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] 32/495*I*(-48*I*A*exp(I*(f*x+e))+30*I*B*exp(3*I*(f*x+e))+495*B*exp(8*I*(f*x
+e))-102*I*B*exp(5*I*(f*x+e))+528*A*exp(6*I*(f*x+e))+18*I*B*exp(I*(f*x+e))-
198*B*exp(6*I*(f*x+e))+272*I*A*exp(5*I*(f*x+e))+400*A*exp(4*I*(f*x+e))+880*
I*A*exp(7*I*(f*x+e))-150*B*exp(4*I*(f*x+e))-330*I*B*exp(7*I*(f*x+e))+80*A*e
xp(2*I*(f*x+e))-80*I*A*exp(3*I*(f*x+e))-30*B*exp(2*I*(f*x+e))-8*A+3*B)/(exp
(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^11/f/c^6/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$=\frac{16\,(8\,A-3\,B)\cos{(fx+e)}^{8}-72\,(8\,A-3\,B)\cos{(fx+e)}^{6}+30\,(8\,A-3\,B)\cos{(fx+e)}^{4}+7\,(8\,A-3\,B)}{495\,\left(3\,a^{3}c^{6}f\cos{(fx+e)}^{7}-4\,a^{3}\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/495*(16*(8*A - 3*B)*cos(f*x + e)^8 - 72*(8*A - 3*B)*cos(f*x + e)^6 + 30*(
8*A - 3*B)*cos(f*x + e)^4 + 7*(8*A - 3*B)*cos(f*x + e)^2 + (48*(8*A - 3*B)*
cos(f*x + e)^6 - 40*(8*A - 3*B)*cos(f*x + e)^4 - 14*(8*A - 3*B)*cos(f*x + e
)^2 - 72*A + 27*B)*sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*cos(f*x + e)^7
- 4*a^3*c^6*f*cos(f*x + e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(
f*x + e)^5)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11011 vs. 2(187) = 374.

Time = 100.07 (sec) , antiderivative size = 11011, normalized size of antiderivative = 53.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-990*A*tan(e/2 + f*x/2)**15/(495*a**3*c**6*f*tan(e/2 + f*x/2)**1
6 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/
2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2
+ f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*t
an(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c*
*6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a*
*3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 495
0*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 +
2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 2970*A*tan(e/2 + f*x
/2)**14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 +
f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e
/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*
f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**
3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 3267
0*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 +
 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)*
*3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/
2) - 495*a**3*c**6*f) - 3630*A*tan(e/2 + f*x/2)**13/(495*a**3*c**6*f*tan(e/
2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*t
an(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c*
*6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*
a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 +
54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)*
*6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*
x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 +
 f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 4950*A*
tan(e/2 + f*x/2)**12/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6
*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3
```

*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 168
30*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**1
0 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x
/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2
+ f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(
e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*t
an(e/2 + f*x/2) - 495*a**3*c**6*f) + 9834*A*tan(e/2 + f*x/2)**11/(495*a**3*
c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 -
24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)
11 + 32670*a3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 +
f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(
e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f
*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**
6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6
*f) + 66*A*tan(e/2 + f*x/2)**10/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 297
0*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14
+ 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2
)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2
+ f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*ta
n(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6
*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*
c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a*
*3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 23430*A*tan(e/2 + f*x/2)**9
/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)*
*15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*
x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e
/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*
f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*
c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*
a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 49
50*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 49
5*a**3*c**6*f) + 17490*A*tan(e/2 + f*x/2)**8/(495*a**3*c**6*f*tan(e/2 + f*x
/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2
+ f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*ta
n(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c*
*6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a
3*c6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16
830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4
 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)
2 + 2970*a3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 4070*A*tan(e/2
 + f*x/2)**7/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e
/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*
tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*
c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 5445

0*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 16434*A*tan(e/2 + f*x/2)**6/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 1334*A*tan(e/2 + f*x/2)**5/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 7550*A*tan(e/2 + f*x/2)**4/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 6130*A*tan(e/2 + f*x/2)**3/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 470*A*tan(e/2 + f*x/2)**2/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 2475

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0*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 -
4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) -
495*a**3*c**6*f) + 510*A*tan(e/2 + f*x/2)/(495*a**3*c**6*f*tan(e/2 + f*x/2)
**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f
*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e
/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*
f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3
*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830
*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 -
4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2
 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 250*A/(495*a**3*c
**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a
**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 2
4750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)*
*11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 +
f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e
/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*
tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6
*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*
f) - 990*B*tan(e/2 + f*x/2)**14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 297
0*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14
+ 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2
)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2
+ f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*ta
n(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6
*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*
c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a*
*3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 1980*B*tan(e/2 + f*x/2)**13
/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)*
*15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*
x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e
/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*
f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*
c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*
a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 49
50*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 49
5*a**3*c**6*f) - 2970*B*tan(e/2 + f*x/2)**12/(495*a**3*c**6*f*tan(e/2 + f*x
/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2
+ f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*ta
n(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c*
*6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a
**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16
830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4
 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)
**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 1584*B*tan(e/2
```

+ f*x/2)**11/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(
e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f
*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3
*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 544
50*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7
- 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2
)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f
*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2
+ f*x/2) - 495*a**3*c**6*f) + 594*B*tan(e/2 + f*x/2)**10/(495*a**3*c**6*f*t
an(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**
6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a*
*3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 3
2670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)*
*9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*
x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2
+ f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(
e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 19
80*B*tan(e/2 + f*x/2)**9/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*
c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 +
16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2
)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 +
f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(
e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*
tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6
*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 10890*B*tan(e/2 + f*x/2)**8/(495*a
3*c6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4
950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**1
3 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*
x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e
/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*
tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c*
*6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3
*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*
c**6*f) + 5280*B*tan(e/2 + f*x/2)**7/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16
- 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)
14 + 4950*a3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 +
f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan
(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6
*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3
*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 29
70*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 1386*B*tan(e/2 + f*x/2
)**6/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x
/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2

+ f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*t
an(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c
6*f*tan(e/2 + f*x/2)9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a
3*c6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24
750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3
- 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2)
- 495*a**3*c**6*f) - 2604*B*tan(e/2 + f*x/2)**5/(495*a**3*c**6*f*tan(e/2 +
f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e
/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f
*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3
*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 5445
0*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 -
16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)
4 - 4950*a3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x
/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 1470*B*tan(
e/2 + f*x/2)**4/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*ta
n(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6
*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a*
*3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 5
4450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**
7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x
/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 +
f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/
2 + f*x/2) - 495*a**3*c**6*f) + 1680*B*tan(e/2 + f*x/2)**3/(495*a**3*c**6*f
*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c
6*f*tan(e/2 + f*x/2)14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*
a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 +
32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2
)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 +
f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e
/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*ta
n(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) -
1290*B*tan(e/2 + f*x/2)**2/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**
3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 495
0*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12
+ 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x
/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2
+ f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*ta
n(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*
f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c*
*6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 180*B*tan(e/2 + f*x/2)/(495*a**3
*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950
*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 -
24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2
)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2

```
+ f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan
(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*
f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c*
*6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**
6*f) - 30*B/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/
2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*t
an(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c
**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450
*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 -
32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)*
*5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x
/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 +
f*x/2) - 495*a**3*c**6*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*
(-c*sin(e) + c)**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(196) = 392$.

Time = 0.27 (sec) , antiderivative size = 1387, normalized size of antiderivative = 6.77

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/495*(A*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f
*x + e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 -
1815*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x
+ e) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/(a^3*c^6 -
6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c
^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*
c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(
f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16) + 3*B*(30
```

```
*sin(f*x + e)/(cos(f*x + e) + 1) - 215*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 245*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 434*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 880*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1815*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 + 330*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 99*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 264*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 - 495*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 330*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - 165*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 5)/(a^3*c
^6 - 6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*
a^3*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^
3*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^
3*c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(c
os(f*x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10
*a^3*c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(196) = 392$.

Time = 0.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.17

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^6}\,dx = -\frac{\frac{33\left(555\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-315\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+1920\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-1020\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2710\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1410\,B\tan}{a^3c^6\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/63360*(33*(555*A*tan(1/2*f*x + 1/2*e)^4 - 315*B*tan(1/2*f*x + 1/2*e)^4 +
 1920*A*tan(1/2*f*x + 1/2*e)^3 - 1020*B*tan(1/2*f*x + 1/2*e)^3 + 2710*A*tan
(1/2*f*x + 1/2*e)^2 - 1410*B*tan(1/2*f*x + 1/2*e)^2 + 1760*A*tan(1/2*f*x +
1/2*e) - 900*B*tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(tan(1/2*f*x
+ 1/2*e) + 1)^5) + (108405*A*tan(1/2*f*x + 1/2*e)^10 + 10395*B*tan(1/2*f*x
+ 1/2*e)^10 - 784080*A*tan(1/2*f*x + 1/2*e)^9 - 5940*B*tan(1/2*f*x + 1/2*e)
^9 + 2901195*A*tan(1/2*f*x + 1/2*e)^8 - 79695*B*tan(1/2*f*x + 1/2*e)^8 - 66
52800*A*tan(1/2*f*x + 1/2*e)^7 + 388080*B*tan(1/2*f*x + 1/2*e)^7 + 10407474
*A*tan(1/2*f*x + 1/2*e)^6 - 816354*B*tan(1/2*f*x + 1/2*e)^6 - 11435424*A*ta
n(1/2*f*x + 1/2*e)^5 + 1114344*B*tan(1/2*f*x + 1/2*e)^5 + 8949270*A*tan(1/2
*f*x + 1/2*e)^4 - 990990*B*tan(1/2*f*x + 1/2*e)^4 - 4899840*A*tan(1/2*f*x +
```

```
 1/2*e)^3 + 609840*B*tan(1/2*f*x + 1/2*e)^3 + 1816265*A*tan(1/2*f*x + 1/2*e
)^2 - 235785*B*tan(1/2*f*x + 1/2*e)^2 - 411664*A*tan(1/2*f*x + 1/2*e) + 563
64*B*tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(tan(1/2*f*x + 1/2*e
) - 1)^11))/f
```

Mupad [B] (verification not implemented)

Time = 14.67 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.31

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{2\left(\frac{165\,B\,\sin(e+f\,x)}{4}-\frac{6875\,A\,\cos(e+f\,x)}{64}-\frac{825\,B\,\cos(e+f\,x)}{64}-110\,A\,\sin\left(e+f\,x\right)-\frac{495\,B}{8}-66\,A\,\cos\left(2\,e+2\,f\,x\right)-\right.}{}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x)

[Out] (2*((165*B*sin(e + f*x))/4 - (6875*A*cos(e + f*x))/64 - (825*B*cos(e + f*x)
)/64 - 110*A*sin(e + f*x) - (495*B)/8 - 66*A*cos(2*e + 2*f*x) - (2125*A*cos
(3*e + 3*f*x))/64 - 50*A*cos(4*e + 4*f*x) + (625*A*cos(5*e + 5*f*x))/64 - 1
0*A*cos(6*e + 6*f*x) + (375*A*cos(7*e + 7*f*x))/64 + A*cos(8*e + 8*f*x) + (
99*B*cos(2*e + 2*f*x))/4 - (255*B*cos(3*e + 3*f*x))/64 + (75*B*cos(4*e + 4*
f*x))/4 + (75*B*cos(5*e + 5*f*x))/64 + (15*B*cos(6*e + 6*f*x))/4 + (45*B*co
s(7*e + 7*f*x))/64 - (3*B*cos(8*e + 8*f*x))/8 + (4125*A*sin(2*e + 2*f*x))/6
4 - 34*A*sin(3*e + 3*f*x) + (3125*A*sin(4*e + 4*f*x))/64 + 10*A*sin(5*e + 5
*f*x) + (625*A*sin(6*e + 6*f*x))/64 + 6*A*sin(7*e + 7*f*x) - (125*A*sin(8*e
 + 8*f*x))/128 + (495*B*sin(2*e + 2*f*x))/64 + (51*B*sin(3*e + 3*f*x))/4 +
(375*B*sin(4*e + 4*f*x))/64 - (15*B*sin(5*e + 5*f*x))/4 + (75*B*sin(6*e + 6
*f*x))/64 - (9*B*sin(7*e + 7*f*x))/4 - (15*B*sin(8*e + 8*f*x))/128))/(495*a
^3*c^6*f*((5*cos(5*e + 5*f*x))/32 - (17*cos(3*e + 3*f*x))/32 - (55*cos(e +
f*x))/32 + (3*cos(7*e + 7*f*x))/32 + (33*sin(2*e + 2*f*x))/32 + (25*sin(4*e
 + 4*f*x))/32 + (5*sin(6*e + 6*f*x))/32 - sin(8*e + 8*f*x)/64))
```

3.81 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 198

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c&-c\sin(e+fx))^{7/2}\,dx = \frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}\\&+\frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f}\\&+\frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f}\\&-\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}\end{aligned}$$

```
[Out] 256/3465*a*(11*A-5*B)*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/99*a*(11*
A-5*B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f-2/11*a*B*c*cos(f*x+e)^3*(c
-c*sin(f*x+e))^(5/2)/f+64/1155*a*(11*A-5*B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x
+e))^(1/2)+8/231*a*(11*A-5*B)*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2935, 2753, 2752}

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{256ac^5(11A-5B)\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}} + \frac{64ac^4(11A-5B)\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8ac^3(11A-5B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2ac^2(11A-5B)\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*(11*A - 5*B)*c^5*Cos[e + f*x]^3)/(3465*f*(c - c*Sin[e + f*x])^(3/2))
 + (64*a*(11*A - 5*B)*c^4*Cos[e + f*x]^3)/(1155*f*Sqrt[c - c*Sin[e + f*x]])
 + (8*a*(11*A - 5*B)*c^3*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(231*f) +
 (2*a*(11*A - 5*B)*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(99*f) -
(2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
```

```
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$$

$$= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f} + \frac{1}{11}(a(11A-5B)c)\int \cos^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx$$

$$= \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f} + \frac{1}{33}\left(4a(11A-5B)c^2\right)\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx$$

$$= \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f} + \frac{1}{231}\left(32a(11A-5B)c^3\right)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx$$

$$= \frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f} + \frac{(128a(11A-5B)c^4)\int\frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{1155}$$

$$= \frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}} + \frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}$$

Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{ac^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}(-35332A+27085B+60(121A-202B)\cos(}{13860f\,(\cos}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/13860*(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e +
f*x]]*(-35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*Cos
[4*(e + f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[3*(
e + f*x)] + 2870*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]))
```

Maple [A] (verified)

Time = 7.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^2a(315B(\cos^4(fx+e))+(-385A+1435B)(\cos^2(fx+e))\sin(fx+e)+(1815A-3345B)(\cos^2(fx+e)}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2aA(\sin(fx+e)-1)c^4(1+\sin(fx+e))(5(\sin^3(fx+e))-27(\sin^2(fx+e))+71\sin(fx+e)-177)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2Ba(\sin(fx+e)-1)c^4(1+\sin(fx+e)}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3465*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*(315*B*cos(f*x+e)^4+(-385*A+14
35*B)*cos(f*x+e)^2*sin(f*x+e)+(1815*A-3345*B)*cos(f*x+e)^2+(3916*A-4300*B)*
sin(f*x+e)-5324*A+4940*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\frac{2\left(315\,Bac^3\cos\left(fx+e\right)^6-35\left(11\,A-32\,B\right)ac^3\cos\left(fx+e\right)^5+5\left(209\,A-221\,B\right)ac^3\cos\left(fx\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5
+ 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(f*
x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3*co
s(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B*a*c^3*cos(f*x + e)^5 + 35*(11*
A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)^3 -
96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x + e)
- 256*(11*A - 5*B)*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(-c\sin(fx + e) + c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.49

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = -\frac{\sqrt{2}\left(6930\,Bac^3\cos\left(-\frac{3}{4}\pi + \frac{3}{2}fx + \frac{3}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 315\,Bac^3\cos\left(-\frac{11}{4}\pi + \frac{11}{2}fx + \frac{11}{2}e\right)\cdots}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
[Out] -1/55440*sqrt(2)*(6930*B*a*c^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 315*B*a*c^3*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 48510*(2*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 693*(16*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*
a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) +
 495*(10*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 9*B*a*c^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 385*(2*A*a*c^3*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (A + B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2), x
)
```

3.82 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 157

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}$$

```
[Out] 64/315*a*(3*A-B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/9*a*B*c*cos(f*
x+e)^3*(c-c*sin(f*x+e))^(3/2)/f+16/105*a*(3*A-B)*c^3*cos(f*x+e)^3/f/(c-c*si
n(f*x+e))^(1/2)+2/21*a*(3*A-B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{3046, 2935, 2753, 2752\}$

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64ac^4(3A-B)\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}} + \frac{16ac^3(3A-B)\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} + \frac{2ac^2(3A-B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*(3*A - B)*c^4*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (1
6*a*(3*A - B)*c^3*Cos[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*(
3*A - B)*c^2*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(21*f) - (2*a*B*c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx \\
&= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f} \\
&\quad +\frac{1}{3}(a(3A-B)c)\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx \\
&= \frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} \\
&\quad -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f} \\
&\quad +\frac{1}{21}\left(8a(3A-B)c^2\right)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx \\
&= \frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} \\
&\quad -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f} \\
&\quad +\frac{1}{105}\left(32a(3A-B)c^3\right)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= \frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} \\
&\quad +\frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} \\
&\quad -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{ac^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}(-942A+664B+30(3A-8B)\cos(2(e+fx$$
$$630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/630*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*
x]]*(-942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*Sin
[e + f*x] + 35*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
))
```

Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^2a(-35B(\cos^2(fx+e))\sin(fx+e)+(-45A+120B)(\cos^2(fx+e))+(-162A+194B)\sin(fx+e)+}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2aA(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3(\sin^2(fx+e))-14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2Ba(\sin(fx+e)-1)c^3(1+\sin(fx+e))(35(\sin^4(fx+e}{315\cos(fx-}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(-35*B*cos(f*x+e)^2*sin(f*x+e)
+(-45*A+120*B)*cos(f*x+e)^2+(-162*A+194*B)*sin(f*x+e)+258*A-226*B)/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.55

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{2\left(35\,Bac^2\cos\left(fx+e\right)^5+5\left(9\,A-10\,B\right)ac^2\cos\left(fx+e\right)^4+\left(117\,A-109\,B\right)ac^2\cos\left(fx+e\right)$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (1
17*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 32*
(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x + e
)^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x + e)
^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$
\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c \\
&-c\sin(e+fx))^{5/2}\,dx = a\left(\int Ac^2\sqrt{-c\sin{(e+fx)}+c}\,dx\right. \\
&+\int \left(-Ac^2\sqrt{-c\sin{(e+fx)}+c}\sin{(e+fx)}\right)\,dx \\
&+\int \left(-Ac^2\sqrt{-c\sin{(e+fx)}+c}\sin^2{(e+fx)}\right)\,dx \\
&+\int Ac^2\sqrt{-c\sin{(e+fx)}+c}\sin^3{(e+fx)}\,dx \\
&+\int Bc^2\sqrt{-c\sin{(e+fx)}+c}\sin{(e+fx)}\,dx \\
&+\int \left(-Bc^2\sqrt{-c\sin{(e+fx)}+c}\sin^2{(e+fx)}\right)\,dx \\
&+\int \left(-Bc^2\sqrt{-c\sin{(e+fx)}+c}\sin^3{(e+fx)}\right)\,dx \\
&\left.+\int Bc^2\sqrt{-c\sin{(e+fx)}+c}\sin^4{(e+fx)}\,dx\right)
\end{aligned}
$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c**2*sqrt(-c
*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c**2*sqrt(-c*sin(e + f*x)
 + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e
 + f*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x)
 + Integral(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integra
l(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*c**2*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x))
```

Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(-c\sin(fx + e) + c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.67

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = -\frac{\sqrt{2}\left(35\,Bac^2\cos\left(-\frac{9}{4}\,\pi + \frac{9}{2}\,fx + \frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 630\left(5\,Aac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(35*B*a*c^2*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 630*(5*A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
2*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 210*(A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 126*(3*A*a*c^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*A*a*c^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) - 3*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi
 + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2), x
)
```

3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 116

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] 8/105*a*(7*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/35*a*(7*A-B)*c^
2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)-2/7*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x
+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2935, 2753, 2752}

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8ac^3(7A-B)\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2(7A-B)\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a*(7*A - B)*c^3*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (2*
a*(7*A - B)*c^2*Cos[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*B*c*
Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx\\ &= -\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\\ &\quad+\frac{1}{7}(a(7A-B)c)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\end{aligned}$$

$$= \frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} - \frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f} + \frac{1}{35}(4a(7A-B)c^2)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= \frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}} + \frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} - \frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{ac\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(98A-59B+15B\cos(2(e+fx))+(-42A+66B)}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e +
 f*x)] + (-42*A + 66*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^2 a\left(-15B\left(\cos^2(fx+e)\right)+\sin(fx+e)(21A-33B)-49A+37B\right)}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2aA(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2Ba(\sin(fx+e)-1)c^2(1+\sin(fx+e))\left(15\left(\sin^3(fx+e)\right)-39\left(\sin^2(fx+e)\right)+52\right.}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(-15*B*cos(f*x+e)^2+sin(f*x+e)*
(21*A-33*B)-49*A+37*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{2\left(15\,Bac\cos\left(fx+e\right)^{4}-3\,(7\,A-6\,B)ac\cos\left(fx+e\right)^{3}+(7\,A-B)ac\cos\left(fx+e\right)^{2}-4\,(7\,A-B)ac\cos\left(f\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A -
 B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c - (
15*B*a*c*cos(f*x + e)^3 + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a*c*
cos(f*x + e) + 8*(7*A - B)*a*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = a\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx + \int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)\right)dx + \int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx + \int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)\right)dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x), x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3,
x))
```

Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(-c\sin(fx + e) + c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \frac{\sqrt{2}\left(15\,Bac\cos\left(-\frac{7}{4}\,\pi + \frac{7}{2}\,fx + \frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) - 105\left(4\,Aac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi\right.\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
[Out] 1/420*sqrt(2)*(15*B*a*c*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) - 105*(4*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) - 35*(2
*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a*c*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - B*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi
 + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))\,(c - c\,\sin(e + f\,x))^{3/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2), x
)
```

3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal result

Integrand size = 36, antiderivative size = 73

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2/15*a*(5*A+B)*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/5*a*B*c*cos(f*x+
e)^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2935, 2752}

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2ac^2(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(5*A + B)*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2*a
*B*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
```

```
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= -\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{1}{5}(a(5A+B)c)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= \frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(73) = 146.

Time = 1.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.62

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(32B-30\sqrt{2}A\sqrt{1+\cos(e+fx)}\right)+\sqrt{2}\sqrt{1+\cos(e+fx)}(5(2A+B)\cos\left(\frac{3}{2}(e+\right.}{30\sqrt{2}f\sqrt{1+\cos(e}}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/30*(a*(Cos[(e + f*x)/2]*(32*B - 30*Sqrt[2]*A*Sqrt[1 + Cos[e + f*x]]) + S
qrt[2]*Sqrt[1 + Cos[e + f*x]]*(5*(2*A + B)*Cos[(3*(e + f*x))/2] + 3*B*Cos[(
5*(e + f*x))/2] - 2*(20*A + B + 2*(5*A + B)*Cos[e + f*x] - 3*B*Cos[2*(e + f
*x)])*Sin[(e + f*x)/2]))*Sqrt[c - c*Sin[e + f*x]])/(Sqrt[2]*f*Sqrt[1 + Cos[
e + f*x]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^2 a(3B\sin(fx+e)+5A-2B)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$
parts	$-\frac{2aA(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}-\frac{2Ba(\sin(fx+e)-1)c(1+\sin(fx+e))(3(\sin^2(fx+e))-4\sin(fx+e)+8)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}-\frac{2a(A+B)(\sin(fx}{3\cos}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{2\left(3\,Ba\cos\left(fx+e\right)^{3}+(5\,A+4\,B)a\cos\left(fx+e\right)^{2}-(5\,A+B)a\cos\left(fx+e\right)-2\,(5\,A+B)a+\left(3\,Ba\cos\right.\right.}{15\left(f\cos\left(fx+e\right)-f\sin\left(\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2/15*(3*B*a*cos(f*x + e)^3 + (5*A + 4*B)*a*cos(f*x + e)^2 - (5*A + B)*a*co
s(f*x + e) - 2*(5*A + B)*a + (3*B*a*cos(f*x + e)^2 - (5*A + B)*a*cos(f*x +
e) - 2*(5*A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
 - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = a\left(\int A\sqrt{-c\sin{(e+fx)}+c}\,dx+\int A\sqrt{-c\sin{(e+fx)}+c}\sin{(e+fx)}\,dx + \int B\sqrt{-c\sin{(e+fx)}+c}\sin{(e+fx)}\,dx + \int B\sqrt{-c\sin{(e+fx)}+c}\sin^2{(e+fx)}\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
[Out] a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)\sqrt{-c\sin{(fx+e)}+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{\sqrt{2}\left(30\,Aa\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Ba\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}(\mathrm{si}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/30*sqrt(2)*(30*A*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3*B*a*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 5*(2*A*a*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2), x
)
```

3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 122

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\sqrt{2}a(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

```
[Out] 2*a*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^
(1/2)/f/c^(1/2)-2/3*a*(3*A+5*B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)+2/3*a*B
*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2937, 2830, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\sqrt{2}a(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin
[e + f*x]])])/(Sqrt[c]*f) - (2*a*(3*A + 5*B)*Cos[e + f*x])/(3*f*Sqrt[c - c*
Sin[e + f*x]]) + (2*a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2937

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*Cos[e + f*x]*((
a + b*Sin[e + f*x])^(m + 2)/(b^2*f*(m + 3))), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$= \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} - \frac{(2a)\int \frac{-\frac{3Ac}{2}-\frac{Bc}{2}+\left(-\frac{3Ac}{2}-\frac{5Bc}{2}\right)\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{3c}$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} + (2a(A+B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} - \frac{(4a(A+B))\text{Subst}\left(\int \frac{1}{2c-x^2}dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}$$

$$= \frac{2\sqrt{2}a(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sqrt{-c(1+\sin(e+fx))}+\right.}{3\sqrt{c}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] -1/3*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*Sqrt[2]*(A + B)*ArcTan[Sqr
t[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x]))]
 + Sqrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x])))/
(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

method	result
default	$-\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a\left(3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A+3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B-B(c(1+\right.}{3c^{2}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{aA(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{Ba(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\right.}{3c^{2}\cos(fx+e)\sqrt{c-}}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(3*c^(3/2)*2^(1/2)*arctanh(1
/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+3*c^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-B*(c*(1+sin(f*x+e)))^(3/2)-3*
(c*(1+sin(f*x+e)))^(1/2)*A*c-3*(c*(1+sin(f*x+e)))^(1/2)*B*c)/c^2/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(105) = 210.

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.08

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= \frac{\frac{3\sqrt{2}((A+B)ac\cos(fx+e)-(A+B)ac\sin(fx+e)+(A+B)ac)\log\left(-\frac{\cos(fx+e)^{2}+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)-}{\sqrt{c}}}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{\sqrt{c}}}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A +
B)*a*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos
(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2))/sqrt(c) + 2*(B*a*cos(f*x + e)^2 - (3*A + 4*B)*a*cos(f*x + e) - (
3*A + 5*B)*a - (B*a*cos(f*x + e) + (3*A + 5*B)*a)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = a\left(\int \frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx + \int \frac{A\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx + \int \frac{B\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx + \int \frac{B\sin^{2}\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt(
-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c
), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) +
c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(105) = 210$.

Time = 0.33 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.46

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\frac{3\sqrt{2}(Aa\sqrt{c}+Ba\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{4\sqrt{2}\left(3Aa\sqrt{c}+5Ba\sqrt{c}-\frac{6Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+}}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^{3}\mathrm{s}}}{3f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/3*(3*sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) - 4*sqrt(2)*(3*A*a*sqrt(c) + 5*B*a*sqrt(c) - 6*A*a*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 6*
B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 3*A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2 + 9*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c*((cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2),
 x)
```

3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{a(A+5B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*(A+5*B)*arctanh(1/2*cos(f
*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)+2*a*B*cos(f
*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2936, 2830, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{a(A+5B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((a*(A + 5*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[2]*c^(3/2)*f)) + (a*(A + B)*Cos[e + f*x])/(f*(c - c*Sin[e + f
*x])^(3/2)) + (2*a*B*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{a\int \frac{-Ac-3Bc-2Bc\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(a(A+5B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+5B))\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}$$

$$= -\frac{a(A+5B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\sec(e+fx)\left(\sqrt{2}(A+5B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)(\cos$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sq
rt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin[e
 + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B - 2
*B*Sin[e + f*x])))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 1.69 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.97

method	result
default	$\frac{a\left(A\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+5B\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-A\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx}{2\sqrt{c}}\right.}{2}$
parts	$\frac{aA\left(-\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\sin(fx+e)+2\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}+\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\right)\sqrt{c(1+\sin(f}}{4c^{\frac{7}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/c^(5/2)*a*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/
2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)-5*B*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+2*(c*(1+sin(f*x+e)))^(
1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B)*(c*(1+sin(f*x+e)))^(1/
2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.77

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\Big((A+5\,B)ac\cos(fx+e)^2-(A+5\,B)ac\cos(fx+e)-2\,(A+5\,B)ac+((A+5\,B)ac\cos\cdots}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) - 2
*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x + e
))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x
+ e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
- 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) + (A + B
)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x
+ e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = a\Bigg(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+\cdots}}\cdots$$

$$+\int \frac{A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$

$$+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$

$$+\int \frac{B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\Bigg)$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e
+ f*x) + c)), x) + Integral(A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c
*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x)
+ Integral(B*sin(e + f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c
*sqrt(-c*sin(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)}{(-c\sin{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(102) = 204$.

Time = 0.36 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.39

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{2\sqrt{2}(Aa\sqrt{c}+5Ba\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{\sqrt{2}\left(\frac{Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(Aa\sqrt{}\right.}{}}{8f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -1/8*(2*sqrt(2)*(A*a*sqrt(c) + 5*B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) + sqrt(2)*(A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) - 28*B*a*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
```

```
+ 1/2*e) + 1)^2 - 5*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - (cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2),
x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2),
 x)
```

3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 126

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a(A-7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*(A+9*B)*cos(f*x+e)/c/
f/(c-c*sin(f*x+e))^(3/2)-1/16*a*(A-7*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1
/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2936, 2829, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a(A-7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -1/8*(a*(A - 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(Sqrt[2]*c^(5/2)*f) + (a*(A + B)*Cos[e + f*x])/(2*f*(c - c*Sin[e
```

```
 + f*x])^(5/2)) - (a*(A + 9*B)*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} + \frac{a\int \frac{-Ac-5Bc-4Bc\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} - \frac{(a(A-7B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{16c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + \frac{(a(A-7B))\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{8c^2f}$$

$$= -\frac{a(A-7B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 4.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(\sqrt{2}(A-7B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin(e}{16c^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c\,\mathrm{s}}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/16*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[S
qrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1 +
Sin[e + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A - 5
*B + (A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5))/(c^
(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(107) = 214.

Time = 2.64 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.12

method	result
default	$-\frac{a\left(\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}(A-7B)(\cos^{2}(fx+e))+2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}(A-7B)+2A(c+c\sin}{}$
parts	$\frac{aA\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^{2}(fx+e))c^{2}+6\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}\sin(fx+e)+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}}{32c^{\frac{9}{2}}(\sin(fx+e)-1)\cos(fx+e)\sqrt{c-c\sin(f}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/c^(9/2)*a*(arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2
)*c^2*(A-7*B)*cos(f*x+e)^2+2*sin(f*x+e)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^2*(A-7*B)+2*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+4*A
*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-2*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*c^2+18*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-28*B*(c+c*sin(
f*x+e))^(1/2)*c^(3/2)+14*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/
2)/c^(1/2))*c^2)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*si
n(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.13

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}((A-7\,B)a\cos{(fx+e)}^{3}+3\,(A-7\,B)a\cos{(fx+e)}^{2}-2\,(A-7\,B)a\cos{(fx+e)}-4\,(A-7\,B)a-(}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) - 4*(A +
B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f
)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(107) = 214$.

Time = 0.38 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.33

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{4\sqrt{2}(Aa-7Ba)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa\sqrt{c}+Ba\sqrt{c}+\frac{16Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\cdots}{c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\cdots\right.\right.}}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/128*(4*sqrt(2)*(A*a - 7*B*a)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) + sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) + 16*B*a*sqrt(c)*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 6*A*a*sqrt(c
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^2 + 42*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(
```

```
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (1
6*sqrt(2)*B*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + sqrt(2)*A*a*c^(7
/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + sqrt(2)*B*a*c^(7/2)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2)/c^6)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2),
 x)
```

3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 163

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a(A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} + \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*(A+13*B)*cos(f*x+e)/
c/f/(c-c*sin(f*x+e))^(5/2)-1/32*a*(A-3*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))
^(3/2)-1/64*a*(A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e
))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3046, 2936, 2829, 2729, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a(A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} - \frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} + \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] -1/32*(a*(A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(Sqrt[2]*c^(7/2)*f) + (a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[
```

```
e + f*x])^(7/2)) - (a*(A + 13*B)*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A - 3*B)*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$
$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} + \frac{a\int \frac{-Ac-7Bc-6Bc\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c^2}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{(a(A-3B))\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{16c^2}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(a(A-3B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{64c^3}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}} + \frac{(a(A-3B))\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{32c^3f}$$
$$= -\frac{a(A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} + \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 6.37 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(3\sqrt{2}(A-3B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin(\ldots}{192c^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\ldots\right.\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -1/192*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*Sqrt[2]*(A - 3*B)*ArcTa
n[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1
 + Sin[e + f*x]))] + (Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*A -
 13*B + 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7))/(c^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(140) = 280.

Time = 2.97 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.16

method	result
default	$-\frac{a\left(3\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^4(A-3B)\left(\cos^2(fx+e)\right)\sin(fx+e)-9\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^4(A-3B)\left(\cos^2(fx+\right.\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/c^(15/2)*a*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^4*(A-3*B)*cos(f*x+e)^2*sin(f*x+e)-9*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*cos(f*x+e)^2-12*arctanh(1/2*(c+c*s
in(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*sin(f*x+e)+6*A*(c+c*s
in(f*x+e))^(5/2)*c^(3/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-24*A*(c+c*sin(
f*x+e))^(1/2)*c^(7/2)-18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-32*B*(c+c*sin(f*x
+e))^(3/2)*c^(5/2)+72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+12*A*2^(1/2)*arctanh
(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-36*B*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f
*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(140) = 280.

Time = 0.28 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.01

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((A-3\,B)a\cos\left(fx+e\right)^4-3\,(A-3\,B)a\cos\left(fx+e\right)^3-8\,(A-3\,B)a\cos\left(fx+e\right)^2+4\,(A-3\,B)a\cos\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((A - 3*B)*a*cos(f*x + e)^4 - 3*(A - 3*B)*a*cos(f*x + e)^
3 - 8*(A - 3*B)*a*cos(f*x + e)^2 + 4*(A - 3*B)*a*cos(f*x + e) + 8*(A - 3*B)
*a + ((A - 3*B)*a*cos(f*x + e)^3 + 4*(A - 3*B)*a*cos(f*x + e)^2 - 4*(A - 3*
B)*a*cos(f*x + e) - 8*(A - 3*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x +
e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x
+ e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(
```

```
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
3*(A - 3*B)*a*cos(f*x + e)^3 - (7*A + 43*B)*a*cos(f*x + e)^2 + 2*(11*A - B)
*a*cos(f*x + e) + 32*(A + B)*a + (3*(A - 3*B)*a*cos(f*x + e)^2 + 2*(5*A + 1
7*B)*a*cos(f*x + e) + 32*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 +
 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x +
 e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(140) = 280$.

Time = 0.45 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.85

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
[Out] -1/1536*(12*sqrt(2)*(A*a*sqrt(c) - 3*B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) - 3*A*a*sqrt(c)*
```

```
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) +
 9*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 3*A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 22*A*a*sqrt(c)*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 6
6*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3/(c^4*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3
*A*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 3*B*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 9*B*a*c^(17/2)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - A*a*
c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)^3 - B*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/
f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
 x)
```

3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 210

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c&-c\sin(e+fx))^{7/2}\,dx = \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}\\&+\frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}\\&+\frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}\\&-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\end{aligned}$$

```
[Out] 256/15015*a^2*(13*A-3*B)*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/3003*
a^2*(13*A-3*B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^2*B*c^2*cos
(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/f+8/429*a^2*(13*A-3*B)*c^4*cos(f*x+e)^5/f/
(c-c*sin(f*x+e))^(1/2)+2/143*a^2*(13*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e
))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2935, 2753, 2752}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{256a^2c^6(13A-3B)\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2c^5(13A-3B)\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2c^4(13A-3B)\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c^3(13A-3B)\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f} - \frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^2*(13*A - 3*B)*c^6*Cos[e + f*x]^5)/(15015*f*(c - c*Sin[e + f*x])^(5/
2)) + (64*a^2*(13*A - 3*B)*c^5*Cos[e + f*x]^5)/(3003*f*(c - c*Sin[e + f*x])
^(3/2)) + (8*a^2*(13*A - 3*B)*c^4*Cos[e + f*x]^5)/(429*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*a^2*(13*A - 3*B)*c^3*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/
(143*f) - (2*a^2*B*c^2*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx\\
&= -\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\\
&\quad+\frac{1}{13}\left(a^2(13A-3B)c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx\\
&= \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}\\
&\quad-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\\
&\quad+\frac{1}{143}\left(12a^2(13A-3B)c^3\right)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\\
&= \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}\\
&\quad-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\\
&\quad+\frac{1}{429}\left(32a^2(13A-3B)c^4\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f} \\
&\quad - \frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f} \\
&\quad + \frac{(128a^2(13A-3B)c^5)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{3003} \\
&= \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f} \\
&\quad - \frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1355 vs. $2(210) = 420$.

Time = 10.72 (sec) , antiderivative size = 1355, normalized size of antiderivative = 6.45

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{(7A-2B)\cos\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$-\frac{(4A+B)\cos\left(\frac{3}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{32f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(22A-7B)\cos\left(\frac{5}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{160f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(A-4B)\cos\left(\frac{7}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{A\cos\left(\frac{9}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{48f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(2A-3B)\cos\left(\frac{11}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{352f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{B\cos\left(\frac{13}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{416f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(7A-2B)\sin\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(4A+B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\sin\left(\frac{3}{2}(e+fx)\right)}{32f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(22A-7B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\sin\left(\frac{5}{2}(e+fx)\right)}{160f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$-\frac{(A-4B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\sin\left(\frac{7}{2}(e+fx)\right)}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{A(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\sin\left(\frac{9}{2}(e+fx)\right)}{48f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$-\frac{(2A-3B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\sin\left(\frac{11}{2}(e+fx)\right)}{352f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{B(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\sin\left(\frac{13}{2}(e+fx)\right)}{416f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(
7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x))/2]
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((A - 4*
B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/
(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e
 + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[
e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos[(13*(e + f*x)
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((7*
A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
)/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2)*Sin[(5*(e + f*x))/2])/(160*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((A - 4*B)*(
a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(112
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4) + (A*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e +
 f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4) - ((2*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*(a + a*Sin[e + f*x])^
2*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] (verified)

Time = 36.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^3a^2\left(1155B\left(\cos^4(fx+e)\right)+(-1365A+4935B)\left(\cos^2(fx+e)\right)\sin(fx+e)+(5915A-10605B)\left(\cos^2(fx+\right.\right.}{15015\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2A\,a^2(\sin(fx+e)-1)c^4(1+\sin(fx+e))\left(5\left(\sin^3(fx+e)\right)-27\left(\sin^2(fx+e)\right)+71\sin(fx+e)-177\right)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2B\,a^2(\sin(fx+e)-1)c^4(1+\sin(fx+}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15015*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*(1155*B*cos(f*x+e)^4+(-1365
*A+4935*B)*cos(f*x+e)^2*sin(f*x+e)+(5915*A-10605*B)*cos(f*x+e)^2+(11180*A-1
1820*B)*sin(f*x+e)-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{2\left(1155\,Ba^2c^3\cos\left(fx+e\right)^7+105\,(13\,A-14\,B)a^2c^3\cos\left(fx+e\right)^6+35\,(91\,A-87\,B)a^2c^3\cos$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/15015*(1155*B*a^2*c^3*cos(f*x + e)^7 + 105*(13*A - 14*B)*a^2*c^3*cos(f*x
+ e)^6 + 35*(91*A - 87*B)*a^2*c^3*cos(f*x + e)^5 - 20*(13*A - 3*B)*a^2*c^3*
cos(f*x + e)^4 + 32*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^3 - 64*(13*A - 3*B)*a
^2*c^3*cos(f*x + e)^2 + 256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A -
 3*B)*a^2*c^3 + (1155*B*a^2*c^3*cos(f*x + e)^6 - 105*(13*A - 25*B)*a^2*c^3*
cos(f*x + e)^5 + 140*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^4 + 160*(13*A - 3*B)
*a^2*c^3*cos(f*x + e)^3 + 192*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^2 + 256*(13
*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A - 3*B)*a^2*c^3)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\, dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2 (-c\sin(fx + e) + c)^{\frac{7}{2}}\, dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.78

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\, dx = -\frac{\sqrt{2}\left(10010\, Aa^2c^3 \cos\left(-\frac{9}{4}\pi + \frac{9}{2}fx + \frac{9}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 1155\, Ba^2c^3 \cos\left(-\frac{13}{4}\pi + \frac{13}{2}fx + \right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] -1/480480*sqrt(2)*(10010*A*a^2*c^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 1155*B*a^2*c^3*cos(-13/4*pi + 13/2*f*x + 13/2*
e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 60060*(7*A*a^2*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(4*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) + B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/
2*f*x + 3/2*e) - 3003*(22*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7
*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2
*e) + 4290*(A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^2*c^3*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 1365*(2*
A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\, dx = \int (A + B\, \sin(e + f\, x))\, (a + a\, \sin(e + f\, x))^2\, (c - c\, \sin(e + f\, x))^{7/2}\, dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
 x)
```

3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 167

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

```
[Out] 64/3465*a^2*(11*A-B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/693*a^2*(
11*A-B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/99*a^2*(11*A-B)*c^3*cos
(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)-2/11*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x
+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used

$= \{3046, 2935, 2753, 2752\}$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{64a^2c^5(11A-B)\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}} + \frac{16a^2c^4(11A-B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2c^3(11A-B)\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}} - \frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^2*(11*A - B)*c^5*Cos[e + f*x]^5)/(3465*f*(c - c*Sin[e + f*x])^(5/2))
+ (16*a^2*(11*A - B)*c^4*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^(3/2))
 + (2*a^2*(11*A - B)*c^3*Cos[e + f*x]^5)/(99*f*Sqrt[c - c*Sin[e + f*x]]) -
(2*a^2*B*c^2*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx \\
&= -\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} \\
&\quad+\frac{1}{11}\left(a^2(11A-B)c^2\right)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx \\
&= \frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} \\
&\quad+\frac{1}{99}\left(8a^2(11A-B)c^3\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= \frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}} \\
&\quad-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} \\
&\quad+\frac{1}{693}\left(32a^2(11A-B)c^4\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx \\
&= \frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}} \\
&\quad+\frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1173 vs. $2(167) = 334$.

Time = 12.62 (sec) , antiderivative size = 1173, normalized size of antiderivative = 7.02

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{(6A-B)\cos\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$-\frac{(4A+B)\cos\left(\frac{3}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(8A-3B)\cos\left(\frac{5}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{80f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$-\frac{(2A+3B)\cos\left(\frac{7}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(2A-B)\cos\left(\frac{9}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{144f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$-\frac{B\cos\left(\frac{11}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{176f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(6A-B)\sin\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(4A+B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\sin\left(\frac{3}{2}(e+fx)\right)}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(8A-3B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\sin\left(\frac{5}{2}(e+fx)\right)}{80f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(2A+3B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\sin\left(\frac{7}{2}(e+fx)\right)}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{(2A-B)(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\sin\left(\frac{9}{2}(e+fx)\right)}{144f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$
$$+\frac{B(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\sin\left(\frac{11}{2}(e+fx)\right)}{176f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((6*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c
- c*Sin[e + f*x])^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*Cos[(5*(e + f*x))/2]*(a
 + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(80*f*(Cos[(e + f*x)/2] -
```

```
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((2*A + 3*B)
*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(1
12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + ((2*A - B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^4) - (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e +
 f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((6*A - B)*Sin[(e + f*x
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A
+ B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e + f*x))/2]
)/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4) + ((8*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5
/2)*Sin[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A + 3*B)*(a + a*Sin[e + f*x])^2
*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - B)
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(1
44*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + (B*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(
e + f*x))/2])/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] (verified)

Time = 36.99 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^3a^2(-315B(\cos^2(fx+e))\sin(fx+e)+(-385A+980B)(\cos^2(fx+e))+(-1210A+1370B)\sin(fx+}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2Aa^2(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3(\sin^2(fx+e))-14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2Ba^2(\sin(fx+e)-1)c^3(1+\sin(fx+e))(63(\sin^5(fx+}{6}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)

[Out] -2/3465*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(-315*B*cos(f*x+e)^2*sin(f*
x+e)+(-385*A+980*B)*cos(f*x+e)^2+(-1210*A+1370*B)*sin(f*x+e)+1562*A-1402*B)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(151) = 302.

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{2\left(315\,Ba^2c^2\cos\left(fx+e\right)^6-35\left(11\,A-10\,B\right)a^2c^2\cos\left(fx+e\right)^5+5\left(11\,A-B\right)a^2c^2\cos\left(fx+e\right)^4-8\left(11\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*c^2*cos(f*x + e)^6 - 35*(11*A - 10*B)*a^2*c^2*cos(f*x +
e)^5 + 5*(11*A - B)*a^2*c^2*cos(f*x + e)^4 - 8*(11*A - B)*a^2*c^2*cos(f*x +
 e)^3 + 16*(11*A - B)*a^2*c^2*cos(f*x + e)^2 - 64*(11*A - B)*a^2*c^2*cos(f*
x + e) - 128*(11*A - B)*a^2*c^2 - (315*B*a^2*c^2*cos(f*x + e)^5 + 35*(11*A
- B)*a^2*c^2*cos(f*x + e)^4 + 40*(11*A - B)*a^2*c^2*cos(f*x + e)^3 + 48*(11
*A - B)*a^2*c^2*cos(f*x + e)^2 + 64*(11*A - B)*a^2*c^2*cos(f*x + e) + 128*(
11*A - B)*a^2*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
- f*sin(f*x + e) + f)
```

Sympy [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c&\\ -c\sin(e+fx))^{5/2}\,dx = a^2\Bigg(\int Ac^2\sqrt{-c\sin\left(e+fx\right)+c}\,dx&\\ +\int\left(-2Ac^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)\right)\,dx&\\ +\int Ac^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^4\left(e+fx\right)dx&\\ +\int Bc^2\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx&\\ +\int\left(-2Bc^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)\,dx&\\ +\int Bc^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^5\left(e+fx\right)dx\Bigg)&\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-2*A*c**2*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin(e
```

```
 + f*x) + c)*sin(e + f*x)**4, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c
)*sin(e + f*x), x) + Integral(-2*B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f
*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**5, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(151) = 302$.

Time = 0.62 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.04

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{\sqrt{2}\left(315\,Ba^2c^2\cos\left(-\frac{11}{4}\,\pi+\frac{11}{2}\,fx+\frac{11}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+6930\left(6\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -1/55440*sqrt(2)*(315*B*a^2*c^2*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 6930*(6*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 2310*(4*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^
2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) -
 693*(8*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 495*(2*A*a^2
*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) + 385*(2*A*a^2*c^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
)*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (A + B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),
x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),
 x)
```

3.91 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 120

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 8/315*a^2*(9*A+B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/63*a^2*(9*A+B
)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/9*a^2*B*c^2*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2935, 2753, 2752}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^2*(9*A + B)*c^4*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (
2*a^2*(9*A + B)*c^3*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) - (2*
a^2*B*c^2*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{9}\left(a^2(9A+B)c^2\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= \frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{63}\left(4a^2(9A+B)c^3\right)\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$= \frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 9.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{a^2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(162A-87B+35B\cos(2(e+fx))+(-90A+130B}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*(
e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^3a^2\left(-35B\left(\cos^2(fx+e)\right)+\sin(fx+e)(45A-65B)-81A+61B\right)}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2A\,a^2(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f} + \frac{2B\,a^2(\sin(fx+e)-1)c^2(1+\sin(fx+e))\left(35\left(\sin^4(fx+e)\right)-85\left(\sin^3(fx+e)\right)+1\right.}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/315*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(-35*B*cos(f*x+e)^2+sin(f*x+e
)*(45*A-65*B)-81*A+61*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(108) = 216$.

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = -\frac{2\left(35\,Ba^2c\cos\left(fx+e\right)^5 + 5\,(9\,A+8\,B)a^2c\cos\left(fx+e\right)^4 - (9\,A+B)a^2c\cos\left(fx+e\right)^3 + 2\,(9\,A+B)a^2c\cos\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (9
*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A +
B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4 - 5
*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A
 + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\begin{aligned}\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c&\\ - c\sin(e + fx))^{3/2}\,dx &= a^2\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx\right.\\ &+\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx\\ &+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)\right)dx\\ &+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)dx\\ &+\int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx\\ &+\int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)dx\\ &+\int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)dx\\ &\left.+\int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^4\left(e+fx\right)\right)dx\right)\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*c*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**
3, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(
B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-B*c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(-B*c*sqrt(-c*sin(e + f*x) +
 c)*sin(e + f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(108) = 216$.

Time = 0.65 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.98

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{\sqrt{2}\left(1890\,Aa^2c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-35\,Ba^2c\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\operatorname{sgn}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(1890*A*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 35*B*a^2*c*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 210*(3*A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x +
 3/2*e) - 126*(A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 45*(2*A*a^2
*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),
x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),
 x)
```

3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+f$

Optimal result

Integrand size = 38, antiderivative size = 81

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 2/35*a^2*(7*A+3*B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/7*a^2*B*c^2*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3046, 2935, 2752}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^2c^3(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(7*A + 3*B)*c^3*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) -
(2*a^2*B*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
```

```
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx \\
&= -\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{1}{7}\left(a^2(7A+3B)c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx \\
&= \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx \\
&= \frac{2a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(7A-2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(7*A - 2*B + 5*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^3 a^2(5B\sin(fx+e)+7A-2B)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$
parts	$-\frac{2A\,a^2(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}-\frac{2B\,a^2(\sin(fx+e)-1)c(1+\sin(fx+e))(5(\sin^3(fx+e))-6(\sin^2(fx+e))+8\sin(fx+e)-16)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/35*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.38

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\frac{2\left(5\,Ba^2\cos\left(fx+e\right)^4-(7\,A+8\,B)a^2\cos\left(fx+e\right)^3-(21\,A+19\,B)a^2\cos\left(fx+e\right)^2+2\,(7\,A+3\,B)a^2\cos\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/35*(5*B*a^2*cos(f*x + e)^4 - (7*A + 8*B)*a^2*cos(f*x + e)^3 - (21*A + 19*
B)*a^2*cos(f*x + e)^2 + 2*(7*A + 3*B)*a^2*cos(f*x + e) + 4*(7*A + 3*B)*a^2
- (5*B*a^2*cos(f*x + e)^3 + (7*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(7*A + 3*B)
*a^2*cos(f*x + e) - 4*(7*A + 3*B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= a^2\left(\int A\sqrt{-c\sin(e+fx)+c}\,dx+\int 2A\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx\right.$$
$$+\int A\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx+\int B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$+\int 2B\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx$$
$$\left.+\int B\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Int
egral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2\sqrt{-c\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(73) = 146$.

Time = 0.54 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.47

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}\left(5\,Ba^2\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+35\left(4\,Aa^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/140*sqrt(2)*(5*B*a^2*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 35*(4*A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(2*
A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 7*(2*A*a^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 3*B*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi
 + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),
 x)
```

3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 161

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{4\sqrt{2}a^2(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2/5*a^2*B*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/3*a^2*(A+B)*c*cos(f*
x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*
2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-4*a^2*(A+B)*cos(f*x+e)/f/
(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3046, 2939, 2758, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{4\sqrt{2}a^2(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (4*Sqrt[2]*a^2*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^2*B*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[
e + f*x])^(5/2)) - (2*a^2*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x
])^(3/2)) - (4*a^2*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}+\left(a^2(A+B)c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}\\
&\quad+\left(2a^2(A+B)c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}\\
&\quad-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\left(4a^2(A+B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}\\
&\quad-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{\left(8a^2(A+B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}\\
&= \frac{4\sqrt{2}a^2(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\\
&\quad-\frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\$\text{Aborted}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
[Out] $Aborted
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
default	$-\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^2\left(30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A+30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B-3B(}{15c^3\cos(fx+e)\sqrt{c-}}$
parts	$-\frac{A\,a^2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{B\,a^2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\left(15c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arc}\right.}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(30*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+30*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-3*B*(c*(1+sin(f*x+e)))^(
5/2)-5*A*(c*(1+sin(f*x+e)))^(3/2)*c-5*B*(c*(1+sin(f*x+e)))^(3/2)*c-30*(c*(1
+sin(f*x+e)))^(1/2)*A*c^2-30*(c*(1+sin(f*x+e)))^(1/2)*B*c^2)/c^3/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(142) = 284.

Time = 0.29 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.93

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\frac{2\left(\frac{15\sqrt{2}((A+B)a^2c\cos(fx+e)-(A+B)a^2c\sin(fx+e)+(A+B)a^2c)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+}\right.}{\sqrt{c}}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*sqrt(2)*((A + B)*a^2*c*cos(f*x + e) - (A + B)*a^2*c*sin(f*x + e) +
 (A + B)*a^2*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*
sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c)
 + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2))/sqrt(c) + (3*B*a^2*cos(f*x + e)^3 + (5*A + 14*B)*a^2*cos
(f*x + e)^2 - (35*A + 41*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2 + (3*B*a
^2*cos(f*x + e)^2 - (5*A + 11*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2)*si
n(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e)
 + c*f)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= a^2\left(\int \frac{A}{\sqrt{-c\sin{(e+fx)}+c}}\,dx + \int \frac{2A\sin{(e+fx)}}{\sqrt{-c\sin{(e+fx)}+c}}\,dx + \int \frac{A\sin^2{(e+fx)}}{\sqrt{-c\sin{(e+fx)}+c}}\,dx + \int \frac{B\sin{(e+fx)}}{\sqrt{-c\sin{(e+fx)}+c}}\,dx + \int \frac{2B\sin^2{(e+fx)}}{\sqrt{-c\sin{(e+fx)}+c}}\,dx + \int \frac{B\sin^3{(e+fx)}}{\sqrt{-c\sin{(e+fx)}+c}}\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sin(e + f*x)/
sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)**2/sqrt(-c*sin(e +
f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Int
egral(2*B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e
+ f*x)**3/sqrt(-c*sin(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^2}{\sqrt{-c\sin{(fx+e)}+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(142) = 284$.

Time = 0.48 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.98

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$
$$= \frac{2\left(\frac{15\sqrt{2}(Aa^2\sqrt{c}+Ba^2\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{\mathrm{csgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{8\sqrt{2}\left(10Aa^2\sqrt{c}+13Ba^2\sqrt{c}-\frac{35Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{35Ba^2\sqrt{c}\left(\cos\cdots\right.}{\cos\left(-\frac{1}{4}\cdots\right.}\cdots}{\cdots}\cdots\right)}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 2/15*(15*sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) - 8*sqrt(2)*(10*A*a^2*sqrt(c) + 13*B*a^2*sqrt(c) - 35*A*
a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) - 35*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) + 55*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 85*B*a^2*sqrt(c)*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
45*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^3 - 45*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*a^2*sqrt(c)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 30*B*a^2*
sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4)/(c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2}{\sqrt{c - c\,\sin(e + f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}a^2(3A+7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)+1/6*a^2*(3*A+7*B)*c
os(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-a^2*(3*A+7*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+a^2*(3*A+7*B)*cos(
f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3046, 2938, 2758, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}a^2(3A+7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2c^2(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((Sqrt[2]*a^2*(3*A + 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
 c*Sin[e + f*x]])])/(c^(3/2)*f)) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(2*f*(c
 - c*Sin[e + f*x])^(7/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x]^3)/(6*f*(c - c*Si
n[e + f*x])^(3/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e +
 f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{1}{4}\left(a^2(3A+7B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}\\
&\quad -\frac{1}{2}\left(a^2(3A+7B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}\\
&\quad +\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\left(a^2(3A+7B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\\
&\quad +\frac{\left(2a^2(3A+7B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}\\
&= -\frac{\sqrt{2}a^2(3A+7B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}\\
&\quad +\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.02

$$
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*Ar
cTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c -
c*Sin[e + f*x])^(3/2))

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.60

method	result
default	$-\frac{a^2\left(\sin(fx+e)\left(6A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-9A\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2+2B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{c}+18B\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}\cdots\right.\right.}{\cdots}$
parts	$-\frac{A\,a^2\left(\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\sin(fx+e)-\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2-2\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}\right)\sqrt{c(1+\sin(\cdots}{4c^{\frac{7}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)

[Out] -1/3/c^(7/2)*a^2*(sin(f*x+e)*(6*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-9*A*2^(1/2
)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+2*B*(c+c*sin(f*x+
e))^(3/2)*c^(1/2)+18*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-21*B*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2)-12*A*(c+c*sin(f*x+e))^(1/2
)*c^(3/2)+9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c
^2-2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-24*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+2
1*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2)*(c*(1+
sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(157) = 314.

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.19

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}\left((3A+7B)a^2c\cos(fx+e)^2-(3A+7B)a^2c\cos(fx+e)-2(3A+7B)a^2c+((3\cdots\right.}{\cdots}$$

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")

```
[Out] 1/6*(3*sqrt(2)*((3*A + 7*B)*a^2*c*cos(f*x + e)^2 - (3*A + 7*B)*a^2*c*cos(f*
x + e) - 2*(3*A + 7*B)*a^2*c + ((3*A + 7*B)*a^2*c*cos(f*x + e) + 2*(3*A + 7
*B)*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x
+ e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1
)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(B*a^2*cos(f*x + e)^3 + (3*A + 10*B
)*a^2*cos(f*x + e)^2 + 6*(A + 2*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2 + (B*a^
2*cos(f*x + e)^2 - 3*(A + 3*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2)*sin(f*x +
e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) -
 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = a^2\Bigg(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{2A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{2B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^3(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\Bigg)$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin
(e + f*x) + c)), x) + Integral(2*A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) +
c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(A*sin(e + f*x
)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) +
c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(2*B*sin(e + f*x)**2/(-c*sqr
t(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + In
tegral(B*sin(e + f*x)**3/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqr
t(-c*sin(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(157) = 314$.

Time = 0.52 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.22

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(3*A*a^2*sqrt(c) + 7*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a^2*sqrt(c)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c
) + 6*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) + 14*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/
(c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) - 16*sqrt(2)*(3*A*a^2*sqrt(c) + 11*B*a^2*sqrt(c) - 6*A*a^2*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 18*
B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 3*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2
), x)
```

3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{3a^2(A+9B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)-1/8*a^2*(A+9*B)*cos
(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)+3/8*a^2*(A+9*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-3/8*a^2*(A+9*B)*co
s(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3046, 2938, 2759, 2758, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{3a^2(A+9B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (3*a^2*(A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(4*Sqrt[2]*c^(5/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(4*f*(c
- c*Sin[e + f*x])^(9/2)) - (a^2*(A + 9*B)*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e
 + f*x])^(5/2)) - (3*a^2*(A + 9*B)*Cos[e + f*x])/(8*c^2*f*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{1}{8}\left(a^2(A+9B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}+\frac{\left(3a^2(A+9B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{16c}\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}\\
&\quad-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\left(3a^2(A+9B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{8c^2}\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{\left(3a^2(A+9B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{4c^2f}\\
&= \frac{3a^2(A+9B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}\\
&\quad-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.51 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}(\right.\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (
3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*(5*A
+ 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 8*B*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(152) = 304.

Time = 3.30 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.21

method	result
default	$-\frac{a^2\left(3A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2+27B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2-6A\sqrt{2}\,\operatorname{arctan}}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^2*c^2+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)
*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-6*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2-16*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3
/2)*sin(f*x+e)^2-54*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)*c^2+10*A*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*A*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+26*B*(c*(1+sin(f*x
+e)))^(3/2)*c^(1/2)+32*(c*(1+sin(f*x+e)))^(1/2)*B*c^(3/2)*sin(f*x+e)+27*B*2
^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*A*(c*(1
```

```
+sin(f*x+e)))^(1/2)*c^(3/2)-60*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+si
n(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(152) = 304$.

Time = 0.28 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.57

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3\sqrt{2}((A+9B)a^2\cos(fx+e)^3+3(A+9B)a^2\cos(fx+e)}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x + e
)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos(f
*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e))*s
qrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)
*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2
*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*cos(f*x
+ e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos(f*x + e
)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*
sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*
c^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^2}{(-c\sin(fx + e) + c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(152) = 304$.

Time = 0.67 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.43

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -1/64*(256*sqrt(2)*B*a^2/(c^(5/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 - 12*sqrt(2)*(A*a^2*sqrt(c) + 9*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c) + 8*A*a^2*sqrt
(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1) + 24*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 18*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 162*B*a^2*sqrt(c)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (8*sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) + 24*sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 +
 sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/c^6)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2
), x)
```

3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2(A-11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/6*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)+1/24*a^2*(A-11*B)*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-1/16*a^2*(A-11*B)*cos(f*x+e)/c^2/f/(c
-c*sin(f*x+e))^(3/2)+1/32*a^2*(A-11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/
2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3046, 2938, 2759, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2(A-11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2c^2(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*(A - 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(16*Sqrt[2]*c^(7/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(6*f*(c
```

```
- c*Sin[e + f*x])^(11/2)) + (a^2*(A - 11*B)*Cos[e + f*x]^3)/(24*f*(c - c*Si
n[e + f*x])^(7/2)) - (a^2*(A - 11*B)*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e +
 f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{1}{12}(a^2(A-11B)c)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{(a^2(A-11B))\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{16c}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}} + \frac{(a^2(A-11B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{32c^3}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(a^2(A-11B))\,\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{16c^3f}$$
$$= \frac{a^2(A-11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.88 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.95

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(7*A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
+ 3*(A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/
4)*(A - 11*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e
```

```
 + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(7*A + 1
9*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(A + 21*B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*
x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(
7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(152) = 304.

Time = 3.26 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.02

method	result
default	$\frac{a^2\left(3\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^3(A-11B)(\cos^2(fx+e))\sin(fx+e)-9\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^3(A-11B)(\cos^2(fx+\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/96/c^(13/2)*a^2*(3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(
1/2))*c^3*(A-11*B)*cos(f*x+e)^2*sin(f*x+e)-9*2^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*cos(f*x+e)^2-12*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*sin(f*x+e)+6*A*(c+
c*sin(f*x+e))^(5/2)*c^(1/2)+32*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-24*A*(c+c*s
in(f*x+e))^(1/2)*c^(5/2)+126*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-352*B*(c+c*si
n(f*x+e))^(3/2)*c^(3/2)+264*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+12*A*2^(1/2)*a
rctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-132*B*2^(1/2)*arctan
h(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)
/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(152) = 304.

Time = 0.27 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((A-11\,B)a^2\cos\left(fx+e\right)^4-3\,(A-11\,B)a^2\cos\left(fx+e\right)^3-8\,(A-11\,B)a^2\cos\left(fx+e\right)^2+4\,(A-1\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*x
 + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e) +
 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*cos(f
*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x + e))
*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(
c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
 - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A + 13*B)*
a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*a^2 + (3*
(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e) - 32*(A +
B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*
c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^
4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e)
 - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(152) = 304.

Time = 0.74 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.76

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/768*(12*sqrt(2)*(A*a^2*sqrt(c) - 11*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c) + 3*A*a^2
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1) + 15*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) - 3*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 93*B*a^2*sqrt(c)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 22*A
*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^3 + 242*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))) + sqrt(2)*(3*A*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 93*B*a^2*c^(17/2)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*A*a^2*c^(17/2)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^2 - 15*B*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^2 - A*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - B*a^2*c^(17/2)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2
), x)
```

3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 222

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2(3A-13B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/8*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)+1/48*a^2*(3*A-13*B
)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/64*a^2*(3*A-13*B)*cos(f*x+e)/c^2/
f/(c-c*sin(f*x+e))^(5/2)+1/256*a^2*(3*A-13*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x
+e))^(3/2)+1/512*a^2*(3*A-13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c
*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3046, 2938, 2759, 2729, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2(3A-13B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}} + \frac{a^2c^2(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} - \frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (a^2*(3*A - 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(256*Sqrt[2]*c^(9/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(8*f*
(c - c*Sin[e + f*x])^(13/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x]^3)/(48*f*(c -
 c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 13*B)*Cos[e + f*x])/(64*c^2*f*(c - c*
Sin[e + f*x])^(5/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x])/(256*c^3*f*(c - c*Si
n[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{1}{16}\left(a^2(3A-13B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}\\
&\quad-\frac{\left(a^2(3A-13B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx}{32c}\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}\\
&\quad-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{\left(a^2(3A-13B)\right)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{128c^3}\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}\\
&\quad-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}\\
&\quad+\frac{\left(a^2(3A-13B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{512c^4}\\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}\\
&\quad+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}-\frac{\left(a^2(3A-13B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{256c^4f}\\
&= \frac{a^2(3A-13B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f}\\
&\quad+\frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}\\
&\quad-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.35 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.61

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Cos
[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 791*
B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f*x)
)/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*I)*(
-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] + 1517*
B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e + f*x))/2
] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Sin[(7*(e
+ f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(195) = 390.

Time = 3.72 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.98

method	result
default	$-\frac{a^2\left(3\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)\left(\cos^4(fx+e)\right)+12\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)\left(\cos^2(fx+e)\right)\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/1536*a^2*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*
c^5*(3*A-13*B)*cos(f*x+e)^4+12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c
^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^2*sin(f*x+e)-24*arctanh(1/2*(c+c*
sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^2-24*a
rctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*s
in(f*x+e)-18*A*(c+c*sin(f*x+e))^(7/2)*c^(3/2)+132*A*(c+c*sin(f*x+e))^(5/2)*
c^(5/2)+264*A*(c+c*sin(f*x+e))^(3/2)*c^(7/2)-144*A*(c+c*sin(f*x+e))^(1/2)*c
^(9/2)+78*B*(c+c*sin(f*x+e))^(7/2)*c^(3/2)+452*B*(c+c*sin(f*x+e))^(5/2)*c^(
```

```
5/2)-1144*B*(c+c*sin(f*x+e))^(3/2)*c^(7/2)+624*B*(c+c*sin(f*x+e))^(1/2)*c^(
9/2)+72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5-3
12*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1
+sin(f*x+e)))^(1/2)/c^(19/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(195) = 390$.

Time = 0.29 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.95

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{3\sqrt{2}\left((3A-13B)a^2\cos(fx+e)^5+5(3A-13B)a^2\cos(fx+e)^4-8(3A-13B)a^2\cos(fx+e)^3-20\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/3072*(3*sqrt(2)*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*co
s(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*cos(
f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3*A
- 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*A -
13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B
)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*cos(f*x +
 e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2*cos(f*x +
 e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3*(3*A - 13*
B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 12*(19*A + 3*
B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^
3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(
f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos
(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(9/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(195) = 390$.

Time = 0.54 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.36

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
[Out] 1/24576*(24*sqrt(2)*(3*A*a^2*sqrt(c) - 13*B*a^2*sqrt(c))*log(-(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^5*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*a^2*sqrt(c) + 3*B*a^2*sqrt(c) +
 32*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 24*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 72*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 96*B*a^2*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^3 + 150*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 650*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
```

```
 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^4/(c^5*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - (96*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) + 24*sqrt(2)*A*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2 - 72*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
32*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 3*sqrt(2
)*A*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 3*sqrt(2)*B*a^2*c
^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/c^20)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2
), x)
```

3.98 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 210

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}$$

```
[Out] 256/45045*a^3*(15*A-B)*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/6435*a^
3*(15*A-B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+8/715*a^3*(15*A-B)*c^5
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)+2/195*a^3*(15*A-B)*c^4*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^(1/2)-2/15*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(1/2)
/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used

= {3046, 2935, 2753, 2752}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{256a^3c^7(15A-B)\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}} + \frac{64a^3c^6(15A-B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}} + \frac{8a^3c^5(15A-B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3c^4(15A-B)\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}} - \frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^3*(15*A - B)*c^7*Cos[e + f*x]^7)/(45045*f*(c - c*Sin[e + f*x])^(7/2)
) + (64*a^3*(15*A - B)*c^6*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^(5/
2)) + (8*a^3*(15*A - B)*c^5*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^(3/
2)) + (2*a^3*(15*A - B)*c^4*Cos[e + f*x]^7)/(195*f*Sqrt[c - c*Sin[e + f*x]]
) - (2*a^3*B*c^3*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx\\
&= -\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\\
&\quad+\frac{1}{15}\left(a^3(15A-B)c^3\right)\int \cos^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\\
&= \frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\\
&\quad+\frac{1}{65}\left(4a^3(15A-B)c^4\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= \frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\\
&\quad+\frac{1}{715}\left(32a^3(15A-B)c^5\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= \frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}\\
&\quad+\frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\\
&\quad+\frac{\left(128a^3(15A-B)c^6\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{6435}\\
&= \frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}\\
&\quad+\frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1569 vs. $2(210) = 420$.

Time = 11.61 (sec) , antiderivative size = 1569, normalized size of antiderivative = 7.47

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{5(8A-B)\cos\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{64f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$-\frac{5(6A+B)\cos\left(\frac{3}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{192f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{3(10A-3B)\cos\left(\frac{5}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{320f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$-\frac{3(4A+3B)\cos\left(\frac{7}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{448f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{(12A-5B)\cos\left(\frac{9}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{576f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$-\frac{(2A+5B)\cos\left(\frac{11}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{704f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{(2A-B)\cos\left(\frac{13}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{832f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$-\frac{B\cos\left(\frac{15}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{5(8A-B)\sin\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}}{64f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{5(6A+B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{3}{2}(e+fx)\right)}{192f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{3(10A-3B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{5}{2}(e+fx)\right)}{320f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{3(4A+3B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{7}{2}(e+fx)\right)}{448f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{(12A-5B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{9}{2}(e+fx)\right)}{576f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{(2A+5B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{11}{2}(e+fx)\right)}{704f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{(2A-B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{13}{2}(e+fx)\right)}{832f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$
$$+\frac{B(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\sin\left(\frac{15}{2}(e+fx)\right)}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(
7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e + f*
x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (3
*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x
])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)*Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(704*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(6*A + B)*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(19
2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6) + (3*(10*A - 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/
2)*Sin[(5*(e + f*x))/2])/(320*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(4*A + 3*B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(448*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((12*A -
 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2
])/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6) + ((2*A + 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^
(7/2)*Sin[(11*(e + f*x))/2])/(704*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(832*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*(a +
 a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(15*(e + f*x))/2])/(960*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^6)
```

Maple [A] (verified)

Time = 167.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^4a^3(3003B(\cos^4(fx+e))+(-3465A+12243B)(\cos^2(fx+e))\sin(fx+e)+(14175A-24969B)(\cos^2(f}{45045\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2Aa^3(\sin(fx+e)-1)c^4(1+\sin(fx+e))(5(\sin^3(fx+e))-27(\sin^2(fx+e))+71\sin(fx+e)-177)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2Ba^3(\sin(fx+e)-1)c^4(1+\sin(fx+}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/45045*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*(3003*B*cos(f*x+e)^4+(-3465
*A+12243*B)*cos(f*x+e)^2*sin(f*x+e)+(14175*A-24969*B)*cos(f*x+e)^2+(24780*A
-25676*B)*sin(f*x+e)-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(190) = 380.

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.93

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{2\left(3003\,Ba^3c^3\cos\left(fx+e\right)^8-231\,(15\,A-14\,B)a^3c^3\cos\left(fx+e\right)^7+21\,(15\,A-B)a^3c^3\cos\left(fx+e\right)^6-28\,(\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -2/45045*(3003*B*a^3*c^3*cos(f*x + e)^8 - 231*(15*A - 14*B)*a^3*c^3*cos(f*x
 + e)^7 + 21*(15*A - B)*a^3*c^3*cos(f*x + e)^6 - 28*(15*A - B)*a^3*c^3*cos(
f*x + e)^5 + 40*(15*A - B)*a^3*c^3*cos(f*x + e)^4 - 64*(15*A - B)*a^3*c^3*c
os(f*x + e)^3 + 128*(15*A - B)*a^3*c^3*cos(f*x + e)^2 - 512*(15*A - B)*a^3*
c^3*cos(f*x + e) - 1024*(15*A - B)*a^3*c^3 - (3003*B*a^3*c^3*cos(f*x + e)^7
 + 231*(15*A - B)*a^3*c^3*cos(f*x + e)^6 + 252*(15*A - B)*a^3*c^3*cos(f*x +
 e)^5 + 280*(15*A - B)*a^3*c^3*cos(f*x + e)^4 + 320*(15*A - B)*a^3*c^3*cos(
f*x + e)^3 + 384*(15*A - B)*a^3*c^3*cos(f*x + e)^2 + 512*(15*A - B)*a^3*c^3
*cos(f*x + e) + 1024*(15*A - B)*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(190) = 380$.

Time = 0.56 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.18

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{\sqrt{2}\left(3003\,Ba^3c^3\cos\left(-\frac{15}{4}\pi+\frac{15}{2}fx+\frac{15}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-225225\left(8\,Aa^3c^3\mathrm{sg}\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
[Out] 1/2882880*sqrt(2)*(3003*B*a^3*c^3*cos(-15/4*pi + 15/2*f*x + 15/2*e)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) - 225225*(8*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e) - 75075*(6*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) +
 B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2
*e) + 27027*(10*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^3*c^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) + 1930
```

```
5*(4*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 5005*(12*A*a^3*
c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a^3*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) - 4095*(2*A*a^3*c^3*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))*cos(-11/4*pi + 11/2*f*x + 11/2*e) + 3465*(2*A*a^3*c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-13
/4*pi + 13/2*f*x + 13/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^3\,\left(c-c\,\sin(e+f\,x)\right)^{7/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),
x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),
 x)
```

3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 161

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 64/9009*a^3*(13*A+B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/1287*a^3*
(13*A+B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/143*a^3*(13*A+B)*c^4*c
os(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{3046, 2935, 2753, 2752\}$

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64a^3c^6(13A+B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}} + \frac{16a^3c^5(13A+B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3c^4(13A+B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^3*(13*A + B)*c^6*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^(7/2))
+ (16*a^3*(13*A + B)*c^5*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^(5/2)
) + (2*a^3*(13*A + B)*c^4*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)
) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (a^3c^3)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{1}{13}\left(a^3(13A+B)c^3\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= \frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\\
&\quad+\frac{1}{143}\left(8a^3(13A+B)c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= \frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}\\
&\quad-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{\left(32a^3(13A+B)c^5\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{1287}\\
&= \frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}\\
&\quad+\frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{a^3c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}(-949}{18018f\left(\cos\left(\frac{1}{2}(e+fx\right)\right.}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/18018*(a^3*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])
^2*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]*(-9490*A + 6200*B + 126*(1
3*A - 32*B)*Cos[2*(e + f*x)] + (9464*A - 9667*B)*Sin[e + f*x] + 693*B*Sin[3
*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 168.77 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^4a^3(-693B(\cos^2(fx+e))\sin(fx+e)+(-819A+2016B)(\cos^2(fx+e))+(-2366A+2590B)\sin(fx-}{9009\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2Aa^3(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3(\sin^2(fx+e))-14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2Ba^3(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3465(\sin^6(fx}{}$

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/9009*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(-693*B*cos(f*x+e)^2*sin(f*x+e)+(-819*A+2016*B)*cos(f*x+e)^2+(-2366*A+2590*B)*sin(f*x+e)+2782*A-2558*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2\left(693\,Ba^3c^2\cos\left(fx+e\right)^7+63\left(13\,A+12\,B\right)a^3c^2\cos\left(fx+e\right)^6-7\left(13\,A+B\right)a^3c^2\cos\left(fx+e\right)^5+10\left(13\,A\right.\right.}{}$$

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -2/9009*(693*B*a^3*c^2*cos(f*x + e)^7 + 63*(13*A + 12*B)*a^3*c^2*cos(f*x + e)^6 - 7*(13*A + B)*a^3*c^2*cos(f*x + e)^5 + 10*(13*A + B)*a^3*c^2*cos(f*x + e)^4 - 16*(13*A + B)*a^3*c^2*cos(f*x + e)^3 + 32*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^2 + (693*B*a^3*c^2*cos(f*x + e)^6 - 63*(13*A + B)*a^3*c^2*cos(f*x + e)^5 - 70*(13*A + B)*a^3*c^2*cos(f*x + e)^4 - 80*(13*A + B)*a^3*c^2*cos(f*x + e)^3 - 96*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(145)=290$.

Time = 0.59 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.31

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=-\frac{\sqrt{2}\left(180180\,Aa^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+693\,Ba^3c^2\cos\left(-\frac{13}{4}\pi+\frac{13}{2}fx\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
[Out] -1/288288*sqrt(2)*(180180*A*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 693*B*a^3*c^2*cos(-13/4*pi + 13/2*f*x + 13/2*
e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15015*(4*A*a^3*c^2*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-
3/4*pi + 3/2*f*x + 3/2*e) - 9009*(2*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)) - B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f
*x + 5/2*e) - 2574*(5*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a
```

```
^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e)
+ 2002*(A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^3*c^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) + 819*(2*A*a^3
*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),
 x)
```

3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 124

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 8/693*a^3*(11*A+3*B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/99*a^3*(11
*A+3*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)-2/11*a^3*B*c^3*cos(f*x+e)
^7/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2935, 2753, 2752}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a^3c^5(11A+3B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4(11A+3B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^3*(11*A + 3*B)*c^5*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2))
+ (2*a^3*(11*A + 3*B)*c^4*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2))
 - (2*a^3*B*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx \\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{1}{11}\left(a^3(11A+3B)c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\\
&\quad+\frac{1}{99}\left(4a^3(11A+3B)c^4\right)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&=\frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1157 vs. $2(124) = 248$.

Time = 12.55 (sec) , antiderivative size = 1157, normalized size of antiderivative = 9.33

$$
\begin{aligned}
&\int(a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{(6A+B)\cos\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{(8A+3B)\cos\left(\frac{3}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{B\cos\left(\frac{5}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{16f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{(6A+B)\cos\left(\frac{7}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{(2A+3B)\cos\left(\frac{9}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{144f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&+\frac{B\cos\left(\frac{11}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{176f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&+\frac{(6A+B)\sin\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&+\frac{(8A+3B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\sin\left(\frac{3}{2}(e+fx)\right)}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{B(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\sin\left(\frac{5}{2}(e+fx)\right)}{16f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&+\frac{(6A+B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\sin\left(\frac{7}{2}(e+fx)\right)}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{(2A+3B)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\sin\left(\frac{9}{2}(e+fx)\right)}{144f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}\\
&-\frac{B(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\sin\left(\frac{11}{2}(e+fx)\right)}{176f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Cos[(7*(e
 + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
 - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e +
f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3
*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((8*A + 3*B)*(
a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e + f*x))/2])/(24*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(5*(e +
f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6) + ((6*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(3/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 3*B)*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(11*(e + f*x))/2])/(1
76*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6)
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^4a^3\left(-63B\left(\cos^2(fx+e)\right)+\sin(fx+e)(77A-105B)-121A+93B\right)}{693\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2A\,a^3(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2B\,a^3(\sin(fx+e)-1)c^2(1+\sin(fx+e))\left(15\left(\sin^5(fx+e)\right)-35\left(\sin^4(fx+e)\right)+4\right.}{165\cos(fx+e)\sqrt{c-c\sin(fx}}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/693*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(-63*B*cos(f*x+e)^2+sin(f*x+e
```

```
)*(77*A-105*B)-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(112) = 224$.

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.31

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{2\left(63\,Ba^3c\cos\left(fx+e\right)^6-7\left(11\,A+12\,B\right)a^3c\cos\left(fx+e\right)^5-\left(187\,A+177\,B\right)a^3c\cos\left(fx+e\right)^4\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
[Out] 2/693*(63*B*a^3*c*cos(f*x + e)^6 - 7*(11*A + 12*B)*a^3*c*cos(f*x + e)^5 - (
187*A + 177*B)*a^3*c*cos(f*x + e)^4 + 2*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 -
 4*(11*A + 3*B)*a^3*c*cos(f*x + e)^2 + 16*(11*A + 3*B)*a^3*c*cos(f*x + e) +
 32*(11*A + 3*B)*a^3*c - (63*B*a^3*c*cos(f*x + e)^5 + 7*(11*A + 21*B)*a^3*c
*cos(f*x + e)^4 - 10*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 - 12*(11*A + 3*B)*a^
3*c*cos(f*x + e)^2 - 16*(11*A + 3*B)*a^3*c*cos(f*x + e) - 32*(11*A + 3*B)*a
^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x +
 e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = a^3\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx + \int 2Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx + \int\left(-2Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)dx + \int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^4\left(e+fx\right)\right)dx + \int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx + \int 2Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)dx + \int\left(-2Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^4\left(e+fx\right)\right)dx + \int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^5\left(e+fx\right)\right)dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x), x) + Integral(-2*A*c*sqrt(-c*sin(e + f*x) +
c)*sin(e + f*x)**3, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*
x)**4, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integ
ral(2*B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-2*B*c*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x) + Integral(-B*c*sqrt(-c*sin(e
+ f*x) + c)*sin(e + f*x)**5, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(112) = 224$.

Time = 0.49 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.37

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{\sqrt{2}\big(693\,Ba^3c\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\big(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\big)-63\,Ba^3c\cos\left(-\frac{11}{4}\,\pi+\frac{11}{2}\,fx+\frac{11}{2}\,e\right)\mathrm{s}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/11088*sqrt(2)*(693*B*a^3*c*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 63*B*a^3*c*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) + 1386*(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 462*(8*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*
c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 99*
(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 77*(2*A*a^3*c*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),
 x)
```

3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

Optimal result

Integrand size = 38, antiderivative size = 81

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 2/63*a^3*(9*A+5*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/9*a^3*B*c^3*
cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3046, 2935, 2752}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^3c^4(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(9*A + 5*B)*c^4*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) -
(2*a^3*B*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
```

```
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}+\frac{1}{9}\left(a^3(9A+5B)c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$
\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx
= \frac{2a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(9A-2B+7B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(9*A - 2*B + 7*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^4 a^3 (7B\sin(fx+e)+9A-2B)}{63\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$
parts	$-\frac{2A\, a^3(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f} - \frac{2B\, a^3(\sin(fx+e)-1)c(1+\sin(fx+e))(35(\sin^4(fx+e))-40(\sin^3(fx+e))+48(\sin^2(fx+e))}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/63*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.86

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\, dx$$
$$= \frac{2\left(7\, Ba^3 \cos\left(fx + e\right)^5 + (9\, A + 26\, B)a^3 \cos\left(fx + e\right)^4 - (27\, A + 29\, B)a^3 \cos\left(fx + e\right)^3 - 4\,(18\, A + 17\, B)a^3\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/63*(7*B*a^3*cos(f*x + e)^5 + (9*A + 26*B)*a^3*cos(f*x + e)^4 - (27*A + 29
*B)*a^3*cos(f*x + e)^3 - 4*(18*A + 17*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)
*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3 + (7*B*a^3*cos(f*x + e)^4 - (9*A + 19
*B)*a^3*cos(f*x + e)^3 - 12*(3*A + 4*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)*
a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = a^3\left(\int A\sqrt{-c\sin(e+fx)+c}\,dx + \int 3A\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int 3A\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx + \int A\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx + \int B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int 3B\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx + \int 3B\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx + \int B\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(3*A*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x)**2, x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x)
+ Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*B*sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(3*B*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x)**3, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e +
f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3\sqrt{-c\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(73) = 146.

Time = 0.44 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}\left(7\,Ba^3\cos\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+126\left(5\,Aa^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/504*sqrt(2)*(7*B*a^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 126*(5*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a^
3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 42*
(9*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(A*a^3*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4
*pi + 5/2*f*x + 5/2*e) + 9*(2*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5
*B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e))
*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
 x)
```

3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{8\sqrt{2}a^3(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2/7*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/5*a^3*(A+B)*c^2*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*(A+B)*c*cos(f*x+e)^3/f/(c-c*sin(f
*x+e))^(3/2)+8*a^3*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*
x+e))^(1/2))*2^(1/2)/f/c^(1/2)-8*a^3*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3046, 2939, 2758, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{8\sqrt{2}a^3(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^3c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (8*Sqrt[2]*a^3*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[
e + f*x])^(7/2)) - (2*a^3*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f
*x])^(5/2)) - (4*a^3*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (8*a^3*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx \\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}+\left(a^3(A+B)c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx \\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} \\
&\quad+\left(2a^3(A+B)c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx \\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} \\
&\quad-\frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\left(4a^3(A+B)c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx \\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} \\
&\quad-\frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad+\left(8a^3(A+B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} \\
&\quad-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{\left(16a^3(A+B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f} \\
&= \frac{8\sqrt{2}a^3(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} \\
&\quad-\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} \\
&\quad-\frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left((6720+6720i)\sqrt[4]{-1}(A+B)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\right.\right.}{}$$

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] -1/420*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((6720 + 6720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B + 6*(7*A + 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin[3*(e + f*x)])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin[e + f*x]])

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.16

method	result
default	$\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^3\left(-420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A-420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B+15\right.}{}$
parts	$-\frac{A\,a^3(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{B\,a^3(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(105c^{\frac{7}{2}}\sqrt{2}\,\mathrm{ar}\right.}{}$

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/105*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(-420*c^(7/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-420*c^(7/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+15*B*(c*(1+sin(f*x+e)))^(7/2)+21*A*(c*(1+sin(f*x+e)))^(5/2)*c+21*B*(c*(1+sin(f*x+e)))^(5/2)*c+70*A*(c*(1+sin(f*x+e)))^(3/2)*c^2+70*B*(c*(1+sin(f*x+e)))^(3/2)*c^2+420*(c*(1+sin(f*x+e)))^(1/2)*A*c^3+420*(c*(1+sin(f*x+e)))^(1/2)*B*c^3)/c^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{2\left(\frac{210\sqrt{2}((A+B)a^3c\cos(fx+e)-(A+B)a^3c\sin(fx+e)+(A+B)a^3c)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+\cdots}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(f\cdots}\right.}{\sqrt{c}}\cdots\right.}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(210*sqrt(2)*((A + B)*a^3*c*cos(f*x + e) - (A + B)*a^3*c*sin(f*x + e)
 + (A + B)*a^3*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(
c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(c) - (15*B*a^3*cos(f*x + e)^4 - 3*(7*A + 22*B)*a^
3*cos(f*x + e)^3 - (133*A + 253*B)*a^3*cos(f*x + e)^2 + 4*(133*A + 148*B)*a
^3*cos(f*x + e) + 4*(161*A + 191*B)*a^3 - (15*B*a^3*cos(f*x + e)^3 + 3*(7*A
 + 27*B)*a^3*cos(f*x + e)^2 - 4*(28*A + 43*B)*a^3*cos(f*x + e) - 4*(161*A +
 191*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c
*f*sin(f*x + e) + c*f)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = a^3\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^4(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)/
sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)**2/sqrt(-c*sin(e
+ f*x) + c), x) + Integral(A*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x)
+ Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(e + f*x)**3/sq
rt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**4/sqrt(-c*sin(e + f*
x) + c), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(177) = 354.

Time = 0.39 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.24

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 4/105*(105*(sqrt(2)*A*a^3*sqrt(c) + sqrt(2)*B*a^3*sqrt(c))*log(-(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*(161*A*a^3*sqrt(c) + 191*B*a^3*sqr
t(c) - 812*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) - 812*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 2121*A*a^3*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 2751*B
*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2 - 3080*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 3080*B*a^3*sqrt(c)*(cos(-1/4*pi +
```

```
 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 2555*A*a^
3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4 + 3605*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1260*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 1260*B*a^3*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^5 + 315*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^6/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^6 + 525*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^6/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^6)/(c*((cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^7*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^3}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 218

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{2}a^3(5A+9B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(9/2)+1/10*a^3*(5*A+9*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+1/3*a^3*(5*A+9*B)*cos(f*x+e)^3/f/(c
-c*sin(f*x+e))^(3/2)-2*a^3*(5*A+9*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)
/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+2*a^3*(5*A+9*B)*cos(f*x+e)/c/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used

= {3046, 2938, 2758, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{2}a^3(5A+9B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3c^3(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3c(5A+9B)\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (-2*Sqrt[2]*a^3*(5*A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c
- c*Sin[e + f*x]])])/(c^(3/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(2*f*(c
 - c*Sin[e + f*x])^(9/2)) + (a^3*(5*A + 9*B)*c*Cos[e + f*x]^5)/(10*f*(c - c
*Sin[e + f*x])^(5/2)) + (a^3*(5*A + 9*B)*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e
+ f*x])^(3/2)) + (2*a^3*(5*A + 9*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{1}{4}\left(a^3(5A+9B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}\\
&\quad -\frac{1}{2}\left(a^3(5A+9B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}\\
&\quad +\frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\left(a^3(5A+9B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}\\
&\quad +\frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\\
&\quad -\frac{\left(2a^3(5A+9B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}+\frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}\\
&\quad +\frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}+\frac{\left(4a^3(5A+9B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}
\end{aligned}$$

$$= -\frac{2\sqrt{2}a^3(5A+9B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.75 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.04

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A + 9
*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 5*(2*A + 9*B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2] - 3*B*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^2*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.62

method	result
default	$\frac{2a^3\left(\sin(fx+e)\left(-60A\sqrt{c+c\sin(fx+e)}\,c^{\frac{5}{2}}-5A(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}-120B\sqrt{c+c\sin(fx+e)}\,c^{\frac{5}{2}}-15B(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}-3B(c+\right.\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*a^3*(sin(f*x+e)*(-60*A*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-5*A*(c+c*sin(f*x
+e))^(3/2)*c^(3/2)-120*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-15*B*(c+c*sin(f*x+e
))^(3/2)*c^(3/2)-3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)+75*A*2^(1/2)*arctanh(1/
2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+135*B*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)+90*A*(c+c*sin(f*x+e))^(1/2)*c^(5/
2)+5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+150*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+
15*B*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-75*A
*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-135*B*2^(1
/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+
e)))^(1/2)/c^(9/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(195) = 390$.

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.97

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}\Big((5A+9B)a^3c\cos(fx+e)^2-(5A+9B)a^3c\cos(fx+e)-2(5A+9B)a^3c+((5}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((5*A + 9*B)*a^3*c*cos(f*x + e)^2 - (5*A + 9*B)*a^3*c*cos(
f*x + e) - 2*(5*A + 9*B)*a^3*c + ((5*A + 9*B)*a^3*c*cos(f*x + e) + 2*(5*A +
 9*B)*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*
x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) +
 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2))/sqrt(c) + 2*(3*B*a^3*cos(f*x + e)^4 - (5*A +
18*B)*a^3*cos(f*x + e)^3 - (65*A + 141*B)*a^3*cos(f*x + e)^2 - 30*(3*A + 5*
B)*a^3*cos(f*x + e) - 30*(A + B)*a^3 - (3*B*a^3*cos(f*x + e)^3 + (5*A + 21*
B)*a^3*cos(f*x + e)^2 - 60*(A + 2*B)*a^3*cos(f*x + e) + 30*(A + B)*a^3)*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x
 + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(195) = 390$.

Time = 0.43 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.35

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
[Out] -1/30*(30*sqrt(2)*(5*A*a^3*sqrt(c) + 9*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) + 15*sqrt(2)*(A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a^3*sqrt(c)*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqr
t(c) + 10*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 18*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1
```

```
/2*e))) - 16*sqrt(2)*(35*A*a^3*sqrt(c) + 81*B*a^3*sqrt(c) - 130*A*a^3*sqrt(
c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
) - 270*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 200*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 480*B*a^3*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 150*A*
a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^3 - 330*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 135*B*a^3*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^4)/(c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^3}{\left(c-c\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2
), x)
```

3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 225

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5a^3(3A+11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^3*(3*A+11*B)
*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)-5/24*a^3*(3*A+11*B)*cos(f*x+e)^3/c
/f/(c-c*sin(f*x+e))^(3/2)+5/4*a^3*(3*A+11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)
*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-5/4*a^3*(3*A+11*B)*cos(f
*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3046, 2938, 2759, 2758, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5a^3(3A+11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3c^3(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} - \frac{a^3c(3A+11B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (5*a^3*(3*A + 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[
e + f*x]])])/(2*Sqrt[2]*c^(5/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(4*f*
(c - c*Sin[e + f*x])^(11/2)) - (a^3*(3*A + 11*B)*c*Cos[e + f*x]^5)/(8*f*(c
- c*Sin[e + f*x])^(7/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x]^3)/(24*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x])/(4*c^2*f*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}\left(a^3(3A+11B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} \\
&\quad + \frac{1}{16}\left(5a^3(3A+11B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} \\
&\quad - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} + \frac{\left(5a^3(3A+11B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{8c} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} \\
&\quad - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{\left(5a^3(3A+11B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{4c^2} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\left(5a^3(3A+11B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2c^2f}
\end{aligned}$$

$$= \frac{5a^3(3A+11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.93

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\Big($$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(12*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 3*(9*A + 17*B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(3*A + 11*B)*ArcTan[(1/2 + I/
2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - 6*(2*A + 11*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4 + 2*B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 24*(
A + B)*Sin[(e + f*x)/2] - 6*(9*A + 17*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^2*Sin[(e + f*x)/2] - 6*(2*A + 11*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^4*Sin[(e + f*x)/2] - 2*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(3*
(e + f*x))/2]))/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
 f*x])^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(198) = 396.

Time = 3.98 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.93

method	result
default	$-\frac{a^3\left(\left(24A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-45A\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2+8B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{c}+120B\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-165\right.\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/12/c^(9/2)*a^3*((24*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-45*A*2^(1/2)*arctan
h(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+8*B*(c+c*sin(f*x+e))^(3/2
)*c^(1/2)+120*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-165*B*2^(1/2)*arctanh(1/2*(c
+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2)*cos(f*x+e)^2+sin(f*x+e)*(48*A*(c
+c*sin(f*x+e))^(1/2)*c^(3/2)-90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*c^2+16*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+240*B*(c+c*sin(f
*x+e))^(1/2)*c^(3/2)-330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/
2)/c^(1/2))*c^2)+54*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-132*A*(c+c*sin(f*x+e))
^(1/2)*c^(3/2)+90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*c^2+86*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-420*B*(c+c*sin(f*x+e))^(1/2)*c
^(3/2)+330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^
2)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(198) = 396.

Time = 0.29 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.24

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{15\sqrt{2}((3A+11B)a^3\cos(fx+e)^3+3(3A+11B)a^3\cos$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(15*sqrt(2)*((3*A + 11*B)*a^3*cos(f*x + e)^3 + 3*(3*A + 11*B)*a^3*cos(
f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B)*a^3 - ((3*A +
 11*B)*a^3*cos(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B
)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*B*a^3*cos(f*x + e)^4 - 4*(
3*A + 14*B)*a^3*cos(f*x + e)^3 + 3*(13*A + 37*B)*a^3*cos(f*x + e)^2 + 3*(13
*A + 53*B)*a^3*cos(f*x + e) - 12*(A + B)*a^3 - (4*B*a^3*cos(f*x + e)^3 + 12
*(A + 5*B)*a^3*cos(f*x + e)^2 + 3*(17*A + 57*B)*a^3*cos(f*x + e) + 12*(A +
B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(198) = 396$.

Time = 0.45 (sec) , antiderivative size = 778, normalized size of antiderivative = 3.46

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
[Out] 1/96*(60*sqrt(2)*(3*A*a^3*sqrt(c) + 11*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqrt(c) + 16*A
*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 32*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 90*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 330*B*a^3*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 3*(16*sqrt(2)*A*a^3*c^(7/2)
```

```
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 32*sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2 + sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/
c^6 - 128*sqrt(2)*(3*A*a^3*sqrt(c) + 17*B*a^3*sqrt(c) - 6*A*a^3*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 30
*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) + 3*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 21*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^3*((cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2
), x)
```

3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 217

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{5a^3(A+13B)\,\mathrm{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/6*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(13/2)-1/24*a^3*(A+13*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)+5/48*a^3*(A+13*B)*cos(f*x+e)^3/c/f/
(c-c*sin(f*x+e))^(5/2)-5/16*a^3*(A+13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(
1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)+5/16*a^3*(A+13*B)*cos(f*x+e)
/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

$= \{3046, 2938, 2759, 2758, 2728, 212\}$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{5a^3(A+13B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3c^3(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}} - \frac{a^3c(A+13B)\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (-5*a^3*(A + 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(8*Sqrt[2]*c^(7/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(6*f*(
c - c*Sin[e + f*x])^(13/2)) - (a^3*(A + 13*B)*c*Cos[e + f*x]^5)/(24*f*(c -
c*Sin[e + f*x])^(9/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x]^3)/(48*c*f*(c - c*S
in[e + f*x])^(5/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x])/(16*c^3*f*Sqrt[c - c*
Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{1}{12}\left(a^3(A+13B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \\
&\quad+\frac{1}{48}\left(5a^3(A+13B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \\
&\quad+\frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}}-\frac{\left(5a^3(A+13B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{32c^2}
\end{aligned}
$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}} - \frac{(5a^3(A+13B))\int\frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{16c^3}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}} + \frac{(5a^3(A+13B))\,\text{Subst}\left(\int\frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{8c^3f}$$

$$= -\frac{5a^3(A+13B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.66 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.94

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-
1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A + 25*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A + 47*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(24*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(190) = 380.

Time = 4.36 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.41

method	result
default	$\frac{a^3\left(15A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^3(fx+e))c^3+195B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^3(fx+e))c^3-45A\sqrt{2}\,\operatorname{arctanh}\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*sin(f*x+e)^3*c^3+195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3-45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-96*B*(c*(1+sin(f*x+e)))^(1/2
)*c^(5/2)*sin(f*x+e)^3-585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*A
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^(1/2)*c^
(5/2)*sin(f*x+e)^2+585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*A*2^(1
/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3-928*B*(c*(1+s
in(f*x+e)))^(3/2)*c^(3/2)-288*(c*(1+sin(f*x+e)))^(1/2)*B*c^(5/2)*sin(f*x+e)
-195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3+12
0*A*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)+888*B*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)
)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(190) = 380.

Time = 0.29 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{15\,\sqrt{2}\big((A+13\,B)a^3\cos{(fx+e)}^4-3\,(A+13\,B)a^3\cos{(fx}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x
+ e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e) +
```

```
8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos(f*
x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)*a^3*cos
(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)*a^3*cos(
f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A + 79*B)*a^3*
cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x +
 e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*co
s(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin
(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(190) = 380.

Time = 0.52 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.31

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/384*(1536*sqrt(2)*B*a^3/(c^(7/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - 60*sqrt(2)*(A*a^3*sqrt(c) + 13*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqrt(c) + 9*A*a^3*sq
rt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1) + 21*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 45*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 237*B*a^3*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 110*A
*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^3 + 1430*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))) - sqrt(2)*(45*A*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 237*B*a^3*c^(17/2)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*a^3*c^(17/
2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2 + 21*B*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2 + A*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + B*a^3*c^(17/2)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2
), x)
```

3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 217

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{5a^3(A-15B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}} + \frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/8*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(15/2)+1/48*a^3*(A-15*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/192*a^3*(A-15*B)*cos(f*x+e)^3/c/
f/(c-c*sin(f*x+e))^(7/2)+5/128*a^3*(A-15*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e
))^(3/2)-5/256*a^3*(A-15*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin
(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used

$= \{3046, 2938, 2759, 2728, 212\}$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{5a^3(A-15B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3c^3(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}} + \frac{a^3c(A-15B)\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (-5*a^3*(A - 15*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(128*Sqrt[2]*c^(9/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(8*f
*(c - c*Sin[e + f*x])^(15/2)) + (a^3*(A - 15*B)*c*Cos[e + f*x]^5)/(48*f*(c
- c*Sin[e + f*x])^(11/2)) - (5*a^3*(A - 15*B)*Cos[e + f*x]^3)/(192*c*f*(c -
 c*Sin[e + f*x])^(7/2)) + (5*a^3*(A - 15*B)*Cos[e + f*x])/(128*c^3*f*(c - c
*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{1}{16}\left(a^3(A-15B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \\
&\quad -\frac{1}{96}\left(5a^3(A-15B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \\
&\quad -\frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}}+\frac{\left(5a^3(A-15B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{128c^2} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \\
&\quad -\frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}} \\
&\quad -\frac{\left(5a^3(A-15B)\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{256c^4} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}} \\
&\quad +\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{\left(5a^3(A-15B)\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{128c^4f}
\end{aligned}
$$

$$
= -\frac{5a^3(A-15B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}} + \frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.41 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.64

$$
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\Big(
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Cos
[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 2703*
B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e + f*x
))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 + 120
*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2] + 40
5*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e + f*x)
)/2] - 397*A*Sin[(5*(e + f*x))/2] + 579*B*Sin[(5*(e + f*x))/2] - 15*A*Sin[(
7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2]))/(3072*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(190) = 380.

Time = 4.52 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.99

method	result
default	$-\frac{a^3\left(-15\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^4(A-15B)(\cos^4(fx+e))-60\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^4(A-15B)(\cos^2(fx+e))\right.}{}$
parts	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/768/c^(17/2)*a^3*(-15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^4-60*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))
^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^2*sin(f*x+e)+120*2^(1/2)*ar
ctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^2
+120*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*
B)*sin(f*x+e)+30*A*(c+c*sin(f*x+e))^(7/2)*c^(1/2)+292*A*(c+c*sin(f*x+e))^(5
/2)*c^(3/2)-440*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)+240*A*(c+c*sin(f*x+e))^(1/
2)*c^(7/2)+1086*B*(c+c*sin(f*x+e))^(7/2)*c^(1/2)-4380*B*(c+c*sin(f*x+e))^(5
/2)*c^(3/2)+6600*B*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-3600*B*(c+c*sin(f*x+e))^(
1/2)*c^(7/2)-120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^4+1800*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
c^4)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^
(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(190) = 380$.

Time = 0.30 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.92

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\, dx =$$

$$-\frac{15\sqrt{2}((A - 15B)a^3\cos(fx + e)^5 + 5(A - 15B)a^3\cos(fx + e)^4 - 8(A - 15B)a^3\cos(fx + e)^3 - 20(A}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/1536*(15*sqrt(2)*((A - 15*B)*a^3*cos(f*x + e)^5 + 5*(A - 15*B)*a^3*cos(f
*x + e)^4 - 8*(A - 15*B)*a^3*cos(f*x + e)^3 - 20*(A - 15*B)*a^3*cos(f*x + e
)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3 - ((A - 15*B)*a^3*c
os(f*x + e)^4 - 4*(A - 15*B)*a^3*cos(f*x + e)^3 - 12*(A - 15*B)*a^3*cos(f*x
 + e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(5*A + 181*B)*a^3*cos(f*x + e)^4 - (191*A - 561
*B)*a^3*cos(f*x + e)^3 - 2*(169*A + 537*B)*a^3*cos(f*x + e)^2 + 12*(21*A -
59*B)*a^3*cos(f*x + e) + 384*(A + B)*a^3 - (3*(5*A + 181*B)*a^3*cos(f*x + e
)^3 + 2*(103*A - 9*B)*a^3*cos(f*x + e)^2 - 12*(11*A + 91*B)*a^3*cos(f*x + e
) - 384*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*
x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f
*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5
*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5
*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(9/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(190) = 380.

Time = 0.49 (sec) , antiderivative size = 928, normalized size of antiderivative = 4.28

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
[Out] -1/12288*(120*sqrt(2)*(A*a^3 - 15*B*a^3)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(9/2)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^3*sqrt(c) + 3*B*a^3*sqrt(c) + 16*A*a^3*sq
rt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1) + 48*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 24*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 312*B*a^3*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 48*A*
a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^3 + 1392*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/
```

```
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 250*A*a^3*sqrt(c)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 3750*B*a^3*
sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4/(c^5*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (48*sqrt(2)*A*a^
3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1392*sqrt(2)*B*a^3*c^(31/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 24*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^2 - 312*sqrt(2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)^2 - 16*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^3 - 48*sqrt(2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^3 - 3*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 3*
sqrt(2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/c^20)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2
),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2
), x)
```

3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 266

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{a^3(3A-17B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}} + \frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}} - \frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/10*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(17/2)+1/80*a^3*(3*A-17*
B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-1/96*a^3*(3*A-17*B)*cos(f*x+e)^
3/c/f/(c-c*sin(f*x+e))^(9/2)+1/128*a^3*(3*A-17*B)*cos(f*x+e)/c^3/f/(c-c*sin
(f*x+e))^(5/2)-1/512*a^3*(3*A-17*B)*cos(f*x+e)/c^4/f/(c-c*sin(f*x+e))^(3/2)
-1/1024*a^3*(3*A-17*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+
e))^(1/2))/c^(11/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

= {3046, 2938, 2759, 2729, 2728, 212}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{a^3(3A-17B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f} - \frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}} + \frac{a^3c^3(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}} + \frac{a^3c(3A-17B)\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/
2),x]
```

```
[Out] -1/512*(a^3*(3*A - 17*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c
*Sin[e + f*x]])])/(Sqrt[2]*c^(11/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(
10*f*(c - c*Sin[e + f*x])^(17/2)) + (a^3*(3*A - 17*B)*c*Cos[e + f*x]^5)/(80
*f*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x]^3)/(96*c*f
*(c - c*Sin[e + f*x])^(9/2)) + (a^3*(3*A - 17*B)*Cos[e + f*x])/(128*c^3*f*(
c - c*Sin[e + f*x])^(5/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x])/(512*c^4*f*(c
- c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
```

```
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{1}{20}\left(a^3(3A-17B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} \\
&\quad -\frac{1}{32}\left(a^3(3A-17B)\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} \\
&\quad -\frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}}+\frac{\left(a^3(3A-17B)\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx}{64c^2}
\end{aligned}
$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}} + \frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}} - \frac{(a^3(3A-17B))\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{256c^4}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}} + \frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}} - \frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}} - \frac{(a^3(3A-17B))\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{1024c^5}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}} + \frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}} - \frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}} + \frac{(a^3(3A-17B))\,\text{Subst}\left(\int\frac{1}{2c-x^2}dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{512c^5f}$$

$$= -\frac{a^3(3A-17B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}} + \frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}} - \frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.20 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.54

$$\int\frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(56370*A*Cos[(e + f*x)/2] + 38970*B*Cos[(e + f*x)/2] - 31140*A*Cos[(3*(e + f*x))/2] - 38580*B*Cos[(3*(e + f*x))/2] - 10404*A*Cos[(5*(e + f*x))/2] - 12724*B*Cos[(5*(e + f*x))/2] + 435*A*Cos[(7*(e + f*x))/2] + 7775*B*Cos[(7*(e + f*x))/2] - 45*A*Cos[(9*(e + f*x))/2] + 255*B*Cos[(9*(e + f*x))/2] + (240 + 240*I)*(-1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10 + 56370*A*Sin[(e + f*x)/2] + 38970*B*Sin[(e + f*x)/2] + 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(3*(e + f*x))/2] - 10404*A*Sin[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))/2] - 435*A*Sin[(7*(e + f*x))/2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[(9*(e + f*x))/2] + 255*B*Sin[(9*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(11/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(235) = 470$.

Time = 5.05 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.98

method	result
default	$\frac{a^3\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^6(3A-17B)(\cos^4(fx+e))\sin(fx+e)-75\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^6(3A-17B)(\cos^4\right.}{}$
parts	Expression too large to display

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)

[Out] 1/15360*a^3*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^4*sin(f*x+e)-75*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^4-180*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2*sin(f*x+e)+300*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2+240*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin(f*x+e)-90*A*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x+e))^(5/2)*c^(7/2)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f*x+e))^(1/2)*c^(11/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin(f*x+e))^(7/2)*c^(5/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c*sin(f*x+e))^(3/2)*c^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/c^(23/2)/(sin(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(235) = 470$.

Time = 0.29 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.86

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx =$$
$$-\frac{15\sqrt{2}((3A - 17B)a^3\cos(fx + e)^6 - 5(3A - 17B)a^3\cos(fx + e)^5 - 18(3A - 17B)a^3\cos(fx + e)^4 +}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3*
cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3*c
os(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3*co
s(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6*(3*
A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32*(3*A
 - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(3*A -
17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c
*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x
+ e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B)*a^3*cos(
f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 389*B)*a^3*co
s(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*A + 43*B)*a^
3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 8
0*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*cos(f*x + e)^2 -
 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin(f*x + e))*sqrt
(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*
c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 -
16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x
+ e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f
*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(11/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(235) = 470$.

Time = 0.56 (sec) , antiderivative size = 990, normalized size of antiderivative = 3.72

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] -1/245760*(120*sqrt(2)*(3*A*a^3*sqrt(c) - 17*B*a^3*sqrt(c))*log(-(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^6*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(6*A*a^3*sqrt(c) + 6*B*a^3*sqrt(c
) + 15*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1) + 75*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 30*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 290*B*a^3*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 - 120*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 360*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 60*A*a^3*sqrt(c)*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4
 - 900*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^4 + 822*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 4658*B*a^3*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5/(c^6*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(60*A*a^3*c^(49/2)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 900
*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 120*A*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/
```

```
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 360*B*a^3*c^(49/2)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 30*A*a^3*c
^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^3 + 290*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 75*B*a^3*c^(49/
2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^4 + 6*A*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^5 + 6*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5)/(c^30*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^3}{(c-c\,\sin(e+f\,x))^{11/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/
2),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/
2), x)
```

3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = -\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}-\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}-\frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf}$$

```
[Out] -12/35*(7*A-9*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-1/7*(7*A-9*B)*c*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(9/
2)/a/c/f-128/35*(7*A-9*B)*c^4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-32/35*(
7*A-9*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

= {3046, 2934, 2726, 2725}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = -\frac{128c^4(7A-9B)\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}} - \frac{32c^3(7A-9B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af} - \frac{12c^2(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} - \frac{c(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-128*(7*A - 9*B)*c^4*Cos[e + f*x])/(35*a*f*Sqrt[c - c*Sin[e + f*x]]) - (32
*(7*A - 9*B)*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(35*a*f) - (12*(7*A
 - 9*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(35*a*f) - ((7*A - 9*B
)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*a*f) - ((A - B)*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(9/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{ac} \\
&= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} - \frac{(7A-9B)\int(c-c\sin(e+fx))^{7/2}\,dx}{2a} \\
&= -\frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} \\
&\quad - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} \\
&\quad - \frac{(6(7A-9B)c)\int(c-c\sin(e+fx))^{5/2}\,dx}{7a} \\
&= -\frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} \\
&\quad - \frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} \\
&\quad - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} \\
&\quad - \frac{(48(7A-9B)c^2)\int(c-c\sin(e+fx))^{3/2}\,dx}{35a} \\
&= -\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af} \\
&\quad - \frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} \\
&\quad - \frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} \\
&\quad - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} \\
&\quad - \frac{(64(7A-9B)c^3)\int\sqrt{c-c\sin(e+fx)}\,dx}{35a}
\end{aligned}$$

$$= -\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}} - \frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}$$
$$- \frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}$$
$$- \frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af}$$
$$- \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf}$$

Mathematica [A] (verified)

Time = 14.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(4900A-6125B+196(A-2B)\cos(2(e+fx)}{140af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{si}\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -1/140*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(4900*A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] +
2450*A*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B*Si
n[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*
x]))
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

method	result
default	$\frac{2c^4(\sin(fx+e)-1)(5B(\cos^4(fx+e))+(-7A+29B)(\cos^2(fx+e))\sin(fx+e)+(49A-103B)(\cos^2(fx+e))+(308A-436B)\sin(fx+e)}{35a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/35*c^4/a*(sin(f*x+e)-1)*(5*B*cos(f*x+e)^4+(-7*A+29*B)*cos(f*x+e)^2*sin(f*
x+e)+(49*A-103*B)*cos(f*x+e)^2+(308*A-436*B)*sin(f*x+e)+588*A-716*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = -\frac{2\left(5\,Bc^3\cos\left(fx+e\right)^4+(49\,A-103\,B)c^3\cos\left(fx+e\right)^2+4\,(147\,A-179\,B)c^3-\left((7\,A-29\,B)c^3\cos\left(fx+\right.\right.\right.}{35\,af\cos\left(fx+e\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147*
A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(182) = 364.

Time = 0.34 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(\frac{7\left(91\,c^{\frac{7}{2}}+\frac{86\,c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{336\,c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{266\,c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{490\,c^{\frac{7}{2}}\sin}{(\cos(fx}\right.}{\left(a+\frac{a\sin(fx+}{\cos(fx+e)}\right.}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7
```

```
/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*x + e)
 + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(7/2) +
407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 20
30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(182) = 364$.

Time = 0.51 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.88

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 16/35*sqrt(2)*sqrt(c)*(35*(A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c^
3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (77*A*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 109*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 504*
A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 728*B*c^3*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 1337*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 2009*B*c
^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1680*A*c^3*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^3 + 2800*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 10
15*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1015*B*c^3*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^4 - 280*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5
 + 280*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 + 35*A*c^3*(cos(-1/4*pi
```

```
 + 1/2*f*x + 1/2*e) - 1)^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^6 - 35*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^
6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
6)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) - 1)^7))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),
x)
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),
 x)
```

3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 159

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} - \frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf}$$

```
[Out] -1/5*(5*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-(A-B)*sec(f*x+e)*(c-
c*sin(f*x+e))^(7/2)/a/c/f-32/15*(5*A-7*B)*c^3*cos(f*x+e)/a/f/(c-c*sin(f*x+e
))^(1/2)-8/15*(5*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

= {3046, 2934, 2726, 2725}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{32c^3(5A-7B)\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8c^2(5A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} - \frac{c(5A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-32*(5*A - 7*B)*c^3*Cos[e + f*x])/(15*a*f*Sqrt[c - c*Sin[e + f*x]]) - (8*(
5*A - 7*B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a*f) - ((5*A - 7*
B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) - ((A - B)*Sec[e + f*
x]*(c - c*Sin[e + f*x])^(7/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{ac} \\
&= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf} - \frac{(5A-7B)\int(c-c\sin(e+fx))^{5/2}\,dx}{2a} \\
&= -\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} \\
&\quad -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf} \\
&\quad -\frac{(4(5A-7B)c)\int(c-c\sin(e+fx))^{3/2}\,dx}{5a} \\
&= -\frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} \\
&\quad -\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} \\
&\quad -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf} \\
&\quad -\frac{(16(5A-7B)c^2)\int\sqrt{c-c\sin(e+fx)}\,dx}{15a} \\
&= -\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} \\
&\quad -\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} \\
&\quad -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(450A-600B+2(5A-16B)\cos(2(e+fx))+}{30af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x]),x]
[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
450*A - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e + f
*x] + 3*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
+ Sin[e + f*x]))
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2c^3(\sin(fx+e)-1)\left(-3B\left(\cos^2(fx+e)\right)\sin(fx+e)+(-5A+16B)\left(\cos^2(fx+e)\right)+(-50A+82B)\sin(fx+e)-110A+142B\right)}{15a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	95

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
[Out] -2/15*c^3/a*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2*sin(f*x+e)+(-5*A+16*B)*cos(f*
x+e)^2+(-50*A+82*B)*sin(f*x+e)-110*A+142*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{2\left((5A-16B)c^2\cos(fx+e)^2+2(55A-71B)c^2+\left(3Bc^2\cos(fx+e)^2+2(25A-41B)c^2\right)\sin(fx+e\right.}{15\,af\cos(fx+e)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*cos
(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/
(a*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(145) = 290$.

Time = 0.32 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.43

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(\frac{5\left(23\,c^{\frac{5}{2}}+\frac{20\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{65\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{40\,c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{65\,c^{\frac{5}{2}}\sin\cdots}{(\cos(fx+\cdots}\right.}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^{\cdots}}\right.}\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/
2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(5/2)) - 2*(79*c^(5/2) + 79*c^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) + 205*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 170*c^(5/
2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 205*c^(5/2)*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 79*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 79*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(145) = 290.

Time = 0.44 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 8/15*sqrt(2)*sqrt(c)*(15*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (25*A*c^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - 41*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 110*A*
c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 190*B*c^2*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 160*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 320*B*c^2*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^3 + 90*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*c^2*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 1)^5))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}{a+a\,\sin(e+f\,x)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),
 x)
```

3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

```
[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/c/f-4/3*(3*A-5*B)*c^2*cos(f*x+e)
/a/f/(c-c*sin(f*x+e))^(1/2)-1/3*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/
2)/a/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2726, 2725}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{4c^2(3A-5B)\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{c(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-4*(3*A - 5*B)*c^2*Cos[e + f*x])/(3*a*f*Sqrt[c - c*Sin[e + f*x]]) - ((3*A
- 5*B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) - ((A - B)*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(5/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf} - \frac{(3A-5B)\int (c-c\sin(e+fx))^{3/2}\,dx}{2a}$$

$$= -\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf} - \frac{(2(3A-5B)c)\int\sqrt{c-c\sin(e+fx)}dx}{3a}$$

$$= -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

Mathematica [A] (verified)

Time = 9.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-18A+27B+B}{3af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{si}\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)]
 + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2c^2(\sin(fx+e)-1)\left(-B\left(\cos^2(fx+e)\right)+\sin(fx+e)(3A-7B)+9A-13B\right)}{3a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	73

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3*c^2/a*(sin(f*x+e)-1)*(-B*cos(f*x+e)^2+sin(f*x+e)*(3*A-7*B)+9*A-13*B)/co
s(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(Bc\cos(fx+e)^2-(3A-7B)c\sin(fx+e)-(9A-1\right.}{3af\cos(fx+e)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{Ac\sqrt{-c\sin(e+fx)+c}}{\sin(e+fx)+1}\,dx+\int\left(-\frac{Ac\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin(e+fx)+1}\right)d}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(-
A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integra
l(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integ
ral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x) + 1), x))/
a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(108) = 216.

Time = 0.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.49

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(\frac{3\left(3c^{\frac{3}{2}}+\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right.}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
```

```
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*A/((a + a*sin(f*x
+ e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) -
 2*(7*c^(3/2) + 7*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 12*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 7*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*B/((a + a*sin(f*x + e
)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(108) = 216$.

Time = 0.42 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.99

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{4\sqrt{2}\sqrt{c}\left(\frac{3\left(Ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right.\right.}{a\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 4/3*sqrt(2)*sqrt(c)*(3*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (3*A*c*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) - 7*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 18*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 1)^3))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),
 x)
```

3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 73

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

```
[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/c/f-(A-3*B)*c*cos(f*x+e)/a/f/(c-
c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3046, 2934, 2725}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{c(A-3B)\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 3*B)*c*Cos[e + f*x])/(a*f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*Sec
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\ &= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf} - \frac{(A-3B)\int \sqrt{c-c\sin(e+fx)}\,dx}{2a}\\ &= -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\sec(e+fx)(-A+2B+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{af}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x]),x]
[Out] (2*Sec[e + f*x]*(-A + 2*B + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(-B\sin(fx+e)+A-2B)}{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	53

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2*c/a*(sin(f*x+e)-1)*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\left(B\sin\left(fx+e\right)-A+2B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2*(B*sin(f*x + e) - A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{A\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx + \int \frac{B\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(69) = 138$.

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.38

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}-\frac{A\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqr
t(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqrt(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(69) = 138$.

Time = 0.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.40

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{2\sqrt{2}\left(A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{A\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}(\sin}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.}\right.}{af\left(\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-1\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -2*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - A*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - B*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1))*sqrt(c)/(a*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$$
$$=\frac{2\sqrt{-c\,(\sin(e+f\,x)-1)}\,\left(2\,B\,\sin(2\,e+2\,f\,x)-2\,A\,\sin(2\,e+2\,f\,x)-4\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+7\,B\,\left(\right.\right.}{a\,f\,\left(4\sin(e+f\,x)^2+\sin(e+f\,x)+\sin(3\,e+3\,f\,x)-\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] (2*(-c*(sin(e + f*x) - 1))^(1/2)*(2*B*sin(2*e + 2*f*x) - 2*A*sin(2*e + 2*f*
x) - 4*A*(2*sin(e/2 + (f*x)/2)^2 - 1) + 7*B*(2*sin(e/2 + (f*x)/2)^2 - 1) +
B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a*f*(sin(e + f*x) + sin(3*e + 3*f*x
) + 4*sin(e + f*x)^2 - 4))
```

3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] 1/2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/
f*2^(1/2)/c^(1/2)-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(Sqrt[2]*a*Sqrt[c]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\
&= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}+\frac{(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a}\\
&= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}-\frac{(A+B)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{af}\\
&= \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+B-(1+i)\sqrt[4]{-1}(A+B\right.}{af(1+\sin(e+fx))\sqrt{c-c\sin}}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e +
 f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

method	result
default	$-\frac{(\sin(fx+e)-1)\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}\,A+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}\,B\right.}{2a\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)*B)
/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.78

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\sqrt{2}(A+B)\sqrt{c}\cos(fx+e)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{4\,acf\cos(fx+e)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x +
 e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
 sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
 + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c)*
(A - B))/(a*c*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
+ c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + sqrt(-c*sin(e + f*x) + c)), x))/a
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\frac{\sqrt{2}(A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{4\sqrt{2}(A\sqrt{c}-B\sqrt{c})}{ac\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{4f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/4*(sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))) + 4*sqrt(2)*(A*sqrt(c) - B*sqrt(c))/(a*c*((cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
 x)
```

3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 136

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(3A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} + \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+1/8*(3*A-B)*arctanh(1/2*c
os(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-(A-B)
*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3046, 2934, 2729, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(3A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} + \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] ((3*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(4*Sqrt[2]*a*c^(3/2)*f) + ((3*A - B)*Cos[e + f*x])/(4*a*f*(c - c*Sin[e
+ f*x])^(3/2)) - ((A - B)*Sec[e + f*x])/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}\\
&= -\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}\\
&= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac}
\end{aligned}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} - \frac{(3A-B)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{4acf}$$

$$= \frac{(3A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} + \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.09

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (1 + I)*
(-1)^(1/4)*(3*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]) + 2*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(4*
a*f*(1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\sin(fx+e)\left(3A\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c-B\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c-6Ac^{\frac{3}{2}}\right.}{8c^{\frac{5}{2}}\,a\cos(fx+}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*
arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-6*A*c^(3/2)+2*B*c^(3/
2))-3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2
```

```
^(1/2)/c^(1/2))*c+B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x
+e))^(1/2)*2^(1/2)/c^(1/2))*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}((3\,A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-(3\,A-B)\cos\left(fx+e\right))\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\,\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{}}{\cos(fx+}\right.}{16\left(ac^2f\cos\left(fx+e\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x + e
))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqr
t(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e)
 - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c*sin(
f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{A}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{}{-c\sqrt{-c}}}{a}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] (Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
 + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{(a\sin\left(fx+e\right)+a)(-c\sin\left(fx+e\right)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(119) = 238.

Time = 0.36 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.98

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx=\frac{\frac{2\sqrt{2}(3A\sqrt{c}-B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}f\cdots\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\cdots\right.}\cdots\right.}{ac^2\mathrm{sg}\cdots}\cdots}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/32*(2*sqrt(2)*(3*A*sqrt(c) - B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))) + sqrt(2)*(A + B + 14*A*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 14*B*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*A*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + B*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a*c^(3/2)*(
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) +
 (cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),
 x)
```

3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 180

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3(5A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 3/32*(5*A-3*B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A+B)*sec(f*x+e)
/a/c/f/(c-c*sin(f*x+e))^(3/2)+3/64*(5*A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)
*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(5/2)/f*2^(1/2)-1/8*(5*A-3*B)*sec(f*x+
e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3046, 2938, 2766, 2729, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3(5A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} + \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (3*(5*A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(32*Sqrt[2]*a*c^(5/2)*f) + (3*(5*A - 3*B)*Cos[e + f*x])/(32*a*c*f*(
```

c - c*Sin[e + f*x])^(3/2)) + ((A + B)*Sec[e + f*x])/(4*a*c*f*(c - c*Sin[e + f*x])^(3/2)) - ((5*A - 3*B)*Sec[e + f*x])/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*x]])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2766

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2938

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 3046

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac} \\
&= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{(5A-3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2} \\
&= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} + \frac{(3(5A-3B))\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{16ac} \\
&= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} + \frac{(3(5A-3B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{64ac^2} \\
&= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} - \frac{(3(5A-3B))\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{32ac^2f} \\
&= \frac{3(5A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.24

$$
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\right.}{}
$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e +
```

```
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*A -
 B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
+ Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]) + 8*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]) + 2*(7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(32*a*f*(1 + Sin[e + f*
x])*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(157) = 314$.

Time = 1.01 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(-15A\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2+30A\,c^{\frac{5}{2}}+9B\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2-$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/64/c^(9/2)/a*((-15*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin
(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+30*A*c^(5/2)+9*B*(c+c*sin(f*x+e))^(1/2)
*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*c^(5/
2))*cos(f*x+e)^2+sin(f*x+e)*(-30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*A*c^(5/2)+18*B*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^
2-24*B*c^(5/2))+30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-24*A*c^(5/2)-18*B*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*B*c^(5/2)
)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((5A-3B)\cos(fx+e)^3+2(5A-3B)\cos(fx+e)\sin(fx+e)-2(5A-3B)\cos(fx+e)\right)\sqrt{c}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/128*(3*sqrt(2)*((5*A - 3*B)*cos(f*x + e)^3 + 2*(5*A - 3*B)*cos(f*x + e)*
sin(f*x + e) - 2*(5*A - 3*B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 -
 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) +
 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*
x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(5*A
 - 3*B)*cos(f*x + e)^2 + 4*(5*A - 3*B)*sin(f*x + e) - 12*A + 20*B)*sqrt(-c*
sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x
 + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(157) = 314$.

Time = 0.62 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\frac{12\sqrt{2}\left(5A\sqrt{c}-3B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}-\frac{16\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/512*(12*sqrt(2)*(5*A*sqrt(c) - 3*B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 16*A*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 90*A*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2 - 54*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(a*c^3*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 + 128*sqrt(2)*(A*sqrt(c) - B*sqrt(c))/(a*c^3*((cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) - (16*sqrt(2)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 sqrt(2)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - sqrt(2)*B*a*c
^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),
 x)
```

3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 242

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f} - \frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}$$

```
[Out] -512/105*(7*A-13*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-64/105*(7*A
-13*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-16/105*(7*A-13*B)*c*sec(
f*x+e)*(c-c*sin(f*x+e))^(7/2)/a^2/f-1/21*(7*A-13*B)*sec(f*x+e)*(c-c*sin(f*x
+e))^(9/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(13/2)/a^2/c^2/f+2
048/105*(7*A-13*B)*c^4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2753, 2752}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx=\frac{2048c^4(7A-13B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f}-\frac{512c^3(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}-\frac{64c^2(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f}-\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f}-\frac{16c(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (2048*(7*A - 13*B)*c^4*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*a^2*f) -
 (512*(7*A - 13*B)*c^3*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(105*a^2*f)
 - (64*(7*A - 13*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(105*a^2*f
) - (16*(7*A - 13*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*a^2*f)
 - ((7*A - 13*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(21*a^2*f) - ((A
- B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}\,dx}{a^2c^2}\\
&= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}\\
&\quad -\frac{(7A-13B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{6a^2c}\\
&= -\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f}\\
&\quad -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}\\
&\quad -\frac{(8(7A-13B))\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{21a^2}\\
&= -\frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f}\\
&\quad -\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f}\\
&\quad -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}\\
&\quad -\frac{(32(7A-13B)c)\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{35a^2}
\end{aligned}$$

$$= -\frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f} - \frac{(256(7A-13B)c^2)\int\sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{105a^2}$$

$$= -\frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f} - \frac{(1024(7A-13B)c^3)\int\sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{105a^2}$$

$$= \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f} - \frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}$$

Mathematica [A] (verified)

Time = 12.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}{}$$

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^2,x]

[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(95550*A - 179340*B - 72*(203*A - 402*B)*Cos[2*(e + f*x)] + 6*(7*A - 38*B)*Cos[4*(e + f*x)] + 119952*A*Sin[e + f*x] - 219618*B*Sin[e + f*x] + 784*A*Sin[3*(e + f*x)] - 2131*B*Sin[3*(e + f*x)] + 15*B*Sin[5*(e + f*x)]))/(840*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)

Maple [A] (verified)

Time = 20.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)(15B(\cos^4(fx+e))\sin(fx+e)+(21A-114B)(\cos^4(fx+e))+(196A-544B)(\cos^2(fx+e))\sin(fx+e)+(-1848A+}{105a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] -2/105*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(15*B*cos(f*x+e)^4*sin(f*x+e)+(21*A-114*B)*cos(f*x+e)^4+(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(-1848*A+3732*B)*cos(f*x+e)^2+(7448*A-13592*B)*sin(f*x+e)+6888*A-13032*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(3\,(7\,A-38\,B)c^4\cos\left(fx+e\right)^4-12\,(154\,A-311\,B)c^4\,\mathrm{c}\right.}{}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x + e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136*B

```
)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{9/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(218) = 436.

Time = 0.33 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.15

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{9/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
[Out] -2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 537
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(
9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^2 + 3*a^2*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2
)) - 2*(4707*c^(9/2) + 14121*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3525
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 68549*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 99549*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 134802*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 138012*c^(9/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 134802*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 99549*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 68549*c^(9/2
)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 35250*c^(9/2)*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 14121*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 47
```

```
07*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*B/((a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)^(9/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(218) = 436$.

Time = 0.84 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.04

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -16/105*sqrt(2)*sqrt(c)*(35*(11*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 17*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*c^4*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 36*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*c^4*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 1)^3) - (511*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 1069*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3262*A*c^4*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) + 6958*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 842
1*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 18459*B*c^4*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^2 - 10780*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^3 + 24220*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 7105*A*c^4*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 13195*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4 - 2310*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 + 3990*B*c^4
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 + 315*A*c^4*(cos(-1/4*pi + 1/2*f*x
```

```
+ 1/2*e) - 1)^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^6 - 525*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^6*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^6)/(a^2*((
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
1)^7))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
2, x)
```

3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 201

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}$$

```
[Out] -32/15*(5*A-11*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-4/15*(5*A-11*
B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/15*(5*A-11*B)*sec(f*x+e)*(c-
c*sin(f*x+e))^(7/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^
2/c^2/f+128/15*(5*A-11*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{3046, 2934, 2753, 2752\}$

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{128c^3(5A-11B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{32c^2(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{4c(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (128*(5*A - 11*B)*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a^2*f) - (
32*(5*A - 11*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (
4*(5*A - 11*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*a^2*f) - ((5*
A - 11*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*a^2*f) - ((A - B)*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}\,dx}{a^2c^2}\\
&= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}\\
&\quad-\frac{(5A-11B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{6a^2c}\\
&= -\frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f}\\
&\quad-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}\\
&\quad-\frac{(2(5A-11B))\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{5a^2}\\
&= -\frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f}\\
&\quad-\frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f}\\
&\quad-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}\\
&\quad-\frac{(16(5A-11B)c)\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{15a^2}
\end{aligned}
$$

$$= -\frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{(64(5A-11B)c^2)\int\sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{15a^2}$$

$$= \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}$$

Mathematica [A] (verified)

Time = 8.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-2100A+4725B+12(25A-62B)\cos(2(e+}{60a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
-2100*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)]
 - 2730*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 46*B
*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
 + f*x])^2)
```

Maple [A] (verified)

Time = 20.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

method	result
default	$\frac{2c^4(\sin(fx+e)-1)(3B(\cos^4(fx+e))+(-5A+23B)(\cos^2(fx+e))\sin(fx+e)+(75A-189B)(\cos^2(fx+e))+(-340A+724B)\sin(fx+e)}{15a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x,method=_RE TURNVERBOSE)

[Out] 2/15*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*cos(f*x+e)^4+(-5*A+23*B)*co s(f*x+e)^2*sin(f*x+e)+(75*A-189*B)*cos(f*x+e)^2+(-340*A+724*B)*sin(f*x+e)-3 00*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{2\left(3\,Bc^3\cos\left(fx+e\right)^4+3\,(25\,A-63\,B)c^3\cos\left(fx+e\right)^2-12\,(25\,A-57\,B)c^3-\left((5\,A-23\,B)c^3\cos\left(fx+\right.\right.\right.}{15\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+\right.\right.}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg orithm="fricas")

[Out] -2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(25 *A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)*s in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a ^2*f*cos(f*x + e))

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(181) = 362$.

Time = 0.34 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.33

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^
(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^
(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2896*c
^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 361
2*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/2)*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(181) = 362$.

Time = 0.75 (sec) , antiderivative size = 774, normalized size of antiderivative = 3.85

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -16/15*sqrt(2)*sqrt(c)*(5*(4*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*
B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*A*c^3*(cos(-1/4*pi + 1/2*f*x
```

```
+ 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 6*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 1)^3) - (20*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 53*B*
c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 85*A*c^3*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 235*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 125*A*c^3*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 365*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2 - 75*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 165*B*c^3*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4 - 30*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^2*((cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5))
/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{7/2}}{\left(a+a\,\sin(e+f\,x)\right)^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
2, x)
```

3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 154

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}$$

```
[Out] -8/3*(A-3*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-3*B)*sec(f*x+
e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/
2)/a^2/c^2/f+32/3*(A-3*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2753, 2752}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} + \frac{32c^2(A-3B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{8c(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (32*(A - 3*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - (8*(A
 - 3*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - 3*B)*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(9/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
```

```
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{a^2c^2} \\
&= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} \\
&\quad -\frac{(A-3B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{2a^2c} \\
&= -\frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} \\
&\quad -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} \\
&\quad -\frac{(4(A-3B))\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{3a^2} \\
&= -\frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} \\
&\quad -\frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} \\
&\quad -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} \\
&\quad -\frac{(16(A-3B)c)\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{3a^2}
\end{aligned}
$$

$$= \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}$$

Mathematica [A] (verified)

Time = 7.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-50A+160B+6(A-4B)\cos(2(e+fx))+(-}{6a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -1/6*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-
50*A + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x]
+ B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Si
n[e + f*x])^2)
```

Maple [A] (verified)

Time = 19.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{2c^3(\sin(fx+e)-1)\left(-B\left(\cos^2(fx+e)\right)\sin(fx+e)+(-3A+12B)\left(\cos^2(fx+e)\right)+(18A-50B)\sin(fx+e)+14A-46B\right)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	105

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(-B*cos(f*x+e)^2*sin(f*x+e)+(-3*
A+12*B)*cos(f*x+e)^2+(18*A-50*B)*sin(f*x+e)+14*A-46*B)/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(3\,(A-4\,B)c^2\cos\left(fx+e\right)^2-2\,(7\,A-23\,B)c^2+\left(Bc^2\cos\left(fx+e\right)^2-2\,(9\,A-25\,B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{\cdots}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x
+ e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f
*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(138) = 276.

Time = 0.33 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.75

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{90\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{11\cdots}{(\cdots}\right.}{\left(a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}\cdots\right)}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(17
*c^(5/2) + 51*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 92*c^(5/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 149*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 150*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 149*c^(5/2)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 92*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 + 51*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(5/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(138) = 276$.

Time = 0.61 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.90

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{32\sqrt{2}\Big(Ac^2\mathrm{sgn}\big(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\big)-3Bc^2\mathrm{sgn}\big(\sin\left(\right.$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 32/3*sqrt(2)*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 3*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
 + 9*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 2*A*c^2*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^3 + 2*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*s
qrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^2 - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
2, x)
```

3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 115

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx=\frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}$$

```
[Out] -1/3*(A-7*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3
*(c-c*sin(f*x+e))^(7/2)/a^2/c^2/f+4/3*(A-7*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))
^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2753, 2752}

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx &= \\ &-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f} \\ &-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} \\ &+\frac{4c(A-7B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (4*(A - 7*B)*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - 7*B
)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x
]^3*(c - c*Sin[e + f*x])^(7/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^2c^2}\\ &= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}\\ &\quad-\frac{(A-7B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{6a^2c}\end{aligned}$$

$$= -\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f} - \frac{(2(A-7B))\int\sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{3a^2}$$

$$= \frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}$$

Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(2A-23B+3B\cos}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)]
 + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{2c^2(\sin(fx+e)-1)(3B(\cos^2(fx+e))+\sin(fx+e)(3A-15B)+A-13B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	81

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*cos(f*x+e)^2+sin(f*x+e)*(3*
A-15*B)+A-13*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(3\,Bc\cos\left(fx+e\right)^2+3\left(A-5\,B\right)c\sin\left(fx+e\right)+(A-\right.}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\,c\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e)
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(103) = 206.

Time = 0.35 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.19

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx =$$

$$-\frac{2\left(\frac{\left(c^{\frac{3}{2}}+\frac{6\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{12\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6\,c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)A}{\left(a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}-\frac{2\left(5\,c^{\frac{3}{2}}+\frac{15\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}\right.}{\left(a^2\right.}\right.}{3\,f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
```

```
e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*A
/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*sin(f*x + e)/(
cos(f*x + e) + 1) + 21*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*c^(
3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 21*c^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^(3/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(103) = 206$.

Time = 0.54 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.46

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{4\sqrt{2}\sqrt{c}\left(\frac{3\,Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)}+\frac{Ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\,Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{3\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\cdots}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}f\cdots\right.}}{\cdots}\right)}{3\,f\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -4/3*sqrt(2)*sqrt(c)*(3*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^2*((cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1))
+ (A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + 3*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 9*B*c*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) - 3*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2
*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
 + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.28

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2\,B\,c}{a^2\,f}-\frac{B\,c\,e^{e\,1i+f\,x\,1i}\,2i}{a^2\,f}\right)}{e^{e\,1i+f\,x\,1i}-i} + \frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2\,B\,c}{3\,a^2\,f}-\frac{c\,(2\,A-3\,B)}{3\,a^2\,f}-\frac{2\,c\,(3\,A-2\,B)}{3\,a^2\,f}+\frac{c\,(A\,2i-B\,3i)\,1i}{3\,a^2\,f}+\frac{c\,(A\,3i-B\,2i)\,2i}{3\,a^2\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^3} - \frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{c\,(A-B)\,4i}{a^2\,f}+\frac{c\,(A\,1i-B\,2i)}{a^2\,f}+\frac{c\,(A\,1i+B\,2i)}{3\,a^2\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^2} - \frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{4\,B\,c}{a^2\,f}+\frac{c\,(A\,1i-B\,2i)\,4i}{a^2\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((2*B*c)/(3*a^2*f) - (c*(2*A - 3*B))/(3*a^2*f) - (2*c*(3*A
 - 2*B))/(3*a^2*f) + (c*(A*2i - B*3i)*1i)/(3*a^2*f) + (c*(A*3i - B*2i)*2i)/
(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - ((c -
 c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((2*B*c
)/(a^2*f) - (B*c*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i) - 1i)
 - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*((c*(A - B)*4i)/(a^2*f) + (c*(A*1i - B*2i))/(a^2*f) + (
c*(A*1i + B*2i))/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
 + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2)*((4*B*c)/(a^2*f) + (c*(A*1i - B*2i)*4i)/(a^2*f
)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^2/c^2/f-1/3*(A+5*B)*sec(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3046, 2934, 2752}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f} - \frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] -1/3*((A + 5*B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a^2*f) - ((A - B)*S
ec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2} \\
&= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f} \\
&\quad + \frac{(A+5B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{6a^2c} \\
&= -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$
\begin{aligned}
&\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx \\
&= -\frac{2(A+2B+3B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}
\end{aligned}
$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^2,x]
```

```
[Out] (-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(3B\sin(fx+e)+A+2B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	63

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\,(3\,B\sin\left(fx+e\right)+A+2\,B)\sqrt{-c\sin\left(fx+e\right)+c}}{3\,(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right))}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x
+ e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{A\sqrt{-c\sin(e+fx)+c}}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**2
+ 2*sin(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(70) = 140.

Time = 0.31 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.40

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(\frac{2B\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{A\left(\sqrt{c}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\cdots}}\right)}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] 2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sq
rt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(70) = 140$.

Time = 0.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.86

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\sqrt{2}\left(A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{12B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right.}{3a^2f\left(\frac{\cos\left(-\frac{1}{4}\pi\right.}{\cos\left(-\frac{1}{4}\pi\right.}\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/3*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 12*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2 + 3*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)
*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(B\,3\mathrm{i}+2\,A\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+4\,B\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-B\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}\right)}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^3\,\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*(B*3i + 2*A*exp(e*1i + f*x*1i) + 4*B*exp(e*1i + f*x*1i)
- B*exp(e*2i + f*x*2i)*3i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*1i
+ f*x*1i)*1i + 1))
```

3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 135

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f+1/4*(A+B)*arctanh(
1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/f*2^(1/2)/c^(1/2
)-1/2*(A+B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2934, 2754, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(2*Sqrt[2]*a^2*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(2*a^2*c*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2
*c^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{(A+B)\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{2a^2c}$$
$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{4a^2}$$
$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} - \frac{(A+B)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2a^2f}$$
$$= \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(-A+B)-3(A+B)\left(\cos\left(\right.\right.\right.}{6a^2}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 + 3*
I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sq
rt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

method	result
default	$-\frac{(\sin(fx+e)-1)\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))^{\frac{3}{2}}cA+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))}{12a^2c^{\frac{5}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/12/a^2*(sin(f*x+e)-1)/c^(5/2)/(1+sin(f*x+e))*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A+3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*B-10*A*c^(5/2)-6*A*c^(5/2)*sin(f*x+e)-2*B*c^(5/2)-6*B*c^(5/2)*sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.61

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{3\sqrt{2}((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos}{\cos(fx+e)^2}\right.}{24\left(a^2cf\cos(fx+e)\right.}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*sin(f*x + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e\cdots}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(\cdots}}}{a^2}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e +
f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e
+ f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x))/a**2
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^2\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(116) = 232.

Time = 0.41 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.92

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\frac{3\sqrt{2}(A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{8\sqrt{2}\left(2A\sqrt{c}+B\sqrt{c}+\frac{3A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\cdots}{a^2c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\cdots\right.\right.}}{24f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))) + 8*sqrt(2)*(2*A*sqrt(c) + B*sqrt(c) + 3*A*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*B*sqrt(
c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
) + 3*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^2)/(a^2*c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}} \,d x$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
), x)
```

3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(5A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}$$

```
[Out] 1/8*(5*A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)+1/16*(5*A+B)*arctanh(1/
2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(3/2)/f*2^(1/2)-
1/6*(5*A+B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^
3*(c-c*sin(f*x+e))^(1/2)/a^2/c^2/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3046, 2934, 2766, 2729, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(5A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((5*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(8*Sqrt[2]*a^2*c^(3/2)*f) + ((5*A + B)*Cos[e + f*x])/(8*a^2*f*(c - c*Si
n[e + f*x])^(3/2)) - ((5*A + B)*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e +
 f*x]]) - ((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}\\
&= -\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\int\frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{6a^2c}\\
&= -\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}\\
&\quad+\frac{(5A+B)\int\frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{4a^2}\\
&= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{16a^2c}\\
&= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}\\
&\quad-\frac{(5A+B)\text{Subst}\left(\int\frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{8a^2cf}\\
&= \frac{(5A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}\\
&\quad-\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.71

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\right.}{}$$

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(3/2)),x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^3))/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
f*x])^(3/2))

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.60

method	result
default	$-\frac{15A(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+30A\,c^{\frac{5}{2}}\left(\cos^2(fx+e)\right)+3B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-}}{}\right.}{}$

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)

[Out] -1/48/c^(7/2)/a^2*(15*A*(c+c*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(c+c*sin
(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c+30*A*c^(5/2)*cos(f*x+e)^2+3*B*
(c+c*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)*c+6*B*c^(5/2)*cos(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c+c*s
in(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A-20*A*c^(5/2)*s
in(f*x+e)-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+
c*sin(f*x+e))^(3/2)*c*B-4*B*c^(5/2)*sin(f*x+e)-4*A*c^(5/2)-20*B*c^(5/2))/(1
+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}(5A+B)\sqrt{c}\cos(fx+e)^3\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c}\cdots}{\cdots}\right)\cdots}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2
*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1
) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x
+ e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A +
 B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(152) = 304$.

Time = 0.45 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.85

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\,dx = \frac{\frac{6\sqrt{2}(5A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\cdots\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\cdots\right.}\cdots\right.}{a^2c\cdots}\cdots}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/192*(6*sqrt(2)*(5*A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^2*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 10*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
2*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*c^2*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 16*sqrt(2)*(7*A*
sqrt(c) - B*sqrt(c) + 12*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c
^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\,dx = \int \frac{A + B\,\sin\left(e + f\,x\right)}{{\left(a + a\,\sin\left(e + f\,x\right)\right)}^2\,{\left(c - c\,\sin\left(e + f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
), x)
```

3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 225

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5(7A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} + \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 5/64*(7*A-B)*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+1/24*(7*A-B)*sec(f*x
+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+5/128*(7*A-B)*arctanh(1/2*cos(f*x+e)*c^(
1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-5/48*(7*A-B)*sec
(f*x+e)/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^3/a^2/c^2/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3046, 2934, 2760, 2766, 2729, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5(7A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (5*(7*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(64*Sqrt[2]*a^2*c^(5/2)*f) + (5*(7*A - B)*Cos[e + f*x])/(64*a^2*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((7*A - B)*Sec[e + f*x])/(24*a^2*c*f*(c - c*Si
n[e + f*x])^(3/2)) - (5*(7*A - B)*Sec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Si
n[e + f*x]]) - ((A - B)*Sec[e + f*x]^3)/(3*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(7A-B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{6a^2c}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(5(7A-B))\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{48a^2c^2}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(5(7A-B))\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{32a^2c}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(5(7A-B))\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{128a^2c^2}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(5(7A-B))\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{64a^2c^2f}$$

$$= \frac{5(7A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} + \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(
7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(A
 + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(11*A +
3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3))/(192*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(198) = 396.

Time = 0.92 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.89

method	result
default	$-\frac{-210A\left(\sin^3(fx+e)\right)c^{\frac{7}{2}}+30B\left(\sin^3(fx+e)\right)c^{\frac{7}{2}}-46B\sin(fx+e)c^{\frac{7}{2}}+70A\left(\sin^2(fx+e)\right)c^{\frac{7}{2}}-10B\left(\sin^2(fx+e)\right)c^{\frac{7}{2}}+322A\sin(fx+e)c^{\frac{7}{2}}}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/384/c^(11/2)/a^2*(-210*A*sin(f*x+e)^3*c^(7/2)+30*B*sin(f*x+e)^3*c^(7/2)-
46*B*sin(f*x+e)*c^(7/2)+70*A*sin(f*x+e)^2*c^(7/2)-10*B*sin(f*x+e)^2*c^(7/2)
+322*A*sin(f*x+e)*c^(7/2)-15*B*sin(f*x+e)^2*2^(1/2)*arctanh(1/2*(c*(1+sin(f
*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c^2-210*A*sin(f*x+e
)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f
*x+e)))^(3/2)*c^2+30*B*sin(f*x+e)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c^2+105*A*sin(f*x+e)^2*2^(1/2
)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^
(3/2)*c^2-86*A*c^(7/2)+122*B*c^(7/2)+105*A*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-15*B*(c*(1+sin(f
*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*c^2)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{15\sqrt{2}\left((7A-B)\cos(fx+e)^3\sin(fx+e)-(7A-B)\cos(fx+e)^3\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/768*(15*sqrt(2)*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(f
*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e)
+ c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(
f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*(7*A
- B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*sin(f*
x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(198) = 396$.

Time = 0.57 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.33

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/3072*(60*sqrt(2)*(7*A*sqrt(c) - B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^3*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 24*A*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 8*B
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1) + 210*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2 - 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2/(a^2*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(5*A*sqrt(c) - 2*B*sqrt(c) + 9*A*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
```

```
3*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 6*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^3*((cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - 3*(24*sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 8*sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
 - sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - sqrt(2)*B
*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^4*c^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
), x)
```

3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 242

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = &-\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}\\ &+\frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}\\ &-\frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f}\\ &-\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f}\\ &-\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf}\\ &-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}\end{aligned}$$

```
[Out] -2048/15*(A-3*B)*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+512/5*(A-3*B
)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-64/5*(A-3*B)*c*sec(f*x+e)^3
*(c-c*sin(f*x+e))^(7/2)/a^3/f-16/15*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(
9/2)/a^3/f-1/5*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^3/c/f-1/5*(A-
B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(15/2)/a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2753, 2752}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{2048c^3(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{512c^2(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{64c(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-2048*(A - 3*B)*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f)
+ (512*(A - 3*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) -
 (64*(A - 3*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (16
*(A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*f) - ((A - 3*
B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c*f) - ((A - B)*Sec[e
 + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
```

```
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}\,dx}{a^3c^3}\\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(A-3B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{2a^3c^2}\\
&= -\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(8(A-3B))\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{5a^3c}\\
&= -\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(32(A-3B))\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{5a^3}
\end{aligned}
$$

$$= -\frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(256(A-3B)c)\int\sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{5a^3}$$

$$= \frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} + \frac{(1024(A-3B)c^2)\int\sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{5a^3}$$

$$= -\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}$$

Mathematica [A] (verified)

Time = 9.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{c^4(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}(11298A-33516B-40(137A-402B)\cos(2(e+fx))-10(A-}{120a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/120*(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 33516
*B - 40*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)] +
15600*A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1335
*B*Sin[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 120.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)(3B(\cos^4(fx+e))\sin(fx+e)+(5A-30B)(\cos^4(fx+e))+(100A-336B)(\cos^2(fx+e))\sin(fx+e)+(680A-1980B}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(3*B*cos(f*x+e)^4*sin(f*x+e)+
(5*A-30*B)*cos(f*x+e)^4+(100*A-336*B)*cos(f*x+e)^2*sin(f*x+e)+(680*A-1980*B
)*cos(f*x+e)^2+(-1000*A+3048*B)*sin(f*x+e)-1048*A+3096*B)/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{2\left(5\,(A-6\,B)c^4\cos{(fx+e)}^4+20\,(34\,A-99\,B)c^4\cos{(fx+e)}^2-8\,(131\,A-387\,B)c^4+\left(3\,Bc^4\cos{(fx+\right.}{15\left(a^3f\cos{(fx+e)}^3-2\,a^3f\cos{(fx+e)}\,\mathrm{s}\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^2
 - 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*co
s(f*x + e)^2 - 8*(125*A - 381*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(
f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(218) = 436$.

Time = 0.33 (sec) , antiderivative size = 945, normalized size of antiderivative = 3.90

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 30
200*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9
/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*A/
((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2)) - 6*(181*c^(9/2) + 905*c^(9/
```

```
2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2627*c^(9/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 5870*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10521*c^(9/
2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15351*c^(9/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 19695*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 20772
*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 19695*c^(9/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15351*c^(9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
 10521*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 5870*c^(9/2)*sin(f*x
 + e)^11/(cos(f*x + e) + 1)^11 + 2627*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e)
 + 1)^12 + 905*c^(9/2)*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 181*c^(9/2)*
sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(9/2)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{1024\sqrt{2}\left(Ac^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3Bc^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{5Ac^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right.}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1024/15*sqrt(2)*(A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^4*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^2 + 15*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*A*c^4*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^4 - 30*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^4 - 6*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 6*B*c^4*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^5)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{9/2}}{\left(a+a\,\sin(e+f\,x)\right)^3}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3, x)
```

3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 209

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}$$

```
[Out] -128/15*(3*A-13*B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+32/5*(3*A-
13*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-4/5*(3*A-13*B)*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-1/15*(3*A-13*B)*sec(f*x+e)^3*(c-c*sin(f*x+e
))^(9/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(13/2)/a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2753, 2752}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f} - \frac{128c^2(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} + \frac{32c(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-128*(3*A - 13*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f
) + (32*(3*A - 13*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f)
 - (4*(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (
(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*c*f) - ((A
- B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}\,dx}{a^3c^3}\\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}\\
&\quad -\frac{(3A-13B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{10a^3c^2}\\
&= -\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf}\\
&\quad -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}\\
&\quad -\frac{(2(3A-13B))\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{5a^3c}\\
&= -\frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f}\\
&\quad -\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf}\\
&\quad -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}\\
&\quad -\frac{(16(3A-13B))\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{5a^3}
\end{aligned}$$

$$= \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f} + \frac{(64(3A-13B)c)\int\sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{5a^3}$$

$$= -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}$$

Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(1092A-4557B+(-540A+2200B)\cos(2(e-}{60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
1092*A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)]
 + 1410*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 170*
B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[
e + f*x])^3)
```

Maple [A] (verified)

Time = 121.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2c^4(\sin(fx+e)-1)(5B(\cos^4(fx+e))+(-15A+85B)(\cos^2(fx+e))\sin(fx+e)+(-135A+545B)(\cos^2(fx+e))+(180A-820B)\sin(fx+\cdots}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 2/15*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*cos(f*x+e)^4+(-15*A+85*B)*cos(f*x+e)^2*sin(f*x+e)+(-135*A+545*B)*cos(f*x+e)^2+(180*A-820*B)*sin(f*x+e)+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\left(5\,Bc^3\cos\left(fx+e\right)^4-5\,(27\,A-109\,B)c^3\cos\left(fx+e\right)^2-\cdots\right.}{15\,\left(a^3f\cos\left(\cdots\right.\right.}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51*A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(189) = 378$.

Time = 0.33 (sec) , antiderivative size = 854, normalized size of antiderivative = 4.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(3*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(
7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 106
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
110*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
 - 2*(147*c^(7/2) + 735*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1992*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4015*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 6605*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8370*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9520*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 8370*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 660
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4015*c^(7/2)*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 + 1992*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
+ 735*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 147*c^(7/2)*sin(f*x +
 e)^12/(cos(f*x + e) + 1)^12)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/
2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(189) = 378.

Time = 0.56 (sec) , antiderivative size = 774, normalized size of antiderivative = 3.70

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -4/15*sqrt(2)*sqrt(c)*(5*(3*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 19*
B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c^3*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 42*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^2)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) - 1)^3) - (33*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 113*
B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 150*A*c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 490*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 240*A*c^3*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 740*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^2 + 90*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 390*B*c^3*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*c^3*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^4 - 75*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*((cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^
5))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{7/2}}{\left(a+a\,\sin(e+f\,x)\right)^3}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
3, x)
```

3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}$$

```
[Out] -32/15*(A-11*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+8/5*(A-11*B)*se
c(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-1/5*(A-11*B)*sec(f*x+e)^3*(c-c*sin(
f*x+e))^(7/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c^
3/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used

= {3046, 2934, 2753, 2752}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f} - \frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} + \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{32c(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-32*(A - 11*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) + (
8*(A - 11*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) - ((A - 1
1*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^3}\\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}\\
&\quad -\frac{(A-11B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{10a^3c^2}\\
&= -\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}\\
&\quad -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}\\
&\quad -\frac{(4(A-11B))\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{5a^3c}\\
&= \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}\\
&\quad -\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}\\
&\quad -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}\\
&\quad +\frac{(16(A-11B))\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{5a^3}\\
&= -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}\\
&\quad +\frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}\\
&\quad -\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}\\
&\quad -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(58A-488B-30(A-8B)\cos(2(e+fx))+5}{30a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
58*A - 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*x]
 + 15*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
+ Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 118.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2c^3(\sin(fx+e)-1)(15B(\cos^2(fx+e))\sin(fx+e)+(-15A+120B)(\cos^2(fx+e))+(10A-170B)\sin(fx+e)+22A-182B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	105

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)+
(-15*A+120*B)*cos(f*x+e)^2+(10*A-170*B)*sin(f*x+e)+22*A-182*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{2\left(15(A-8B)c^2\cos(fx+e)^2-2(11A-91B)c^2-5\left(3Bc^2\cos(fx+e)^2+2(A-17B)c^2\right)\sin(fx+}{15\left(a^3f\cos(fx+e)^3-2a^3f\cos(fx+e)\sin(fx+e)-2a^3f\cos(fx+e)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*c
os(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(
a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x
+ e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(144) = 288$.

Time = 0.31 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(31*c^(5/2) + 155*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 395*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 680*
c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1030*c^(5/2)*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 1050*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10
30*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 680*c^(5/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 395*c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1
55*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 31*c^(5/2)*sin(f*x + e)^10
```

```
/(cos(f*x + e) + 1)^10)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(144) = 288$.

Time = 0.51 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.81

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}}{(a + a\sin(e + fx))^3}\,dx = \frac{4\sqrt{2}\sqrt{c}\left(\frac{15\,Bc^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)} + \frac{4\,Ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,f\right.\right.}{}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 4/15*sqrt(2)*sqrt(c)*(15*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^3*((c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1
)) + (4*A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 29*B*c^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 20*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 130*B
*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 40*A*c^2*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2 - 200*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*B*c^
2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 15*B*c^2*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^4)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 904, normalized size of antiderivative = 5.65

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*B*c^2)/(a^3*f) - (B*c^2*exp(e*1i + f*x*1i)*2i)/(a^3*f)))/(exp(e*1i + f*x*
1i) - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*7i)*1i)/(3*a^3*f) - (2*c^2*(7*
A - 12*B))/(3*a^3*f) + (c^2*(A*23i - B*28i)*2i)/(3*a^3*f) - (c^2*(42*A - 67
*B))/(15*a^3*f) + (2*B*c^2)/(3*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1
i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i
)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i)*4i)/(a^3*f) + (
4*B*c^2)/(a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)) -
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((8*c^2*(A*1i - B*1i))/(a^3*f) + (c^2*(A*1i - B*3i))/(2*a^
3*f) + (c^2*(A*11i - B*1i))/(10*a^3*f) + (c^2*(12*A - 17*B)*1i)/(4*a^3*f) +
 (c^2*(52*A - 47*B)*1i)/(4*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i +
f*x*1i) + 1i)^4) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i))/(a^3*f) + (c^2*(A*
5i - B*4i))/(3*a^3*f) + (c^2*(A - 2*B)*8i)/(a^3*f)))/((exp(e*1i + f*x*1i) -
 1i)*(exp(e*1i + f*x*1i) + 1i)^2) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*5i)*1i
)/(5*a^3*f) - (c^2*(4*A - 3*B))/(a^3*f) - (c^2*(2*A - 5*B))/(5*a^3*f) + (c^
2*(A*4i - B*3i)*1i)/(a^3*f) - (c^2*(10*A - 11*B))/(5*a^3*f) + (c^2*(A*10i -
 B*11i)*1i)/(5*a^3*f) + (2*B*c^2)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)^5)
```

3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 121

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx=\frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}$$

```
[Out] 4/15*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f-1/5*(A+9*B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))
^(9/2)/a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3046, 2934, 2753, 2752}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}+\frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (4*(A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) - ((A +
9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^3}\\ &= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}\\ &\quad+\frac{(A+9B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{10a^3c^2}\end{aligned}$$

$$= -\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f} - \frac{(2(A+9B))\int\sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{5a^3c}$$

$$= \frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}$$

Mathematica [A] (verified)

Time = 6.83 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2A+27B-15B}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*x
)] + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{2c^2(\sin(fx+e)-1)\left(-15B\left(\cos^2(fx+e)\right)+\sin(fx+e)(5A+15B)-A+21B\right)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	83

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(-15*B*cos(f*x+e)^2+sin(f*x+e
)*(5*A+15*B)-A+21*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\left(15\,Bc\cos\left(fx+e\right)^2 - 5\,(A+3\,B)c\sin\left(fx+e\right) + (A-\right.}{15\left(a^3f\cos\left(fx+e\right)^3 - 2\,a^3f\cos\left(fx+e\right)\sin\left(fx+\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*sq
rt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*
x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.48

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
```

```
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 6
*(c^(3/2) + 5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 14*c^(3/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 15*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 26*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 14*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 5*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(109) = 218$.

Time = 0.45 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{2\sqrt{2}\left(Ac\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+9\,Bc\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{5\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}(s}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -2/15*sqrt(2)*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 45*B*c
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 + 75*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*A*c*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^
3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
3)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 683, normalized size of antiderivative = 5.64

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((4*B*c)/(5*a^3*f) - (2*c*(2*A - 3*B))/(5*a^3*f) - (4*c*(3
*A - 2*B))/(5*a^3*f) + (c*(A*2i - B*3i)*2i)/(5*a^3*f) + (c*(A*3i - B*2i)*4i
)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5) - (ex
p(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*
1i)/2))^(1/2)*((2*B*c)/(3*a^3*f) + (c*(A*2i - B*5i)*2i)/(3*a^3*f) - (2*c*(1
0*A - 13*B))/(3*a^3*f) + (c*(A*8i - B*13i)*2i)/(15*a^3*f)))/((exp(e*1i + f*
x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp
(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(2*A - 3*B)
*1i)/(3*a^3*f) - (B*c*1i)/(a^3*f) + (2*c*(A*1i - B*3i))/(a^3*f)))/((exp(e*1
i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c
*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(A -
B)*4i)/(a^3*f) - (B*c*1i)/(2*a^3*f) + (c*(8*A - 3*B)*1i)/(10*a^3*f) + (c*(A
*1i - B*2i))/(a^3*f) + (c*(A*7i - B*6i))/(a^3*f)))/((exp(e*1i + f*x*1i) - 1
i)*(exp(e*1i + f*x*1i) + 1i)^4) + (4*B*c*exp(e*1i + f*x*1i)*(c - c*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i
+ f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}$$

```
[Out] -1/15*(3*A+7*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c/f-1/5*(A-B)*sec(f
*x+e)^5*(c-c*sin(f*x+e))^(7/2)/a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3046, 2934, 2752}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f} - \frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] -1/15*((3*A + 7*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(a^3*c*f) - (
(A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}\\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}\\
&\quad+\frac{(3A+7B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{10a^3c^2}\\
&= -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{2(3A+2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^3,x]
```

```
[Out] (-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(5B\sin(fx+e)+3A+2B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	65

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\,(5\,B\sin(fx+e)+3\,A+2\,B)\sqrt{-c\sin(fx+e)+c}}{15\,\left(a^3f\cos(fx+e)^3-2\,a^3f\cos(fx+e)\sin(fx+e)-2\,a^3f\cos(fx+e)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*
x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 505, normalized size of antiderivative = 5.94

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\left(\frac{2B\left(\sqrt{c}+\frac{5\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5\sqrt{c}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{\cdots}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a}{(\cos}\cdots\right.}\right.}{15f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
[Out] 2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(77) = 154$.

Time = 0.41 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.33

$$\int \frac{(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}}{(a + a\sin(e + fx))^3}\,dx$$
$$= \frac{\sqrt{2}\left(3\,A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 7\,B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + \frac{20\,B\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right)\mathrm{sgn}(\sin}{\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 20*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*A*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^2 + 60*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^4 + 15*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)*sqrt(c
)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 18.18 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.64

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{8\,B}{5\,a^3\,f}-\frac{16\,A-8\,B}{10\,a^3\,f}+\frac{(A\,16i-B\,8i)\,1i}{10\,a^3\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^5}$$
$$-\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{4\,B}{3\,a^3\,f}-\frac{16\,A-16\,B}{30\,a^3\,f}+\frac{(A\,80i-B\,120i)\,1i}{30\,a^3\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^3}$$
$$-\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(-\frac{B\,1i}{a^3\,f}+\frac{A\,16i-B\,16i}{40\,a^3\,f}+\frac{A\,80i-B\,80i}{40\,a^3\,f}+\frac{(160\,A-120\,B)\,1i}{40\,a^3\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^4}$$
$$-\frac{B\,e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,8i}{3\,a^3\,f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^2}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((8*B)/(5*a^3*f) - (16*A - 8*B)/(10*a^3*f) + ((A*16i - B*8
i)*1i)/(10*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5)
 - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*((4*B)/(3*a^3*f) - (16*A - 16*B)/(30*a^3*f) + ((A*80i -
 B*120i)*1i)/(30*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) +
1i)^3) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2)*((A*16i - B*16i)/(40*a^3*f) - (B*1i)/(a^3*f) + (A
*80i - B*80i)/(40*a^3*f) + ((160*A - 120*B)*1i)/(40*a^3*f)))/((exp(e*1i + f
*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (B*exp(e*1i + f*x*1i)*(c - c*((
exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*8i)/(3*a^3*f
*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2)
```

3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 174

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} \\ &\quad - \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} \\ &\quad - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}\end{aligned}$$

```
[Out] -1/6*(A+B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*(A-B)*sec(f*x+
e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^3/f+1/8*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1
/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/f*2^(1/2)/c^(1/2)-1/4*(A+B)*sec(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a^3/c/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used

$= \{3046, 2934, 2754, 2728, 212\}$

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}$$
$$- \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(4*Sqrt[2]*a^3*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(4*a^3*c*f) - ((A + B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3
*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{(A+B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{2a^3c^2}$$

$$= -\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{(A+B)\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{4a^3c}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{8a^3}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} - \frac{(A+B)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{4a^3f}$$

$$= \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(-A+B)-10(A+B)\left(\cos\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(A
 + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(A +
 B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^5))/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e +
f*x]])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.15

method	result
default	$-\frac{(\sin(fx+e)-1)\left(15\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))^{\frac{5}{2}}c^2A-74c^{\frac{9}{2}}A-80Ac^{\frac{9}{2}}\sin(fx+e)-30Ac^{\frac{9}{2}}(\sin^2(fx+e))+\right.}{120a^3c^{\frac{9}{2}}(1+\sin(fx+e))^2\cos(fx+e)}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/120/a^3*(sin(f*x+e)-1)/c^(9/2)/(1+sin(f*x+e))^2*(15*2^(1/2)*arctanh(1/2*
(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*A-74
*c^(9/2)*A-80*A*c^(9/2)*sin(f*x+e)-30*A*c^(9/2)*sin(f*x+e)^2+15*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)
```

```
*c^2*B-26*c^(9/2)*B-80*B*c^(9/2)*sin(f*x+e)-30*B*c^(9/2)*sin(f*x+e)^2)/cos(
f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{15\sqrt{2}\left((A+B)\cos\left(fx+e\right)^3-2\left(A+B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\left(A+B\right)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c}{}\right.}{24}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x
+ e) - 2*(A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*co
s(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (
cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(A + B)*cos(f*x
 + e)^2 - 40*(A + B)*sin(f*x + e) - 52*A - 28*B)*sqrt(-c*sin(f*x + e) + c))
/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*
cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^3\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
 + c)), x)
```

Giac [B] (verification not implemented)

Leave count of result is larger than twice the leaf count of optimal. 450 vs. $2(151) = 302$.

Time = 0.46 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.59

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\frac{15\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{4\sqrt{2}\left(23A\sqrt{c}+17B\sqrt{c}+\frac{70A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{70B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-\cdots\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/240*(15*sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) + 4*sqrt(2)*(23*A*sqrt(c) + 17*B*sqrt(c) + 70*A*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 70
*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + 140*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2 + 80*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 90*A*sqrt(c)*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 90*B*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^3 + 45*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^4 + 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*c*((cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
), x)
```

3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 224

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(7A+3B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}+\frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}}-\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}-\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}$$

```
[Out] 1/16*(7*A+3*B)*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*(A-B)*sec(f*x+e)
^5*(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f+1/32*(7*A+3*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-1/12*(7*A+3*B)
*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-1/30*(7*A+3*B)*sec(f*x+e)^3*(c-c
*sin(f*x+e))^(1/2)/a^3/c^2/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used

= {3046, 2934, 2754, 2766, 2729, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(7A+3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} + \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((7*A + 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + ((7*A + 3*B)*Cos[e + f*x])/(16*a^3*f*(c
- c*Sin[e + f*x])^(3/2)) - ((7*A + 3*B)*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c -
c*Sin[e + f*x]]) - ((7*A + 3*B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3
0*a^3*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c
^3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
```

```
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^3c^3}\\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}\\
&\quad+\frac{(7A+3B)\int \sec^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{10a^3c^2}\\
&= -\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}\\
&\quad-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}+\frac{(7A+3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{12a^3c}
\end{aligned}
$$

$$= -\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{(7A+3B)\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{8a^3}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{(7A+3B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{32a^3c}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} - \frac{(7A+3B)\text{Subst}\left(\int\frac{1}{2c-x^2}dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{16a^3cf}$$

$$= \frac{(7A+3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} + \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.59

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+s\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-40*A*Cos[e + f*x]^2 + 24*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
```

```
^2 - 30*(3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^4 + 15*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (15 + 15*I)*(-1)^(1/4)*(7*A + 3*B)
*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(A + B)*Si
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(240*a^3*f*(1 + Si
n[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.38

method	result
default	$-\frac{105A(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+45B(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right.}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/480/c^(9/2)/a^3*(105*A*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c+45*B*(c*(1+sin(f*x+e)))^
(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x
+e)*c-210*A*sin(f*x+e)^3*c^(7/2)-90*B*sin(f*x+e)^3*c^(7/2)+18*B*sin(f*x+e)*
c^(7/2)-350*A*sin(f*x+e)^2*c^(7/2)-150*B*sin(f*x+e)^2*c^(7/2)+42*A*sin(f*x+
e)*c^(7/2)+278*A*c^(7/2)-18*B*c^(7/2)-45*B*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-105*A*(c*(1+sin(f*
x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*
c)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}\left((7\,A+3\,B)\cos\left(fx+e\right)^3\sin\left(fx+e\right)+(7\,A+3\,B\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/960*(15*sqrt(2)*((7*A + 3*B)*cos(f*x + e)^3*sin(f*x + e) + (7*A + 3*B)*co
s(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*c
```

```
os(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(25*(7*A + 3*B)*cos(f*x + e)^2 + 3*(
5*(7*A + 3*B)*cos(f*x + e)^2 - 28*A - 12*B)*sin(f*x + e) - 36*A - 84*B)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*
cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(197) = 394$.

Time = 0.46 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
[Out] 1/1920*(30*sqrt(2)*(7*A*sqrt(c) + 3*B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^2*sgn(sin(
```

```
-1/4*pi + 1/2*f*x + 1/2*e))) + 15*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 14*A*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1) - 6*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*c^2*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 32*sqrt(2)
*(29*A*sqrt(c) + 6*B*sqrt(c) + 100*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*B*sqrt(c)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 170*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
2 + 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^2 + 120*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*A*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)
/(a^3*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
), x)
```

3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 258

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{7(9A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} \\ &+ \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} \\ &- \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} \\ &- \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}\end{aligned}$$

```
[Out] 7/128*(9*A+B)*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/240*(9*A+B)*sec(f
*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/256*(9*A+B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(5/2)/f*2^(1/2)-7/96*(9*A+B)*s
ec(f*x+e)/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/30*(9*A+B)*sec(f*x+e)^3/a^3/c^
2/f/(c-c*sin(f*x+e))^(1/2)-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/a^
3/c^3/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used

$= \{3046, 2934, 2766, 2760, 2729, 2728, 212\}$

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{7(9A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} + \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (7*(9*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(128*Sqrt[2]*a^3*c^(5/2)*f) + (7*(9*A + B)*Cos[e + f*x])/(128*a^3*c*f
*(c - c*Sin[e + f*x])^(3/2)) + (7*(9*A + B)*Sec[e + f*x])/(240*a^3*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (7*(9*A + B)*Sec[e + f*x])/(96*a^3*c^2*f*Sqrt[c -
 c*Sin[e + f*x]]) - ((9*A + B)*Sec[e + f*x]^3)/(30*a^3*c^2*f*Sqrt[c - c*Sin
[e + f*x]]) - ((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*c^3*
f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
```

```
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}\\
&= -\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}+\frac{(9A+B)\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{10a^3c^2}\\
&= -\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}\\
&\quad+\frac{(7(9A+B))\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{60a^3c}
\end{aligned}
$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{(7(9A+B))\int\frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{96a^3c^2}$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{(7(9A+B))\int\frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{64a^3c}$$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{(7(9A+B))\int\frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{256a^3c^2}$$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} - \frac{(7(9A+B))\text{Subst}\left(\int\frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{128a^3c^2f}$$

$$= \frac{7(9A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} + \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\mathrm{s}\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (105 + 10
5*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(1920*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\left(-1890c^{\frac{9}{2}}A-210c^{\frac{9}{2}}B\right)\left(\cos^4(fx+e)\right)+\left(1260c^{\frac{9}{2}}A+140c^{\frac{9}{2}}B\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+\left(-945\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right.}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3840/c^(13/2)/a^3*((-1890*c^(9/2)*A-210*c^(9/2)*B)*cos(f*x+e)^4+(1260*c^
(9/2)*A+140*c^(9/2)*B)*cos(f*x+e)^2*sin(f*x+e)+(-945*2^(1/2)*arctanh(1/2*(c
+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+252*c^(9
/2)*A-105*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*
sin(f*x+e))^(5/2)*c^2*B+28*c^(9/2)*B)*cos(f*x+e)^2+(-1890*2^(1/2)*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+864
*c^(9/2)*A-210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
(c+c*sin(f*x+e))^(5/2)*c^2*B+96*c^(9/2)*B)*sin(f*x+e)+1890*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+96
```

```
*c^(9/2)*A+210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
(c+c*sin(f*x+e))^(5/2)*c^2*B+864*c^(9/2)*B)/(1+sin(f*x+e))^2/(sin(f*x+e)-1)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{105\sqrt{2}(9A+B)\sqrt{c}\cos(fx+e)^5\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-}}{}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2
 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e)
 + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(
f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(105*
(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A + B)*co
s(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin(f*x +
 e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(227) = 454$.

Time = 0.56 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.49

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3 (c - c\sin(e + fx))^{5/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/30720*(420*sqrt(2)*(9*A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^3*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 32*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
16*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 378*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 42*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2/(a^3*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(54*A*sqrt(c) - 4*B*sqrt(c) + 195*A*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) - 5*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) + 315*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 25*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 225*A*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3
- 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^3 + 75*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*c^3*((cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*(32*sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) + 16*sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) - sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
 sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^6*c^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{5/2}}\,dx = \int \frac{A + B\,\sin\left(e + f\,x\right)}{{\left(a + a\,\sin\left(e + f\,x\right)\right)}^3\,{\left(c - c\,\sin\left(e + f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
), x)
```

3.131 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/4*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)+1/5
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] -1/4*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin[
e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A+B)\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx - \frac{B\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}\,dx}{c}\\ &= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(4(-60A+23B)\sin(e+fx)+4\cos(2(e+fx))(-3}{160f}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] -1/160*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]
]*(4*(-60*A + 23*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-35*A + 25*B + 4*(5*
A - 6*B)*Sin[e + f*x]) + Cos[4*(e + f*x)]*(5*A - 15*B + 4*B*Sin[e + f*x])))
/f
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

method	result
default	$\frac{c^3\tan(fx+e)(4B(\sin^2(fx+e))(\cos^2(fx+e))+5A\sin(fx+e)(\cos^2(fx+e))+15B(\sin^3(fx+e))-20A(\cos^2(fx+e))-24B(\sin^2(fx+e}{20f}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,c^3(\cos^3(fx+e)+4\cos(fx+e)\sin(fx+e)-8\cos(fx+e)-8\tan(fx+e)+7\sec(fx+e))}{4f}-$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/20*c^3/f*tan(f*x+e)*(4*B*sin(f*x+e)^2*cos(f*x+e)^2+5*A*sin(f*x+e)*cos(f*x
+e)^2+15*B*sin(f*x+e)^3-20*A*cos(f*x+e)^2-24*B*sin(f*x+e)^2-35*A*sin(f*x+e)
+10*B*sin(f*x+e)+40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.49

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{(5\,(A-3\,B)c^3\cos{(fx+e)}^4-40\,(A-B)c^3\cos{(fx+e)}^2+5\,(7\,A-5\,B)c^3+4\,\left(Bc^3\cos{(fx+e)}^4+(5\,A}{20\,f\cos{(fx}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/20*(5*(A - 3*B)*c^3*cos(f*x + e)^4 - 40*(A - B)*c^3*cos(f*x + e)^2 + 5*(
7*A - 5*B)*c^3 + 4*(B*c^3*cos(f*x + e)^4 + (5*A - 7*B)*c^3*cos(f*x + e)^2 -
 2*(5*A - 3*B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{4\left(8\,Bc^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-5\,Ac^{3}\mathrm{sgn}(\cdots}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/5*(8*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 5*A*c^3*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^8 - 5*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 16.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{c^{3}\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(100\,B\,\cos(e+f\,x)-140\,A\,\cos(e+f\,x)-135\,A\,\cos(\cdots}{\cdots}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] -(c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(100*B*cos
(e + f*x) - 140*A*cos(e + f*x) - 135*A*cos(3*e + 3*f*x) + 5*A*cos(5*e + 5*f
*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x
) + 40*A*sin(4*e + 4*f*x) + 90*B*sin(2*e + 2*f*x) - 48*B*sin(4*e + 4*f*x) +
 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

3.132 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/3*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+1/4
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}} - \frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/3*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin[
e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A+B)\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx \\ &\quad - \frac{B\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx}{c} \\ &= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(3B\cos(4(e+fx))+16(7A-2B)\sin}{96f}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A - 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(-12*A
+ 9*B + 4*(A - 2*B)*Sin[e + f*x])))/(96*f)
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
default	$-\frac{c^{2} \tan(fx+e)\left(-3B\left(\sin^{3}(fx+e)\right)+4A\left(\cos^{2}(fx+e)\right)+8B\left(\sin^{2}(fx+e)\right)+12A\sin(fx+e)-6B\sin(fx+e)-16A\right)\sqrt{-c(\sin(fx+e)-1)}}{12f}$
parts	$-\frac{A c^{2} \sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)\sin(fx+e)-3\cos(fx+e)-4\tan(fx+e)+3\sec(fx+e))}{3f}+\frac{B\sec(fx+e)(3(c}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/12*c^2/f*tan(f*x+e)*(-3*B*sin(f*x+e)^3+4*A*cos(f*x+e)^2+8*B*sin(f*x+e)^2
+12*A*sin(f*x+e)-6*B*sin(f*x+e)-16*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f
*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{\left(3\,Bc^{2}\cos\left(fx+e\right)^{4}+12\,(A-B)c^{2}\cos\left(fx+e\right)^{2}-3\,(4\,A-3\,B)c^{2}-4\,\left((A-2\,B)c^{2}\cos\left(fx\right.\right.}{12\,f\cos\left(fx+e\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*c^2*cos(f*x + e)^4 + 12*(A - B)*c^2*cos(f*x + e)^2 - 3*(4*A - 3*B
)*c^2 - 4*((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(2*A - B)*c^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{4\left(3\,Bc^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-2\,Ac^2\mathrm{sgn}(\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/3*(3*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*c^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^6 - 2*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)-36\,B\,\cos\left(e+f\,x\right)+\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(e
 + f*x) - 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) - 33*B*cos(3*e + 3*f*x)
 + 3*B*cos(5*e + 5*f*x) + 112*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x) - 3
2*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1)
)
```

3.133 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+1/3
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/2*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[
e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A+B)\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx \\ &\quad -\frac{B\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx}{c} \\ &= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(2(6A-B)\sin(e+fx)+\cos(2(e+fx))}{12f}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2*(6*A
 - B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*A - 3*B + 2*B*Sin[e + f*x])))/(12*
f)
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c\tan(fx+e)\left(2B\left(\sin^2(fx+e)\right)+3A\sin(fx+e)-3B\sin(fx+e)-6A\right)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{6f}$
parts	$\frac{Ac\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)+2\tan(fx+e)-\sec(fx+e))}{2f}+\frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+e))(2$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6*c/f*tan(f*x+e)*(2*B*sin(f*x+e)^2+3*A*sin(f*x+e)-3*B*sin(f*x+e)-6*A)*(-
c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{\left(3\,(A-B)c\cos\left(fx+e\right)^2-3\,(A-B)c+2\left(Bc\cos\left(fx+e\right)^2+(3\,A-B)c\right)\sin\left(fx+e\right)\right)\sqrt{}}{6\,f\cos\left(fx+e\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(3*(A - B)*c*cos(f*x + e)^2 - 3*(A - B)*c + 2*(B*c*cos(f*x + e)^2 + (3*
A - B)*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/
(f*cos(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*(A + B*s
in(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2\left(4\,Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-3\,Ac\mathrm{sgn}\left(\cos\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
[Out] -2/3*(4*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 3*A*c*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^4 - 3*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\,A\,\cos\left(e+f\,x\right)-3\,B\,\cos\left(e+f\,x\right)+3\,A\,\mathrm{c}\right.}{12\,f\,\left(\mathrm{c}\right.}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
f*x) - 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) - 3*B*cos(3*e + 3*f*x) + 12*
A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(cos
(2*e + 2*f*x) + 1))
```

3.134 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

Optimal result

Integrand size = 40, antiderivative size = 92

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 1/2*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)-a*(A+B
)*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Si
n[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A+B)\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx \\ &\quad - \frac{B\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx}{c} \\ &= -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\begin{aligned}&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx \\ &= \frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}(A+B\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}{2Bf}\end{aligned}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]],x]
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(A + B*Sin[e + f*x])^2*Sqrt[c - c*
Sin[e + f*x]])/(2*B*f)
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\tan(fx+e)(B\sin(fx+e)+2A)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{2f}$	49
parts	$\frac{A\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{f}+\frac{B\sin(fx+e)\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{2f}$	81

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*tan(f*x+e)*(B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x
+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=-\frac{(B\cos(fx+e)^2-2A\sin(fx+e)-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{2f\cos(fx+e)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int \sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(
e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{2\left(B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4 - A\mathrm{sgn}\left(\cos\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -2*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - B*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(B\,\cos\left(e+f\,x\right)+B\,\cos\left(3\,e+3\,f\,x\right)-4\,A\,\sin\left(2\,e+\right.\right.}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(B*cos(e + f*x
) + B*cos(3*e + 3*f*x) - 4*A*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1)
)
```

3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 100

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -a*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)+a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3050, 2817, 2816, 2746, 31}

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\text{integral} = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx - \frac{B\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{c}$$

$$= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} + \frac{(a(A+B)c\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} - \frac{(a(A+B)\cos(e+fx))\text{Subst}\left(\int\frac{1}{c+x}\,dx,x,-c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int\frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left((A+B)\left(-ifx+2\log\left(i-e^{i(e+fx)}\right)\right)+B\sin\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -((((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*((A + B
)*((-I)*f*x + 2*Log[I - E^(I*(e + f*x))]) + B*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(92) = 184.

Time = 2.53 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.00

method	result
parts	$\frac{A\left(2\ln(\csc(fx+e)-\cot(fx+e)-1)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sqrt{a(1+\sin(fx+e))}\,(-\cos(fx+e)+\sin(fx+e)-1)}{f(\cos(fx+e)+\sin(fx+e)+1)\sqrt{-c(\sin(fx+e)-1)}} - \frac{B\left(2\ln(\csc(fx+e)-\cot(f\right.}{}$
default	$-\frac{\left(2A\cos(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-2A\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] A/f*(2*ln(csc(f*x+e)-cot(f*x+e)-1)-ln(2/(1+cos(f*x+e))))*(a*(1+sin(f*x+e)))
^(1/2)*(-cos(f*x+e)+sin(f*x+e)-1)/(cos(f*x+e)+sin(f*x+e)+1)/(-c*(sin(f*x+e)
-1))^(1/2)-B/f*(2*ln(csc(f*x+e)-cot(f*x+e)-1)*cos(f*x+e)-2*ln(csc(f*x+e)-co
```

```
t(f*x+e)-1)*sin(f*x+e)-ln(2/(1+cos(f*x+e)))*cos(f*x+e)+ln(2/(1+cos(f*x+e)))
*sin(f*x+e)+cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)+2*ln(csc(f*x+e)-cot(f*x+e)-1
)-ln(2/(1+cos(f*x+e)))+sin(f*x+e)-1)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)+s
in(f*x+e)+1)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{\sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),
x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{B\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}} + \frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(c+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) + A\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}\right.}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
+ A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{2\sqrt{2}B\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(A\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(\right.}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2f}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*B*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))/(sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*
(A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sqrt(c)*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2)/(c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 99

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+a*B*cos(
f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3050, 2816, 2746, 31, 2817}

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + (a*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}\\ &= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{(aB\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(}{f\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + B -
I*B*f*x + 2*B*Log[I - E^(I*(e + f*x))] + I*B*(f*x + (2*I)*Log[I - E^(I*(e +
 f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin
[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

method	result
default	$\frac{\sec(fx+e)\left(-2B\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)+B\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(fx+e)+B\sin(fx+e)+2B\ln(\csc(f\right.}{cf\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A\tan(fx+e)\sqrt{a(1+\sin(fx+e))}}{fc\sqrt{-c(\sin(fx+e)-1)}} - \frac{B\sec(fx+e)\left(2\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-\sin(fx+\right.}{fc\sqrt{-c(\sin(fx+e)-1}}$

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/c/f*sec(f*x+e)*(-2*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+B*sin(f*x+e)*
ln(2/(1+cos(f*x+e)))+A*sin(f*x+e)+B*sin(f*x+e)+2*B*ln(csc(f*x+e)-cot(f*x+e)
-1)-B*ln(2/(1+cos(f*x+e))))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1
/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(4\,B\sqrt{c}\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{A\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+B\sqrt{c}\mathrm{sgn}(c}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}}{2\,c^{2}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(4*B*sqrt(c)*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + (A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sq
rt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
)*sqrt(a)/(c^2*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.137 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 92

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-a*B*
cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx-\frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}\\ &= \frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{a(1+\sin(e+fx))}(A-B+2B\sin(e+fx))\sqrt{c-}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx\right)\right.}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\tan(fx+e)(A\sin(fx+e)-B\sin(fx+e)-2A)\sqrt{a(1+\sin(fx+e))}}{2c^2f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}}$	71
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)+2\tan(fx+e)-\sec(fx+e))}{2f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}\,c^2}+\frac{B\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)-\sec(fx+e))}{2f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}\,c^2}$	128

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2/f*tan(f*x+e)*(A*sin(f*x+e)-B*sin(f*x+e)-2*A)*(a*(1+sin(f*x+e)))^(1/
2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(2\,B\sin(fx+e)+A-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{2\left(c^3f\cos(fx+e)^3+2\,c^3f\cos(fx+e)\sin(fx+e)-2\,c^3f\cos(fx+e)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f
*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\left(\sin(e+fx)+1\right)}(A+B\sin(e+fx))}{\left(-c\left(\sin(e+fx)-1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\left(4\,B\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\right.\right.}{8\,c^3 f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\right.\right.}$$

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] 1/8*(4*B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^3*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2), x)

3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)-1/2*
a*B*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\text{integral} = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{a(1+\sin(e+fx))}(2A-B+3B\sin(e+fx))\sqrt{c}}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
 f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

method	result
default	$\frac{\tan(fx+e)\left(2A\left(\cos^2(fx+e)\right)+B\left(\sin^2(fx+e)\right)+6A\sin(fx+e)-3B\sin(fx+e)-8A\right)\sqrt{a(1+\sin(fx+e))}}{6c^3f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)\sin(fx+e)-3\cos(fx+e)-4\tan(fx+e)+3\sec(fx+e))}{3f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}\,c^3} - \frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+e)}{6f(\cos^2(fx+e)+2\sin(fx+e)-2}$

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/c^3/f*tan(f*x+e)*(2*A*cos(f*x+e)^2+B*sin(f*x+e)^2+6*A*sin(f*x+e)-3*B*si
n(f*x+e)-8*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*(s
in(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{(3\,B\sin(fx+e)+2\,A-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{6\left(3\,c^4f\cos(fx+e)^3-4\,c^4f\cos(fx+e)-\left(c^4f\cos(fx+e)^3-4\,c^4f\cos(fx+e)\right)\sin(fx+e)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x +
 e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(3\,B\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\right.\right.}{24\,c^4 f\mathrm{sgn}\left(\sin\left(-\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 1/24*(3*B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(c)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^4*f*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{2\,A\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}-B\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}+3\,B\,\sin\left(e\right.}{\frac{9\,c^4\,f\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{21\,c^4\,f\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{3\,c^4\,f\,\sin\left(4\,e+4\,f\,x\right)}{4}-\frac{21\,c^4\,f\,\mathrm{co}}{2}}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] -(2*A*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) - B*(a + a*sin(
e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*B*sin(e + f*x)*(a + a*sin(e
+ f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e + 3*f*x))/2 + (
21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4 - (21*c^4*f*cos
(e + f*x))/2)
```

3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a+a\sin(e+fx)}} - \frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{15f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

```
[Out] -1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/30*a^2*
(3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-1/15*a*(
3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used

$= \{3052, 2819, 2817\}$

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = -\frac{a^2(3A - B)\cos(e + fx)(c - c\sin(e + fx))^{7/2}}{30f\sqrt{a\sin(e + fx) + a}} - \frac{a(3A - B)\cos(e + fx)\sqrt{a\sin(e + fx) + a}(c - c\sin(e + fx))^{7/2}}{15f} - \frac{B\cos(e + fx)(a\sin(e + fx) + a)^{3/2}(c - c\sin(e + fx))^{7/2}}{6f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/30*(a^2*(3*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a
*Sin[e + f*x]]) - (a*(3*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(7/2))/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(
c - c*Sin[e + f*x])^(7/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f} \\
&\quad+\frac{1}{3}(3A-B)\int(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{15f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f} \\
&\quad+\frac{1}{15}(2a(3A-B))\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{15f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.40

$$
\int(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =
$$

$$
-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(15(16A-11B)\cos(2(e+fx))+30(2A-}{960f\,(\mathrm{co}}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*S
in[e + f*x]]*(15*(16*A - 11*B)*Cos[2*(e + f*x)] + 30*(2*A - B)*Cos[4*(e + f
*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] - 240*B*Sin[e + f*x] + 60*
A*Sin[3*(e + f*x)] + 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x)] + 24*B*S
in[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a\,c^3\tan(fx+e)\left(5B\left(\sin^5(fx+e)\right)+6A\left(\cos^4(fx+e)\right)-12\left(\sin^4(fx+e)\right)B+15A\sin(fx+e)\left(\cos^2(fx+e)\right)-12A\left(\cos^2(fx+e)\right)+20B(\mathrm{s}}{30f}$
parts	$-\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c^3a\left(2\left(\cos^3(fx+e)\right)\sin(fx+e)-5\left(\cos^3(fx+e)\right)-4\cos(fx+e)\sin(fx+e)-8\tan(fx+e)+\right.}{10f}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/30*a*c^3/f*tan(f*x+e)*(5*B*sin(f*x+e)^5+6*A*cos(f*x+e)^4-12*sin(f*x+e)^4
*B+15*A*sin(f*x+e)*cos(f*x+e)^2-12*A*cos(f*x+e)^2+20*B*sin(f*x+e)^2+15*A*si
n(f*x+e)-15*B*sin(f*x+e)-24*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\frac{\left(5\,Bac^3\cos\left(fx+e\right)^6+15\,(A-B)ac^3\cos\left(fx+e\right)^4-5\,(3\,A-2\,B)ac^3-2\,\left(3\,(A-\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a*c^3*cos(f*x + e)^6 + 15*(A - B)*a*c^3*cos(f*x + e)^4 - 5*(3*A -
 2*B)*a*c^3 - 2*(3*(A - 2*B)*a*c^3*cos(f*x + e)^4 - 2*(3*A - B)*a*c^3*cos(f
*x + e)^2 - 4*(3*A - B)*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{8\left(20\,Bac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 8/15*(20*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 12*A*a*c^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^10 - 36*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 15*A*a*c^3*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^8 + 15*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sq
rt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{\mathrm{e}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin(e+f\,x)}}{96\,f}-\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}}{}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((B*a*c^3*exp(e*6i + f*x*6
i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) - (a*c^3*exp(e*6i +
f*x*6i)*sin(e + f*x)*(A*7i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f) + (
a*c^3*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A - B)*(a + a*sin(e + f*x))^(1
/2))/(16*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A - 11*B)*(a +
 a*sin(e + f*x))^(1/2))/(32*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(A*3i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) + (a*c^3*exp(e*6i + f*
x*6i)*sin(5*e + 5*f*x)*(A*1i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f))
)/(2*cos(e + f*x))
```

3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a+a\sin(e+fx)}} - \frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{20f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

```
[Out] -1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/30*a^2*
(5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/20*a*(
5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used

$= \{3052, 2819, 2817\}$

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = -\frac{a^2(5A - B)\cos(e + fx)(c - c\sin(e + fx))^{5/2}}{30f\sqrt{a\sin(e + fx) + a}} - \frac{a(5A - B)\cos(e + fx)\sqrt{a\sin(e + fx) + a}(c - c\sin(e + fx))^{5/2}}{20f} - \frac{B\cos(e + fx)(a\sin(e + fx) + a)^{3/2}(c - c\sin(e + fx))^{5/2}}{5f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/30*(a^2*(5*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a
*Sin[e + f*x]]) - (a*(5*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(5/2))/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(
c - c*Sin[e + f*x])^(5/2))/(5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad+\frac{1}{5}(5A-B)\int(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx \\
&= -\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{20f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad+\frac{1}{10}(a(5A-B))\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx \\
&= -\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{20f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18

$$
\int(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{c^2(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(4(100A-11B)\sin(}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(4*(100*A - 11*B)*Sin[e + f*x] + 3*Cos[4*(e + f*x)]*(5*A - 5*B + 4*B*
Sin[e + f*x]) + 4*Cos[2*(e + f*x)]*(15*(A - B) + 4*(5*A + 2*B)*Sin[e + f*x]
)))/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^3)
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a\,c^{2}\tan(fx+e)\left(12B\left(\sin^{2}(fx+e)\right)\left(\cos^{2}(fx+e)\right)+15A\sin(fx+e)\left(\cos^{2}(fx+e)\right)+15B\left(\sin^{3}(fx+e)\right)-20A\left(\cos^{2}(fx+e)\right)+8B\left(\sin^{2}(\right.\right.}{60f}$
parts	$\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c^{2}a\left(3\left(\cos^{3}(fx+e)\right)+4\cos(fx+e)\sin(fx+e)+8\tan(fx+e)-3\sec(fx+e)\right)}{12f}+\frac{B\sqrt{-c(\sin(}{}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/60*a*c^2/f*tan(f*x+e)*(12*B*sin(f*x+e)^2*cos(f*x+e)^2+15*A*sin(f*x+e)*co
s(f*x+e)^2+15*B*sin(f*x+e)^3-20*A*cos(f*x+e)^2+8*B*sin(f*x+e)^2+15*A*sin(f*
x+e)-30*B*sin(f*x+e)-40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{\left(15\,(A-B)ac^{2}\cos\left(fx+e\right)^{4}-15\,(A-B)ac^{2}+4\,\left(3\,Bac^{2}\cos\left(fx+e\right)^{4}+(5\,A-B\right.\right.}{60\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/60*(15*(A - B)*a*c^2*cos(f*x + e)^4 - 15*(A - B)*a*c^2 + 4*(3*B*a*c^2*cos
(f*x + e)^4 + (5*A - B)*a*c^2*cos(f*x + e)^2 + 2*(5*A - B)*a*c^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{4\left(24\,Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 4/15*(24*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 15*A*a*c^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^8 - 45*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 20*A*a*c^2*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^6 + 20*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt
(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(60\,A\,\cos\left(e+f\,x\right)-60\,B\,\cos\left(e\right.\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*cos
(e + f*x) - 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*f*
x) - 75*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) + 400*A*sin(2*e + 2*f*x)
 + 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 134

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}} - \frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

```
[Out] -1/4*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/f-1/3*a^2*A
*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*A*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used

= {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}} - \frac{aA\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -1/3*(a^2*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[e +
f*x]]) - (a*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2))/(3*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(3/2))/(4*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{4f} \\
&\quad + A\int(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx \\
&= -\frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{4f} \\
&\quad + \frac{1}{3}(2aA)\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx \\
&= -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.74

$$
\int(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\sec^3(e+fx)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(9B+12.}{96f}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]

[Out] (c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c -
 c*Sin[e + f*x]]*(9*B + 12*B*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)] - 72*A
*Sin[e + f*x] - 8*A*Sin[3*(e + f*x)]))/(96*f)
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

method	result
default	$\frac{ac\tan(fx+e)\left(3B\left(\cos^2(fx+e)\right)\sin(fx+e)+4A\left(\cos^2(fx+e)\right)+3B\sin(fx+e)+8A\right)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{12f}$
parts	$\frac{A\tan(fx+e)ca\left(\cos^2(fx+e)+2\right)\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}}{3f}+\frac{B\sin(fx+e)\tan(fx+e)ca\left(\cos^2(fx+e)+1\right)\sqrt{-c(\sin(fx}{4f}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/12*a*c/f*tan(f*x+e)*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+3*B*sin
(f*x+e)+8*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{\left(3\,Bac\cos\left(fx+e\right)^{4}-3\,Bac-4\left(Aac\cos\left(fx+e\right)^{2}+2\,Aac\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(\right.}}{12\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/12*(3*B*a*c*cos(f*x + e)^4 - 3*B*a*c - 4*(A*a*c*cos(f*x + e)^2 + 2*A*a*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(116) = 232.

Time = 0.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.77

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{4\left(3\,Bac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 4/3*(3*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*a*c*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^6 - 6*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 3*A*a*c*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^4 + 3*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx =$$

$$-\frac{a\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,B\,\cos\left(e+f\,x\right)+15\,B\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(\right.\right.}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*B*cos(
e + f*x) + 15*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) - 80*A*sin(2*e + 2*
f*x) - 8*A*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)+1/3*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx}{a}\\ &\quad -(-A+B)\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx\\ &= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{a\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(-2(6A+B)\sin(e+fx)+\cos(2(e+fx))(3(A+B}{12f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -1/12*(a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-2*(6*A + B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*(A + B) + 2*B*Sin[e + f*x])
))/f
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

method	result
default	$\frac{a\tan(fx+e)\left(2B\left(\sin^2(fx+e)\right)+3A\sin(fx+e)+3B\sin(fx+e)+6A\right)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{6f}$
parts	$-\frac{Aa\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)-2\tan(fx+e)-\sec(fx+e))}{2f}-\frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+e))(}{}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
[Out] 1/6*a/f*tan(f*x+e)*(2*B*sin(f*x+e)^2+3*A*sin(f*x+e)+3*B*sin(f*x+e)+6*A)*(-c
*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{\left(3\,(A+B)a\cos{(fx+e)}^2-3\,(A+B)a+2\left(Ba\cos{(fx+e)}^2-(3\,A+B)a\right)\sin{(fx+e)}\right)\sqrt{a\sin{(fx+e)}}}{6\,f\cos{(fx+e)}}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
[Out] -1/6*(3*(A + B)*a*cos(f*x + e)^2 - 3*(A + B)*a + 2*(B*a*cos(f*x + e)^2 - (3
*A + B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
/(f*cos(f*x + e))
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int \left(a(\sin{(e+fx)}+1)\right)^{\frac{3}{2}}\sqrt{-c\,(\sin{(e+fx)}-1)}(A+B\sin{(e+fx)})\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*(A + B*s
in(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}\,d$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.50

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{2\left(4\,Ba\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Aa\cos\left(\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -2/3*(4*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*A*a*cos(-1/4*pi + 1/2*f*x + 1
/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)) - 3*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{a\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\,A\,\cos\left(e+f\,x\right)+3\,B\,\cos\left(e+f\,x\right)+3\,A\,\cos\left(3\,e+3\right.\right.}{12\,f\,\left(\cos\left(2\,e+2\right.\right.}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
 f*x) + 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12
*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(co
s(2*e + 2*f*x) + 1))
```

3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 145

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-2*a^2*(A+
B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)-a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3052, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a^2*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + (2a(A+B))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \frac{(2a^2(A+B)c\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{(2a^2(A+B)\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\left(-B\cos(2(e+fx))+16(A+B)\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-
(B*Cos[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 4*(A + 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqr
t[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(131) = 262.

Time = 2.66 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.92

method	result
default	$-\frac{a\left(-B\left(\cos^{3}(fx+e)\right)+B\left(\cos^{2}(fx+e)\right)\sin(fx+e)+2A\left(\cos^{2}(fx+e)\right)+2A\sin(fx+e)\cos(fx+e)-4A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+8A}{}$
parts	$-\frac{A\left(\cos(fx+e)\sin(fx+e)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-4\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)+\cos^{2}(fx+e)-2\ln\left(\frac{2}{1+\cos(fx+e}\right.}{f(\cos(fx+e)+\mathrm{si}}$

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method =_RETURNVERBOSE)

[Out] -1/2*a/f*(-B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2+2*A*si n(f*x+e)*cos(f*x+e)-4*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+8*A*cos(f*x+e)*ln(c sc(f*x+e)-cot(f*x+e)-1)+4*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-8*A*sin(f*x+e)* ln(csc(f*x+e)-cot(f*x+e)-1)+3*B*cos(f*x+e)^2+4*B*cos(f*x+e)*sin(f*x+e)-4*B* cos(f*x+e)*ln(2/(1+cos(f*x+e)))+8*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+ 4*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-8*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e) -1)+2*A*sin(f*x+e)-4*A*ln(2/(1+cos(f*x+e)))+8*A*ln(csc(f*x+e)-cot(f*x+e)-1) +cos(f*x+e)*B+3*B*sin(f*x+e)-4*B*ln(2/(1+cos(f*x+e)))+8*B*ln(csc(f*x+e)-cot (f*x+e)-1)-2*A-3*B)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)+sin(f*x+e)+1)/(-c* (sin(f*x+e)-1))^(1/2)

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(a(\sin{(e+fx)}+1))^{\frac{3}{2}}\,(A+B\sin{(e+fx)})}{\sqrt{-c\,(\sin{(e+fx)}-1)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(-c*(sin(e
+ f*x) - 1)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^{\frac{3}{2}}}{\sqrt{-c\sin{(fx+e)}+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.68

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(Aa\sqrt{c}\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+Ba\sqrt{c}\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}f}{c\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] sqrt(2)*sqrt(a)*(sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-cos(-1/4*pi + 1/2*f*
x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (sqrt(2)*B*a*c^
(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*A*a*c^(3/2)*cos(-1/4*pi + 1/2
*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) + sqrt(2)*B*a*c^(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^2)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{3/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 158

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*(A
+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+1/2*a*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (a^2*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a +
```

```
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 3*B)*Cos[e + f*x]*Sqrt[
a + a*Sin[e + f*x]])/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\int\frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}} - \frac{(a(A+3B))\int\frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}} - \frac{(a^2(A+3B)\cos(e+fx))\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{(a^2(A+3B)\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{c+x}\,dx,x,-c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 8.56 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.33

$$\int\frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(4A+3B+B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{2cf\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/2*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
*A + 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.50

method	result
default	$-\frac{a\sec(fx+e)\left(2A\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+6B\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(2\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-2\ln(\csc(fx+e)-\cot(fx+e)-1)+\ln\left(\frac{2}{1+\cos(\right.\right.}{fc\sqrt{-c(\sin(fx+e)-1)}}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -a/c/f*sec(f*x+e)*(2*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-A*sin(f*x+e)*
ln(2/(1+cos(f*x+e)))+6*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-3*B*sin(f*x
+e)*ln(2/(1+cos(f*x+e)))+B*sin(f*x+e)^2-2*A*ln(csc(f*x+e)-cot(f*x+e)-1)+A*l
n(2/(1+cos(f*x+e)))-2*A*sin(f*x+e)-6*B*ln(csc(f*x+e)-cot(f*x+e)-1)+3*B*ln(2
/(1+cos(f*x+e)))-3*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1
))^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(
f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(a(\sin{(e+fx)}+1))^{\frac{3}{2}}\,(A+B\sin{(e+fx)})}{(-c\,(\sin{(e+fx)}-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f*
x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(142) = 284$.

Time = 0.33 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.32

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=$$
$$-\frac{B\left(\frac{6\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{2\left(\frac{3\,a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}-\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\,a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c^{\frac{3}{2}}-\frac{2\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
[Out] -(B*(6*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 3*a^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 2*(3*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) - 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c^(3/2) - 2*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) + A*(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/c^(3/2) - a^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4
*a^(3/2)*sqrt(c)*sin(f*x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}Ba\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{3}{2}}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ba\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right.}{2f}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))/(c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(
2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-8*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 8)/
(c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{3/2}}{\left(c-c\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a*B*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^2*B*cos(f*x+e)
*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2818, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f
*x])^(3/2)) - (a^2*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\int\frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{(aB)\int\frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{(a^2B\cos(e+fx))\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{(a^2B\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 9.00 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.33

$$\int\frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos[
2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.45

method	result
default	$-\frac{a\sec(fx+e)\left(-B\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+2B\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)+2B\left(\sin^2(fx+e)\right)-2B\sin(fx+e$
parts	$-\frac{A\tan(fx+e)\sqrt{a(1+\sin(fx+e))}\,a}{f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}\,c^2}+\frac{B\sec(fx+e)\left(\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -a/c^2/f*sec(f*x+e)*(-B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*B*cos(f*x+e)^2*
ln(csc(f*x+e)-cot(f*x+e)-1)+2*B*sin(f*x+e)^2-2*B*sin(f*x+e)*ln(2/(1+cos(f*x
+e)))+4*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+A*sin(f*x+e)-B*sin(f*x+e)+
2*B*ln(2/(1+cos(f*x+e)))-4*B*ln(csc(f*x+e)-cot(f*x+e)-1))*(a*(1+sin(f*x+e))
)^(1/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 -
(c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f*
x) - 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{2}\sqrt{a}\left(\frac{4\sqrt{2}Ba\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx-\right.\right.}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*sqrt(a)*(4*sqrt(2)*B*a*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
+ 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))) + sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*
B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(A*a*sqrt(c)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}+\frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*(
A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3051, 2821}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(5/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 9.78 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(A+4B-3B\cos(2(e+fx))+3(A-B)\sin(}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] -1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A
 + 4*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(c^3*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

method	result
default	$\frac{a \tan(fx+e)\left(A(\cos^2(fx+e))-B(\sin^2(fx+e))+3A\sin(fx+e)-3B\sin(fx+e)-7A\right)\sqrt{a(1+\sin(fx+e))}}{6c^3 f(\cos^2(fx+e)+2\sin(fx+e)-2)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a(\cos(fx+e)\sin(fx+e)-3\cos(fx+e)-7\tan(fx+e)+3\sec(fx+e))}{6f(\cos^2(fx+e)+2\sin(fx+e)-2)\sqrt{-c(\sin(fx+e)-1)}\,c^3}+\frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+e}{6f(\cos^2(fx+e)+2\sin(fx+e)-}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6*a/c^3/f*tan(f*x+e)*(A*cos(f*x+e)^2-B*sin(f*x+e)^2+3*A*sin(f*x+e)-3*B*si
n(f*x+e)-7*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*(s
in(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(6\,Ba\cos\left(fx+e\right)^2-3\,(A-B)a\sin\left(fx+e\right)-(A+7\,B\right.}{6\,\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(f\right.\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4
*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}}{(-c\sin(fx + e) + c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(84) = 168.

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.91

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx =$$
$$-\frac{\left(12\,Ba\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^4 - 3\,Aa\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -1/24*(12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^4 - 3*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 - 9*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 2*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(
c^4*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^{3/2}}{(c - c\,\sin(e + f\,x))^{7/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(9/2)+1/24*(
A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/96*(A
-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}} + \frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
```

```
c*Sin[e + f*x])^(7/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)
)/(96*c^2*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\int\frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}} + \frac{(A-3B)\int\frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{24c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(}$$

[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2),x]

[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A + 3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.87

method	result
default	$\frac{a\tan(fx+e)\left(A\sin(fx+e)\left(\cos^2(fx+e)\right)-4A\left(\cos^2(fx+e)\right)-7A\sin(fx+e)+3B\sin(fx+e)+10A\right)\sqrt{a(1+\sin(fx+e))}}{6c^4f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-3\left(\cos^2(fx+e)\right)-4\sin(fx+e)+4\right)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a\left(\cos^3(fx+e)+4\cos(fx+e)\sin(fx+e)-8\cos(fx+e)-10\tan(fx+e)+7\sec(fx+e)\right)}{6f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-3\left(\cos^2(fx+e)\right)-4\sin(fx+e)+4\right)\sqrt{-c(\sin(fx+e)-1)}\,c^4} - \frac{Ba}{2f\left(\left(\cos^2(fx+e)\right)\sin(f\right.}$

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method =_RETURNVERBOSE)

[Out] 1/6*a/c^4/f*tan(f*x+e)*(A*sin(f*x+e)*cos(f*x+e)^2-4*A*cos(f*x+e)^2-7*A*sin(f*x+e)+3*B*sin(f*x+e)+10*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{\left(3\,Ba\cos\left(fx+e\right)^{2}-2\,Aa\sin\left(fx+e\right)-(A+3\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+}}{6\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\cos\left(fx+e\right)\right)}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(
f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f
*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{3}{2}}}{(-c\sin\left(fx+e\right)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{\left(12\,Ba\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-4\,Aa\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{si}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] -1/96*(12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^4 - 4*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 - 12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 3*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/
(c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8
)
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.68

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\sin(e}}{3\,c^5\,f}\right.}{84\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((8*a*exp(e*5i + f*x*5i)*(2*A + 3*B)*(a + a*sin
(e + f*x))^(1/2))/(3*c^5*f) + (32*A*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a +
a*sin(e + f*x))^(1/2))/(3*c^5*f) - (8*B*a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*
x)*(a + a*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i
) - 54*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e +
 5*f*x) - 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*si
n(4*e + 4*f*x))
```

3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 154

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{40cf(c-c\sin(e+fx))^{9/2}}-\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(11/2)-1/12
0*a^2*(3*A-7*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1
/2)+1/40*a*(3*A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))
^(9/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2818, 2817}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + (a*(3*A - 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(40*c*f*(
c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 7*B)*Cos[e + f*x])/(120*c^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-7B)\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{40cf(c-c\sin(e+fx))^{9/2}} - \frac{(a(3A-7B))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx}{40c^2}$$

$$
= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{40cf(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}
$$

Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$
\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(9(A+B)-10B\cos(2(e+fx))+5(3A+B)}{60c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] -1/60*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
9*(A + B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(c^5*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f
*x]])
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.22

method	result
default	$\frac{a\tan(fx+e)\left(9A\left(\cos^4(fx+e)\right)+B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+45A\sin(fx+e)\left(\cos^2(fx+e)\right)+5B\left(\sin^3(fx+e)\right)-108A\left(\cos^2(fx+e)\right)}{60c^5f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a\left(3\left(\cos^3(fx+e)\right)\sin(fx+e)-15\left(\cos^3(fx+e)\right)-36\cos(fx+e)\sin(fx+e)+60\cos(fx+e)+53\tan(fx+e)-45\sec(}{20f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+8)\sqrt{-c(\sin(fx+e)-1)}\,c^5}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/60*a/c^5/f*tan(f*x+e)*(9*A*cos(f*x+e)^4+B*sin(f*x+e)^2*cos(f*x+e)^2+45*A*
sin(f*x+e)*cos(f*x+e)^2+5*B*sin(f*x+e)^3-108*A*cos(f*x+e)^2-11*B*sin(f*x+e)
^2-135*A*sin(f*x+e)+30*B*sin(f*x+e)+159*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*
x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-8*sin(f*x+e)+8)/(-c*(sin(f*
x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{\left(20\,Ba\cos\left(fx+e\right)^2-5\,(3\,A+B)a\sin\left(fx+e\right)-(9\,A+19\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{\cdots}}{60\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+\cdots\right.\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a)
*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5
 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5
- 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{3}{2}}}{(-c\sin\left(fx+e\right)+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$
$$-\frac{\left(40\,Ba\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,Aa\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] -1/960*(40*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4 - 15*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 - 45*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 12*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt
(a)/(c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^10)
```

Mupad [B] (verification not implemented)

Time = 21.01 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(\right.}{}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,2}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(A + B)*(a + a*sin(e + f
*x))^(1/2)*48i)/(5*c^6*f) - (B*a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2)*32i)/(3*c^6*f) + (16*a*exp(e*6i + f*x*6i)*sin(e + f*x)
*(A*3i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f)))/(cos(e + f*x)*exp(e*
6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f
*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + ex
p(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)
*2i)
```

3.149 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 198

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = &-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a+a\sin(e+fx)}}\\ &-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{105f}\\ &-\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{42f}\\ &-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\end{aligned}$$

```
[Out] -1/42*a*(7*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-
1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/105*a^3*
(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a^
2*(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a\sin(e+fx)+a}} - \frac{2a^2(7A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{105f} - \frac{a(7A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{42f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/105*(a^3*(7*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a^2*(7*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(
c - c*Sin[e + f*x])^(7/2))/(105*f) - (a*(7*A - B)*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(42*f) - (B*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(7*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad+\frac{1}{7}(7A-B)\int(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{42f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad+\frac{1}{21}(2a(7A-B))\int(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{105f} \\
&\quad-\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{42f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad+\frac{1}{105}\left(4a^2(7A-B)\right)\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{105f} \\
&\quad-\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{42f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx =$$

$$-\frac{c^3(-1 + \sin(e + fx))^3(a(1 + \sin(e + fx)))^{5/2}\sqrt{c - c\sin(e + fx)}(525(A - B)\cos(2(e + fx)) + 210(A - \ldots}{\ldots}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*
Sin[e + f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)]
+ 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] - 525
*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A*Sin
[5*(e + f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 73.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a^2c^3\tan(fx+e)(30B(\cos^4(fx+e))(\sin^2(fx+e))+35A(\cos^4(fx+e))\sin(fx+e)+35B(\cos^2(fx+e))(\sin^3(fx+e))-42A(\cos^4(fx+\ldots}{\ldots}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^2c^3(5(\cos^5(fx+e))+6(\cos^3(fx+e))\sin(fx+e)+8\cos(fx+e)\sin(fx+e)+16\tan(fx+e)-\ldots}{30f}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/210*a^2*c^3/f*tan(f*x+e)*(30*B*cos(f*x+e)^4*sin(f*x+e)^2+35*A*cos(f*x+e)
^4*sin(f*x+e)+35*B*cos(f*x+e)^2*sin(f*x+e)^3-42*A*cos(f*x+e)^4+24*B*sin(f*x
+e)^2*cos(f*x+e)^2+35*A*sin(f*x+e)*cos(f*x+e)^2+70*B*sin(f*x+e)^3-56*A*cos(
f*x+e)^2+16*B*sin(f*x+e)^2+35*A*sin(f*x+e)-105*B*sin(f*x+e)-112*A)*(-c*(sin
(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{\left(35\,(A-B)a^2c^3\cos\left(fx+e\right)^6-35\,(A-B)a^2c^3+2\left(15\,Ba^2c^3\cos\left(fx+e\right)^6+3\,(7\,A\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^2
*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^2*
c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{5}{2}}(-c\sin\left(fx+e\right)+c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(174) = 348$.

Time = 0.49 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.79

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx =$$
$$-\frac{16\left(120\,Ba^2c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{14} - 70\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -16/105*(120*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 70*A*a^2*c^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^12 - 350*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 1
68*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 336*B*a^2*c^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^10 - 105*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 105*B*a^2*c
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.93

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{\mathrm{e}^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\,\sqrt{c - c\,\sin\left(e + f\,x\right)}\,\left(-\frac{a^2\,c^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}\,5\mathrm{i}}{32\,f} - \right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^2*c^3*exp(e*7i + f*x*7
i)*sin(5*e + 5*f*x)*(4*A + 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^2*
c^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^
(1/2)*1i)/(16*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i - B*1
i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(
```

```
2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (5*a^2*c
^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A - B)*(a + a*sin(e + f*x))^(1/2))/(3
2*f) + (a^2*c^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A + B)*(a + a*sin(e
 + f*x))^(1/2))/(96*f) + (B*a^2*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a
+ a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 180

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = & -\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} \\ & -\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5f} \\ & -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} \\ & -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}\end{aligned}$$

```
[Out] -1/5*a*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/6*B*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f-2/15*a^3*A*cos(f*
x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*A*cos(f*x+e)*(
c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}} - \frac{a^2A\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{5f} - \frac{aA\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (-2*a^3*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a^2*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2))/(5*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2))/(5*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(5/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f} \\
&\quad + A\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx \\
&= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f} \\
&\quad + \frac{1}{5}(4aA)\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx \\
&= -\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad - \frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f} \\
&\quad + \frac{1}{5}(2a^2A)\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx \\
&= -\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad - \frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(50B+75B\cos(2(e+fx))+30B\cos(4(e+fx))}{960f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -1/960*(a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e +
f*x]]*(50*B + 75*B*Cos[2*(e + f*x)] + 30*B*Cos[4*(e + f*x)] + 5*B*Cos[6*(e
+ f*x)] - 600*A*Sin[e + f*x] - 100*A*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x
)]))/f
```

Maple [A] (verified)

Time = 72.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

method	result
default	$\frac{a^2c^2\tan(fx+e)\left(5B\left(\cos^4(fx+e)\right)\sin(fx+e)+6A\left(\cos^4(fx+e)\right)+5B\left(\cos^2(fx+e)\right)\sin(fx+e)+8A\left(\cos^2(fx+e)\right)+5B\sin(fx+e)+16A\right)}{30f}$
parts	$\frac{A\tan(fx+e)a^2c^2\left(3\left(\cos^4(fx+e)\right)+4\left(\cos^2(fx+e)\right)+8\right)\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}}{15f}-\frac{B\sec(fx+e)a^2c^2\left(\cos^4(fx+e)+\cos\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/30*a^2*c^2/f*tan(f*x+e)*(5*B*cos(f*x+e)^4*sin(f*x+e)+6*A*cos(f*x+e)^4+5*B
*cos(f*x+e)^2*sin(f*x+e)+8*A*cos(f*x+e)^2+5*B*sin(f*x+e)+16*A)*(-c*(sin(f*x
+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{\left(5\,Ba^2c^2\cos\left(fx+e\right)^6-5\,Ba^2c^2-2\left(3\,Aa^2c^2\cos\left(fx+e\right)^4+4\,Aa^2c^2\cos\left(fx+e\right)^2+8\,Aa^2c^2\right)\sin\left(fx+e\right.}{30\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/30*(5*B*a^2*c^2*cos(f*x + e)^6 - 5*B*a^2*c^2 - 2*(3*A*a^2*c^2*cos(f*x +
e)^4 + 4*A*a^2*c^2*cos(f*x + e)^2 + 8*A*a^2*c^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(156) = 312$.

Time = 0.44 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.97

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=-\frac{16\left(10\,Ba^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}-6\,Ac\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
[Out] -16/15*(10*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 6*A*a^2*c^2*sgn(cos(-
```

```
1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^10 - 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 15*A*
a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^8 - 10*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 10*B*a^2*c^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,B\,\cos\left(e+f\,x\right)+105\,B\,\cos\left(3\,e+3\,f\,x\right)+35\,B\,\mathrm{c}\right.}{960\,f\,\left(\cos\left(\right.\right.}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] -(a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*B*
cos(e + f*x) + 105*B*cos(3*e + 3*f*x) + 35*B*cos(5*e + 5*f*x) + 5*B*cos(7*e
 + 7*f*x) - 700*A*sin(2*e + 2*f*x) - 112*A*sin(4*e + 4*f*x) - 12*A*sin(6*e
+ 6*f*x)))/(960*f*(cos(2*e + 2*f*x) + 1))
```

3.151 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 142

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{20f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

```
[Out] -1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(5*A
+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/20*(5*
A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used

$= \{3052, 2819, 2817\}$

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c^2(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{c(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{20f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] ((5*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((5*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c
- c*Sin[e + f*x]])/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5f} \\
&\quad+\frac{1}{5}(5A+B)\int(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx \\
&= \frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{20f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5f} \\
&\quad+\frac{1}{10}((5A+B)c)\int(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx \\
&= \frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{30f\sqrt{c-c\sin(e+fx)}} \\
&\quad+\frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{20f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.16

$$\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{c(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}(4(100A+11B)\sin(e+fx)+4\cos(2(e+}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/480*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]*(4*(100*A + 11*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-15*(A + B) +
 4*(5*A - 2*B)*Sin[e + f*x]) - 3*Cos[4*(e + f*x)]*(5*(A + B) + 4*B*Sin[e +
f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

method	result
default	$\frac{a^2 c \tan(fx+e)(12B(\sin^2(fx+e))(\cos^2(fx+e))+15A\sin(fx+e)(\cos^2(fx+e))-15B(\sin^3(fx+e))+20A(\cos^2(fx+e))+8B(\sin^2(fx+e}{60f}$
parts	$\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c\,a^2(-3(\cos^3(fx+e))+4\cos(fx+e)\sin(fx+e)+8\tan(fx+e)+3\sec(fx+e))}{12f} - \frac{B\sqrt{-c(\sin(}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/60*a^2*c/f*tan(f*x+e)*(12*B*sin(f*x+e)^2*cos(f*x+e)^2+15*A*sin(f*x+e)*cos
(f*x+e)^2-15*B*sin(f*x+e)^3+20*A*cos(f*x+e)^2+8*B*sin(f*x+e)^2+15*A*sin(f*x
+e)+30*B*sin(f*x+e)+40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{(15\,(A+B)a^2c\cos{(fx+e)}^4 - 15\,(A+B)a^2c + 4\,(3\,Ba^2c\cos{(fx+e)}^4 - (5\,A+B)a^2c\cos{(fx+e)}^2 - 2\,(}{60\,f\cos{(fx+e)}}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/60*(15*(A + B)*a^2*c*cos(f*x + e)^4 - 15*(A + B)*a^2*c + 4*(3*B*a^2*c*co
s(f*x + e)^4 - (5*A + B)*a^2*c*cos(f*x + e)^2 - 2*(5*A + B)*a^2*c)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(124) = 248$.

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.40

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{4\left(24\,Ba^2c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-15\,Aa^2\cdots}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -4/15*(24*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 15*A*a^2*c*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^8 - 75*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 40*A*a^2*c*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^6 + 80*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 30
*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 30*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^4)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{a^2\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(60\,A\,\cos\left(e+f\,x\right)+60\,B\,\cos\left(e+f\,x\right)+75\,A\,\cos\left(3\,e\right.\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*co
s(e + f*x) + 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*f
*x) + 75*B*cos(3*e + 3*f*x) + 15*B*cos(5*e + 5*f*x) - 400*A*sin(2*e + 2*f*x
) - 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\int(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx}{a} \\ &\quad -(-A+B)\int(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx \\ &= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(3B\cos(4(e+$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A + 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(12*A +
 9*B + 4*(A + 2*B)*Sin[e + f*x])))/(96*f)
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

method	result
default	$-\frac{a^{2} \tan(fx+e)\left(-3B\left(\sin^{3}(fx+e)\right)+4A\left(\cos^{2}(fx+e)\right)-8B\left(\sin^{2}(fx+e)\right)-12A\sin(fx+e)-6B\sin(fx+e)-16A\right)\sqrt{-c(\sin(fx+e)-1}}{12f}$
parts	$-\frac{A\,a^{2}\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)\sin(fx+e)+3\cos(fx+e)-4\tan(fx+e)-3\sec(fx+e))}{3f}+\frac{B\sec(fx+e)(3}{}$

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method =_RETURNVERBOSE)

[Out] -1/12*a^2/f*tan(f*x+e)*(-3*B*sin(f*x+e)^3+4*A*cos(f*x+e)^2-8*B*sin(f*x+e)^2 -12*A*sin(f*x+e)-6*B*sin(f*x+e)-16*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f *x+e)))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{\left(3\,Ba^{2}\cos\left(fx+e\right)^{4}-12\,(A+B)a^{2}\cos\left(fx+e\right)^{2}+3\,(4\,A+3\,B)a\right.}{}$$

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/12*(3*B*a^2*cos(f*x + e)^4 - 12*(A + B)*a^2*cos(f*x + e)^2 + 3*(4*A + 3*B)*a^2 - 4*((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(2*A + B)*a^2)*sin(f*x + e))*sq rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\text{Timed out}$$

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2), x)

[Out] Timed out

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=-\frac{4\left(3\,Ba^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,Aa^2\cos\left(\right.$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/3*(3*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - 2*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.55

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=-\frac{a^2\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)+36\,B\,\cos\left(e+f\,x\right)+48\,A\,\cos\left(3\,e\right.\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(
e + f*x) + 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) + 33*B*cos(3*e + 3*f*x
) - 3*B*cos(5*e + 5*f*x) - 112*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x) -
32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1
))
```

3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 193

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-4*a^3*(A+B)*c
os(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-
2*a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

= {3052, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-4*a^3*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*(a + a*S
in[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
```

```
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (A+B)\int\frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (2a(A+B))\int\frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \left(4a^2(A+B)\right)\int\frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \frac{(4a^3(A+B)c\cos(e+fx))\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} - \frac{(4a^3(A+B)\cos(e+fx))\operatorname{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.68 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-3(A+3B)\cos(2(e+fx))+96A\log\left(\cos\right.\right.}{12f\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(
-3*(A + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] + 96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e +
f*x] - B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt
[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(173) = 346.

Time = 3.76 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.55

method	result
default	$-\frac{a^2\left(15A\sin(fx+e)-15A-17B-2B\left(\cos^3(fx+e)\right)\sin(fx+e)+3A\cos(fx+e)+17B\sin(fx+e)+48A\cos(fx+e)\ln(\csc(fx+e)-\cot(f\right.}{}$
parts	$\frac{A\left(\cos^3(fx+e)-\left(\cos^2(fx+e)\right)\sin(fx+e)-5\left(\cos^2(fx+e)\right)-16\ln(\csc(fx+e)-\cot(fx+e)-1)\cos(fx+e)+8\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos(fx+e\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/f*(15*A*sin(f*x+e)-15*A-17*B+7*B*cos(f*x+e)^2*sin(f*x+e)+3*A*cos(f
*x+e)+17*B*sin(f*x+e)+19*B*cos(f*x+e)^2+15*A*cos(f*x+e)^2+48*A*cos(f*x+e)*l
n(csc(f*x+e)-cot(f*x+e)-1)+3*A*sin(f*x+e)*cos(f*x+e)^2+18*A*sin(f*x+e)*cos(
f*x+e)+26*B*cos(f*x+e)*sin(f*x+e)-24*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+24*A
*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*B*cos(f*x+e)^4-24*B*ln(2/(1+cos(f*x+e)))
+9*cos(f*x+e)*B+48*A*ln(csc(f*x+e)-cot(f*x+e)-1)+48*B*ln(csc(f*x+e)-cot(f*x
+e)-1)-3*A*cos(f*x+e)^3-48*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+48*B*co
s(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-48*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+
e)-1)-24*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+24*B*sin(f*x+e)*ln(2/(1+cos(f*x+
e)))-9*B*cos(f*x+e)^3-24*A*ln(2/(1+cos(f*x+e)))-2*B*cos(f*x+e)^3*sin(f*x+e)
)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)+sin(f*x+e)+1)/(-c*(sin(f*x+e)-1))^(1
/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.57

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.\right.}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx\right.\right.}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(a)*(6*(sqrt(2)*A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + sqrt(2)*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-co
s(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 + sqrt(2)*(4*B*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + 3*A*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a^2*c^(5/2)*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2*c^(5/2)
*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(c^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 210

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}+\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a*
(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+4*a^3*
(A+2*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+2*a^2*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x
+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3051, 2819, 2816, 2746, 31\}$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2(A+2B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{a(A+2B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^3*(A + 2*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(A + 2*B)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 2*B)*Co
s[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
```

```
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{(2a(A+2B))\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} - \frac{(4a^2(A+2B))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{(4a^3(A+2B)\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{(4a^3(A+2B)\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{c+x}\,dx,x,-c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.77 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.10

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(28A+16B+2(2A+7B)\cos(2(e+fx))+64}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/8*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(28*A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8*A
+ 31*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x
] + B*Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

method	result
default	$-\frac{a^2 \sec(fx+e)\left(B\left(\sin^3(fx+e)\right)+2\left(\sin^2(fx+e)\right)A-8A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+16A\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-}{}$
parts	$-\frac{A\sec(fx+e)\left(8\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)-4\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-\left(\cos^2(fx+e)\right)-8\ln(\csc(fx+e)-\cot(fx-}{fc\sqrt{-c(\sin(fx+e)-1)}}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/c/f*sec(f*x+e)*(B*sin(f*x+e)^3+2*sin(f*x+e)^2*A-8*A*sin(f*x+e)*ln(
2/(1+cos(f*x+e)))+16*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+7*B*sin(f*x+e
)^2-16*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+32*B*sin(f*x+e)*ln(csc(f*x+e)-cot(
f*x+e)-1)-10*A*sin(f*x+e)+8*A*ln(2/(1+cos(f*x+e)))-16*A*ln(csc(f*x+e)-cot(f
*x+e)-1)-16*B*sin(f*x+e)+16*B*ln(2/(1+cos(f*x+e)))-32*B*ln(csc(f*x+e)-cot(f
*x+e)-1))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad A
rgument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 212

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}}-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-1/4*a*
(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*(A
+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)-1/2*a^2*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used

$= \{3051, 2818, 2819, 2816, 2746, 31\}$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (a^3*(A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]]
)/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + 5*B
)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} \\
&\quad - \frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(a(A+5B))\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(a^2(A+5B))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}
\end{aligned}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} - \frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(a^3(A+5B)\cos(e+fx))\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} - \frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(a^3(A+5B)\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{c+x}\,dx,x,-c\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} - \frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.41 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98

$$\int\frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5}}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A +
 B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.74

method	result
default	$-\frac{a^2\sec(fx+e)\left(2A(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)-A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-B(\sin^3(fx+e))+10B(\cos^2(fx\cdots\right.}{\cdots}$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-4\ln(\mathrm{cs}\cdots\right.}{f(\sin(fx+e)-1)\sqrt{\cdots}}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -a^2/c^2/f*sec(f*x+e)*(2*A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-A*cos(f
*x+e)^2*ln(2/(1+cos(f*x+e)))-B*sin(f*x+e)^3+10*B*cos(f*x+e)^2*ln(csc(f*x+e)
-cot(f*x+e)-1)-5*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*sin(f*x+e)^2*A+4*A*s
in(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+8
*B*sin(f*x+e)^2+20*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-10*B*sin(f*x+e)
*ln(2/(1+cos(f*x+e)))-4*A*ln(csc(f*x+e)-cot(f*x+e)-1)+2*A*ln(2/(1+cos(f*x+e
)))-5*B*sin(f*x+e)-20*B*ln(csc(f*x+e)-cot(f*x+e)-1)+10*B*ln(2/(1+cos(f*x+e)
)))*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin
(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(190) = 380$.

Time = 0.32 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.39

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}}\,dx =$$

$$-\frac{\left(\frac{8\,a^{\frac{5}{2}}\sqrt{c}\sin(fx+e)^2}{\left(c^3 - \frac{4\,c^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{6\,c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{4\,c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2} - \frac{2\,a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{c^{\frac{5}{2}}} + \frac{a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1}\right.}{c^{\frac{5}{2}}}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] -((8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e
) + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^
(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))*A - B*(10*a^(5/
2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 5*a^(5/2)*log(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + 2*(5*a^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) - 16*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*a^(5/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 16*a^(5/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 5*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(5/2) - 4*c^
(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 8*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*c^(5/2)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{4\sqrt{2}Ba^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\sqrt{2}\cdots}{\cdots}\cdots\right)}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(4*sqrt(2)*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*sq
rt(2)*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sqrt(c)*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-32*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 32)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^2*sqrt(
c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a^2*sqrt(c)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) - 4*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
2*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 196

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^2*B*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^3*B*cos(f*x+e)*l
n(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2818, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a^2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e
 + f*x])^(5/2)) + (a^2*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c -
 c*Sin[e + f*x])^(3/2)) + (a^3*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{(aB)\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(a^2B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^3}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(a^3B\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{(a^3B\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(4(A+B)-6(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] ((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.63

method	result
default	$\frac{a^2\sec(fx+e)\left(-6B\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)+3B\sin(fx+e)\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(fx\right.}{}$
parts	$\frac{A\tan(fx+e)a^2\left(\cos^2(fx+e)-4\right)\sqrt{a(1+\sin(fx+e))}}{3f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}\,c^3}-\frac{B\sec(fx+e)\left(6\left(\cos^2(fx+e)\right)\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2/c^3/f*sec(f*x+e)*(-6*B*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*
x+e)-1)+3*B*sin(f*x+e)*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+A*sin(f*x+e)*cos(f
*x+e)^2-7*B*sin(f*x+e)^3+18*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-9*B*
cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+6*B*sin(f*x+e)^2+24*B*sin(f*x+e)*ln(csc(f
*x+e)-cot(f*x+e)-1)-12*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-4*A*sin(f*x+e)-3*B
*sin(f*x+e)-24*B*ln(csc(f*x+e)-cot(f*x+e)-1)+12*B*ln(2/(1+cos(f*x+e))))*(a*
(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/
2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")

[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(7/2), x)

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{6\sqrt{2}Ba^2\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{7}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3\left(Aa^2\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5Ba^2\sqrt{c}\operatorname{sgn}\right.\right.}{}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*sqrt(a)*(6*sqrt(2)*B*a^2*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) - sqrt(2)*(3*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 5*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1
/2*f*x + 1/2*e)^4 + A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*
B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*(A*a^2*sqrt(c)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)^3*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{(A-7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*(
A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3051, 2821}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c -
c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 12.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A
- 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[3*
(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

method	result
default	$\frac{a^2 \tan(fx+e)(A\sin(fx+e)(\cos^2(fx+e))+B(\sin^3(fx+e))-4A(\cos^2(fx+e))+2B(\sin^2(fx+e))-4A\sin(fx+e)+3B\sin(fx+e)+10A)}{6c^4 f((\cos^2(fx+e))\sin(fx+e)-3(\cos^2(fx+e))-4\sin(fx+e)+4)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^2(\cos^3(fx+e)+4\cos(fx+e)\sin(fx+e)-5\cos(fx+e)-10\tan(fx+e)+4\sec(fx+e))}{6f((\cos^2(fx+e))\sin(fx+e)-3(\cos^2(fx+e))-4\sin(fx+e)+4)\sqrt{-c(\sin(fx+e)-1)}\,c^4}+\frac{B\sec(fx+e)(\cos(fx-}{6f((\cos^2(fx+e)}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2/c^4/f*tan(f*x+e)*(A*sin(f*x+e)*cos(f*x+e)^2+B*sin(f*x+e)^3-4*A*cos(
f*x+e)^2+2*B*sin(f*x+e)^2-4*A*sin(f*x+e)+3*B*sin(f*x+e)+10*A)*(a*(1+sin(f*x
+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/(-c*(si
n(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.72

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{\left(3\,(A-B)a^2\cos\left(fx+e\right)^2-4\,(A-B)a^2+2\,\left(3\,Ba^2\cos\left(fx+e\right)^2-(A+5\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx\right.}}{6\,\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\,\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(\right.\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x + e
)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x
 + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(84) = 168$.

Time = 0.40 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.79

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$

$$-\frac{\left(24\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+6\,Aa^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4\operatorname{sg}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] -1/48*(24*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 6*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - 42*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*A*a^2*sqrt(c)*cos(-1/4*pi +
1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 28*B*a^2*sqrt(c)*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*a^2
*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*sqrt(c)*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^4*
c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(9/2),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(9/2), x)
```

3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/40
*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/240
*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2822, 2821}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}} + \frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
```

```
- c*Sin[e + f*x])^(9/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2))/(240*c^2*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{(A-4B)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{40c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 13.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{a^2\big(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\big)\sqrt{a(1+\sin(e+f}}{120c^5f\,\big(\cos\left(\frac{1}{2}(e+f\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36*
A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 15*
B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^2\tan(fx+e)\big(4A(\cos^4(fx+e))+B(\sin^2(fx+e))(\cos^2(fx+e))+20A\sin(fx+e)(\cos^2(fx+e))+5B(\sin^3(fx+e))-48A(\cos^2(fx+e))}{30c^5f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a^2\big(2(\cos^3(fx+e))\sin(fx+e)-10(\cos^3(fx+e))-24\cos(fx+e)\sin(fx+e)+35\cos(fx+e)+37\tan(fx+e)-25\sec}{15f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+8)\sqrt{-c(\sin(fx+e)-1)}\,c^5}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/30*a^2/c^5/f*tan(f*x+e)*(4*A*cos(f*x+e)^4+B*sin(f*x+e)^2*cos(f*x+e)^2+20*
A*sin(f*x+e)*cos(f*x+e)^2+5*B*sin(f*x+e)^3-48*A*cos(f*x+e)^2+4*B*sin(f*x+e)
^2-50*A*sin(f*x+e)+15*B*sin(f*x+e)+74*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+
e)^4+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-8*sin(f*x+e)+8)/(-c*(sin(f*x+
e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = -\frac{\left(5\,(2\,A + B)a^2\cos\left(fx + e\right)^2 - 2\,(7\,A + 2\,B)a^2 + 5\,\left(3\,Ba^2\cos\left(fx + e\right)^2 - 2\,(A + 2\,B)a^2\right)\sin\left(fx + e\right)\right)\sqrt{a}}{30\,\left(5\,c^6f\cos\left(fx + e\right)^5 - 20\,c^6f\cos\left(fx + e\right)^3 + 16\,c^6f\cos\left(fx + e\right) - \left(c^6f\cos\left(fx + e\right)^5 - 12\,c^6f\cos\left(fx + \right.\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos(
f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*
c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c
^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = \int \frac{(B\sin\left(fx + e\right) + A)(a\sin\left(fx + e\right) + a)^{\frac{5}{2}}}{(-c\sin\left(fx + e\right) + c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(128) = 256$.

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.84

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\left(30\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] 1/240*(30*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 10*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) - 40*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1
/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*a^2*sqrt(c)*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*sqrt(c)*
cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*a^
2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^2*sqrt(c)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^5
*c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 21.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{16\,a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(A\,6\mathrm{i}+B\,1\mathrm{i}\right)\,\sqrt{a+a\,\sin(e+f\,x)}}{5\,c^6\,f}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((16*a^2*exp(e*6i + f*x*6i)*(A*6i + B*1i)*(a +
a*sin(e + f*x))^(1/2))/(5*c^6*f) - (16*a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f
*x)*(A*2i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f) + (a^2*exp(e*6i + f
*x*6i)*sin(e + f*x)*(8*A + 13*B)*(a + a*sin(e + f*x))^(1/2)*8i)/(3*c^6*f) -
 (B*a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*8i)/
(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*
e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*
6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e
*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 196

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(13/2)+1/40
*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/16
0*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)+1/
960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^3/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2822, 2821}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx=\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5
/2))/(160*c^2*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
a*Sin[e + f*x])^(5/2))/(960*c^3*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\int\frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\int\frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{20c^2}$$

$$
\begin{aligned}
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}} \\
&\quad + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\int\frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{160c^3} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} \\
&\quad + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}} \\
&\quad + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}} \\
&\quad + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 14.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$
\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx}}{120c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + 
f*x])^(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*A
 + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B*S
in[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.11

method	result
default	$\frac{a^2\tan(fx+e)\left(7A\left(\cos^4(fx+e)\right)\sin(fx+e)-B\left(\sin^5(fx+e)\right)-42A\left(\cos^4(fx+e)\right)+6\left(\sin^4(fx+e)\right)B-119A\sin(fx+e)\left(\cos^2(fx+e)\right)-15\right.}{60c^6f\left(\left(\cos^4(fx+e)\right)\sin(fx+e)-5\left(\cos^4(fx+e)\right)-12\left(\cos^2(fx+e)\right)\sin(fx+e)+20(\cos\right.}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^2\left(7\left(\cos^5(fx+e)\right)+42\left(\cos^3(fx+e)\right)\sin(fx+e)-126\left(\cos^3(fx+e)\right)-224\cos(fx+e)\sin(fx+e)+321\cos(fx+e)\right.}{60f\left(\left(\cos^4(fx+e)\right)\sin(fx+e)-5\left(\cos^4(fx+e)\right)-12\left(\cos^2(fx+e)\right)\sin(fx+e)+20\left(\cos^2(fx+e)\right)+16\sin(fx+e)-16\right)\sqrt{-}}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/60*a^2/c^6/f*tan(f*x+e)*(7*A*cos(f*x+e)^4*sin(f*x+e)-B*sin(f*x+e)^5-42*A*
cos(f*x+e)^4+6*sin(f*x+e)^4*B-119*A*sin(f*x+e)*cos(f*x+e)^2-15*B*sin(f*x+e)
```

```
^3+224*A*cos(f*x+e)^2+202*A*sin(f*x+e)-30*B*sin(f*x+e)-242*A)*(a*(1+sin(f*x
+e)))^(1/2)/(cos(f*x+e)^4*sin(f*x+e)-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x
+e)+20*cos(f*x+e)^2+16*sin(f*x+e)-16)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(15\,(A+B)a^2\cos{(fx+e)}^2 - 2\,(11\,A+7\,B}{60\,(c^7 f\cos{(fx+e)}^7 - 18\,c^7 f\cos{(fx+e)}^5 + 48\,c^7 f\cos}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] 1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos(
f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c
^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 1
6*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^{\frac{5}{2}}}{(-c\sin{(fx+e)}+c)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(13/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(40\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+15\,Aa^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4\operatorname{sg}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] -1/960*(40*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 15*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 45*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x +
1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*a^2*sqrt(c)*cos(-1/4*pi
+ 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a^2*sqrt(c)
*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*a
^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*sqrt(c)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^
6*c^7*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.82

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\left(A\,29\mathrm{i}+B\,13\mathrm{i}\right)\,\sqrt{a}}{15\,c^7\,f}\right.}{-858\,\cos\left(e+f\,x\right)\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\,f\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(13/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*7i + f*x*7i)*(A*29i + B*13i)*(a + a
*sin(e + f*x))^(1/2)*16i)/(15*c^7*f) - (a^2*exp(e*7i + f*x*7i)*cos(2*e + 2*
f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^7*f) - (32*a^2*exp(e*
7i + f*x*7i)*sin(e + f*x)*(6*A + 7*B)*(a + a*sin(e + f*x))^(1/2))/(5*c^7*f)
 + (32*B*a^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)
)/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858*cos(e + f*x)*e
xp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x) + 2*exp(e*7i +
f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e + 2*f*x) - 416*e
xp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*sin(6*e + 6*f*x)
)
```

3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 250

$$\begin{aligned}\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = &-\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a+a\sin(e+fx)}}\\ &-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{126f}\\ &-\frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{84f}\\ &-\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f}\\ &-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\end{aligned}$$

```
[Out] -1/84*a^2*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/
f-1/72*a*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/f
-1/9*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/f-1/315*a^4
*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1/2)-1/126*a
^3*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = -\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a\sin(e+fx)+a}} - \frac{a^3(9A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{126f} - \frac{a^2(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{84f} - \frac{a(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9/2}}{72f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] -1/315*(a^4*(9*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(f*Sqrt[a +
a*Sin[e + f*x]]) - (a^3*(9*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(9/2))/(126*f) - (a^2*(9*A - B)*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(84*f) - (a*(9*A - B)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(9/2))/(72*f) - (B*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(9*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f} \\
&\quad+\frac{1}{9}(9A-B)\int(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx \\
&= -\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f} \\
&\quad+\frac{1}{12}(a(9A-B))\int(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}\,dx \\
&= -\frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{84f} \\
&\quad-\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f} \\
&\quad+\frac{1}{21}\left(a^2(9A-B)\right)\int(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}\,dx \\
&= -\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{126f} \\
&\quad-\frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{84f} \\
&\quad-\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f} \\
&\quad+\frac{1}{63}\left(a^3(9A-B)\right)\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}\,dx
\end{aligned}$$

$$= -\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a+a\sin(e+fx)}} - \frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{126f} - \frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{84f} - \frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

Mathematica [A] (verified)

Time = 11.51 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.08

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \frac{a^3c^4(-1+\sin(e+fx))^4(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(1$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(9/2),x]
```

```
[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(17640*(A - B)*Cos[2*(e + f*x)] + 8820*(A - B)
*Cos[4*(e + f*x)] + 2520*A*Cos[6*(e + f*x)] - 2520*B*Cos[6*(e + f*x)] + 315
*A*Cos[8*(e + f*x)] - 315*B*Cos[8*(e + f*x)] + 176400*A*Sin[e + f*x] - 1764
0*B*Sin[e + f*x] + 35280*A*Sin[3*(e + f*x)] + 7056*A*Sin[5*(e + f*x)] + 201
6*B*Sin[5*(e + f*x)] + 720*A*Sin[7*(e + f*x)] + 900*B*Sin[7*(e + f*x)] + 14
0*B*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A] (verified)

Time = 74.96 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.01

method	result
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^3c^4\left(35\left(\cos^7(fx+e)\right)+40\left(\cos^5(fx+e)\right)\sin(fx+e)+48\left(\cos^3(fx+e)\right)\sin(fx+e)+64\cos(fx+\right.}{280f}$
default	$-\frac{a^3c^4\tan(fx+e)\left(280B\left(\sin^2(fx+e)\right)\left(\cos^6(fx+e)\right)+315A\sin(fx+e)\left(\cos^6(fx+e)\right)+315B\left(\sin^3(fx+e)\right)\left(\cos^4(fx+e)\right)-360A\left(\cos^6(f\right.\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/280*A/f*(a*(1+sin(f*x+e)))^(1/2)*(-c*(sin(f*x+e)-1))^(1/2)*a^3*c^4*(35*co
s(f*x+e)^7+40*cos(f*x+e)^5*sin(f*x+e)+48*cos(f*x+e)^3*sin(f*x+e)+64*cos(f*x
+e)*sin(f*x+e)+128*tan(f*x+e)-35*sec(f*x+e))+1/2520*B/f*sec(f*x+e)*(280*cos
(f*x+e)^6*sin(f*x+e)-315*cos(f*x+e)^6+240*cos(f*x+e)^4*sin(f*x+e)-315*cos(f
*x+e)^4+192*cos(f*x+e)^2*sin(f*x+e)-315*cos(f*x+e)^2+128*sin(f*x+e)-315)*(-
c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*c^4*a^3*(cos(f*x+e)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \frac{\left(315\,(A-B)a^3c^4\cos\left(fx+e\right)^8-315\,(A-B)a^3c^4+8\,\left(35\,Ba^3c^4\cos\left(fx+e\right)^8+5\,(\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] 1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B*
a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)*a
^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B)*a
^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{9}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(220) = 440.

Time = 0.55 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.81

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \frac{32\left(560\,Ba^3c^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
[Out] 32/315*(560*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^18 - 315*A*a^3*c^4*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^16 - 2205*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^16 +
1080*A*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 + 3240*B*a^3*c^4*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^14 - 1260*A*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 2100*B
*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 504*A*a^3*c^4*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^10 + 504*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)*sqrt(a)*sqrt(c)
/f
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.93

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx=\frac{\mathrm{e}^{-e\,9\mathrm{i}-f\,x\,9\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(-\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,1\mathrm{i}-B\,1\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,7\mathrm{i}}{64\,f}-\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9
/2),x)
```

```
[Out] (exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^4*exp(e*9i + f*x*9
i)*sin(5*e + 5*f*x)*(7*A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^3*
c^4*exp(e*9i + f*x*9i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^
(1/2)*7i)/(128*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos(6*e + 6*f*x)*(A*1i - B*
1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(64*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos
(8*e + 8*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(512*f) - (a^3*c
^4*exp(e*9i + f*x*9i)*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(
1/2)*7i)/(64*f) + (a^3*c^4*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(4*A + 5*B)*
(a + a*sin(e + f*x))^(1/2))/(896*f) + (7*A*a^3*c^4*exp(e*9i + f*x*9i)*sin(3
*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*9i + f*x*
9i)*sin(e + f*x)*(10*A - B)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (B*a^3*c^4
*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/2))/(1152*f)))
/(2*cos(e + f*x))
```

3.161 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 226

$$\begin{aligned}
&\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \\
&-\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} \\
&-\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35f} \\
&-\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&-\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}
\end{aligned}$$

```
[Out] -1/7*a^2*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a
*A*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/8*B*cos(f*x
+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2)/f-2/35*a^4*A*cos(f*x+e)*(
c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^3*A*cos(f*x+e)*(c-c*s
in(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}} - \frac{4a^3A\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35f} - \frac{a^2A\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} - \frac{aA\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (-2*a^4*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^3*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(7/2))/(35*f) - (a^2*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(7/2))/(7*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
*(c - c*Sin[e + f*x])^(7/2))/(8*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f} \\
&\quad + A\int(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f} \\
&\quad + \frac{1}{7}(6aA)\int(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad - \frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f} \\
&\quad + \frac{1}{7}\left(4a^2A\right)\int(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx \\
&= -\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35f} \\
&\quad - \frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad - \frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f} \\
&\quad + \frac{1}{35}\left(8a^3A\right)\int\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx
\end{aligned}$$

$$= -\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} - \frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35f} - \frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} - \frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{a^3c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(1225B+1960B\cos(2(e+fx))+980B\cos(4(e$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/35840*(a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
+ f*x]]*(1225*B + 1960*B*Cos[2*(e + f*x)] + 980*B*Cos[4*(e + f*x)] + 280*B*
Cos[6*(e + f*x)] + 35*B*Cos[8*(e + f*x)] - 19600*A*Sin[e + f*x] - 3920*A*Si
n[3*(e + f*x)] - 784*A*Sin[5*(e + f*x)] - 80*A*Sin[7*(e + f*x)]))/f
```

Maple [A] (verified)

Time = 74.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.62

method	result
default	$\frac{a^3c^3\tan(fx+e)(35B\sin(fx+e)(\cos^6(fx+e))+40A(\cos^6(fx+e))+35B(\cos^4(fx+e))\sin(fx+e)+48A(\cos^4(fx+e))+35B(\cos^2(fx}{280f}$
parts	$\frac{A\tan(fx+e)a^3c^3(5(\cos^6(fx+e))+6(\cos^4(fx+e))+8(\cos^2(fx+e))+16)\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}{35f} - \frac{B\sec(fx+e)a^3}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/280*a^3*c^3/f*tan(f*x+e)*(35*B*sin(f*x+e)*cos(f*x+e)^6+40*A*cos(f*x+e)^6+
35*B*cos(f*x+e)^4*sin(f*x+e)+48*A*cos(f*x+e)^4+35*B*cos(f*x+e)^2*sin(f*x+e)
```

```
+64*A*cos(f*x+e)^2+35*B*sin(f*x+e)+128*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+s
in(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.59

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{\left(35\,Ba^3c^3\cos\left(fx+e\right)^8-35\,Ba^3c^3-8\left(5\,Aa^3c^3\cos\left(fx+e\right)^6+6\,Aa^3c^3\cos\left(fx+e\right)^4+8\,Aa^3c^3\cos\left(fx+\right.\right.\right.}{280\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*x
 + e)^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*a^
3*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{7}{2}}(-c\sin\left(fx+e\right)+c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(196) = 392$.

Time = 0.54 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.00

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{7/2}\, dx = \frac{32\left(35\, Ba^3c^3\text{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\text{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi + \right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 32/35*(35*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^16 - 20*A*a^3*c^3*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^14 - 140*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 + 70*A
*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 210*B*a^3*c^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^12 - 84*A*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 140*B*a^3*c^3*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^10 + 35*A*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8
+ 35*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.70

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{7/2}\, dx = \frac{e^{-e\,8i-f\,x\,8i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,A\,a^3\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin(e+f\,x)}}{32\,f} - \frac{7\,B\,a^3\,c^3\,e^{e\,8i+}}{}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*A*a^3*c^3*exp(e*8i +
f*x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*B*a^3*c^3*exp(
```

```
e*8i + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*B*a
^3*c^3*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128
*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(
1/2))/(64*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(512*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(
a + a*sin(e + f*x))^(1/2))/(32*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(5*e
 + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (A*a^3*c^3*exp(e*8i + f*x*8
i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.162 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 192

$$\begin{aligned}\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c&-c\sin(e+fx))^{5/2}\,dx = \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}\\&+\frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}\\&+\frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{42f}\\&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}\end{aligned}$$

```
[Out] 1/42*(7*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f-1
/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2)/f+1/105*(7*A+
B)*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/105*(7*
A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{c^3(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{105f\sqrt{c-c\sin(e+fx)}} + \frac{2c^2(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{105f} + \frac{c(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{42f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] ((7*A + B)*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(105*f*Sqrt[c - c*S
in[e + f*x]]) + (2*(7*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sq
rt[c - c*Sin[e + f*x]])/(105*f) + ((7*A + B)*c*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f} \\
&\quad+\frac{1}{7}(7A+B)\int(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx \\
&= \frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{42f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f} \\
&\quad+\frac{1}{21}(2(7A+B)c)\int(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx \\
&= \frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{105f} \\
&\quad+\frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{42f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f} \\
&\quad+\frac{1}{105}\left(4(7A+B)c^2\right)\int(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx \\
&= \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{105f\sqrt{c-c\sin(e+fx)}} \\
&\quad+\frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{105f} \\
&\quad+\frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{42f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.21

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{a^3c^2(-1 + \sin(e + fx))^2(1 + \sin(e + fx))^3\sqrt{a(1 + \sin(e + fx))}\sqrt{c - c\sin(e + fx)}(-$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)*C
os[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Si
n[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(e +
f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e + f*x
)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^7)
```

Maple [A] (verified)

Time = 75.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

method	result
default	$\frac{a^3c^2\tan(fx+e)\left(30B\left(\cos^4(fx+e)\right)\left(\sin^2(fx+e)\right)+35A\left(\cos^4(fx+e)\right)\sin(fx+e)-35B\left(\cos^2(fx+e)\right)\left(\sin^3(fx+e)\right)+42A\left(\cos^4(fx+e)\right)+\right.}{}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^3c^2\left(-5\left(\cos^5(fx+e)\right)+6\left(\cos^3(fx+e)\right)\sin(fx+e)+8\cos(fx+e)\sin(fx+e)+16\tan(fx+e)+\right.}{30f}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/210*a^3*c^2/f*tan(f*x+e)*(30*B*cos(f*x+e)^4*sin(f*x+e)^2+35*A*cos(f*x+e)^
4*sin(f*x+e)-35*B*cos(f*x+e)^2*sin(f*x+e)^3+42*A*cos(f*x+e)^4+24*B*sin(f*x+
e)^2*cos(f*x+e)^2+35*A*sin(f*x+e)*cos(f*x+e)^2-70*B*sin(f*x+e)^3+56*A*cos(f
*x+e)^2+16*B*sin(f*x+e)^2+35*A*sin(f*x+e)+105*B*sin(f*x+e)+112*A)*(-c*(sin(
f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{(35\,(A+B)a^3c^2\cos{(fx+e)}^6-35\,(A+B)a^3c^2+2\,(15\,Ba^3c^2\cos{(fx+e)}^6-3\,(7\,A+B)a^3c^2\cos{(fx+}}{210\,f\,c}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a^
3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a^3
*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^{\frac{7}{2}}(-c\sin{(fx+e)}+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(168) = 336$.

Time = 0.56 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.36

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{16\left(120\,Ba^3c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi + \right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 16/105*(120*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 70*A*a^3*c^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^12 - 490*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 25
2*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 756*B*a^3*c^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^10 - 315*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 525*B*a^3*c^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 140*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^6 + 140*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.99

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{e^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\,\sqrt{c - c\,\sin\left(e + f\,x\right)}\left(\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}\,5\mathrm{i}}{32\,f} + \frac{a^3\,c}{}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^2*exp(e*7i + f*x*7
i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (
```

```
a^3*c^2*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*
x))^(1/2)*1i)/(16*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i +
 B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) + (a^3*c^2*exp(e*7i + f*x*7i)*
sin(5*e + 5*f*x)*(4*A - 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^2
*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A - B)*(a + a*sin(e + f*x))^(1/2))
/(96*f) + (5*a^3*c^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A + B)*(a + a*sin(e
 + f*x))^(1/2))/(32*f) - (B*a^3*c^2*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a
+ a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.163 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 142

$$\begin{aligned}\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c&-c\sin(e+fx))^{3/2}\,dx=\frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{30f\sqrt{c-c\sin(e+fx)}}\\&+\frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}\\&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}\end{aligned}$$

```
[Out] -1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(3*A
+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/15*(3*
A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used

$= \{3052, 2819, 2817\}$

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c^2(3A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{c(3A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] ((3*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((3*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c -
 c*Sin[e + f*x])^(3/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f} \\
&\quad+\frac{1}{3}(3A+B)\int(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx \\
&= \frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f} \\
&\quad+\frac{1}{15}(2(3A+B)c)\int(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx \\
&= \frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{30f\sqrt{c-c\sin(e+fx)}} \\
&\quad+\frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f} \\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$
\int(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =
$$

$$
-\frac{a^3c(-1+\sin(e+fx))(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(-15(16A+11B)\cos(2(e
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/960*(a^3*c*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-15*(16*A + 11*B)*Cos[2*(e + f*x)] - 30*(2
*A + B)*Cos[4*(e + f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] + 240*
B*Sin[e + f*x] + 60*A*Sin[3*(e + f*x)] - 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5
*(e + f*x)] - 24*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a^3 c \tan(fx+e)\left(5B\left(\sin^5(fx+e)\right)+6A\left(\cos^4(fx+e)\right)+12\left(\sin^4(fx+e)\right)B-15A\sin(fx+e)\left(\cos^2(fx+e)\right)-12A\left(\cos^2(fx+e)\right)-20B(\text{s}}{30f}$
parts	$-\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c\,a^3\left(2\left(\cos^3(fx+e)\right)\sin(fx+e)+5\left(\cos^3(fx+e)\right)-4\cos(fx+e)\sin(fx+e)-8\tan(fx+e)-\right.}{10f}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/30*a^3*c/f*tan(f*x+e)*(5*B*sin(f*x+e)^5+6*A*cos(f*x+e)^4+12*sin(f*x+e)^4
*B-15*A*sin(f*x+e)*cos(f*x+e)^2-12*A*cos(f*x+e)^2-20*B*sin(f*x+e)^2-15*A*si
n(f*x+e)-15*B*sin(f*x+e)-24*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{\left(5\,Ba^3c\cos\left(fx+e\right)^6-15\,(A+B)a^3c\cos\left(fx+e\right)^4+5\,(3\,A+2\,B)a^3c-2\,(3\,(A+\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a^3*c*cos(f*x + e)^6 - 15*(A + B)*a^3*c*cos(f*x + e)^4 + 5*(3*A +
 2*B)*a^3*c - 2*(3*(A + 2*B)*a^3*c*cos(f*x + e)^4 - 2*(3*A + B)*a^3*c*cos(f
*x + e)^2 - 4*(3*A + B)*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.74

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{8\left(20\,Ba^3c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 8/15*(20*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c*cos(-1/4*pi +
1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - 36*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^3*c*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sq
rt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.26

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{e^{-e\,6i-f\,x\,6i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos(4\,e+4\,f\,x)\,(2\,A+B)\,\sqrt{a+a\,\sin(e+f\,x)}}{16\,f}-\frac{B\,a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+}}{96\,f}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c*exp(e*6i + f*x*6i
)*cos(4*e + 4*f*x)*(2*A + B)*(a + a*sin(e + f*x))^(1/2))/(16*f) - (B*a^3*c*
exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (a
^3*c*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i + B*2i)*(a + a*sin(e + f*x))^(1/
2)*1i)/(4*f) + (a^3*c*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A + 11*B)*(a
+ a*sin(e + f*x))^(1/2))/(32*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x
)*(A*3i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) - (a^3*c*exp(e*6i + f
*x*6i)*sin(5*e + 5*f*x)*(A*1i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f)
))/(2*cos(e + f*x))
```

3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{B\int (a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A+B)\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.26

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{a^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(4(60A+23$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(6
0*A + 23*B)*Sin[e + f*x] + Cos[4*(e + f*x)]*(5*A + 15*B + 4*B*Sin[e + f*x])
 - 4*Cos[2*(e + f*x)]*(5*(7*A + 5*B) + 4*(5*A + 6*B)*Sin[e + f*x])))/(160*f
)
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^{3}\tan(fx+e)\left(4B\left(\sin^{2}(fx+e)\right)\left(\cos^{2}(fx+e)\right)+5A\sin(fx+e)\left(\cos^{2}(fx+e)\right)-15B\left(\sin^{3}(fx+e)\right)+20A\left(\cos^{2}(fx+e)\right)-24B\left(\sin^{2}(fx+e\right.\right.}{20f}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^{3}\left(\cos^{3}(fx+e)-4\cos(fx+e)\sin(fx+e)-8\cos(fx+e)+8\tan(fx+e)+7\sec(fx+e)\right)}{4f}+\frac{B\,\mathrm{se}}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/20*a^3/f*tan(f*x+e)*(4*B*sin(f*x+e)^2*cos(f*x+e)^2+5*A*sin(f*x+e)*cos(f*
x+e)^2-15*B*sin(f*x+e)^3+20*A*cos(f*x+e)^2-24*B*sin(f*x+e)^2-35*A*sin(f*x+e
)-10*B*sin(f*x+e)-40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.45

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(5\,(A+3\,B)a^{3}\cos{(fx+e)}^{4}-40\,(A+B)a^{3}\cos{(fx+e)}^{2}+5\,(7\,A+}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(f*x + e)^4 - 40*(A + B)*a^3*cos(f*x + e)^2 + 5*(7
*A + 5*B)*a^3 + 4*(B*a^3*cos(f*x + e)^4 - (5*A + 7*B)*a^3*cos(f*x + e)^2 +
2*(5*A + 3*B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}\sqrt{-c\sin(fx+e)+c}\,d$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{4\left(8\,Ba^3\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,Aa^3\,\mathrm{co}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
[Out] -4/5*(8*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a^3*cos(-1/4*pi + 1/2*f*
x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 5*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.80

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(140\,A\,\cos\left(e+f\,x\right)+100\,B\,\cos\left(e+f\,x\right)+135\,A\,\cos\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(140*A*cos
(e + f*x) + 100*B*cos(e + f*x) + 135*A*cos(3*e + 3*f*x) - 5*A*cos(5*e + 5*f
*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x
) + 40*A*sin(4*e + 4*f*x) - 90*B*sin(2*e + 2*f*x) + 48*B*sin(4*e + 4*f*x) -
 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 239

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3*a
*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-1/4*B*cos
(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)-8*a^4*(A+B)*cos(f*x
+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^3*
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-8*a^4*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (4*a^3*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + B)*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x
]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f
*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+(A+B)\int\frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+(2a(A+B))\int\frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\\
&\quad-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\left(4a^2(A+B)\right)\int\frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + \left(8a^3(A+B)\right)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{\left(8a^4(A+B)c\cos(e+fx)\right)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{\left(8a^4(A+B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 12.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

$$
\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =
$$

$$
-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\left(-12(8A+15B)\cos(2(e+\right.}{96f\left(\cos\left(\frac{1}{2}(\right.\right.}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[
a*(1 + Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f
*x)] + 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A + 3
6*B)*Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(215) = 430.

Time = 3.30 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.30

method	result
default	$\frac{a^3\left(-64A\sin(fx+e)+64A+67B+16B\left(\cos^3(fx+e)\right)\sin(fx+e)-24A\cos(fx+e)-67B\sin(fx+e)-192A\cos(fx+e)\ln(\csc(fx+e)-c\right.}{}$
parts	$\frac{A\left(\cos^4(fx+e)+\left(\cos^3(fx+e)\right)\sin(fx+e)+6\left(\cos^3(fx+e)\right)-5\left(\cos^2(fx+e)\right)\sin(fx+e)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos(fx+e)-24\ln\left(\frac{}{1+\cos}\right.\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/12*a^3/f*(-64*A*sin(f*x+e)+64*A+67*B-32*B*cos(f*x+e)^2*sin(f*x+e)-24*A*co
s(f*x+e)-67*B*sin(f*x+e)-80*B*cos(f*x+e)^2-68*A*cos(f*x+e)^2-192*A*cos(f*x+
e)*ln(csc(f*x+e)-cot(f*x+e)-1)-20*A*sin(f*x+e)*cos(f*x+e)^2-88*A*sin(f*x+e)
*cos(f*x+e)-112*B*cos(f*x+e)*sin(f*x+e)+96*A*cos(f*x+e)*ln(2/(1+cos(f*x+e))
)-96*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+13*B*cos(f*x+e)^4+96*B*ln(2/(1+cos(f
*x+e)))+3*B*cos(f*x+e)^4*sin(f*x+e)-45*cos(f*x+e)*B+4*A*cos(f*x+e)^4-192*A*
ln(csc(f*x+e)-cot(f*x+e)-1)-192*B*ln(csc(f*x+e)-cot(f*x+e)-1)+24*A*cos(f*x+
e)^3+192*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-192*B*cos(f*x+e)*ln(csc(f
*x+e)-cot(f*x+e)-1)+192*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+96*B*cos(f
*x+e)*ln(2/(1+cos(f*x+e)))-96*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+48*B*cos(f*
x+e)^3+96*A*ln(2/(1+cos(f*x+e)))-3*B*cos(f*x+e)^5+4*A*cos(f*x+e)^3*sin(f*x+
e)+16*B*cos(f*x+e)^3*sin(f*x+e))*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)+sin(f
*x+e)+1)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(215) = 430.

Time = 0.78 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^3\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^3\sqrt{c}\mathrm{sgn}\left(\cos\left(-\right.\right.\right.}{c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*sqrt(a)*(6*(sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-co
s(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 + (3*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*A*a^3
*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 3*sqrt(2)*A*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
 + 3*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*A*a^3*
c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)))/c^4)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(1/2),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}+\frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a^
2*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+1/
6*a*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)+
4*a^4*(3*A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)+2*a^3*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c
-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{a^2(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{a(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^4*(3*A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^3*(3*A + 5*B)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a^2*(3*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*
x]]) + (a*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{(a(3A+5B))\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{(2a^2(3A+5B))\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&+ \frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}} \\
&- \frac{(4a^3(3A+5B))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&+ \frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}} \\
&- \frac{(4a^4(3A+5B)\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} \\
&+ \frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}} \\
&+ \frac{(4a^4(3A+5B)\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 12.86 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.08

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-132
*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 960*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] - 13*B*Sin[3*(e + f*x)
]))/(24*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[
c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

method	result
default	$-\frac{a^3\sec(fx+e)\left(-2B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+3\left(\sin^3(fx+e)\right)A+13B\left(\sin^3(fx+e)\right)+27\left(\sin^2(fx+e)\right)A+144A\sin(fx+e)\ln(\csc\right.}{}$
parts	$\frac{A\sec(fx+e)\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-48\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+9\left(\cos^2(fx+e\right.\right.}{2fc\sqrt{-c(\sin(fx+e)-1)}}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

[Out] -1/6*a^3/c/f*sec(f*x+e)*(-2*B*sin(f*x+e)^2*cos(f*x+e)^2+3*sin(f*x+e)^3*A+13*B*sin(f*x+e)^3+27*sin(f*x+e)^2*A+144*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-72*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+59*B*sin(f*x+e)^2+240*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-120*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-78*A*sin(f*x+e)-144*A*ln(csc(f*x+e)-cot(f*x+e)-1)+72*A*ln(2/(1+cos(f*x+e)))-120*B*sin(f*x+e)-240*B*ln(csc(f*x+e)-cot(f*x+e)-1)+120*B*ln(2/(1+cos(f*x+e))))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(3/2), x)

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.45

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(\frac{6\left(3\sqrt{2}Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\sqrt{2}Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c^2\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{6\left(\sqrt{2}A\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(a)*(6*(3*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 5*sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*lo
g(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) - 6*(sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sq
rt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/((cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(4
*B*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 3*A*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 9*B*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*B*a^3*c^(9/2)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(c^6*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}\\
&-\frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}}\\
&-\frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&-\frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}}\\
&-\frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(5/2)-1/2*a*
(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)-3/4*a^
2*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)-6*
a^4*(A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)-3*a^3*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c
*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}} - \frac{3a^2(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (6*a^4*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x
]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*(A +
 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]
]) - (3*a^2*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c^2*f*Sqr
t[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c}$$

$$
\begin{aligned}
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} \\
&\quad - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3a(A+3B))\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(3a^2(A+3B))\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} \\
&\quad - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{(6a^3(A+3B))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} \\
&\quad - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{(6a^4(A+3B)\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} - \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(6a^4(A+3B)\cos(e+fx))\operatorname{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} - \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 12.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A
+ B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(e
+ f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(A
 + 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.54

method	result
default	$-\frac{a^3 \sec(fx+e)\left(B(\sin^2(fx+e))(\cos^2(fx+e))+24A(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)-12A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e}\right.\right.}{}$
parts	$-\frac{A\sec(fx+e)\left((\cos^2(fx+e))\sin(fx+e)+12(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)-6(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)+24\ln(\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2*a^3/c^2/f*sec(f*x+e)*(B*sin(f*x+e)^2*cos(f*x+e)^2+24*A*cos(f*x+e)^2*ln
(csc(f*x+e)-cot(f*x+e)-1)-12*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-2*sin(f*x+
e)^3*A+72*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-36*B*cos(f*x+e)^2*ln(2
/(1+cos(f*x+e)))-10*B*sin(f*x+e)^3+20*sin(f*x+e)^2*A+48*A*sin(f*x+e)*ln(csc
(f*x+e)-cot(f*x+e)-1)-24*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+54*B*sin(f*x+e)^
2+144*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-72*B*sin(f*x+e)*ln(2/(1+cos(
f*x+e)))-10*A*sin(f*x+e)-48*A*ln(csc(f*x+e)-cot(f*x+e)-1)+24*A*ln(2/(1+cos(
f*x+e)))-36*B*sin(f*x+e)-144*B*ln(csc(f*x+e)-cot(f*x+e)-1)+72*B*ln(2/(1+cos
(f*x+e))))*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2
)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3
- (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.57

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{6\sqrt{2}\left(Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\right.\right.\right.}{c^3\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\right.\right.}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(6*sqrt(2)*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-cos(-1/4
*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 2
*(sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*A*a^3*c^(7/
2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
```

```
f*x + 1/2*e)))/c^6 + (5*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 9*sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(3
*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*B*a^
3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}} dx$

Optimal result

Integrand size = 40, antiderivative size = 264

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}} dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(7/2)-1/12*a
*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)+1/4*a
^2*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a
^4*(A+7*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*si
n(f*x+e))^(1/2)+1/2*a^3*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-
c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{a^3(A+7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^3f\sqrt{c-c\sin(e+fx)}} + \frac{a^2(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} - \frac{a(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*c*f*(c
- c*Sin[e + f*x])^(5/2)) + (a^2*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(4*c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^4*(A + 7*B)*Cos[e + f*x]*
Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (a^3*(A + 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^3*f*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{(A+7B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} \\
&\quad - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} \\
&\quad + \frac{(a(A+7B))\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^2}
\end{aligned}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(a^2(A+7B))\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^3}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}} - \frac{(a^3(A+7B))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^3}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}} - \frac{(a^4(A+7B)\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}} + \frac{(a^4(A+7B)\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 12.49 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7}}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A + B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(236) = 472.

Time = 4.48 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.06

method	result
default	$\frac{a^3\sec(fx+e)\left(3A\sin(fx+e)(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-6A\sin(fx+e)(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)+21B\sin(fx+\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(6(\cos^2(fx+e))\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-3(\cos^2(fx+e))\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-18(\cos^2(fx+e\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^3/c^3/f*sec(f*x+e)*(3*A*sin(f*x+e)*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-
6*A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+21*B*sin(f*x+e)*cos
(f*x+e)^2*ln(2/(1+cos(f*x+e)))-42*B*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-c
ot(f*x+e)-1)-3*B*sin(f*x+e)^2*cos(f*x+e)^2-9*A*cos(f*x+e)^2*ln(2/(1+cos(f*x
+e)))+18*A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-8*sin(f*x+e)^3*A-63*B*c
os(f*x+e)^2*ln(2/(1+cos(f*x+e)))+126*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e
)-1)-41*B*sin(f*x+e)^3-12*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+24*A*sin(f*x+e)
*ln(csc(f*x+e)-cot(f*x+e)-1)+6*sin(f*x+e)^2*A-84*B*sin(f*x+e)*ln(2/(1+cos(f
*x+e)))+168*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+54*B*sin(f*x+e)^2+12*A
*ln(2/(1+cos(f*x+e)))-24*A*ln(csc(f*x+e)-cot(f*x+e)-1)-6*A*sin(f*x+e)+84*B*
ln(2/(1+cos(f*x+e)))-168*B*ln(csc(f*x+e)-cot(f*x+e)-1)-21*B*sin(f*x+e))*(a*
(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/
2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos
(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(236) = 472$.

Time = 0.36 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.84

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] -1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/2)
*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2)*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(7/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7/2)*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x + e)/(cos(f*x
 + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 22*a^(
7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2)*sqrt(c)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^4*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
5*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.36

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{12\sqrt{2}Ba^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{7}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{6\sqrt{2}\left(Aa^3\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7Ba^3\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\right.\right.\right.}{c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right.\right.}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*(12*sqrt(2)*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 6
*sqrt(2)*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a^3*sqrt(
c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-192*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^2 + 192)/(c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(11*A*a^3
*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 41*B*a^3*sqrt(c)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 18*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e)^4 - 3*(9*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 31*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*c^4*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 247

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} - \frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)-1/3*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*B*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)-a^3*B*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)-a^4*B*cos(f*x+e)*ln(
1-sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2818, 2816, 2746, 31}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a^3B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}} + \frac{a^2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e
 + f*x])^(7/2)) + (a^2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*
(c - c*Sin[e + f*x])^(5/2)) - (a^3*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(c^3*f*(c - c*Sin[e + f*x])^(3/2)) - (a^4*B*Cos[e + f*x]*Log[1 - Sin[e + f
*x]])/(c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\int\frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} \\
&\quad - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{(aB)\int\frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c^2} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} \\
&\quad + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{(a^2B)\int\frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^3} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} \\
&\quad - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} \\
&\quad - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} + \frac{(a^3B)\int\frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^4}
\end{aligned}
$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} + \frac{(a^4B\cos(e+fx))\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c^3\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} - \frac{(a^4B\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{c^4f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} - \frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 12.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\left(6(A+B)-4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] ((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A
+ 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(9/2))
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.66

method	result
default	$-\frac{a^{3}\sec(fx+e)\left(-3B\left(\cos^{4}(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+6B\left(\cos^{4}(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)+8B\left(\sin^{2}(fx+e)\right)\left(\cos^{2}(fx+e)\right.\right.}{}$
parts	$-\frac{A\tan(fx+e)a^{3}\left(\cos^{2}(fx+e)-2\right)\sqrt{a(1+\sin(fx+e))}}{f\left(\left(\cos^{2}(fx+e)\right)\sin(fx+e)-3\left(\cos^{2}(fx+e)\right)-4\sin(fx+e)+4\right)\sqrt{-c(\sin(fx+e)-1)}\,c^{4}}+\frac{B\sec(fx+e)\left(3\left(\cos^{4}(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+\right.\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/3*a^3/c^4/f*sec(f*x+e)*(-3*B*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))+6*B*cos(f
*x+e)^4*ln(csc(f*x+e)-cot(f*x+e)-1)+8*B*sin(f*x+e)^2*cos(f*x+e)^2-12*B*sin(
f*x+e)*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+24*B*sin(f*x+e)*cos(f*x+e)^2*ln(cs
c(f*x+e)-cot(f*x+e)-1)+3*A*sin(f*x+e)*cos(f*x+e)^2+11*B*sin(f*x+e)^3+24*B*c
os(f*x+e)^2*ln(2/(1+cos(f*x+e)))-48*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)
-1)-20*B*sin(f*x+e)^2+24*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-48*B*sin(f*x+e)*
ln(csc(f*x+e)-cot(f*x+e)-1)-6*A*sin(f*x+e)+3*B*sin(f*x+e)-24*B*ln(2/(1+cos(
f*x+e)))+48*B*ln(csc(f*x+e)-cot(f*x+e)-1))*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*
x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*
cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*
c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\frac{\sqrt{2}\sqrt{a}\left(\frac{24\sqrt{2}Ba^3\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.\right.}{c^{\frac{9}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
[Out] 1/48*sqrt(2)*sqrt(a)*(24*sqrt(2)*B*a^3*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(9/2)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) - sqrt(2)*(12*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 7*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e)^6 - 3*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
47*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*(A*a^3*sqrt(c)*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 4*(3*A*a^3*sqrt(c)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 41*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2 - 1)^4*c^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(9/2), x)
```

3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{(A-9B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/80
*(A-9*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3051, 2821}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c
- c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 434 vs. 2(96) = 192.

Time = 17.07 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.52

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+}$$
$$+\frac{(-3A-5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{2(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2
)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + S
in[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*
(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c
- c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(
1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(84) = 168$.

Time = 4.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.98

method	result
default	$\frac{a^3\tan(fx+e)\left(A(\cos^4(fx+e))-B(\sin^2(fx+e))(\cos^2(fx+e))+5A\sin(fx+e)(\cos^2(fx+e))+5B(\sin^3(fx+e))-17A(\cos^2(fx+e))+6}{10c^5f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+8}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3\left((\cos^3(fx+e))\sin(fx+e)-5(\cos^3(fx+e))-17\cos(fx+e)\sin(fx+e)+15\cos(fx+e)+26\tan(fx+e)-10\sec(f}{10f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+8)\sqrt{-c(\sin(fx+e)-1)}\,c^5}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/10*a^3/c^5/f*tan(f*x+e)*(A*cos(f*x+e)^4-B*sin(f*x+e)^2*cos(f*x+e)^2+5*A*s
in(f*x+e)*cos(f*x+e)^2+5*B*sin(f*x+e)^3-17*A*cos(f*x+e)^2+6*B*sin(f*x+e)^2-
10*A*sin(f*x+e)+5*B*sin(f*x+e)+26*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^4
+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1
))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.

Time = 0.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\left(10\,Ba^3\cos\left(fx+e\right)^4-5\,(A+7\,B)a^3\cos\left(fx+e\right)^2+2\right.}{10\,\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] 1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A + 1
3*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 2
0*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12
*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{7}{2}}}{(-c\sin(fx + e) + c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(84) = 168$.

Time = 0.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.55

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{11/2}}\,dx = \frac{\left(40\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] 1/80*(40*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 10*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(co
```

```
s(-1/4*pi + 1/2*f*x + 1/2*e)) - 90*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^3*sqrt(c)*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*B*a^3*sqrt(c)*
cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*
a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - 45*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*
B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi +
1/2*f*x + 1/2*e)^2 - 1)^5*c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(11/2), x)
```

3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60
*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/48
0*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2822, 2821}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx=\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c
```

```
- c*Sin[e + f*x])^(11/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(480*c^2*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\int\frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-5B)\int\frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{60c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 442 vs. $2(146) = 292$.

Time = 17.13 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.03

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+}$$
$$-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$
$$+\frac{3(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2
)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(13/2)) + (3*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(13/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(13/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(13/2))
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.41

method	result
default	$\frac{a^3 \tan(fx+e)(3A(\cos^4(fx+e))\sin(fx+e)-18A(\cos^4(fx+e))-51A\sin(fx+e)(\cos^2(fx+e))-15B(\sin^3(fx+e))+106A(\cos^2(fx+e}{30c^6 f((\cos^4(fx+e))\sin(fx+e)-5(\cos^4(fx+e))-12(\cos^2(fx+e))\sin(fx+e)+20(\cos^2(fx+e}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3(3(\cos^5(fx+e))+18(\cos^3(fx+e))\sin(fx+e)-54(\cos^3(fx+e))-106\cos(fx+e)\sin(fx+e)+129\cos(fx+e}{30f((\cos^4(fx+e))\sin(fx+e)-5(\cos^4(fx+e))-12(\cos^2(fx+e))\sin(fx+e)+20(\cos^2(fx+e))+16\sin(fx+e)-16)\sqrt{-}}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/30*a^3/c^6/f*tan(f*x+e)*(3*A*cos(f*x+e)^4*sin(f*x+e)-18*A*cos(f*x+e)^4-51
*A*sin(f*x+e)*cos(f*x+e)^2-15*B*sin(f*x+e)^3+106*A*cos(f*x+e)^2-10*B*sin(f*
x+e)^2+78*A*sin(f*x+e)-15*B*sin(f*x+e)-118*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos
(f*x+e)^4*sin(f*x+e)-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)+20*cos(f*x+e
)^2+16*sin(f*x+e)-16)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.47

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(15\,Ba^3\cos\left(fx+e\right)^4-15\,(A+3\,B)a^3\cos\left(fx+e\right)^2+6\,(3\,A+5\,B)a^3-2\left(5\,(A+B)a^3\cos\left(fx+e\right)^2-\right.}{30\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(15*B*a^3*cos(f*x + e)^4 - 15*(A + 3*B)*a^3*cos(f*x + e)^2 + 6*(3*A +
 5*B)*a^3 - 2*(5*(A + B)*a^3*cos(f*x + e)^2 - (11*A + 5*B)*a^3)*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7
 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e
) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x
+ e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(13/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(128) = 256.

Time = 0.55 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(60\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+20\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\operatorname{sg}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] -1/480*(60*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 20*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 100*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^3*sqrt(c)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 75*B*a^3*sqrt(
c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6
```

```
*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) - 30*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 5*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2 - 1)^6*c^7*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.78

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = -\frac{\sqrt{c-c\sin(e+fx)}\left(\frac{56\,a^3\,e^{e\,7i+f\,x\,7i}\,(4\,A+5\,B)\,\sqrt{a+a\,\sin(e+f\,x)}}{5\,c^7\,f}+\frac{a^3\,e^{e\,7i+f\,x\,7i}\,\sin(3\,e+3\,f\,x)\,(A\,1i+B\,1i)\,\sqrt{a+a\,\sin(e+f\,x)}\,32i}{3\,c^7\,f} - \cdots\right.}{-858\,\cos(e+f\,x)\,e^{e\,7i+f\,x\,7i}+858\,e^{e\,7i+f\,x\,7i}\,\cos(3\,e+3\,f\,x)-130\,e^{e\,7i+f\,x\,7i}\,\cos(5\,e+5\,f\,x)+\cdots}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((56*a^3*exp(e*7i + f*x*7i)*(4*A + 5*B)*(a + a
*sin(e + f*x))^(1/2))/(5*c^7*f) + (a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*
(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*c^7*f) - (32*a^3*exp(e*7i
+ f*x*7i)*cos(2*e + 2*f*x)*(A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(c^7*f) +
(8*B*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(c
^7*f) - (a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(A*13i + B*5i)*(a + a*sin(e +
f*x))^(1/2)*32i)/(5*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858
*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x)
+ 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e +
 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*s
in(6*e + 6*f*x))
```

3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 202

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/14*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/16
8*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2)+
1/840*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(
11/2)+1/6720*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*
x+e))^(9/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used

$= \{3051, 2822, 2821\}$

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}} + \frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x]
)^(15/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(168*c*f
*(c - c*Sin[e + f*x])^(13/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(7/2))/(840*c^2*f*(c - c*Sin[e + f*x])^(11/2)) + ((3*A - 11*B)*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2))/(6720*c^3*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{14c} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} \\
&\quad + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} \\
&\quad + \frac{(3A-11B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{84c^2} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} \\
&\quad + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-11B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{840c^3} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} \\
&\quad + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} \\
&\quad + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}} \\
&\quad + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 442 vs. 2(202) = 404.

Time = 17.20 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.19

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx &= \frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+} \\
&- \frac{2(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} \\
&+ \frac{6(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} \\
&+ \frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} \\
&+ \frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}
\end{aligned}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(15/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2
)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(15/2))
```

Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.40

method	result
default	$\frac{a^3\tan(fx+e)\left(39A\left(\cos^6(fx+e)\right)+3B\left(\cos^4(fx+e)\right)\left(\sin^2(fx+e)\right)+273A\left(\cos^4(fx+e)\right)\sin(fx+e)+21B\left(\cos^2(fx+e)\right)\left(\sin^3(fx+e)\right)-}{420c^7f(\cos^6(fx+e)+6(\cos^4(fx+e)}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3\left(13\left(\cos^5(fx+e)\right)\sin(fx+e)-91\left(\cos^5(fx+e)\right)-312\left(\cos^3(fx+e)\right)\sin(fx+e)+728\left(\cos^3(fx+e)\right)+1075\cos(fx}{140f(\cos^6(fx+e)+6(\cos^4(fx+e))\sin(fx+e)-18(\cos^4(fx+e))-32(\cos^2(fx+e))\sin(fx+e)+48(\cos^2(fx+e))}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/420*a^3/c^7/f*tan(f*x+e)*(39*A*cos(f*x+e)^6+3*B*cos(f*x+e)^4*sin(f*x+e)^2
+273*A*cos(f*x+e)^4*sin(f*x+e)+21*B*cos(f*x+e)^2*sin(f*x+e)^3-936*A*cos(f*x
+e)^4-69*B*sin(f*x+e)^2*cos(f*x+e)^2-1911*A*sin(f*x+e)*cos(f*x+e)^2-266*B*s
in(f*x+e)^3+3225*A*cos(f*x+e)^2-4*B*sin(f*x+e)^2+2268*A*sin(f*x+e)-210*B*si
n(f*x+e)-2748*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^6+6*cos(f*x+e)^4*sin(
f*x+e)-18*cos(f*x+e)^4-32*cos(f*x+e)^2*sin(f*x+e)+48*cos(f*x+e)^2+32*sin(f*
x+e)-32)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx =$$

$$-\frac{\left(140\,Ba^3\cos\left(fx+e\right)^4-7\,(27\,A+61\,B)a^3\cos\left(fx+e\right)^2+4\,(57\,A+71\,B)a^3-7\,\left(5\,(3\,A+5\,B)a^3\cos\right.\right.}{420\,\left(7\,c^8f\cos\left(fx+e\right)^7-56\,c^8f\cos\left(fx+e\right)^5+112\,c^8f\cos\left(fx+e\right)^3-64\,c^8f\cos\left(fx+e\right)-\left(c^8f\cos\left(f\right.\right.\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="fricas")
```

```
[Out] -1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*(
57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^3)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*
cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^
8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8
*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{15/2}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{7}{2}}}{(-c\sin(fx + e) + c)^{\frac{15}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(15/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.69

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{15/2}}\,dx = \frac{\left(280\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="giac")
```

```
[Out] 1/6720*(280*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 105*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 385*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 63*A*a^3*sqrt(c)*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 231*B*a^3*sq
rt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 21*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) - 77*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - 3*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 11*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(
-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^7*c^8*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))
```

Mupad [B] (verification not implemented)

Time = 25.38 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.09

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(15/2),x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(B*a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
 f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^8*f) + (B*a^3*exp(e*12i + f*x*12i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^
8*f) - (a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) + (a^3*exp(e*11i +
f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^
(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)
/(15*c^8*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)/(15*c^8*f) + (a^3*e
xp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) - (a^3*exp(e*9i + f*x*9i)*(a +
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*43i
+ B*29i)*8i)/(5*c^8*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*
1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(89*A + 82*B)*32i)/(35*c^8*f)
))/(exp(e*1i + f*x*1i)*14i - 90*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*350
i + 910*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*1638i - 2002*exp(e*6i + f*x
*6i) - exp(e*7i + f*x*7i)*1430i - exp(e*9i + f*x*9i)*1430i + 2002*exp(e*10i
 + f*x*10i) + exp(e*11i + f*x*11i)*1638i - 910*exp(e*12i + f*x*12i) - exp(e
*13i + f*x*13i)*350i + 90*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*14i -
 exp(e*16i + f*x*16i) + 1)
```

3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 246

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/16*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(17/2)+1/56
*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(15/2)+1/22
4*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(13/2)+1
/1120*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(11/
2)+1/8960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^4/f/(c-c*sin(f*x+e))^
(9/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2822, 2821}

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx=\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(17/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x]
)^(17/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c
- c*Sin[e + f*x])^(15/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(224*c^2*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^(7/2))/(1120*c^3*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*
B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8960*c^4*f*(c - c*Sin[e + f*x]
)^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(3(A-3B))\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{56c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{112c^3}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{1120c^4}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [A] (verified)

Time = 17.21 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(\cdots}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+\cdots}$$
$$-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(17/2),x]
```

```
[Out] ((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)
)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
 (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x
]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x]
)^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[
e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 +
 Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*
Sin[e + f*x])^(17/2))
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.26

method	result
default	$\frac{a^3\tan(fx+e)\left(12A\sin(fx+e)\left(\cos^6(fx+e)\right)-B\left(\cos^2(fx+e)\right)\left(\sin^5(fx+e)\right)-96A\left(\cos^6(fx+e)\right)+8B\left(\cos^2(fx+e)\right)\left(\sin^4(fx+e)\right)-372\cdots}{140c^8f\left(\left(\cos^6(fx+e)\right)\sin(fx+e)-7\left(\cos^6(fx\cdots\right)\right)}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3\left(3\left(\cos^7(fx+e)\right)+24\left(\cos^5(fx+e)\right)\sin(fx+e)-96\left(\cos^5(fx+e)\right)-240\left(\cos^3(fx+e)\right)\sin(fx+e)+480\left(\cos^3(f\cdots\right)\right)}{35f\left(\left(\cos^6(fx+e)\right)\sin(fx+e)-7\left(\cos^6(fx+e)\right)-24\left(\cos^4(fx+e)\right)\sin(fx+e)+56\left(\cos^4(fx+e)\right)+80\left(\cos^2(fx+e)\right)\sin(\cdots\right)}$

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/140*a^3/c^8/f*tan(f*x+e)*(12*A*sin(f*x+e)*cos(f*x+e)^6-B*cos(f*x+e)^2*sin
(f*x+e)^5-96*A*cos(f*x+e)^6+8*B*cos(f*x+e)^2*sin(f*x+e)^4-372*A*cos(f*x+e)^
4*sin(f*x+e)+29*B*sin(f*x+e)^5+960*A*cos(f*x+e)^4-64*sin(f*x+e)^4*B+1548*A*
sin(f*x+e)*cos(f*x+e)^2+105*B*sin(f*x+e)^3-2332*A*cos(f*x+e)^2-1468*A*sin(f
*x+e)+70*B*sin(f*x+e)+1608*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^6*sin(f*
x+e)-7*cos(f*x+e)^6-24*cos(f*x+e)^4*sin(f*x+e)+56*cos(f*x+e)^4+80*cos(f*x+e
)^2*sin(f*x+e)-112*cos(f*x+e)^2-64*sin(f*x+e)+64)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.99

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{\left(35\,Ba^3\cos\left(fx+e\right)^4-56\,(A+2\,B)a^3\cdots\right.}{140\,\left(c^9f\cos\left(fx+e\right)^9-32\,c^9f\cos\left(fx+e\right)^7+160\,c^9f\cos\cdots\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="fricas")
```

```
[Out] 1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A
+ 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*x +
 e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*cos(
f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f*co
s(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2)
,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{17/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.39

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{17/2}}\,dx =$$

$$-\frac{\left(140\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 56\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^6\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="giac")
```

```
[Out] -1/8960*(140*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 56*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 168*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 28*A*a^3*sqrt(c)*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 84*B*a^3*sqr
t(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 8*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) - 24*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*
pi + 1/2*f*x + 1/2*e)^2 - 1)^8*c^9*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.42

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{17/2}}\, dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(17/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
8*B*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
+ f*x*1i)*1i)/2))^(1/2))/(c^9*f) + (8*B*a^3*exp(e*13i + f*x*13i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^9*f) - (6
4*a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f) - (32*a^3*exp(e*7i + f*x*7i)
*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(8
*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f)
- (32*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*8i + f*
x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/
2)*(A*13i + B*10i))/(7*c^9*f) - (64*a^3*exp(e*10i + f*x*10i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*13i + B*10i))/
(7*c^9*f) + (16*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(64*A + 53*B))/(7*c^9*f)))/(exp(e*1i +
f*x*1i)*16i - 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*544i + 1700*exp(e
*4i + f*x*4i) + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i + f*x*6i) - exp(e*
7i + f*x*7i)*7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(e*10i + f*x*10i) +
exp(e*11i + f*x*11i)*7072i - 6188*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13
i)*3808i + 1700*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*544i - 119*exp(
e*16i + f*x*16i) - exp(e*17i + f*x*17i)*16i + exp(e*18i + f*x*18i) + 1)
```

3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 197

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 1/2*(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*
B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+4*(A-B)*c^3*co
s(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2
*(A-B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3052, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{4c^3(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{2c^2(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}} + \frac{c(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (4*(A - B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*(A - B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[
e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*(c - c*Si
n[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + (A-B)\int\frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + (2(A-B)c)\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + \left(4(A-B)c^2\right)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\left(4a(A-B)c^3\cos(e+fx)\right)\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\left(4(A-B)c^3\cos(e+fx)\right)\text{Subst}\left(\int\frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\left(3(A-3B)\cos(2(e+fx))\right.}{12f\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -1/12*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt
[c - c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (
36*A - 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(177) = 354.

Time = 4.17 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.51

method	result
default	$\frac{c^2\left(15A\sin(fx+e)+15A-17B+2B\left(\cos^3(fx+e)\right)\sin(fx+e)-3A\cos(fx+e)-17B\sin(fx+e)+18A\sin(fx+e)\cos(fx+e)-26B\cos(fx\right.}{}$
parts	$\frac{A\left(\left(\cos^2(fx+e)\right)\sin(fx+e)+\cos^3(fx+e)+6\cos(fx+e)\sin(fx+e)+8\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-16\ln(-\cot(fx+e)+\csc(fx+e)+1)\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2/f*(15*A*sin(f*x+e)+15*A-17*B-7*B*cos(f*x+e)^2*sin(f*x+e)-3*A*cos(f*
x+e)-17*B*sin(f*x+e)+19*B*cos(f*x+e)^2-15*A*cos(f*x+e)^2+3*A*sin(f*x+e)*cos
(f*x+e)^2+18*A*sin(f*x+e)*cos(f*x+e)-26*B*cos(f*x+e)*sin(f*x+e)+24*A*cos(f*
x+e)*ln(2/(1+cos(f*x+e)))+24*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*B*cos(f*x+
e)^4-24*B*ln(2/(1+cos(f*x+e)))+9*cos(f*x+e)*B+3*A*cos(f*x+e)^3-24*B*cos(f*x
+e)*ln(2/(1+cos(f*x+e)))-24*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-48*A*cos(f*x+
e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+48*B*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1
)-48*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+48*B*sin(f*x+e)*ln(-cot(f*x+
e)+csc(f*x+e)+1)-9*B*cos(f*x+e)^3+24*A*ln(2/(1+cos(f*x+e)))+2*B*cos(f*x+e)^
3*sin(f*x+e)-48*A*ln(-cot(f*x+e)+csc(f*x+e)+1)+48*B*ln(-cot(f*x+e)+csc(f*x+
e)+1))*(-c*(sin(f*x+e)-1))^(1/2)/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(1+sin(f*x+e
)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*
x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.53

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{6\sqrt{2}(A\sqrt{a}c^2\text{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-B\sqrt{a}c^2\text{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\log\left(-2\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+2\right)}{a\text{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}-\frac{\sqrt{2}\left(4Ba^{\frac{5}{2}}c^2\text{sgn}\right.}{}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(c)*(6*sqrt(2)*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-2*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 + 2)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sq
rt(2)*(4*B*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^6 - 3*A*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^4 + 3*B*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 6*A*a^(5/2)*c^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 6*B*a^(5/2)*c^2*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{5/2}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+2*(A-B)*c
^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3052, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\frac{2c^2(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*(A - B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e +
```

```
f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (2(A-B)c)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{(2a(A-B)c^2\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{(2(A-B)c^2\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 6.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}\left(B\cos(2(e+fx))-4\left(4(-A\right.\right.}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -1/4*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(132) = 264.

Time = 4.08 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.92

method	result
default	$\frac{c\left(B\left(\cos^3(fx+e)\right)+B\left(\cos^2(fx+e)\right)\sin(fx+e)+2A\left(\cos^2(fx+e)\right)-2A\sin(fx+e)\cos(fx+e)+8A\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+\right.}{}$
parts	$\frac{A\left(2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos(fx+e)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-4\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-4\ln(-\cot(fx+e)+\csc\right.}{f(-\cos(fx+e}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2*c/f*(B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2-2*A*sin(
f*x+e)*cos(f*x+e)+8*A*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-4*A*cos(f*x+e
)*ln(2/(1+cos(f*x+e)))+8*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-4*A*sin(
f*x+e)*ln(2/(1+cos(f*x+e)))-3*B*cos(f*x+e)^2+4*B*cos(f*x+e)*sin(f*x+e)-8*B*
cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))
-8*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*B*sin(f*x+e)*ln(2/(1+cos(f*x
+e)))-2*A*sin(f*x+e)+8*A*ln(-cot(f*x+e)+csc(f*x+e)+1)-4*A*ln(2/(1+cos(f*x+e
)))-cos(f*x+e)*B+3*B*sin(f*x+e)-8*B*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*B*ln(2/(
1+cos(f*x+e)))-2*A+3*B)*(-c*(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)-sin(f*x+e)+1)
/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c*
sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(-c(\sin{(e+fx)}-1))^{\frac{3}{2}}\,(A+B\sin{(e+fx)})}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(a*(sin(e
+ f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(-c\sin{(fx+e)}+c)^{\frac{3}{2}}}{\sqrt{a\sin{(fx+e)}+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{\sqrt{2}\left(A\sqrt{a}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}Ba^{\frac{3}{2}}c\,\mathrm{sgn}(\cos(-\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -sqrt(2)*sqrt(c)*(sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-sin(-1/4*pi + 1/2*f
*x + 1/2*e)^2 + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (sqrt(2)*B*a^(
3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - sqrt(2)*A*a^(3/2)*c*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^2 + sqrt(2)*B*a^(3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/a^2)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(1/2),x)
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] (A-B)*c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)-B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3050, 2817, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{c(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(a(-A+B)c\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{((-A+B)c\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.24

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\left(-ifx+2\log\left(i+e^{i(e+fx)}\right)\right)+B\sin(e+fx)\right)\sqrt{c-c\sin(}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e
+ f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*((-I)*f*x + 2*Log[I + E^(I*
(e + f*x))]) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(88) = 176$.

Time = 3.39 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.19

method	result
parts	$-\frac{A\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)+\sin(fx+e)+1)}{f(-\cos(fx+e)+\sin(fx+e)-1)\sqrt{a(1+\sin(fx+e))}} + \frac{B\left(2\cos(fx+e)\ln(\right.}{}$
default	$\frac{\left(A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2A\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)+A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2A\cos(fx+e)\ln(-\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -A/f*(2*ln(-cot(f*x+e)+csc(f*x+e)+1)-ln(2/(1+cos(f*x+e))))*(-c*(sin(f*x+e)-
1))^(1/2)*(cos(f*x+e)+sin(f*x+e)+1)/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(1+sin(f*
x+e)))^(1/2)+B/f*(2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*ln(-cot(f*x+e
```

```
)+csc(f*x+e)+1)*sin(f*x+e)-ln(2/(1+cos(f*x+e)))*cos(f*x+e)-ln(2/(1+cos(f*x+
e)))*sin(f*x+e)+cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+2*ln(-cot(f*x+e)+csc(f*x
+e)+1)-ln(2/(1+cos(f*x+e)))-sin(f*x+e)-1)*(-c*(sin(f*x+e)-1))^(1/2)/(-cos(f
*x+e)+sin(f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e)
 + a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f
*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.83

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{B\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}} - \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}} - \frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(a+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) - A\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}} - \right.}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
- A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{2}\left(\frac{2\sqrt{2}B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{a}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right.}{2f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*B*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^2/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*
(A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)))*log(-4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 4)/(a*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 113

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)\cos(e+fx)\log(1+\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)+1/2*(A-B)*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/
(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3048, 2816, 2746, 31}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] -1/2*((A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*Cos[e + f*x]*Log[1 + Sin[e + f*x]])
/(2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3048

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist
[(A*b + a*B)/(2*a*b), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] + Dist[(B*c + A*d)/(2*c*d), Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a +
b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a} + \frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2c} \\
&= \frac{(a(A-B)\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)c\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{((A-B)\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{((A+B)\cos(e+fx))\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)\cos(e+fx)\log(1+\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\left(B\log(\cos(e+fx))+A\left(\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(B*Log[Cos[e + f*x]] + A*(Log[1 - Tan[(e + f*x)/2]] - Log[1
 + Tan[(e + f*x)/2]])))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*
x]]))
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(A\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\ln(\csc(fx+e)-\cot(fx+e)-1)-B\ln(-\cot(fx+e)+\csc(fx+e)+1)+B\ln\left(\frac{2}{1+\cos(fx+e)}\right)-B\ln(\right.}{f\sqrt{-c(\sin(fx+e)-1)}\sqrt{a(1+\sin(fx+e))}}$
parts	$\frac{A\cos(fx+e)(\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln(\csc(fx+e)-\cot(fx+e)-1))}{f\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}+\frac{B\left(-\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln(\csc(fx+e\right.}{f\sqrt{a(1+\sin(fx+e))}}$

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(A*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*ln(csc(f*x+e)-cot(f*x+e)-1)-B*ln(-cot
(f*x+e)+csc(f*x+e)+1)+B*ln(2/(1+cos(f*x+e)))-B*ln(csc(f*x+e)-cot(f*x+e)-1))
*cos(f*x+e)/(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(a*c*cos(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{A + B\sin(e + fx)}{\sqrt{a(\sin(e + fx) + 1)}\sqrt{-c(\sin(e + fx) - 1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e +
f*x) - 1))), x)
```

Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{B\sin(fx + e) + A}{\sqrt{a\sin(fx + e) + a}\sqrt{-c\sin(fx + e) + c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}\sqrt{c - c\sin(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*(A
-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A-B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{2c} \\
&= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{((A-B)\cos(e+fx))\int\sec(e+fx)\,dx}{2c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int\frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(A+B+(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)),x]
```

```
[Out] ((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*x])]
*(c - c*Sin[e + f*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(91) = 182$.

Time = 3.12 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.51

method	result
default	$\frac{A\sin(fx+e)\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)\cos(fx+e)-A\left(\cos^2(fx+e)\right)\ln(}{}$
parts	$\frac{A\left(\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)-\cos(fx+e)\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-\left(\cos^2(fx+e)\right)\ln(\csc(fx+}{}$

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/c/f*(A*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*ln(csc(f*x+
e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)-A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*
x+e)+1)+A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*sin(f*x+e)*cos(f*x+e)*
ln(-cot(f*x+e)+csc(f*x+e)+1)+B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f
*x+e)+B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*cos(f*x+e)^2*ln(csc(f*x
+e)-cot(f*x+e)-1)-A*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)^2-A*cos(f*x+e)*ln(-c
ot(f*x+e)+csc(f*x+e)+1)+A*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-B*cos(f*x+
e)*sin(f*x+e)+B*cos(f*x+e)^2+B*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*co
s(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-A*sin(f*x+e)-B*sin(f*x+e)-A-B)/(-cos(f
*x+e)+sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.27

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \left[-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx}{}\right.$$

$$-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right.}{2\left(ac^2f\cos(fx+e)\sin(fx+e)-ac^2f\cos(fx+e)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c)
*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*sin(f*
x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c - c\sin(e + fx))^{3/2}}\,dx = \int \frac{A + B\sin{(e + fx)}}{\sqrt{a(\sin{(e + fx)} + 1)}\,(-c(\sin{(e + fx)} - 1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
 - 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c - c\sin(e + fx))^{3/2}}\,dx = \int \frac{B\sin{(fx + e)} + A}{\sqrt{a\sin{(fx + e)} + a}(-c\sin{(fx + e)} + c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.98

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c - c\sin(e + fx))^{3/2}}\,dx = \frac{\frac{(A\sqrt{a} - B\sqrt{a})\log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{ac^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2(A\sqrt{a} -}{ac^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/4*((A*sqrt(a) - B*sqrt(a))*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a*
c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))) - 2*(A*sqrt(a) - B*sqrt(a))*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/
(a*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))) + (A*sqrt(a) + B*sqrt(a))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*
a*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 153

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx=\frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{4c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A
-B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A-B)*
arctanh(sin(f*x+e))*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3051, 2822, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx=\frac{(A-B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)) + ((A - B)*Cos[e + f*x])/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
```

```
 + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx}{2c}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}dx}{4c^2}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{((A-B)\cos(e+fx))\int \sec(e+fx)\,dx}{4c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{4c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.45

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(A+B+(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 +$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(5/2)),x]
```

```
[Out] ((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[Co
s[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/
2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(135) = 270.

Time = 3.64 (sec) , antiderivative size = 773, normalized size of antiderivative = 5.05

method	result	size
default	Expression too large to display	773
parts	Expression too large to display	815

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/4/c^2/f*(-A*sin(f*x+e)-A-B-B*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f
*x+e)+1)+2*A*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*A*ln(csc(
f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)-2*B*sin(f*x+e)*cos(f*x+e)*ln(-co
t(f*x+e)+csc(f*x+e)+1)+2*B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e
)+2*A*cos(f*x+e)-B*sin(f*x+e)+B*cos(f*x+e)^2+A*cos(f*x+e)^2+2*A*cos(f*x+e)*
ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*sin(f*x+e)*cos(f*x+e)^2-3*A*sin(f*x+e)*cos(
f*x+e)-B*cos(f*x+e)*sin(f*x+e)+A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc
(f*x+e)+1)-B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-A*cos(f*x+e)^3*ln(csc
(f*x+e)-cot(f*x+e)-1)+B*cos(f*x+e)^3*ln(csc(f*x+e)-cot(f*x+e)-1)+A*cos(f*x+
e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*cos(f*x+e)^3+B*sin(f*x+e)*cos(f*x+e)^2
*ln(csc(f*x+e)-cot(f*x+e)-1)-2*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A
*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*B*cos(f*x+e)*ln(-cot(f*x+e)+csc(
f*x+e)+1)-A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+B*cos(f*x+e
)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1
)-B*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)^2*ln(-cot(f*x+e)
+csc(f*x+e)+1))/(cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)-cos(f*x+e)+2*sin(f*x+e)
-2)/(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.77

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \left[-\frac{\left((A-B)\cos\left(fx+e\right)^{3}+2\left(A-B\right)\cos\left(fx+e\right)\sin\left(\right.\right.}{}\right.$$

$$-\frac{\left((A-B)\cos\left(fx+e\right)^{3}+2\left(A-B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\left(A-B\right)\cos\left(fx+e\right)\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-}}{}\right.}{4\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A
 - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
 + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
 e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
+ e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x + e)^3
+ 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt(-a*c)*
arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*c
os(f*x + e)*sin(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\sin{(e+fx)}}{\sqrt{a(\sin{(e+fx)}+1)}\,(-c(\sin{(e+fx)}-1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
 - 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin{(fx+e)}+A}{\sqrt{a\sin{(fx+e)}+a}(-c\sin{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.50

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\frac{2\,(A\sqrt{a}-B\sqrt{a})\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} - \frac{4\,(A\sqrt{a}-B\sqrt{a})\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} + \frac{2}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.}}{16\,f}$$

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/16*(2*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)
/(a*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))) - 4*(A*sqrt(a) - B*sqrt(a))*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))/(a*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) + (2*(A*sqrt(a) - B*sqrt(a))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
+ A*sqrt(a) + B*sqrt(a))/(a*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 271

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\ &\quad -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} \\ &\quad -\frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\ &\quad -\frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} \\ &\quad -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}}\end{aligned}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*(
3*A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-1
/6*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)
-4*(3*A-5*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)-2*(3*A-5*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2819, 2816, 2746, 31}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =\\&-\frac{4c^4(3A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\\&-\frac{2c^3(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}\\&-\frac{c^2(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}\\&-\frac{c(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a\sin(e+fx)+a}}\\&-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(3*A - 5*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(3*A - 5*B)*c^3*Cos[e + f*x]*Sqrt
[c - c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((3*A - 5*B)*c^2*Cos
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((
3*A - 5*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[a + a*Sin
[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(2*f*(a + a
*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(3A-5B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a} \\
&= -\frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((3A-5B)c)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} \\
&= -\frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(2(3A-5B)c^2)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}
\end{aligned}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(4(3A-5B)c^3)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(4(3A-5B)c^4\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(4(3A-5B)c^4\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 12.84 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(132A-45B+2(27A-59B)\cos(2(e+fx))+\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + 
f*x])^(3/2),x]
```

```
[Out] -1/24*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
132*A - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 576*
A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*(e + f
*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

method	result
default	$-\frac{c^3\sec(fx+e)\left(-2B(\sin^2(fx+e))(\cos^2(fx+e))+3(\sin^3(fx+e))A-13B(\sin^3(fx+e))-27(\sin^2(fx+e))A+144A\sin(fx+e)\ln(-\cot\right.}{}$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\sin(fx+e)-48\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-9(\cos^2(fx+e\right.}{2f\sqrt{a(1+\sin(fx+e))}\,a}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6*c^3/a/f*sec(f*x+e)*(-2*B*sin(f*x+e)^2*cos(f*x+e)^2+3*sin(f*x+e)^3*A-13
*B*sin(f*x+e)^3-27*sin(f*x+e)^2*A+144*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e
)+1)-72*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+59*B*sin(f*x+e)^2-240*B*sin(f*x+e
)*ln(-cot(f*x+e)+csc(f*x+e)+1)+120*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-78*A*s
in(f*x+e)+144*A*ln(-cot(f*x+e)+csc(f*x+e)+1)-72*A*ln(2/(1+cos(f*x+e)))+120*
B*sin(f*x+e)-240*B*ln(-cot(f*x+e)+csc(f*x+e)+1)+120*B*ln(2/(1+cos(f*x+e))))
*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.45

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{2}\sqrt{c}\left(\frac{6\left(3\sqrt{2}A\sqrt{a}c^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5\sqrt{2}B\sqrt{a}c^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\cdots\right.\right.\right.}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx\cdots\right.\right.}\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(c)*(6*(3*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - 5*sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log
(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))) - 6*(sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - sqr
t(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/((sin(-1/4*pi + 1/2
*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(4*
B*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^6 - 3*A*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^4 + 9*B*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 12*A*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 24*B*a^(9/2)*c^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^6*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(3/2),x)
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 210

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*(
A-2*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-4*(A-
2*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)-2*(A-2*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*
x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c^3(A-2B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2c^2(A-2B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}} - \frac{c(A-2B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(A - 2*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(A - 2*B)*c^2*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - 2*B)*c*Cos[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*C
os[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-2B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} \\
&= -\frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(2(A-2B)c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} \\
&= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(4(A-2B)c^2)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(4(A-2B)c^3\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(4(A-2B)c^3\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 11.71 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$
\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =
$$
$$
-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(28A-16B+2(2A-7B)\cos(2(e+fx))+64\right.}{}
$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/8*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2
8*A - 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] + S
```

```
in[(e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*A +
 31*B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]
 + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Si
n[e + f*x]))^(3/2))
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

method	result
default	$\frac{c^2\sec(fx+e)\left(B\left(\sin^3(fx+e)\right)+2\left(\sin^2(fx+e)\right)A-16A\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)+8A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-7\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(8\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-4\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+\cos^2(fx+e)+8\ln(-\cot(fx+e)+\csc(fx\right.}{fa\sqrt{a(1+\sin(fx+e))}}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2*c^2/a/f*sec(f*x+e)*(B*sin(f*x+e)^3+2*sin(f*x+e)^2*A-16*A*sin(f*x+e)*ln(
-cot(f*x+e)+csc(f*x+e)+1)+8*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-7*B*sin(f*x+e
)^2+32*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-16*B*sin(f*x+e)*ln(2/(1+co
s(f*x+e)))+10*A*sin(f*x+e)-16*A*ln(-cot(f*x+e)+csc(f*x+e)+1)+8*A*ln(2/(1+co
s(f*x+e)))-16*B*sin(f*x+e)+32*B*ln(-cot(f*x+e)+csc(f*x+e)+1)-16*B*ln(2/(1+c
os(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),
x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\left(B\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+A\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}(\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
[Out] -2*(B*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + A*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 5*B*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - 2*B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(ab
s(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) - B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/cos(-1/4*
pi + 1/2*f*x + 1/2*e)^2)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 159

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(3/2)-(A-3*
B)*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+
e))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {3051, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c^2(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{c(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -(((A - 3*B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])) - ((A - 3*B)*c*Cos[e + f*x]*Sqrt[c - c*S
in[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c -
 c*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
```

```
tQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-3B)\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((A-3B)c)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((A-3B)c^2\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((A-3B)c^2\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 11.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(4A-3B-B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{2f\left(\cos\left(\frac{1}{2}\right.\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + 
f*x])^(3/2),x]
```

```
[Out] -1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*A
 - 3*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
- 12*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.50

method	result
default	$-\frac{c\sec(fx+e)\left(2A\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+B\left(\sin^2(fx+e)\right)-6B\sin(fx+e)\ln(-\mathrm{co}\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+2\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{}{1+}\right.\right.}{fa\sqrt{a(1+\sin(fx+e))}}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -c/a/f*sec(f*x+e)*(2*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*sin(f*x+e)
*ln(2/(1+cos(f*x+e)))+B*sin(f*x+e)^2-6*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+
e)+1)+3*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*A*sin(f*x+e)+2*A*ln(-cot(f*x+e)
+csc(f*x+e)+1)-A*ln(2/(1+cos(f*x+e)))+3*B*sin(f*x+e)-6*B*ln(-cot(f*x+e)+csc
```

```
(f*x+e)+1)+3*B*ln(2/(1+cos(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*
x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),
x)
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f*
x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(143) = 286$.

Time = 0.32 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.31

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{B\left(\frac{6c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{3c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{2\left(\frac{3c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

[Out] -(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) - A*(2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^(3/2) - c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4
*sqrt(a)*c^(3/2)*sin(f*x + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.42

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}Bc\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\sqrt{a}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cdots}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\cdots}{2f}$$

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")

[Out] -1/2*sqrt(2)*(2*sqrt(2)*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^2/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(
2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-8*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 8)/
(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))))*sqrt(c)/f

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(3/2),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(3/2), x)

3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 100

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+B*c*cos
(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3050, 2816, 2746, 31, 2817}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{Bc\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])) + (B*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-}{f\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-A + B - I
*B*f*x + 2*B*Log[I + E^(I*(e + f*x))] + B*((-I)*f*x + 2*Log[I + E^(I*(e + f
*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(3/2))
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

method	result
default	$\frac{\sec(fx+e)\left(2B\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-B\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(fx+e)-B\sin(fx+e)+2B\ln(-\cot\right.}{af\sqrt{a(1+\sin(fx+e))}}$
parts	$\frac{A\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}}{fa\sqrt{a(1+\sin(fx+e))}}+\frac{B\sec(fx+e)\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+2\ln\right.}{fa\sqrt{a(1+\sin(fa}}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/a/f*sec(f*x+e)*(2*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*sin(f*x+e)*
ln(2/(1+cos(f*x+e)))+A*sin(f*x+e)-B*sin(f*x+e)+2*B*ln(-cot(f*x+e)+csc(f*x+e
)+1)-B*ln(2/(1+cos(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(
1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\left(4\,B\sqrt{a}\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{A\sqrt{a}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-B\sqrt{a}\mathrm{sgn}\left(s\right.}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}}{2\,a^{2}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(4*B*sqrt(a)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - (A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sq
rt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*(
A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
+ f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{2a}\\
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int\sec(e+fx)\,dx}{2a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin \right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/
2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(91) = 182$.

Time = 2.68 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.47

method	result
default	$\frac{A(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+A\sin(fx+e)\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A(\cos^2(fx+e))\ln(\csc(fx-}{}$
parts	$\frac{A((\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+\cos(fx+e)\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-(\cos^2(fx+e))\ln(\csc(fx+e)}{}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/a/f*(A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*sin(f*x+e)*cos(f*x+e
)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-A
*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2*ln(-cot(f
*x+e)+csc(f*x+e)+1)+B*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*
cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(
f*x+e)*cos(f*x+e)+A*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)*ln
(csc(f*x+e)-cot(f*x+e)-1)+A*cos(f*x+e)^2+A*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+
e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-B*
cos(f*x+e)^2-B*cos(f*x+e)*sin(f*x+e)+A*sin(f*x+e)-B*sin(f*x+e)-A+B)/(cos(f*
x+e)+sin(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.19

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+\right.$$

$$-\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right.}{2\left(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)*
log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*sin(f*x
 + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{B\sin(fx + e) + A}{(a\sin(fx + e) + a)^{\frac{3}{2}}\sqrt{-c\sin(fx + e) + c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(91) = 182.

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}}\,dx = \frac{\frac{(A+B)\log\left(-512\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+512\right)}{a^{\frac{3}{2}}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2(\cdots}{a^{\frac{3}{2}}\sqrt{c}\,\mathrm{sgn}(\cos(\cdots}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/4*((A + B)*log(-512*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 512)/(a^(3/2)*sqrt
(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - 2*(A + B)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(3/2)*sqrt(c)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (A*
sqrt(a) - B*sqrt(a))/(a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{A + B\sin(e + f\,x)}{\left(a + a\sin(e + f\,x)\right)^{3/2}\sqrt{c - c\sin(e + f\,x)}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 150

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{A\text{arctanh}(\sin(e+fx))\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*A
*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*A*arctanh
(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3051, 2822, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{A\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2acf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(3/2)) + (A*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

integral

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{A\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{2ac}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A\cos(e+fx))\int \sec(e+fx)\,dx}{2ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{A\text{arctanh}(\sin(e+fx))\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\left(2B-A\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right)+A\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)+A\cos(2(e+fx))\left(-\log\right.\right.}{4cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c}}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] -1/4*(Cos[e + f*x]*(2*B - A*Log[1 - Tan[(e + f*x)/2]] + A*Log[1 + Tan[(e +
f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[1 - Tan[(e + f*x)/2]] + Log[1 + Tan[(e
+ f*x)/2]]) + 2*A*Sin[e + f*x]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f
*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

method	result
default	$\frac{A\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\cos(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)+B\tan(fx+e)\sin(fx+e)+\tan(fx+e)A}{2acf\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A(-\ln(\csc(fx+e)-\cot(fx+e)-1)\cos(fx+e)+\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)+\tan(fx+e))}{2fa\sqrt{a(1+\sin(fx+e))}\,c\sqrt{-c(\sin(fx+e)-1)}}+\frac{B\sin(fx+e)\,\mathrm{ta}}{2f\sqrt{a(1+\sin(fx+e))}\,\sqrt{-}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/a/c/f/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)*(A*cos(f*x+e)*
ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+B*tan
(f*x+e)*sin(f*x+e)+tan(f*x+e)*A)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{ac}A\cos{(fx+e)}^3\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\,\sqrt{ac}}{\mathrm{co}}\right.}{}\right.$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x +
 e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*
x + e))/cos(f*x + e)^3) + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a)*s
qrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*A*ar
ctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f
*x) - 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 217

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = &-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\\ &+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\\ &+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\\ &+\frac{(3A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8ac^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+1/8*(
3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A-B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\frac{(3A-B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] -1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A - B)*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
```

```
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx}{2a} \\
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx}{4ac} \\
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad+\frac{(3A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2} \\
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad+\frac{((3A-B)\cos(e+fx))\int\sec(e+fx)\,dx}{8ac^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad + \frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad + \frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8ac^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.41

$$
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\cdots\right.}{\cdots}
$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 +
(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/
2))
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sec(fx+e)\left(3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)-3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)\cdots\right.}{\cdots}$
parts	$\frac{A\sec(fx+e)\left(3\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-3\left(\cos^2(fx+e)\right)\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)+2(\cdots\right.}{8f\,c^2a(\sin(fx+e)-1)\sqrt{-c(\cdots}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/8/a/c^2/f*sec(f*x+e)*(3*A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+
e)-1)-3*A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*sin(f*x+e)
*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+B*sin(f*x+e)*cos(f*x+e)^2*ln(-cot
```

```
(f*x+e)+csc(f*x+e)+1)-2*sin(f*x+e)^3*A-3*A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f
*x+e)-1)+3*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*B*sin(f*x+e)^3+B*c
os(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f
*x+e)+1)-sin(f*x+e)^2*A+3*B*sin(f*x+e)^2+5*A*sin(f*x+e)+B*sin(f*x+e))/(sin(
f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.01

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \left[-\frac{\left((3\,A-B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-(3\,A-B)\,\mathrm{c}\right.}{}\right.$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*
sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
 + 2*((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f
*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8*(((3*A - B)*cos(f*x + e)^3*sin(f*
x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) +
((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*sin(f*
x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x +
e) - a^2*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin{(fx+e)}+A}{(a\sin{(fx+e)}+a)^{\frac{3}{2}}(-c\sin{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.29

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\frac{2\,(3\,A\sqrt{a}-B\sqrt{a})\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{2}c^{\frac{5}{2}}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\,(3\,A}{a^{2}c^{\frac{5}{2}}\,\mathrm{sgn}(\cos}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/32*(2*(3*A*sqrt(a) - B*sqrt(a))*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1
)/(a^2*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) - 4*(3*A*sqrt(a) - B*sqrt(a))*log(abs(cos(-1/4*pi + 1/2*f*x +
1/2*e)))/(a^2*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + (2*(3*A*sqrt(a) - B*sqrt(a))*cos(-1/4*pi + 1/2*f*x +
1/2*e)^4 - 3*(3*A*sqrt(a) - B*sqrt(a))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2
*A*sqrt(a) - 2*B*sqrt(a))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^2*c^(
5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 323

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*(3*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2
)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(5/2)+(3*A
-7*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+1/
3*(3*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1
/2)+8*(3*A-7*B)*c^5*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2
)/(c-c*sin(f*x+e))^(1/2)+4*(3*A-7*B)*c^4*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/
a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{8c^5(3A-7B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{4c^4(3A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}} + \frac{c^3(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}} + \frac{c^2(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}} + \frac{c(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a\sin(e+fx)+a)^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (8*(3*A - 7*B)*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*(3*A - 7*B)*c^4*Cos[e + f*x]*Sqr
t[c - c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^3*
Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 ((3*A - 7*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a
+ a*Sin[e + f*x]]) + ((3*A - 7*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2)
)/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e +
 f*x])^(9/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A-7B)\int\frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a}\\ &= \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}}\\ &\quad-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}}+\frac{((3A-7B)c)\int\frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} \\
&\quad + \frac{(2(3A-7B)c^2)\int\frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} \\
&= \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} \\
&\quad + \frac{(4(3A-7B)c^3)\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} \\
&= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(8(3A-7B)c^4)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}
\end{aligned}
$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(8(3A-7B)c^5\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(8(3A-7B)c^5\cos(e+fx))\operatorname{Subst}\left(\int\frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 14.47 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{9/2}}{}$$

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^(5/2),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2)*(-96*(A - B) + 192*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 3*(A - 7*B)*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 192*(3*A - 7*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 3*(28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x] - B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.38

method	result
default	$-\frac{c^4\sec(fx+e)\left(2B\left(\cos^2(fx+e)\right)\left(\sin^3(fx+e)\right)+3A\left(\cos^2(fx+e)\right)\left(\sin^2(fx+e)\right)-17B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)-144A\left(\cos^2(fx+e)\right)\right.}{}$
parts	$\frac{A\sec(fx+e)\left(\cos^4(fx+e)+12\left(\cos^2(fx+e)\right)\sin(fx+e)-96\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)+48\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(}\right.\right.}{}$

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,method =_RETURNVERBOSE)

[Out] -1/6*c^4/a^2/f*sec(f*x+e)*(2*B*cos(f*x+e)^2*sin(f*x+e)^3+3*A*cos(f*x+e)^2*sin(f*x+e)^2-17*B*sin(f*x+e)^2*cos(f*x+e)^2-144*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+288*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+36*sin(f*x+e)^3*A+336*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-672*B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-106*B*sin(f*x+e)^3+222*sin(f*x+e)^2*A+288*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-576*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-490*B*sin(f*x+e)^2-672*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+1344*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+138*A*sin(f*x+e)+288*A*ln(2/(1+cos(f*x+e)))-576*A*ln(-cot(f*x+e)+csc(f*x+e)+1)-336*B*sin(f*x+e)-672*B*ln(2/(1+cos(f*x+e)))+1344*B*ln(-cot(f*x+e)+csc(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 4*B)*c^4*cos(f*x + e)^4 - 4*(2*A - 3*B)*c^4*cos(f*x + e)^2
+ 8*(A - B)*c^4 + (B*c^4*cos(f*x + e)^4 + 4*(A - 2*B)*c^4*cos(f*x + e)^2 -
8*(A - B)*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x +
 e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{12\left(3\sqrt{2}A\sqrt{a}c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\sqrt{2}B\sqrt{a}c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(7A\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(c)*(12*(3*sqrt(2)*A*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) - 7*sqrt(2)*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*l
og(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))) + 3*sqrt(2)*(7*A*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 11*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*(2*A*sqrt(a)*c^4*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x +
 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(4*B*a^
(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^6 - 3*A*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^4 + 15*B*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 18*A*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 54*B*a^(13/2)*c^4*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^9*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{9/2}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 263

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-
1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(5/2)+3/4*(A
-3*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+6*
(A-3*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+3*(A-3*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a
*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{6c^4(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{3c^3(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{3c^2(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{c(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (6*(A - 3*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (3*(A - 3*B)*c^3*Cos[e + f*x]*Sqrt[c
- c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*(A - 3*B)*c^2*Cos[
e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (
(A - 3*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*a*f*(a + a*Sin[e +
f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*(a +
a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{2a} \\
&= \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3(A-3B)c)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}
\end{aligned}$$

$$= \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3(A-3B)c^2)\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(6(A-3B)c^3)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(6(A-3B)c^4\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(6(A-3B)c^4\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$
\begin{aligned}
&= \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} \\
&\quad - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 12.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.92

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{7/\cdots}}{\cdots}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A - B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4*(A - 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.55

method	result
default	$-\frac{c^3\sec(fx+e)\left(-B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+2\left(\sin^3(fx+e)\right)A-12A\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+24A\left(\cos^2(fx+e)\right)\ln(-\text{co}\cdots}{\cdots}$
parts	$\frac{A\sec(fx+e)\left(6\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-12\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)+\left(\cos^2(fx+e)\right)\sin(fx+e)-12\ln\left(\cdots\right.}{\cdots}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c^3/a^2/f*sec(f*x+e)*(-B*sin(f*x+e)^2*cos(f*x+e)^2+2*sin(f*x+e)^3*A-12
*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+24*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f
*x+e)+1)-10*B*sin(f*x+e)^3+36*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-72*B*cos(
f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+20*sin(f*x+e)^2*A+24*A*sin(f*x+e)*ln(
2/(1+cos(f*x+e)))-48*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-54*B*sin(f*x
+e)^2-72*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+144*B*sin(f*x+e)*ln(-cot(f*x+e)+
csc(f*x+e)+1)+10*A*sin(f*x+e)+24*A*ln(2/(1+cos(f*x+e)))-48*A*ln(-cot(f*x+e)
+csc(f*x+e)+1)-36*B*sin(f*x+e)-72*B*ln(2/(1+cos(f*x+e)))+144*B*ln(-cot(f*x+
e)+csc(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)
))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{6\sqrt{2}\left(A\sqrt{a}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\sqrt{a}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(\sqrt{2}Ba^{\frac{7}{2}}c^3\mathrm{sg}\right.}{}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
[Out] -1/2*sqrt(2)*sqrt(c)*(6*sqrt(2)*(A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - 3*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) -
2*(sqrt(2)*B*a^(7/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - sqrt(2)*A*a^(7/2)*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 5*sqrt(2)*B*a^(7/2)*c^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^2)/a^6 + (5*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - 9*sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(
3*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*sqrt(2)*B*s
qrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 211

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*(A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-
1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(5/2)+(A-5*B
)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+1/2*(A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{c^3(A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{c^2(A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a\sin(e+fx)+a}} + \frac{c(A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a\sin(e+fx)+a)^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A - 5*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - 5*B)*c^2*Cos[e + f*x]*Sqrt[c - c*
Sin[e + f*x]])/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((A - 5*B)*c*Cos[e + f*
x]*(c - c*Sin[e + f*x])^(3/2))/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B
)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
```

```
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(A-5B)\int\frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a}\\
&= \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}}\\
&\quad-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}+\frac{((A-5B)c)\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}\\
&= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}}+\frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}}\\
&\quad-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}+\frac{((A-5B)c^2)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}\\
&= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}}+\frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}}\\
&\quad-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}+\frac{((A-5B)c^3\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}}\\
&\quad+\frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}}\\
&\quad-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}\\
&\quad+\frac{((A-5B)c^3\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

$$= \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 11.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{5/2}}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + 
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A + 2
*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.74

method	result
default	$-\frac{c^2\sec(fx+e)\left(2A(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)-A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-B(\sin^3(fx+e))-10B(\cos^2(fx\right.}{}$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)-2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+4\ln(-\right.}{f(1+\sin(fx+e))}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -c^2/a^2/f*sec(f*x+e)*(2*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(
f*x+e)^2*ln(2/(1+cos(f*x+e)))-B*sin(f*x+e)^3-10*B*cos(f*x+e)^2*ln(-cot(f*x+
e)+csc(f*x+e)+1)+5*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*sin(f*x+e)^2*A-4*A
*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*A*sin(f*x+e)*ln(2/(1+cos(f*x+e))
)-8*B*sin(f*x+e)^2+20*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-10*B*sin(f*
```

```
x+e)*ln(2/(1+cos(f*x+e)))-4*A*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*A*ln(2/(1+cos(
f*x+e)))-5*B*sin(f*x+e)+20*B*ln(-cot(f*x+e)+csc(f*x+e)+1)-10*B*ln(2/(1+cos(
f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin
(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(189) = 378$.

Time = 0.47 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\frac{\left(\frac{8\sqrt{a}c^{\frac{5}{2}}\sin(fx+e)^2}{\left(a^3+\frac{4a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\mathrm{co}}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] ((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c^(5/2
)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)/(cos(f
*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*c^(5/2)*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/2) + 4*a^(
5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^(5/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.41

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{4\sqrt{2}Bc^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2\sqrt{2}(A\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(4*sqrt(2)*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sq
rt(2)*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*sqrt(a)*c^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-32*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 32)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*sqrt(a)*c
^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*sqrt(a)*c^2*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) - 4*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
2*B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(5/2),x)
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)-B*c^2
*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)-B*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2818, 2816, 2746, 31}

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{Bc^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}} - \frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -((B*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) - (B*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{B\int\frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(Bc)\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(Bc^2\cos(e+fx))\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(Bc^2\cos(e+fx))\,\text{Subst}\left(\int\frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.20

$$\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2*
(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(5/2))
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.44

method	result
default	$\frac{c\sec(fx+e)\left(-B\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+2B\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)+2B\left(\sin^2(fx+e)\right)+2B\sin(fx+\right.}{}$
parts	$\frac{A\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}\,c}{f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}\,a^2}+\frac{B\sec(fx+e)\left(2\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)-\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{1+\cos(f\right.\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] c/a^2/f*sec(f*x+e)*(-B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*B*cos(f*x+e)^2*l
n(-cot(f*x+e)+csc(f*x+e)+1)+2*B*sin(f*x+e)^2+2*B*sin(f*x+e)*ln(2/(1+cos(f*x
+e)))-4*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*sin(f*x+e)+B*sin(f*x+e)
+2*B*ln(2/(1+cos(f*x+e)))-4*B*ln(-cot(f*x+e)+csc(f*x+e)+1))*(-c*(sin(f*x+e)
-1))^(1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f*
x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.38

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{2}\sqrt{c}\left(\frac{4\sqrt{2}Bc\log\left(-2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} \cdots\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*sqrt(c)*(4*sqrt(2)*B*c*log(-2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
+ 2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))) - sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*
B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(A*sqrt(a)*c*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\sin(e+fx))\,(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-B*c
*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/2*((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*
Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx \\ &= -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{a(1+\sin(e+fx))}(A+B+2B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[
e + f*x]])/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\tan(fx+e)(A\sin(fx+e)+B\sin(fx+e)+2A)\sqrt{-c(\sin(fx+e)-1)}}{2a^2f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}}$	70
parts	$-\frac{A\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)-2\tan(fx+e)-\sec(fx+e))}{2f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}\,a^2}-\frac{B\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)-\sec(fx+e))}{2f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}\,a^2}$	128

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2/f*tan(f*x+e)*(A*sin(f*x+e)+B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(1
/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{(2\,B\sin\left(fx+e\right)+A+B)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{2\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*
cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}(A+B\sin\left(e+fx\right))}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(4\,B\sqrt{a}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{8\,a^{3}f\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/8*(4*B*sqrt(a)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) + A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/(a^3*f*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(A\,\sin\left(2\,e+2\,f\,x\right)+3\,B\,\sin\left(2\,e+2\,f\,x\right)-2\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-1\right)-3\,B\,\left(2\right.\right.}{a^{2}\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(-8\sin\left(e+f\,x\right)^{2}+4\,\sin\left(e+f\,x\right)+2\sin\left(2\,e+2\,f\,x\right)\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(A*sin(2*e + 2*f*x) + 3*B*sin(2*e + 2*f*x
) - 2*A*(2*sin(e/2 + (f*x)/2)^2 - 1) - 3*B*(2*sin(e/2 + (f*x)/2)^2 - 1) + B
*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(
4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)
^2 + 8))
```

3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 151

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*(
A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*(A+B)
*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3051, 2822, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
+ f*x]]) - ((A + B)*Cos[e + f*x])/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c
- c*Sin[e + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx}{2a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}dx}{4a^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int\sec(e+fx)\,dx}{4a^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\mathrm{s}\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(133) = 266.

Time = 2.94 (sec) , antiderivative size = 772, normalized size of antiderivative = 5.11

method	result	size
default	Expression too large to display	772
parts	Expression too large to display	816

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/f*(A*sin(f*x+e)-A+B+B*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f
*x+e)+1)+2*A*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*A*ln(csc(
f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)+2*B*sin(f*x+e)*cos(f*x+e)*ln(-co
t(f*x+e)+csc(f*x+e)+1)-2*B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e
)+2*A*cos(f*x+e)-B*sin(f*x+e)-B*cos(f*x+e)^2+A*cos(f*x+e)^2-2*A*cos(f*x+e)*
ln(csc(f*x+e)-cot(f*x+e)-1)+2*A*sin(f*x+e)*cos(f*x+e)^2+3*A*sin(f*x+e)*cos(
f*x+e)-B*cos(f*x+e)*sin(f*x+e)+A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc
(f*x+e)+1)-B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+A*cos(f*x+e)^3*ln(csc
(f*x+e)-cot(f*x+e)-1)+B*cos(f*x+e)^3*ln(csc(f*x+e)-cot(f*x+e)-1)-A*cos(f*x+
e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*cos(f*x+e)^3-B*sin(f*x+e)*cos(f*x+e)^2
*ln(csc(f*x+e)-cot(f*x+e)-1)-2*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+2*A
*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*B*cos(f*x+e)*ln(-cot(f*x+e)+csc(
f*x+e)+1)-A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+B*cos(f*x+e
)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1
)-B*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*cos(f*x+e)^2*ln(-cot(f*x+e)
+csc(f*x+e)+1))/(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)-cos(f*x+e)-2*sin(f*x+e)
-2)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{\left((A+B)\cos(fx+e)^3 - 2(A+B)\cos(fx+e)\sin(fx\cdots\right.}{}\right.$$

$$-\frac{\left((A+B)\cos(fx+e)^3 - 2(A+B)\cos(fx+e)\sin(fx+e) - 2(A+B)\cos(fx+e)\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\cdots}{}\right.}{4\left(a^3cf\cos(fx+e)^3 - 2a^3cf\cos(fx+e\cdots\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A
+ B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
- 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
e))/cos(f*x + e)^3) + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x +
 e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x + e)^3 -
 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(-a*c)*a
rctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*co
s(f*x + e)*sin(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f
*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{A + B\sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{5}{2}}\sqrt{-c(\sin(e + fx) - 1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{B\sin(fx + e) + A}{(a\sin(fx + e) + a)^{\frac{5}{2}}\sqrt{-c\sin(fx + e) + c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.40

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\frac{2\,(A+B)\log\left(-4096\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4096\right)}{a^{\frac{5}{2}}\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} - \frac{4\,(\cdots}{a^{\frac{5}{2}}\sqrt{c}\mathrm{sgn}\left(\cos(\cdots\right.}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/16*(2*(A + B)*log(-4096*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 4096)/(a^(5/2)
*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))) - 4*(A + B)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(5/2)*sqrt(c
)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
+ (2*(A*sqrt(a) + B*sqrt(a))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + A*sqrt(a) -
 B*sqrt(a))/(a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 208

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(3A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-1/8*(
3*A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/8*(3*
A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3051, 2822, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(3A+B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{(3A+B)\cos(e+fx)}{8af(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*
x])^(3/2)) - ((3*A + B)*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(3/2)) + ((3*A + B)*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin
[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A + B)*ArcTanh[Sin[e + f*x]]*C
os[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
```

```
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx}{4a} \\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx}{4a^2} \\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{8a^2c} \\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{((3A+B)\cos(e+fx))\int \sec(e+fx)\,dx}{8a^2c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad - \frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad + \frac{(3A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}
$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 +
 (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/
2))
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sec(fx+e)\left(3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)-3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(-3\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)+3\left(\cos^2(fx+e)\right)\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)\right.}{8fca^2(1+\sin(fx+e))\sqrt{}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/8/a^2/c/f*sec(f*x+e)*(3*A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+
e)-1)-3*A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+B*sin(f*x+e)
*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*sin(f*x+e)*cos(f*x+e)^2*ln(-cot
```

```
(f*x+e)+csc(f*x+e)+1)+2*sin(f*x+e)^3*A+3*A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f
*x+e)-1)-3*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*B*sin(f*x+e)^3+B*c
os(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f
*x+e)+1)-sin(f*x+e)^2*A-3*B*sin(f*x+e)^2-5*A*sin(f*x+e)+B*sin(f*x+e))/(1+si
n(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.01

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \left[\frac{\left((3\,A+B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+(3\,A+B)\cos\right.}{}\right.$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*s
qrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
- 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*
x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*(((3*A + B)*cos(f*x + e)^3*sin(f*x
 + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + (
(3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e
) + a^3*c^2*f*cos(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.33

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx=\frac{\frac{2\left(3A\sqrt{a}+B\sqrt{a}\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3c^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(3A\ldots\right.}{a^3c^{\frac{3}{2}}\operatorname{sgn}\left(\cos\ldots\right.}}{\ldots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/32*(2*(3*A*sqrt(a) + B*sqrt(a))*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1
)/(a^3*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) - 4*(3*A*sqrt(a) + B*sqrt(a))*log(abs(cos(-1/4*pi + 1/2*f*x +
1/2*e)))/(a^3*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + (2*(3*A*sqrt(a) + B*sqrt(a))*cos(-1/4*pi + 1/2*f*x +
1/2*e)^4 - (3*A*sqrt(a) + B*sqrt(a))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - A*s
qrt(a) + B*sqrt(a))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^3*c^(3/2)*cos
(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 245

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{3A\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{3A\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*A
*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*A*cos(f*x
+e)/a^2/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*cos(f*x+e)/a^
2/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*arctanh(sin(f*x+e
))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3051, 2822, 2820, 3855}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3A\cos(e+fx)\text{arctanh}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{3A\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} + \frac{3A\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\cos(e+fx)}{2af(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] -1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*
x])^(5/2)) - (A*Cos[e + f*x])/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[
e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (3*A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x
])/(8*a^2*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{A\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx}{a} \\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad-\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A)\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx}{2a^2} \\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad-\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad+\frac{(3A)\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx}{4a^2c}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad -\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad +\frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad +\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad +\frac{(3A)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{8a^2c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad -\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad +\frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad +\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad +\frac{(3A\cos(e+fx))\int \sec(e+fx)\,dx}{8a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad -\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\
&\quad +\frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\
&\quad +\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\
&\quad +\frac{3A\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sec^3(e+fx)\left(16B-9A\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right)-12\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(16*B - 9*A*Log[1 - Tan[(e + f*x)/2]] - 12*A*Cos[2*(e + f*x
)]*(Log[1 - Tan[(e + f*x)/2]] - Log[1 + Tan[(e + f*x)/2]]) - 3*A*Cos[4*(e +
 f*x)]*(Log[1 - Tan[(e + f*x)/2]] - Log[1 + Tan[(e + f*x)/2]]) + 9*A*Log[1
+ Tan[(e + f*x)/2]] + 22*A*Sin[e + f*x] + 6*A*Sin[3*(e + f*x)]))/(64*a^2*c^
2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56

method	result
default	$\frac{3A\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-3A\cos(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-2\cos(fx+e)B+3\tan(fx+e)A+2\tan(}{8a^2c^2f\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A(3\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-3\ln(\csc(fx+e)-\cot(fx+e)-1)\cos(fx+e)+3\tan(fx+e)+2(\sec^2(fx+e))\tan(fx+}{8f\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,a^2c^2}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/8/a^2/c^2/f/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)*(3*A*cos(f
*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-3*A*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-
1)-2*cos(f*x+e)*B+3*tan(f*x+e)*A+2*tan(f*x+e)*sec(f*x+e)^2*A+2*sec(f*x+e)^3
*B)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.25

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \left[\frac{3\sqrt{ac}A\cos(fx+e)^5\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2}{}\right.}{}\right.$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*
x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin
(f*x + e))/cos(f*x + e)^3) + 2*((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2
*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x
+ e)^5), -1/8*(3*sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (
(3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2*B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 36, antiderivative size = 174

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c(B(m-n)+A(1+m+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n),\frac{1}{2}(3+2m),\frac{1}{2}(1}{f(1+2m)(1+m+n)} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)}$$

```
[Out] 2^(1/2+n)*c*(B*(m-n)+A*(1+m+n))*cos(f*x+e)*hypergeom([1/2-n, 1/2+m],[3/2+m]
,1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x
+e))^(-1+n)/f/(1+2*m)/(1+m+n)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+
e))^n/f/(1+m+n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3052, 2824, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx = \frac{c2^{n+\frac{1}{2}}(A(m+n+1)+B(m-n))\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx}{f(2m+1)(m+n+1)} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*(B*(m - n) + A*(1 + m + n))*Cos[e + f*x]*Hypergeometric2F1[(
1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*
x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 +
 2*m)*(1 + m + n)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n)/(f*(1 + m + n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
```

```
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)} + \left(A+\frac{B(m-n)}{1+m+n}\right)\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)} + \left(\left(A+\frac{B(m-n)}{1+m+n}\right)\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c - c\sin(e+fx))^m\right)\int \cos^{2m}(e+fx)(c-c\sin(e+fx))^{-m+n}\,dx$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)} + \frac{\left(c^2\left(A+\frac{B(m-n)}{1+m+n}\right)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\sin(e+f}{f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)} + \frac{\left(2^{-\frac{1}{2}+n}c^2\left(A+\frac{B(m-n)}{1+m+n}\right)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)+m+n}\left(\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n}c\left(A+\frac{B(m-n)}{1+m+n}\right)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(}{f(1+2m)} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)}$$

Mathematica [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
n, x]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^n \, dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^n \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n, x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c - c\sin(e + fx))^n\,dx = \int (a(\sin(e + fx) + 1))^m\,(-c(\sin(e + fx) - 1))^n\,(A + B\sin(e + fx))\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*(A + B*sin(e
+ f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c - c\sin(e + fx))^n\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m(-c\sin(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c - c\sin(e + fx))^n\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m(-c\sin(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

Mupad [F(-1)]

Timed out.

$$
\begin{aligned}
&\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx \\
&= \int \left(A + B\,\sin\left(e + f\,x\right)\right)\,\left(a + a\,\sin\left(e + f\,x\right)\right)^m\,\left(c - c\,\sin\left(e + f\,x\right)\right)^n \, dx
\end{aligned}
$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

3.196 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{2^{\frac{1}{2}+m}a^4c^3(B(3-m)-A(4+m))\cos^7(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1}{2}-m,\frac{9}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+}{7f(4+m)}$$
$$-\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$

```
[Out] 1/7*2^(1/2+m)*a^4*c^3*(B*(3-m)-A*(4+m))*cos(f*x+e)^7*hypergeom([7/2, 1/2-m]
,[9/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
/(4+m)-a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{a^4c^32^{m+\frac{1}{2}}(B(3-m)-A(m+4))\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,\text{Hypergeometr}}{7f(m+4)}$$
$$-\frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f(m+4)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*a^4*c^3*(B*(3 - m) - A*(4 + m))*Cos[e + f*x]^7*Hypergeometric2
F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-4 + m))/(7*f*(4 + m)) - (a^3*B*c^3*Cos[e + f*x]^7*(a +
 a*Sin[e + f*x])^(-3 + m))/(f*(4 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^3c^3)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}(A+B\sin(e+fx))\,dx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)} + \left(a^3c^3\left(A-\frac{B(3-m)}{4+m}\right)\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}\,dx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)} + \frac{\left(a^5c^3\left(A-\frac{B(3-m)}{4+m}\right)\cos^7(e+fx)\right)\text{Subst}\left(\int (a-ax)^{5/2}(a+ax)^{-\frac{1}{2}+m}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))^{7/2}(a+a\sin(e+fx))^{7/2}}$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)} + \frac{\left(2^{-\frac{1}{2}+m}a^5c^3\left(A-\frac{B(3-m)}{4+m}\right)\cos^7(e+fx)(a+a\sin(e+fx))^{-4+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Subst}\left(\int (\ldots\right.}{f(a-a\sin(e+fx))^{7/2}}$$
$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\left(A-\frac{B(3-m)}{4+m}\right)\cos^7(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1}{2}-m,\frac{9}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\ldots}{7f} - \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.95 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.17

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = -\frac{(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^3(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{10(4A-3B)\,\text{Hypergeometric2F1}\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]

[Out] -1/16*((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^3*(Cos[e + f*x] + I*(1
 + Sin[e + f*x]))*((-10*(4*A - 3*B)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Co
s[e + f*x] - Sin[e + f*x]])/m + (2*(15*A - 13*B)*Hypergeometric2F1[1, 2 + m
```

```
, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*x]))
/(-1 + m) + (2*(15*A - 13*B)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x] - S
in[e + f*x]]*(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) + (4*(3*A - 4*B)*Hype
rgeometric2F1[1, -1 + m, -1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e +
 f*x)] - I*Sin[2*(e + f*x)]))/(2 + m) + (4*(3*A - 4*B)*Hypergeometric2F1[1,
 3 + m, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(
e + f*x)]))/(-2 + m) - ((2*I)*(A - 3*B)*Hypergeometric2F1[1, -2 + m, -2 - m
, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]))/(
3 + m) + ((2*I)*(A - 3*B)*Hypergeometric2F1[1, 4 + m, 4 - m, I*Cos[e + f*x]
 - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]))/(-3 + m) + (B*Hyp
ergeometric2F1[1, -3 + m, -3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[4*(e
+ f*x)] - I*Sin[4*(e + f*x)]))/(4 + m) + (B*Hypergeometric2F1[1, 5 + m, 5 -
 m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)]))
/(-4 + m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^3 \, dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3 \, dx = \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*(a*sin
(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= -c^3 \bigg(\int \left(-A(a\sin\left(e + fx\right) + a)^m\right) \, dx + \int 3A(a\sin\left(e + fx\right) + a)^m \sin\left(e + fx\right) dx$$
$$+ \int \left(-3A(a\sin\left(e + fx\right) + a)^m \sin^2\left(e + fx\right)\right) \, dx$$
$$+ \int A(a\sin\left(e + fx\right) + a)^m \sin^3\left(e + fx\right) dx$$
$$+ \int \left(-B(a\sin\left(e + fx\right) + a)^m \sin\left(e + fx\right)\right) \, dx$$
$$+ \int 3B(a\sin\left(e + fx\right) + a)^m \sin^2\left(e + fx\right) dx$$
$$+ \int \left(-3B(a\sin\left(e + fx\right) + a)^m \sin^3\left(e + fx\right)\right) \, dx$$
$$+ \int B(a\sin\left(e + fx\right) + a)^m \sin^4\left(e + fx\right) dx \bigg)$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(3*A*(a*sin(e + f*
x) + a)**m*sin(e + f*x), x) + Integral(-3*A*(a*sin(e + f*x) + a)**m*sin(e +
 f*x)**2, x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3, x) + Int
egral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(3*B*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)**2, x) + Integral(-3*B*(a*sin(e + f*x) + a)**m*s
in(e + f*x)**3, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4, x)
)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \int -(B\sin\left(fx + e\right) + A)(c\sin\left(fx + e\right) - c)^3 (a\sin\left(fx + e\right) + a)^m \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)^3 (a\sin(fx + e) + a)^m \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^3 \, dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

3.197 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{2^{\frac{1}{2}+m}a^3c^2(B(2-m)-A(3+m))\cos^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+}{5f(3+m)}$$
$$-\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)}$$

```
[Out] 1/5*2^(1/2+m)*a^3*c^2*(B*(2-m)-A*(3+m))*cos(f*x+e)^5*hypergeom([5/2, 1/2-m]
,[7/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
/(3+m)-a^2*B*c^2*cos(f*x+e)^5*(a+a*sin(f*x+e))^(-2+m)/f/(3+m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a^3c^22^{m+\frac{1}{2}}(B(2-m)-A(m+3))\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,\text{Hypergeometr}}{5f(m+3)}$$
$$-\frac{a^2Bc^2\cos^5(e+fx)(a\sin(e+fx)+a)^{m-2}}{f(m+3)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a^3*c^2*(B*(2 - m) - A*(3 + m))*Cos[e + f*x]^5*Hypergeometric2
F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-3 + m))/(5*f*(3 + m)) - (a^2*B*c^2*Cos[e + f*x]^5*(a +
 a*Sin[e + f*x])^(-2 + m))/(f*(3 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-2+m}(A+B\sin(e+fx))\,dx$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)} + \left(a^2c^2\left(A-\frac{B(2-m)}{3+m}\right)\right)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-2+m}\,dx$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)} + \frac{\left(a^4c^2\left(A-\frac{B(2-m)}{3+m}\right)\cos^5(e+fx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{-\frac{1}{2}+m}\,dx,x,\sin(e+fx)\right)}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)} + \frac{\left(2^{-\frac{1}{2}+m}a^4c^2\left(A-\frac{B(2-m)}{3+m}\right)\cos^5(e+fx)(a+a\sin(e+fx))^{-3+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Subst}\left(\int \left(\frac{1}{2}\right.\right.}{f(a-a\sin(e+fx))^{5/2}}$$

$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\left(A-\frac{B(2-m)}{3+m}\right)\cos^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+}{5f} - \frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.06

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{ic^2(a(1+\sin(e+fx)))^m(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{4i(3A-2B)\,\text{Hypergeometric2F1}(1,1+m,1-m,i\cos(e+fx)-}{m}\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] ((I/8)*c^2*(a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))*(
((-4*I)*(3*A - 2*B)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Cos[e + f*x] - Sin
[e + f*x]])/m - ((8*A - 7*B)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x] - S
in[e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]))/(1 + m) + ((8*A - 7*B)*Hyperg
```

```
eometric2F1[1, 2 + m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[e + f*x] +
 I*Sin[e + f*x]))/(-1 + m) + ((2*I)*(A - 2*B)*Hypergeometric2F1[1, 3 + m, 3
 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]
))/(-2 + m) + (2*(A - 2*B)*Hypergeometric2F1[1, -1 + m, -1 - m, I*Cos[e + f
*x] - Sin[e + f*x]]*(I*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]))/(2 + m) - (B*H
ypergeometric2F1[1, -2 + m, -2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(
e + f*x)] - I*Sin[3*(e + f*x)]))/(3 + m) + (B*Hypergeometric2F1[1, 4 + m, 4
 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]
))/(-3 + m)))/f
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^2 \, dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2 \, dx = \int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2 \, dx &= c^2\bigg(\int A(a\sin(e+fx)+a)^m \, dx + \int \left(-2A(a\sin(e+fx)+a)^m\sin(e+fx)\right) dx \\ &+ \int A(a\sin(e+fx)+a)^m\sin^2(e+fx) \, dx + \int B(a\sin(e+fx)+a)^m\sin(e+fx) \, dx \\ &+ \int \left(-2B(a\sin(e+fx)+a)^m\sin^2(e+fx)\right) dx \\ &+ \int B(a\sin(e+fx)+a)^m\sin^3(e+fx) \, dx\bigg)\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(A*(a*sin(e + f*x) + a)**m, x) + Integral(-2*A*(a*sin(e + f*x
) + a)**m*sin(e + f*x), x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x
)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-
2*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e + f*x
) + a)**m*sin(e + f*x)**3, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^m\,\left(c-c\,\sin(e+f\,x)\right)^2\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 139

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{2^{\frac{1}{2}+m}a^2c(B(1-m)-A(2+m))\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+}{3f(2+m)}$$
$$-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}$$

```
[Out] 1/3*2^(1/2+m)*a^2*c*(B*(1-m)-A*(2+m))*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],[
5/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(
2+m)-a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3046, 2939, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{a^2c2^{m+\frac{1}{2}}(B(1-m)-A(m+2))\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,\text{Hypergeomet}}{3f(m+2)}$$
$$-\frac{aBc\cos^3(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(m+2)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a^2*c*(B*(1 - m) - A*(2 + m))*Cos[e + f*x]^3*Hypergeometric2F1
[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a +
 a*Sin[e + f*x])^(-2 + m))/(3*f*(2 + m)) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin
[e + f*x])^(-1 + m))/(f*(2 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^{-1+m}(A+B\sin(e+fx))\,dx$$
$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)} + \left(ac\left(A-\frac{B(1-m)}{2+m}\right)\right)\int \cos^2(e+fx)(a+a\sin(e+fx))^{-1+m}\,dx$$
$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)} + \frac{\left(a^3c\left(A-\frac{B(1-m)}{2+m}\right)\cos^3(e+fx)\right)\text{Subst}\left(\int \sqrt{a-ax}(a+ax)^{-\frac{1}{2}+m}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}$$
$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)} + \frac{\left(2^{-\frac{1}{2}+m}a^3c\left(A-\frac{B(1-m)}{2+m}\right)\cos^3(e+fx)(a+a\sin(e+fx))^{-2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Subst}\left(\int (\right.}{f(a-a\sin(e+fx))^{3/2}}$$
$$= -\frac{2^{\frac{1}{2}+m}a^2c\left(A-\frac{B(1-m)}{2+m}\right)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+}{3f} - \frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.37

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$
$$-\frac{2^{-2-m}ce^{ifmx}\left(1-ie^{i(e+fx)}\right)^{-2m}\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^m\left(\frac{iBe^{-i(2e+f(2+m)x)}\,\text{Hypergeometric2F1}(-2-m,-2m}{2+m}\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] -((2^(-2 - m)*c*E^(I*f*m*x)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x
)))^m*((I*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, I*E^(I*(e + f*x))])/(E^
(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*(A - B)*Hypergeometric2F1[-1 - m, -2*
```

```
m, -m, I*E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*x))*(1 + m)) - (2*A*E^(I*(e
 - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*E^(I*(e + f*x))])
/(-1 + m) + (2*B*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2
- m, I*E^(I*(e + f*x))])/(-1 + m) + (I*B*E^((2*I)*e - I*f*(-2 + m)*x)*Hyper
geometric2F1[2 - m, -2*m, 3 - m, I*E^(I*(e + f*x))])/(-2 + m) + ((4*I)*A*Hy
pergeometric2F1[-2*m, -m, 1 - m, I*E^(I*(e + f*x))])/(E^(I*f*m*x)*m) - ((2*
I)*B*Hypergeometric2F1[-2*m, -m, 1 - m, I*E^(I*(e + f*x))])/(E^(I*f*m*x)*m)
)*(-1 + Sin[e + f*x]))/((1 - I*E^(I*(e + f*x)))^(2*m)*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^2))
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c-c\sin(fx+e))\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(f
*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -c\left(\int \left(-A(a\sin(e+fx)+a)^m\right)\,dx + \int A(a\sin(e+fx)+a)^m\sin(e+fx)\,dx + \int \left(-B(a\sin(e+fx)+a)^m\sin(e+fx)\right)\,dx + \int B(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) +
a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x),
x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)(a\sin(fx + e) + a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)(a\sin(fx + e) + a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int (A + B\sin(e + f\,x))\,(a + a\sin(e + f\,x))^m\,(c - c\sin(e + f\,x))\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+f}{f(1+m)}$$

```
[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2830, 2731, 2730}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\right.}{f(m+1)} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{(A+Am+Bm)\int(a+a\sin(e+fx))^m\,dx}{1+m}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{((A+Am+Bm)(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m)\int(1+\sin(e+fx))^m\,dx}{1+m}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\text{s}}{f(1+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\frac{2^m\left((A-B)B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2}+m,\frac{1}{2}\right)+2BB_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{3}{2}+m,\frac{1}{2}\right)\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)(1+\sin(}{f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
[Out] (2^m*((A - B)*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2] + 2*B*Beta[(1 + Sin[
e + f*x])/2, 3/2 + m, 1/2])*Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(a*(1 + Sin[e
 + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

Maple [F]

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx=\int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int \left(a(\sin\left(e+fx\right)+1)\right)^m\left(A+B\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^m\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 123

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{2^{\frac{1}{2}+m}(B+Am+Bm)\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx}{cfm} - \frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm}$$

```
[Out] 2^(1/2+m)*(A*m+B*m+B)*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec
(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/c/f/m-B*sec(f*x+e)*(a+a*s
in(f*x+e))^(1+m)/a/c/f/m
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2768, 72, 71}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-\right.}{cfm} - \frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{acfm}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin
[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x]
)^m)/(c*f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}(A+B\sin(e+fx))\,dx}{ac}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{(B+Am+Bm)\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}\,dx}{acm}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(a(B+Am+Bm)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{3/2}}\,dx\right.}{cfm}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(2^{-\frac{1}{2}+m}a(B+Am+Bm)\sec(e+fx)\sqrt{a-a\sin(e+fx)}(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-}\right.}{cfm}$$

$$= \frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin}{cfm} - \frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm}$$

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
), x]
```

Maple [F]

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{c - c\sin(fx + e)} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c - c\sin(e + fx)} dx = \int -\frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{c\sin(fx + e) - c} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c),
 x)
```

Sympy [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c - c\sin(e + fx)} dx = -\frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin(e+fx)-1} dx + \int \frac{B(a\sin(e+fx)+a)^m \sin(e+fx)}{\sin(e+fx)-1} dx}{c}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
[Out] -(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x) + Integral(B*(a
*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c - c\sin(e + fx)} dx = \int -\frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{c\sin(fx + e) - c} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{c\sin(fx+e)-c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 148

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{2^{\frac{1}{2}+m}(A(1-m)-B(2+m))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+}{3ac^2f(1-m)} + \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)}$$

```
[Out] 1/3*2^(1/2+m)*(A*(1-m)-B*(2+m))*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin(
f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/
(1-m)+B*sec(f*x+e)^3*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2768, 72, 71}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2c^2f(1-m)} + \frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2))\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,\text{Hypergeometri}}{3ac^2f(1-m)}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*(A*(1 - m) - B*(2 + m))*Hypergeometric2F1[-3/2, 1/2 - m, -1/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
```

```
in[e + f*x])^(1 + m))/(3*a*c^2*f*(1 - m)) + (B*Sec[e + f*x]^3*(a + a*Sin[e
+ f*x])^(2 + m))/(a^2*c^2*f*(1 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}(A+B\sin(e+fx))\,dx}{a^2c^2}$$
$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(A-\frac{B(2+m)}{1-m}\right)\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$
$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(\left(A-\frac{B(2+m)}{1-m}\right)\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{5/2}}\right.}{c^2f}$$
$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right)\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)\right.}{c^2f}$$
$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\text{si}}{3ac^2f} + \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)}$$

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2, x]
```

Maple [F]

$$\int \frac{(a + a\sin(fx+e))^m (A + B\sin(fx+e))}{(c - c\sin(fx+e))^2} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c - c\sin(e+fx))^2} dx = \int \frac{(B\sin(fx+e) + A)(a\sin(fx+e) + a)^m}{(c\sin(fx+e) - c)^2} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 +
 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c - c\sin(e+fx))^2} dx = \frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin^2(e+fx)-2\sin(e+fx)+1} dx + \int \frac{B(a\sin(e+fx)+a)^m \sin(e+fx)}{\sin^2(e+fx)-2\sin(e+fx)+1} dx}{c^2}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),
 x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x)**2 - 2*
sin(e + f*x) + 1), x))/c**2
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x
)
```

3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 148

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{2^{\frac{1}{2}+m}(A(2-m)-B(3+m))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^{2+m}}{5a^2c^3f(2-m)} + \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)}$$

```
[Out] 1/5*2^(1/2+m)*(A*(2-m)-B*(3+m))*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin(
f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/
f/(2-m)+B*sec(f*x+e)^5*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3046, 2939, 2768, 72, 71}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{a^3c^3f(2-m)} + \frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3))\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f(2-m)}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*(A*(2 - m) - B*(3 + m))*Hypergeometric2F1[-5/2, 1/2 - m, -3/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
```

```
in[e + f*x])^(2 + m))/(5*a^2*c^3*f*(2 - m)) + (B*Sec[e + f*x]^5*(a + a*Sin[
e + f*x])^(3 + m))/(a^3*c^3*f*(2 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}(A+B\sin(e+fx))\,dx}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(A-\frac{B(3+m)}{2-m}\right)\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(\left(A-\frac{B(3+m)}{2-m}\right)\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{7/2}}\,d\right.}{ac^3f}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right)\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}}\right.}{ac^3f}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin}{5a^2c^3f} + \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)}$$

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3, x]
```

Maple [F]

$$\int \frac{(a + a\sin(fx+e))^{m} (A + B\sin(fx+e))}{(c - c\sin(fx+e))^{3}} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c - c\sin(e+fx))^3} dx = \int -\frac{(B\sin(fx+e) + A)(a\sin(fx+e) + a)^m}{(c\sin(fx+e) - c)^3} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2
 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c - c\sin(e+fx))^3} dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c - c\sin(e+fx))^3} dx = \int -\frac{(B\sin(fx+e) + A)(a\sin(fx+e) + a)^m}{(c\sin(fx+e) - c)^3} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

Giac [F]

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \int -\frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{(c\sin(fx + e) - c)^3}\,dx$$

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m}{(c - c\,\sin(e + f\,x))^3}\,dx$$

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3, x)

3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3052, 2824, 2746, 70}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e+fx))\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 21.88 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)\right)\right.}{2f(1+2m)(3+2}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hypergeom
etric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x]))))/(
2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m(A+B\sin(fx+e))}{\sqrt{c-c\sin(fx+e)}}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] -2*B*cos(f*x+e)*(c+c*sin(f*x+e))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e))
^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3052, 2824, 2746, 70}

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}} - \frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*
x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - (-A-B)\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)\cos(e+fx))\int \sec(e+fx)(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)c\cos(e+fx))\text{Subst}\left(\int \frac{(c+x)^{-\frac{1}{2}+m}}{c-x}\,dx, x, c\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx}{f(1+2m)\sqrt{a-a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 21.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)(c(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)}{2f(1+2m)(3+2m}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hypergeom
etric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x]))))/(
2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))(c+c\sin(fx+e))^m}{\sqrt{a-a\sin(fx+e)}}dx$$

```
[In] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(c\sin(fx+e)+c)^m}{\sqrt{-a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)^m/(a*sin(f*x + e) - a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(c(\sin(e+fx)+1))^m(A+B\sin(e+fx))}{\sqrt{-a(\sin(e+fx)-1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f*
x) - 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(c\sin(fx+e)+c)^m}{\sqrt{-a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
+ a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+c\,\sin(e+f\,x))^m}{\sqrt{a-a\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2
), x)
```

3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 275

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(5+2m)(7+2m)\left(3+8m+4m^2\right)\sqrt{c-c\sin(e+fx)}} - \frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(7+2m)\left(15+16m+4m^2\right)} - \frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)(7+2m)} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)}$$

```
[Out] -2*c*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
3/2)/f/(4*m^2+24*m+35)-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
5/2)/f/(7+2*m)-64*c^3*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f
/(5+2*m)/(7+2*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)-16*c^2*(B*(5-2*m)-A*(
7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(7+2*m)/(4*m
^2+16*m+15)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3052, 2819, 2817}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{64c^3(B(5-2m)-A(2m+7))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)(2m+7)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}} - \frac{16c^2(B(5-2m)-A(2m+7))\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+7)\left(4m^2+16m+15\right)} - \frac{2c(B(5-2m)-A(2m+7))\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)(2m+7)} - \frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-64*c^3*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(
f*(5 + 2*m)*(7 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (16*c^2
*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c
*Sin[e + f*x]])/(f*(7 + 2*m)*(15 + 16*m + 4*m^2)) - (2*c*(B*(5 - 2*m) - A*(
7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(
f*(5 + 2*m)*(7 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Si
n[e + f*x])^(5/2))/(f*(7 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)} \\
&\quad + \frac{\left(Bc\left(-\frac{5}{2}+m\right)+Ac\left(\frac{7}{2}+m\right)\right)\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx}{c\left(\frac{7}{2}+m\right)} \\
&= -\frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)(7+2m)} \\
&\quad - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)} \\
&\quad - \frac{(8c(B(5-2m)-A(7+2m)))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx}{(5+2m)(7+2m)} \\
&= -\frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)(5+2m)(7+2m)} \\
&\quad - \frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)(7+2m)} \\
&\quad - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)} \\
&\quad - \frac{(32c^2(B(5-2m)-A(7+2m)))\int(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx}{(3+2m)(5+2m)(7+2m)} \\
&= -\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)(3+2m)(5+2m)(7+2m)\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)(5+2m)(7+2m)} \\
&\quad - \frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)(7+2m)} \\
&\quad - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.98 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(-1246A+1040B-1140}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/2*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sq
rt[c - c*Sin[e + f*x]]*(-1246*A + 1040*B - 1140*A*m + 664*B*m - 392*A*m^2 +
 256*B*m^2 - 48*A*m^3 + 32*B*m^3 + 2*(3 + 8*m + 4*m^2)*(-4*B*(5 + m) + A*(7
 + 2*m))*Cos[2*(e + f*x)] + (1 + 2*m)*(8*A*(7 + 2*m)^2 - B*(505 + 208*m + 2
8*m^2))*Sin[e + f*x] + 15*B*Sin[3*(e + f*x)] + 46*B*m*Sin[3*(e + f*x)] + 36
*B*m^2*Sin[3*(e + f*x)] + 8*B*m^3*Sin[3*(e + f*x)]))/(f*(1 + 2*m)*(3 + 2*m)
*(5 + 2*m)*(7 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F]

$$\int (a+a\sin(fx+e))^m\,(A+B\sin(fx+e))\,(c-c\sin(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(262) = 524.

Time = 0.31 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.04

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{2\left((8Bc^2m^3+36Bc^2m^2+46Bc^2m+15Bc^2)\cos(fx+e)^4+64(A+B)c^2m-(8(A-2B)c\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 64
*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A -
 86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 + (8
```

```
*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A - 85
*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 + 2*(79
*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^2*m -
(8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3 + 32*(7
*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(31*A - 63
*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*m^3 + 60*
(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(f*x + e))*
sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(16*f*m^4 +
128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f
*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 10
5*f)*sin(f*x + e) + 105*f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(262) = 524$.

Time = 0.34 (sec) , antiderivative size = 725, normalized size of antiderivative = 2.64

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
[Out] -2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)*a^m*c^(
5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e)^3/(co
```

```
s(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^m*c^(5/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 128*m^3
 + 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m + 105)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2)))/f
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e)
+ a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.72

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,c^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}-\frac{c^2\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(2100\,A-1575\,B+1272\,A\right.}{4\,f\,\left(16\,m^4+128\,m^3+34\right.}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((B*c^2*(a + a*sin(e + f*x))^m*(m*46i + m^2*36
i + m^3*8i + 15i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*
exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(2100*A - 1575*B + 1272*A*m - 110
*B*m + 304*A*m^2 + 32*A*m^3 - 68*B*m^2 - 8*B*m^3))/(4*f*(352*m + 344*m^2 +
128*m^3 + 16*m^4 + 105)) + (c^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^m*(
A*2100i - B*1575i + A*m*1272i - B*m*110i + A*m^2*304i + A*m^3*32i - B*m^2*6
8i - B*m^3*8i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*exp
(e*5i + f*x*5i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(350*A - 385*B + 184*A*m -
```

```
 104*B*m + 24*A*m^2 - 12*B*m^2))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
 105)) + (c^2*exp(e*2i + f*x*2i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*350i -
 B*385i + A*m*184i - B*m*104i + A*m^2*24i - B*m^2*12i))/(4*f*(352*m + 344*m
^2 + 128*m^3 + 16*m^4 + 105)) - (B*c^2*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^m*(46*m + 36*m^2 + 8*m^3 + 15))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^
4 + 105)) + (c^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3
)*(14*A - 35*B + 4*A*m - 6*B*m))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
 105)) - (c^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3)*(
A*14i - B*35i + A*m*4i - B*m*6i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4
+ 105))))/(exp(e*4i + f*x*4i) - (exp(e*3i + f*x*3i)*(m*352i + m^2*344i + m^
3*128i + m^4*16i + 105i))/(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105))
```

3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 166

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{4(A-B)c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} - \frac{2(A-3B)c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}} - \frac{2Bc^2\cos(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2f(5+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 4*(A-B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)-
2*(A-3*B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e)
)^(1/2)-2*B*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5+2*m)/(c-c*sin(f*
x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used

$= \{3052, 2819, 2817\}$

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{8c^2(B(3-2m)-A(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}} - \frac{2c(B(3-2m)-A(2m+5))\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)(2m+5)} - \frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-8*c^2*(B*(3 - 2*m) - A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f
*(5 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (2*c*(B*(3 - 2*m)
- A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]]
)/(f*(3 + 2*m)*(5 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)} \\
&\quad + \frac{\left(Bc\left(-\frac{3}{2}+m\right)+Ac\left(\frac{5}{2}+m\right)\right)\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx}{c\left(\frac{5}{2}+m\right)} \\
&= -\frac{2c(B(3-2m)-A(5+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)(5+2m)} \\
&\quad - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)} \\
&\quad - \frac{(4c(B(3-2m)-A(5+2m)))\int(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx}{(3+2m)(5+2m)} \\
&= -\frac{8c^2(B(3-2m)-A(5+2m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)(3+2m)(5+2m)\sqrt{c-c\sin(e+fx)}} \\
&\quad - \frac{2c(B(3-2m)-A(5+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)(5+2m)} \\
&\quad - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05

$$
\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(50A-39B+}{f(1+2m)(3+2m}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3 +
 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)*Si
n[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]))
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^{\frac{3}{2}} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.89

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{2\left((4\,Bcm^2+8\,Bcm+3\,Bc)\cos(fx+e)^3+8\,(A+B)cm+(4\,Acm^2+12\,(A-B)cm+(5\,A$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
[Out] 2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m^
2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4*(
A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A +
B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c - (
4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2
+ 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 3
6*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(160) = 320.

Time = 0.34 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.00

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{2\left(\frac{\left(a^m c^{\frac{3}{2}}(2m+5)-\frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)}{\cos(fx+e)+1}-\frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^m c^{\frac{3}{2}}(2m+5)\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Ae^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+}{(\cos(fx+e}\right.\right.}}{(4m^2+8m+3)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] -2*((a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x +
e) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c
^(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*A*e^(2*m*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1))/((4*m^2 + 8*m + 3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2
*(a^m*c^(3/2)*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2*(2*m^2 +
9*m)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*c^(3/2)*(2*m + 9
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 3
6*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
+ a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.89

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{c-c\sin\left(e+f\,x\right)}\,\left(\frac{c\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\left(a+a\,\sin(e+f\,x)\right)^m\,\left(45\,A-30\,B+28\,A\,m+4\,B\,m+4\,A\,m^2\right)}{f\,\left(m^3\,8\mathrm{i}+m^2\,36\mathrm{i}+m\,46\mathrm{i}+15\mathrm{i}\right)}+\frac{c\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(a+a\,\sin(\cdots}{\cdots}\right)}{\cdots}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(
45*A - 30*B + 28*A*m + 4*B*m + 4*A*m^2))/(f*(m*46i + m^2*36i + m^3*8i + 15i
)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(A*45i - B*30i + A*m*28i
+ B*m*4i + A*m^2*4i))/(f*(m*46i + m^2*36i + m^3*8i + 15i)) + (B*c*(a + a*si
n(e + f*x))^m*(m*8i + m^2*4i + 3i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))
+ (B*c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3))/(2*f*(m
*46i + m^2*36i + m^3*8i + 15i)) + (c*exp(e*1i + f*x*1i)*(2*m + 1)*(a + a*si
n(e + f*x))^m*(10*A - 15*B + 4*A*m - 2*B*m))/(2*f*(m*46i + m^2*36i + m^3*8i
 + 15i)) + (c*exp(e*4i + f*x*4i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*10i -
B*15i + A*m*4i - B*m*2i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))))/(exp(e*3
i + f*x*3i) + (exp(e*2i + f*x*2i)*(46*m + 36*m^2 + 8*m^3 + 15))/(m*46i + m^
2*36i + m^3*8i + 15i))
```

3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

Optimal result

Integrand size = 38, antiderivative size = 104

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+2*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3050, 2817}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 +
 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
```

```
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{a} \\ &\quad -(-A+B)\int (a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx \\ &= \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(-2B+A(3+2m)+B(}{f(1+2m)(3+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*
m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F]

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(A+B\sin \left(fx+e\right)\right) \sqrt{c-c\sin \left(fx+e\right)}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{2\left((2\,Bm+B)\cos\left(fx+e\right)^2-2\,(A+B)m-(2\,Am+3\,A-2\,B)\cos\left(fx+e\right)-(2\,(A+B)m+(2\,Bm\right.}{4\,fm^2+8\,fm+(4\,fm^2+8\,fm+3\,f)\cos\left(fx+e\right.}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x
+ e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) - 3*
A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m +
(4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x + e)
 + 3*f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int \left(a(\sin\left(e+fx\right)+1)\right)^{m}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}(A+B\sin\left(e+fx\right))\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e
 + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(100) = 200$.

Time = 0.34 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.11

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx =$$

$$-\frac{2\left(\frac{2\left(\frac{2a^m\sqrt{c}m\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^m\sqrt{c}m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-a^m\sqrt{c}-\frac{a^m\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(4m^2+8m+\frac{(4m^2+8m+3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+3\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{(\cdots}{\cdots}\right.}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -2*(2*(2*a^m*sqrt(c)*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c)*m*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2 + 8
*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e
) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)))/f
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{-c\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(6\,A\,\cos(e+f\,x)-4\,B\,\cos(e+f\,x)+B\,\sin(2\,e+2\,f}{f\,(\sin(e+f\,x)-1)\,(4\,m^2+8\,m+3)}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(6*A*cos(e + f*x)
- 4*B*cos(e + f*x) + B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*
e + 2*f*x)))/(f*(sin(e + f*x) - 1)*(8*m + 4*m^2 + 3))
```

3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3052, 2824, 2746, 70}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int\sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e+fx))\text{Subst}\left(\int\frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx,x,a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int\frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)\right)\right.}{2f(1+2m)(3+2r}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hypergeom
etric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x]))))/(
2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F]

$$\int\frac{(a+a\sin(fx+e))^m(A+B\sin(fx+e))}{\sqrt{c-c\sin(fx+e)}}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{\sqrt{-c\,(\sin(e+fx)-1)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 134

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(A(1-2m)-B(3+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin}{4cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A*(1-
2*m)-B*(3+2*m))*cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))
*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3051, 2824, 2746, 70}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m}{4cf(2m+1)\sqrt{}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3/
2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 +
```

```
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c*f*(1 + 2*m)*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(-\frac{1}{2}+m\right)\right)\int\frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(-\frac{1}{2}+m\right)\right)\cos(e+fx)\right)\int\sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(a\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(-\frac{1}{2}+m\right)\right)\cos(e+fx)\right)\text{Subst}\left(\int\frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx,x,a\sin(e+fx)\right)}{2c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(A(1-2m)-B(3+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)\right)}{4cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 25.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int\frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(B(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(\right.}{4cf(1+2m)(-1+\text{s}}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] -1/4*(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(B*(3 + 2*m)*Hypergeometric2F1[
1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 2*(B + 2*B
*m + Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(A - A*Si
n[e + f*x]))))/(c*f*(1 + 2*m)*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F]

$$\int\frac{(a+a\sin(fx+e))^m(A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{3}{2}}}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) -
 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,1,1,0,0,0,0,0]%%%}+%%%{1,[0,0,1,1,1,0,0,0,0]%%%} / %%%
{16,[0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2
), x)
```

3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 134

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(A(3-2m)-B(5+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{16c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(5/2)+1/16*(A*(3
-2*m)-B*(5+2*m))*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e)
)*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3051, 2824, 2746, 70}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m \ldots}{16c^2f(2m+1)\ldots} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5/
2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2 +
```

```
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(16*c^2*f*(1 + 2*
m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(-\frac{3}{2}+m\right)\right)\int\frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(-\frac{3}{2}+m\right)\right)\cos(e+fx)\right)\int\sec^3(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{4ac^3\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(a^2\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(-\frac{3}{2}+m\right)\right)\cos(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^2}\,dx,x,a\sin(e+fx)\right)}{4c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(A(3-2m)-B(5+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx\ldots\right.}{16c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 33.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19

$$\int\frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\cos(e+fx)\left(B(5+2m)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{si}\ldots\right.\right.}{16c^2(f+\ldots}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] -1/16*(Cos[e + f*x]*(B*(5 + 2*m)*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1
+ Sin[e + f*x])/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(B + 2*B*m +
 A*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e
 + f*x])^2))*(a*(1 + Sin[e + f*x]))^m)/(c^2*(f + 2*f*m)*(-1 + Sin[e + f*x])
^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F]

$$\int\frac{(a+a\sin(fx+e))^m(A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{5}{2}}}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x +
 e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad A
rgument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^m}{\left(c-c\,\sin(e+f\,x)\right)^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2
), x)
```

3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 267

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)}$$
$$+ \frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{cf(5+2m)(7+2m)}$$
$$+ \frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{c^2f(7+2m)\left(15+16m+4m^2\right)}$$
$$+ \frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^3f(5+2m)(7+2m)\left(3+8m+4m^2\right)}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m)/f/(7+2*m)+(3*A-
2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/c/f/(4*m^2
+24*m+35)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(-2-m)/c^2/f/(8*m^3+60*m^2+142*m+105)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin
(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^3/f/(16*m^4+128*m^3+344*m^2+352*m+105)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3051, 2822, 2821}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^3f(2m+5)(2m+7)\left(4m^2+8m+3\right)}$$
$$+\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{c^2f(2m+7)\left(4m^2+16m+15\right)}$$
$$+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$
$$+\frac{(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{cf(2m+5)(2m+7)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m))
/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-3 - m))/(c*f*(5 + 2*m)*(7 + 2*m)) + (2*(3*A - 2*B*(2
+ m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(c
^2*f*(7 + 2*m)*(15 + 16*m + 4*m^2)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(
a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^3*f*(5 + 2*m)*(7 +
2*m)*(3 + 8*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)} \\
&\quad+\frac{(3A-2B(2+m))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx}{c(7+2m)} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)} \\
&\quad+\frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{cf(5+2m)(7+2m)} \\
&\quad+\frac{(2(3A-2B(2+m)))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx}{c^2(5+2m)(7+2m)} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)} \\
&\quad+\frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{cf(5+2m)(7+2m)} \\
&\quad+\frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{c^2f(3+2m)(5+2m)(7+2m)} \\
&\quad+\frac{(2(3A-2B(2+m)))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx}{c^3(3+2m)(5+2m)(7+2m)} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)} \\
&\quad+\frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{cf(5+2m)(7+2m)} \\
&\quad+\frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{c^2f(3+2m)(5+2m)(7+2m)} \\
&\quad+\frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^3f(1+2m)(3+2m)(5+2m)(7+2m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}\,(B(13+16m+4m^2)-2A(18+41m+24m^2}{ac^4f(1+2m}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-4 - m),x]
```

```
[Out] (Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(B*(13 + 16*m + 4*m^2) - 2*A*(
18 + 41*m + 24*m^2 + 4*m^3) + (13 + 16*m + 4*m^2)*(3*A - 2*B*(2 + m))*Sin[e
 + f*x] + 4*(2 + m)*(-3*A + 2*B*(2 + m))*Sin[e + f*x]^2 + (6*A - 4*B*(2 + m
))*Sin[e + f*x]^3))/(a*c^4*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(-1 +
Sin[e + f*x])^3*(c - c*Sin[e + f*x])^m)
```

Maple [F]

$$\int (a+a\sin(fx+e))^m\,(A+B\sin(fx+e))\,(c-c\sin(fx+e))^{-4-m}\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\frac{(4\,(2\,Bm^2-(3\,A-8\,B)m-6\,A+8\,B)\cos{(fx+e)}^3+(8\,Am^3+12\,(4\,A-B)m^2+2\,(47\,A-24\,B)m+}{}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="fricas")
```

```
[Out] (4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4*
A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m - 3*
A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*m -
 45*A + 60*B)*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*
x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105*f)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-4-m}\, dx$$
$$= \int (a(\sin(e + fx) + 1))^m \left(-c(\sin(e + fx) - 1)\right)^{-m-4} (A + B\sin(e + fx))\, dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-m),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 4)*(A + B
*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-4-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-4}\, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 4), x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-4-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-4}\, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 4), x)
```

Mupad [B] (verification not implemented)

Time = 20.86 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$= -\frac{\sin(4\,e+4\,f\,x)\ (a+a\ \sin(e+f\,x))^m\ (4\,B-3\,A+2\,B\,m)\ 1\mathrm{i}}{4\,f\ (c-c\ \sin(e+f\,x))^{m+4}\ (m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$
$$+\frac{\cos(e+f\,x)\ (a+a\ \sin(e+f\,x))^m\ (A\,168\mathrm{i}-B\,84\mathrm{i}+A\,m\,340\mathrm{i}-B\,m\,96\mathrm{i}+A\,m^2\,192\mathrm{i}+A\,m^3\,32\mathrm{i}-B}{4\,f\ (c-c\ \sin(e+f\,x))^{m+4}\ (m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$
$$+\frac{\sin(2\,e+2\,f\,x)\ (a+a\ \sin(e+f\,x))^m\ (2\,m^2+8\,m+7)\ (4\,B-3\,A+2\,B\,m)\ 1\mathrm{i}}{f\ (c-c\ \sin(e+f\,x))^{m+4}\ (m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$
$$+\frac{\cos(3\,e+3\,f\,x)\ (m+2)\ (a+a\ \sin(e+f\,x))^m\ (-A\,3\mathrm{i}+B\,4\mathrm{i}+B\,m\,2\mathrm{i})}{f\ (c-c\ \sin(e+f\,x))^{m+4}\ (m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 4),x)
```

```
[Out] (cos(e + f*x)*(a + a*sin(e + f*x))^m*(A*168i - B*84i + A*m*340i - B*m*96i +
 A*m^2*192i + A*m^3*32i - B*m^2*24i))/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*
352i + m^2*344i + m^3*128i + m^4*16i + 105i)) - (sin(4*e + 4*f*x)*(a + a*si
n(e + f*x))^m*(4*B - 3*A + 2*B*m)*1i)/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*
352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (sin(2*e + 2*f*x)*(a + a*si
n(e + f*x))^m*(8*m + 2*m^2 + 7)*(4*B - 3*A + 2*B*m)*1i)/(f*(c - c*sin(e + f
*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (cos(3*e +
3*f*x)*(m + 2)*(a + a*sin(e + f*x))^m*(B*4i - A*3i + B*m*2i))/(f*(c - c*sin
(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i))
```

3.212 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 191

$$\begin{aligned}\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx\\ &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)}\\ &\quad+\frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{cf(3+2m)(5+2m)}\\ &\quad+\frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^2f(5+2m)\left(3+8m+4m^2\right)}\end{aligned}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+(2*A-
B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(4*m^2
+16*m+15)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-
1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used

$= \{3051, 2822, 2821\}$

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-3-m}\,dx$$
$$= \frac{(2A - B(2m + 3))\cos(e + fx)(a\sin(e + fx) + a)^m (c - c\sin(e + fx))^{-m-1}}{c^2 f(2m + 5)\left(4m^2 + 8m + 3\right)}$$
$$+ \frac{(A + B)\cos(e + fx)(a\sin(e + fx) + a)^m (c - c\sin(e + fx))^{-m-3}}{f(2m + 5)}$$
$$+ \frac{(2A - B(2m + 3))\cos(e + fx)(a\sin(e + fx) + a)^m (c - c\sin(e + fx))^{-m-2}}{cf(2m + 3)(2m + 5)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))
/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-2 - m))/(c*f*(3 + 2*m)*(5 + 2*m)) + ((2*A - B*(3 + 2*
m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2
*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx}{c(5+2m)} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{cf(3+2m)(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx}{c^2(3+2m)(5+2m)} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{cf(3+2m)(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^2f(1+2m)(3+2m)(5+2m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$
\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx
$$
$$
= \frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}\left(-B(3+2m)+A(7+12m+4m^2)+(3+2m)(\cdots\right.}{ac^3f(1+2m)(3+2m)(5+2m)(-1+\sin(e+\cdots}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-3 - m),x]
```

```
[Out] (Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(-(B*(3 + 2*m)) + A*(7 + 12*m
+ 4*m^2) + (3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x] + (2*A - B*(3 + 2*m)
)*Sin[e + f*x]^2))/(a*c^3*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(-1 + Sin[e + f*x
])^2*(c - c*Sin[e + f*x])^m)
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{-3-m} dx$$

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-3-m} dx$$
$$= \frac{((2Bm - 2A + 3B)\cos(fx + e)^3 + (4Bm^2 - 4(A - 3B)m - 6A + 9B)\cos(fx + e)\sin(fx + e) + (4}{8fm^3 + 36fm^2 + 4}$$

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, algorithm="fricas")

[Out] ((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B)*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-3-m} dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^{-m-3} (A + B\sin(e + fx)) dx$$

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-m),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 3)*(A + B*sin(e + f*x)), x)

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 3), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 3), x)
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.25

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx =$$
$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,(30\,A\,\cos(e+f\,x)-15\,B\,\cos(e+f\,x)-2\,A\,\cos(3\,e+3\,f\,x)+3\,B\,\cos(3\,e+3$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 3),x)
[Out] -((a*(sin(e + f*x) + 1))^m*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*cos
(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2*e
 + 2*f*x) + 8*B*m^2*sin(2*e + 2*f*x) + 48*A*m*cos(e + f*x) - 10*B*m*cos(e +
 f*x) + 16*A*m^2*cos(e + f*x) + 2*B*m*cos(3*e + 3*f*x) - 8*A*m*sin(2*e + 2*
f*x) + 24*B*m*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*m + 3
6*m^2 + 8*m^3 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x
) - 10))
```

3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 114

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)} + \frac{(A-2B(1+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{cf(1+2m)(3+2m)}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+(A-2*
B*(1+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(4*m^2+8
*m+3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3051, 2821}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)} + \frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf(2m+1)(2m+3)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))
/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c
- c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)} \\
&\quad + \frac{(A-2B(1+m))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx}{c(3+2m)} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)} \\
&\quad + \frac{(A-2B(1+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{cf(1+2m)(3+2m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}(B-2A(1+m)+(A-2B(1+m))\sin(e+fx}{ac^2f(1+2m)(3+2m)(-1+\sin(e+fx))}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-2 - m),x]
```

```
[Out] (Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(B - 2*A*(1 + m) + (A - 2*B*(1
 + m))*Sin[e + f*x]))/(a*c^2*f*(1 + 2*m)*(3 + 2*m)*(-1 + Sin[e + f*x])*(c -
 c*Sin[e + f*x])^m)
```

Maple [F]

$$\int (a+a\sin(fx+e))^{m}(A+B\sin(fx+e))(c-c\sin(fx+e))^{-2-m}\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{((2\,Bm-A+2\,B)\cos(fx+e)\sin(fx+e)+(2\,Am+2\,A-B)\cos(fx+e))(a\sin(fx+e)+a)^m(-c\sin}{4\,fm^2+8\,fm+3\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x +
e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m
+ 3*f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \int (a(\sin(e+fx)+1))^{m}(-c(\sin(e+fx)-1))^{-m-2}(A+B\sin(e+fx))\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 2)*(A + B
*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m(-c\sin{(fx+e)}+c)^{-m-2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m(-c\sin{(fx+e)}+c)^{-m-2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx =$$
$$-\frac{(a\,(\sin{(e+f\,x)}+1))^m\,(4\,A\,\cos{(e+f\,x)}-2\,B\,\cos{(e+f\,x)}-A\,\sin{(2\,e+2\,f\,x)}+2\,B\,\sin{(2\,e+2\,f\,x}}{c^2\,f\,(-c\,(\sin{(e+f\,x)}-1))^m\,(4\,m^2+8\,m+3)\,(4\,\sin{(e+f\,x)}+\cos{(}}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*e
 + 2*f*x) + 2*B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2*f
*x)))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m + 4*m^2 + 3)*(4*sin(e + f*x) +
cos(2*e + 2*f*x) - 3))
```

3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)} - \frac{2^{\frac{1}{2}-m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx))\right)(1-\text{s}}{f(1+2m)}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)-2^(1/
2-m)*B*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1-s
in(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3051, 2824, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} - \frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,\text{Hypergeometric2}}{f(2m+1)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))
/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2,
 (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 +
m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
```

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)} - \frac{B\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)} - \frac{(B\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m)\int\cos^{2m}(e+fx)(c-c\sin(e+fx)}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)} - \frac{\left(Bc\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\sin(e+fx))^{\frac{1}{2}(-1-2m)}\right.}{f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)} - \frac{\left(2^{-\frac{1}{2}-m}Bc\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)}\left(\frac{c-c\sin(e+fx)}{c}\right)^{\frac{1}{2}+m}\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)} - \frac{2^{\frac{1}{2}-m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx))\right)}{f(1+2m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.90

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$

$$= \frac{2^{-1-m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-1-m)-2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-1-m}\left(\frac{1}{}\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] (2^(-1 - m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2*(-1 - m) - 2*m)*(a*(1
 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(-1 - m)*((1 - Tan[(e + f*x)/2])/S
qrt[(1 + Cos[e + f*x])^(-1)])^(2*m)*(I*B*(1 + 2*m)*Hypergeometric2F1[1, -2*
m, 1 - 2*m, ((-I)*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Ta
n[(e + f*x)/2]) - I*B*(1 + 2*m)*Hypergeometric2F1[1, -2*m, 1 - 2*m, (I*(-1
+ Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) - 2*(A
 + B)*m*(1 + Tan[(e + f*x)/2])))/(f*m*(1 + 2*m)*((1 - Tan[(e + f*x)/2])/Sqr
t[Sec[(e + f*x)/2]^2])^(2*m)*(-1 + Tan[(e + f*x)/2]))
```

Maple [F]

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-1-m}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{m}(-c\sin\left(fx+e\right)+c)^{-m-1}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{-m-1}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 1)*(A + B
*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m(-c\sin(fx + e) + c)^{-m-1}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m(-c\sin(fx + e) + c)^{-m-1}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m}{(c - c\,\sin(e + f\,x))^{m+1}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 1), x)
```

3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 158

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$= \frac{2^{\frac{1}{2}-m}c(A+2Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx))\right)}{f(1+2m)}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f}$$

```
[Out] 2^(1/2-m)*c*(2*B*m+A)*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*s
in(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m
)/f/(1+2*m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/((c-c*sin(f*x+e))^m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3052, 2824, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$= \frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,\text{Hypergeo}}{f(2m+1)}$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)) - (B*Cos[
e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f} + (A+2Bm)\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f} + \left((A+2Bm)\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{-2m}\,dx$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f} + \frac{\left(c^2(A+2Bm)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\sin(e+fx)\right.}{f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f} + \frac{\left(2^{-\frac{1}{2}-m}c^2(A+2Bm)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)}\left(\frac{c-c\sin(e+}{c}\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}c(A+2Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e\right.}{f(1+2m)} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.54

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx =$$

$$-\frac{i2^{-1-m}\left(a\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)^m(c-c\sin(e+fx))^{-m}(\cosh(m\log(2))+\sinh(m\log(2)}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^m,x]
```

```
[Out] ((-I)*2^(-1 - m)*(a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)^m*(Cosh[m*Log[
2]] + Sinh[m*Log[2]])*((A*Hypergeometric2F1[1, -2*m, 1 - 2*m, -((Cos[e + f*
x] + I*(-1 + Sin[e + f*x]))/(Cos[e + f*x] + I*(1 + Sin[e + f*x])))])/m - (2
*B*Hypergeometric2F1[2, 1 - 2*m, 2 - 2*m, -((Cos[e + f*x] + I*(-1 + Sin[e +
 f*x]))/(Cos[e + f*x] + I*(1 + Sin[e + f*x])))]*(Cos[e + f*x] + I*(-1 + Sin
[e + f*x])))/((-1 + 2*m)*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))) + ((B*Hyper
geometric2F1[2*m, 1 + 2*m, 2 + 2*m, (1 - I*Cos[e + f*x] + Sin[e + f*x])/2]*
(Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x]))^(2*m)*(1 - I*Cos[e + f*x] +
Sin[e + f*x]))/(1 + 2*m) - (A*Hypergeometric2F1[-2*m, -2*m, 1 - 2*m, (1 + I
*Cos[e + f*x] - Sin[e + f*x])/2]*(Cosh[m*Log[16]] + Sinh[m*Log[16]]))/(4^m*
m))/(1 - I*Cos[e + f*x] + Sin[e + f*x])^(2*m)))/(f*(c - c*Sin[e + f*x])^m)
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (-c(\sin(fx+e)-1))^{-m}\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^
m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^m}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m, x
)
```

3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 170

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = \frac{2^{\frac{1}{2}-m}c^2(2A-B(1-2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx))\right)\dots}{f(1+2m)} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f}$$

```
[Out] 2^(1/2-m)*c^2*(2*A-B*(1-2*m))*cos(f*x+e)*hypergeom([1/2+m, -1/2+m],[3/2+m],
1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+
e))^(-1-m)/f/(1+2*m)-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
1-m)/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3052, 2824, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = \frac{c^2 2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\dots}{f(2m+1)} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - m
),x]
```

```
[Out] (2^(1/2 - m)*c^2*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)
) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(2
*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
```

```
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f} + \frac{(2Ac+Bc(-1+2m))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx}{2c}$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f} + \frac{((2Ac+Bc(-1+2m))\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m)\int\cos^{2m}(e+ }{2c}$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f} + \frac{\left(c(2Ac+Bc(-1+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\,\mathrm{s}\right.}{2f}$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f} + \frac{\left(2^{-\frac{1}{2}-m}c^2(2Ac+Bc(-1+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2\right.}{}$$
$$= \frac{2^{\frac{1}{2}-m}c^2(2A-B(1-2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\right.}{f(1+2m)} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f}$$

Mathematica [F]

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = \int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(1 - m),x]
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(1 - m), x]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c-c\sin(fx+e))^{1-m}\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1), x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = \int (a(\sin(e+fx)+1))^m(-c(\sin(e+fx)-1))^{1-m}(A+B\sin(e+fx))\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(1 - m)*(A + B*
sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+1}\, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (A + B\sin(e + f\,x))\,(a + a\sin(e + f\,x))^m\,(c - c\sin(e + f\,x))^{1-m}\, dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m
),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m
), x)
```

3.217 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 173

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$= \frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin\right.}{3f(1+2m)}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f}$$

```
[Out] 1/3*2^(5/2-m)*c^3*(3*A-2*B*(1-m))*cos(f*x+e)*hypergeom([-3/2+m, 1/2+m],[3/2
+m],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(
f*x+e))^(-1-m)/f/(1+2*m)-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e
))^(2-m)/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3052, 2824, 2768, 72, 71}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$= \frac{c^3 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,\text{H}}{3f(2m+1)}$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - m
),x]
```

```
[Out] (2^(5/2 - m)*c^3*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(3*f*(1 + 2*
m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
(3*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
```

```
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f} + \frac{1}{3}(3A-2B(1-m))\int(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f} + \frac{1}{3}\left((3A-2B(1-m))\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c - c\sin(e+fx))^m\right)\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{2-2m}\,dx$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f} + \frac{\left(c^2(3A-2B(1-m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\sin(e\right.}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f} + \frac{\left(2^{\frac{3}{2}-m}c^4(3A-2B(1-m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)}\left(\frac{c-}{}\right.\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}\right.}{3f(1+2m)} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f}$$

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(2 - m),x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(2 - m), x]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m(A+B\sin(fx+e))(c-c\sin(fx+e))^{2-m}\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2), x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (a(\sin{(e + fx)} + 1))^m \left(-c(\sin{(e + fx)} - 1)\right)^{2-m} (A + B\sin{(e + fx)})\, dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(2 - m)*(A + B*
sin(e + f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (B\sin{(fx + e)} + A)(a\sin{(fx + e)} + a)^m (-c\sin{(fx + e)} + c)^{-m+2}\, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (B\sin{(fx + e)} + A)(a\sin{(fx + e)} + a)^m (-c\sin{(fx + e)} + c)^{-m+2}\, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^{2-m}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m
),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m
), x)
```

3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 46, antiderivative size = 34

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{-3+n}}{f}$$

```
[Out] a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(-3+n)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3046, 2933}

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^3c^3)\int \cos^6(e+fx)(c-c\sin(e+fx))^{-3+n}(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$= \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{-3+n}}{f}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$= \frac{a^3B(c-c\sin(e+fx))^n(14\cos(e+fx)-6\cos(3(e+fx))+14\sin(2(e+fx))-\sin(4(e+fx)))}{8f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 +
 n)*Sin[e + f*x]),x]
```

```
[Out] (a^3*B*(c - c*Sin[e + f*x])^n*(14*Cos[e + f*x] - 6*Cos[3*(e + f*x)] + 14*Si
n[2*(e + f*x)] - Sin[4*(e + f*x)]))/(8*f)
```

Maple [A] (verified)

Time = 11.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

method	result	size
parallelrisch	$-\frac{a^3B(-c(\sin(fx+e)-1))^n(6\cos(3fx+3e)+\sin(4fx+4e)-14\sin(2fx+2e)-14\cos(fx+e))}{8f}$	63

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/8*a^3*B*(-c*(sin(f*x+e)-1))^n*(6*cos(3*f*x+3*e)+sin(4*f*x+4*e)-14*sin(2*
f*x+2*e)-14*cos(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = -\frac{\left(3\,Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)+\left(Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(-c\sin}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] -(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) + (B*a^3*cos(f*x + e)^3 - 4
*B*a^3*cos(f*x + e))*sin(f*x + e))*(-c*sin(f*x + e) + c)^n/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(31) = 62.

Time = 65.21 (sec) , antiderivative size = 898, normalized size of antiderivative = 26.41

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Piecewise((-B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*
tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c - 2*c*tan(
e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
 f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/
2 + f*x/2)**2 + f) - 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)*
*2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/
2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*a**3*
(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**5
/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
```

```
 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(
e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*t
an(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
 + 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/
2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/
2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c - 2*c*tan(e/2 +
f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**
8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2
)**2 + f) + B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
+ 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(3 - n) - B*(n + 4)*sin(e)
)*(a*sin(e) + a)**3*(-c*sin(e) + c)**n, True))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$=\int -(B(n+4)\sin(fx+e)+B(n-3))(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9586 vs. $2(34) = 68$.

Time = 48.34 (sec) , antiderivative size = 9586, normalized size of antiderivative = 281.94

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$=\text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] (B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan
(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x +
 e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*ta
n(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f
```

*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x +
1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*
sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f
*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e
)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*
sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 -
4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1
/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e))
- 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4
*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x +
1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f
*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/
4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 + 6*B*a^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) -
4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4
*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1
/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) -
2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan
(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f
*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*
f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*
e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*
x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
- 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(ta
n(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^7 + 14*B*a^3*(sqrt(2*tan(f*x +
e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*t
an(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f
*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)
^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2
*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*
c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1
/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
+ pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)

^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^6 - B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^8 + 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^5 - 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^7 - 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x +

1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan
(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x
 + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x +
 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2
 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^6 - 14*B*a^3*(sqrt(2
*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2
*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*
f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1
/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) -
4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*
tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/
2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) -
2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(
1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*
x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f
*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e
)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(
1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x
 + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*
sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
- 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan
(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^3 - 14*B*a^3*(sqrt(2*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*ta
n(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^5 - 14*B*a^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) -
 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4
*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1
/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) -
 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan
(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f
*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*
f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*

e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*
x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(ta
n(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*(sqrt(2*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*ta
n(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*
e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c
*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/
2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
+ pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^
3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*ta
n(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
- 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*
f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/
2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1
/2*e)))^2*tan(1/2*f*x + 1/2*e) + 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*
x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x
 + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*
tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/
2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 +
 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*
abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*
x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^3 - B*a^3*(sqrt(2*tan(f*x + e)^4*
tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*
x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 -
 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/
2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x +
1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/
2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 +
tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4
 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan
(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)
) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi
*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 +

```
4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/
2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)
^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*
c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x
+ 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*
x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e
)))^2 + 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x
+ e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2
*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2
*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan
(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(
1/2*f*x + 1/2*e)^2 + 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4
- 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x +
1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1
/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^
4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*ta
n(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x +
e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2
+ 1))^n*tan(1/2*f*x + 1/2*e) + B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1
/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/
2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)
^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f
*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^
4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*ta
n(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c
)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1
/2*e)^2 + 1))^n)/(f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(
1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x +
1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*
n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/
4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2
*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2
*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn
(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f
*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) +
 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^
2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1
/2*f*x + 1/2*e)^8 + 4*f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*
tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*
x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4
*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floo
```

r(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan
(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x +
 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1
/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e
)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2
*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*t
an(1/2*f*x + 1/2*e)^6 + f*tan(1/2*f*x + 1/2*e)^8 + 6*f*tan(-1/4*pi*n*sgn(2*
c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1
/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 +
4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(ta
n(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*t
an(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x
 + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan
(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)
) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1
/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n
*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^4 + 4*f*tan(1/2*f*x + 1/
2*e)^6 + 4*f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x
 + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^
3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2
*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x +
1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 +
 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/
2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/
2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) -
 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c
*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*f*tan(1/2*f*x + 1/2*e)^4 + f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f
*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c
)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2
*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x +
 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x
+ 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2
*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x +
1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn
(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8
*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/
2*f*x + 1/2*e)))^2 + 4*f*tan(1/2*f*x + 1/2*e)^2 + f)

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$=\frac{B\,a^3\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

```
[In] int(-(B*(n - 3) + B*sin(e + f*x)*(n + 4))*(a + a*sin(e + f*x))^3*(c - c*sin
(e + f*x))^n,x)
```

```
[Out] (B*a^3*(-c*(sin(e + f*x) - 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 14
*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 45, antiderivative size = 34

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(e+fx))^{-3+n}}{f}$$

```
[Out] -a^3*B*c^3*cos(f*x+e)^7*(c+c*sin(f*x+e))^(-3+n)/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {3046, 2933}

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

```
[In] Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = (a^3c^3)\int \cos^6(e+fx)(c+c\sin(e+fx))^{-3+n}(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(e+fx))^{-3+n}}{f}$$

Mathematica [A] (verified)

Time = 7.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^n}{f}$$

```
[In] Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 +
 n)*Sin[e + f*x]),x]
[Out] -((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)
```

Maple [A] (verified)

Time = 13.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

method	result	size
parallelrisch	$-\frac{a^3B(c(1+\sin(fx+e)))^n(14\cos(fx+e)-6\cos(3fx+3e)+\sin(4fx+4e)-14\sin(2fx+2e))}{8f}$	62

```
[In] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/8*a^3*B*(c*(1+sin(f*x+e)))^n*(14*cos(f*x+e)-6*cos(3*f*x+3*e)+sin(4*f*x+4
*e)-14*sin(2*f*x+2*e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = \frac{\left(3\,Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)-\left(Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(c\sin(}{f}$$

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] (3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4*
B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(32) = 64.

Time = 65.37 (sec) , antiderivative size = 898, normalized size of antiderivative = 26.41

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a-a*sin(f*x+e))**3*(c+c*sin(f*x+e))**n*(B*(3-n)+B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Piecewise((B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*t
an(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*t
an(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c + 2*c*tan(e
/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2
 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**
2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2
)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*a**3*(
c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**5/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
```

```
+ 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e
/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*ta
n(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
- 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2
 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2
 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c + 2*c*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**8
 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)
**2 + f) - B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n/(
f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 +
 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(3 - n) + B*(n + 4)*sin(e))
*(-a*sin(e) + a)**3*(c*sin(e) + c)**n, True))
```

Maxima [F]

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$=\int -(B(n+4)\sin{(fx+e)}-B(n-3))(a\sin{(fx+e)}-a)^3(c\sin{(fx+e)}+c)^n\,dx$$

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9587 vs. 2(34) = 68.

Time = 48.82 (sec) , antiderivative size = 9587, normalized size of antiderivative = 281.97

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$=\text{Too large to display}$$

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] -(B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*ta
n(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x
+ e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*
```

f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 - 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^7 + 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e

)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^6 - B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^8 - 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^7 - 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x +

1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^6 + 14*B*a^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) +
 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4
*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1
/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) -
 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan
(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f
*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*
f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*
e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*
x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(ta
n(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^3 + 14*B*a^3*(sqrt(2*tan(f*x +
e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*t
an(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f
*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x
 + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)
^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^5 - 14*B*a^3*(sqrt
(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1
/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/
2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2
*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
+ 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 +
4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
 tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(
1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e)
- 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*ta
n(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*
f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2
*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2

```
*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f
*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/
4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^
3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(t
an(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*a^3*(sqrt(2*tan(f*x +
e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*t
an(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f
*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x
 + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)
^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2
*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*
c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1
/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
 + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)
^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*t
an(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3
 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2
*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1
/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x +
1/2*e)))^2*tan(1/2*f*x + 1/2*e) - 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f
*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*
tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*
x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2
*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1
/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3
+ 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)
*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f
*x + 1/2*e)^2 + 1))^n*tan(1/2*f*x + 1/2*e)^3 - B*a^3*(sqrt(2*tan(f*x + e)^4
*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f
*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4
+ 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/
2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1
/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x +
 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1
/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 +
 tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^
4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*ta
n(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e
)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + p
i*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 -
```

```
 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1
/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e
)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8
*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x
 + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f
*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*
e)))^2 + 14*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x
 + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 +
3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*ta
n(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan
(1/2*f*x + 1/2*e)^2 - 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4
 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x +
 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(
1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)
^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x +
 e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(
f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2
 + 1))^n*tan(1/2*f*x + 1/2*e) + B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x +
1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1
/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(
f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)
^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*t
an(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(
c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x +
1/2*e)^2 + 1))^n)/(f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan
(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x +
 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi
*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1
/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/
2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/
2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sg
n(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*
f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e))
+ 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)
^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(
1/2*f*x + 1/2*e)^8 + 4*f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c
*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f
*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/
4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*flo
```

or(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*ta
n(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x
+ 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(
1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*
e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/
2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*
tan(1/2*f*x + 1/2*e)^6 + f*tan(1/2*f*x + 1/2*e)^8 + 6*f*tan(-1/4*pi*n*sgn(2
*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x +
1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 +
4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(t
an(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*
tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*
x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*ta
n(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e
)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x +
1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*
n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^4 + 4*f*tan(1/2*f*x + 1
/2*e)^6 + 4*f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*
x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)
^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(
tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(
2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x +
1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2
+ 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1
/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1
/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2)
- 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*
c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*f*tan(1/2*f*x + 1/2*e)^4 + f*tan(-1/4*pi*n*sgn(2*c*tan(1/2*
f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*
c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/
2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x
+ 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x
+ 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^
3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*c*tan(1/2*f*x +
1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) - 1/4*sg
n(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 +
8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1
/2*f*x + 1/2*e)))^2 + 4*f*tan(1/2*f*x + 1/2*e)^2 + f)

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{B\,a^3\,\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)^n\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

```
[In] int(-(B*(n - 3) - B*sin(e + f*x)*(n + 4))*(a - a*sin(e + f*x))^3*(c + c*sin
(e + f*x))^n,x)
```

```
[Out] -(B*a^3*(c*(sin(e + f*x) + 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14
*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

3.220 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 44, antiderivative size = 33

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f}$$

```
[Out] a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3046, 2933}

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}(B(-3+m)-B(4+m)\sin(e+fx))\,dx\\ &= \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 7.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$$
$$=\frac{Bc^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m}{f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4
+ m)*Sin[e + f*x]),x]
[Out] (B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/f
```

Maple [A] (verified)

Time = 13.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
parallelrisch	$\frac{c^3B(a(1+\sin(fx+e)))^m(14\cos(fx+e)-6\cos(3fx+3e)+\sin(4fx+4e)-14\sin(2fx+2e))}{8f}$	62

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*c^3*B*(a*(1+sin(f*x+e)))^m*(14*cos(f*x+e)-6*cos(3*f*x+3*e)+sin(4*f*x+4*
e)-14*sin(2*f*x+2*e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(33) = 66.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = -\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)-\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(a\sin}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 - 4
*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(31) = 62.

Time = 65.47 (sec) , antiderivative size = 898, normalized size of antiderivative = 27.21

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3*(B*(-3+m)-B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Piecewise((-B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*
tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a + 2*a*tan(
e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
 f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/
2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)*
*2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/
2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*c**3*
(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**5
/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
```

```
 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(
e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*t
an(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
 + 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/
2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/
2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a + 2*a*tan(e/2 +
f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**
8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2
)**2 + f) + B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
+ 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(m - 3) - B*(m + 4)*sin(e)
)*(a*sin(e) + a)**m*(-c*sin(e) + c)**3, True))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$$
$$=\int (B(m+4)\sin(fx+e)-B(m-3))(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9586 vs. $2(33) = 66$.

Time = 45.41 (sec) , antiderivative size = 9586, normalized size of antiderivative = 290.48

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$$
$$=\text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] (B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan
(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x +
 e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*ta
n(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f
```

*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 - 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^7 + 14*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)

^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*t
an(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2
*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1
/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x +
1/2*e)))^2*tan(1/2*f*x + 1/2*e)^6 - B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*
x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x
 + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*
tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/
2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 +
 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*
abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*
x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^8 - 14*B*c^3*(sqrt(2*tan(f*x + e)
^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan
(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^
4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan
(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x
 + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x +
 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2
 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e
)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*
tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2
*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) +
 pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3
 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan
(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 +
 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f
*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2
*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/
2*e)))^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*
x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x
 + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*
tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/
2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 +
 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*
abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*
x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^7 - 14*B*c^3*(sqrt(2*tan(f*x + e)
^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan
(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^
4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x +

1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan
(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x
 + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x +
 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2
 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^6 + 14*B*c^3*(sqrt(2
*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2
*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*
f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1
/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) +
4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*
tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/
2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) -
2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(
1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*
x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f
*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e
)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(
1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x
 + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*
sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
+ 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan
(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^3 + 14*B*c^3*(sqrt(2*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*ta
n(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^5 - 14*B*c^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) +
 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4
*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1
/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) -
 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan
(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f
*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*
f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*

e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*
x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(ta
n(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^3*(sqrt(2*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*ta
n(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*
e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a
*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/
2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
+ pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^
3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
+ 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*
f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/
2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1
/2*e)))^2*tan(1/2*f*x + 1/2*e) - 14*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*
x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x
 + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*
tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/
2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 +
 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*
abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*
x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^3 - B*c^3*(sqrt(2*tan(f*x + e)^4*
tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*
x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 +
 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/
2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x +
1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/
2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 +
tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4
 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan
(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)
) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi
*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 -

4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2 + 14*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^2 - 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e) + B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m)/(f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 + 4*f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floo

r(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan
(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x +
 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1
/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e
)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2
*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*t
an(1/2*f*x + 1/2*e)^6 + f*tan(1/2*f*x + 1/2*e)^8 + 6*f*tan(-1/4*pi*m*sgn(2*
a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1
/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 +
4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(ta
n(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*t
an(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x
 + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan
(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)
) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1
/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m
*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^4 + 4*f*tan(1/2*f*x + 1/
2*e)^6 + 4*f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x
 + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^
3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2
*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x +
1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 +
 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/
2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/
2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) -
 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a
*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*f*tan(1/2*f*x + 1/2*e)^4 + f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f
*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a
)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2
*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x +
 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x
+ 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2
*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x +
1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn
(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8
*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/
2*f*x + 1/2*e)))^2 + 4*f*tan(1/2*f*x + 1/2*e)^2 + f)

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$$
$$=\frac{B\,c^3\,\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

```
[In] int((B*(m - 3) - B*sin(e + f*x)*(m + 4))*(a + a*sin(e + f*x))^m*(c - c*sin(
e + f*x))^3,x)
```

```
[Out] (B*c^3*(a*(sin(e + f*x) + 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14*
sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 43, antiderivative size = 35

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{-3+m}}{f}$$

```
[Out] -a^3*B*c^3*cos(f*x+e)^7*(a-a*sin(f*x+e))^(-3+m)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {3046, 2933}

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\text{integral} = \left(a^3c^3\right)\int \cos^6(e+fx)(a-a\sin(e+fx))^{-3+m}(B(-3+m)+B(4+m)\sin(e+fx))\,dx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{-3+m}}{f}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$$
$$= \frac{Bc^3(a-a\sin(e+fx))^m(-14\cos(e+fx)+6\cos(3(e+fx))-14\sin(2(e+fx))+\sin(4(e+fx)))}{8f}$$

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4
+ m)*Sin[e + f*x]),x]
```

```
[Out] (B*c^3*(a - a*Sin[e + f*x])^m*(-14*Cos[e + f*x] + 6*Cos[3*(e + f*x)] - 14*S
in[2*(e + f*x)] + Sin[4*(e + f*x)]))/(8*f)
```

Maple [A] (verified)

Time = 11.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$-\frac{c^3B(-a(\sin(fx+e)-1))^m(-6\cos(3fx+3e)-\sin(4fx+4e)+14\sin(2fx+2e)+14\cos(fx+e))}{8f}$	65

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/8*c^3*B*(-a*(sin(f*x+e)-1))^m*(-6*cos(3*f*x+3*e)-sin(4*f*x+4*e)+14*sin(2
*f*x+2*e)+14*cos(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.20

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)+\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(-a\sin$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] (3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) + (B*c^3*cos(f*x + e)^3 - 4*
B*c^3*cos(f*x + e))*sin(f*x + e))*(-a*sin(f*x + e) + a)^m/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(32) = 64.

Time = 66.13 (sec) , antiderivative size = 898, normalized size of antiderivative = 25.66

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**3*(B*(-3+m)+B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Piecewise((B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*t
an(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*t
an(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a - 2*a*tan(e
/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2
 + f*x/2)**2 + f) + 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**
2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2
)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*c**3*(
a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**5/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
```

```
+ 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e
/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*ta
n(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
- 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2
 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2
 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a - 2*a*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**8
 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)
**2 + f) - B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m/(
f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 +
 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(m - 3) + B*(m + 4)*sin(e))
*(-a*sin(e) + a)**m*(c*sin(e) + c)**3, True))
```

Maxima [F]

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$$
$$=\int (B(m+4)\sin{(fx+e)}+B(m-3))(c\sin{(fx+e)}+c)^3(-a\sin{(fx+e)}+a)^m\,dx$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9587 vs. 2(35) = 70.

Time = 41.81 (sec) , antiderivative size = 9587, normalized size of antiderivative = 273.91

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$$
$$=\text{Too large to display}$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] -(B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*ta
n(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x
+ e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*
```

f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m *sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2* f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2* e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) *sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2* f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1 /4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 + 6*B*c^3*(sqrt (2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1 /2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/ 2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2 *tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*ta n(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2* f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2 *f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2 *e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(ta n(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f *x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/ 4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^ 3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(t an(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^7 + 14*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4* tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2* e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2* f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f* x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/ 2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4 *a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e

)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*
tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x +
1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^
3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/
2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(
1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x +
1/2*e)))^2*tan(1/2*f*x + 1/2*e)^6 - B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f
*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*
tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*
x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2
*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1
/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3
+ 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)
*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f
*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^8 + 14*B*c^3*(sqrt(2*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*ta
n(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*
e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a
*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/
2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
+ pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^
3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
- 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*
f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/
2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1
/2*e)))^2*tan(1/2*f*x + 1/2*e)^5 - 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f
*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*
tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*
x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2
*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1
/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3
+ 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)
*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f
*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^7 - 14*B*c^3*(sqrt(2*tan(f*x + e
)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*ta
n(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x +

1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*
x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
+ 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^
2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^6 - 14*B*c^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) -
 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4
*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1
/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) -
 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan
(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f
*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*
f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*
e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*
x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(ta
n(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^3 - 14*B*c^3*(sqrt(2*tan(f*x +
e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*t
an(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f
*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
 + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)
^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^5 - 14*B*c^3*(sqrt
(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1
/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/
2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2
*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
- 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 -
4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 +
 tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(
1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e)
- 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*ta
n(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*
f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2
*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2

*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f
*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/
4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^
3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(t
an(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c^3*(sqrt(2*tan(f*x +
e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*t
an(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f
*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x
 + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)
^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2
*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*
a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1
/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
 + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)
^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*t
an(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3
 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2
*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1
/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x +
1/2*e)))^2*tan(1/2*f*x + 1/2*e) + 14*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f
*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*
tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*
x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2
*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1
/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3
+ 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)
*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f
*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^3 - B*c^3*(sqrt(2*tan(f*x + e)^4
*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f
*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4
- 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/
2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1
/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x +
 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1
/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 +
 tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^
4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*ta
n(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e
)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + p
i*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 +

4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2 + 14*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(1/2*f*x + 1/2*e) + B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m)/(f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 + 4*f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*flo

or(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*ta
n(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(
1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*
e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/
2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*
tan(1/2*f*x + 1/2*e)^6 + f*tan(1/2*f*x + 1/2*e)^8 + 6*f*tan(-1/4*pi*m*sgn(2
*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x +
1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 +
 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(t
an(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*
tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*
x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*ta
n(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e
)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x +
1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*
m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^4 + 4*f*tan(1/2*f*x + 1
/2*e)^6 + 4*f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*
x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)
^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(
tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(
2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x +
 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2
+ 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1
/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1
/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2)
- 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*
a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x
 + 1/2*e)^2 + 6*f*tan(1/2*f*x + 1/2*e)^4 + f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*
f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*
a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/
2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x
+ 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x
 + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^
3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + 1/4*sgn(4*a*tan(1/2*f*x +
 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*sg
n(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 -
8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1
/2*f*x + 1/2*e)))^2 + 4*f*tan(1/2*f*x + 1/2*e)^2 + f)

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = -\frac{B\,c^3\,\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

```
[In] int((B*(m - 3) + B*sin(e + f*x)*(m + 4))*(a - a*sin(e + f*x))^m*(c + c*sin(
e + f*x))^3,x)
```

```
[Out] -(B*c^3*(-a*(sin(e + f*x) - 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 1
4*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 47, antiderivative size = 36

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f}$$

```
[Out] B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {3049}

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

Rule 3049

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
```

```
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f}$$

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= \frac{B\cos(e+fx)(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 +
 m + n)*Sin[e + f*x]),x]
[Out] (B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f
```

Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{B(a(1+\sin(fx+e)))^m(-c(\sin(fx+e)-1))^n\cos(fx+e)}{f}$	37

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x,
method=_RETURNVERBOSE)
[Out] B/f*(a*(1+sin(f*x+e)))^m*(-c*(sin(f*x+e)-1))^n*cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^nB\cos(fx+e)}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx &= -B\bigg(\int(-m(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n)\,dx \\ &+\int n(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\,dx \\ &+\int(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx \\ &+\int m(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx \\ &+\int n(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx\bigg)\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n*(B*(m-n)-B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] -B*(Integral(-m*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Inte
gral(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Integral((a*s
in(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral(m*
(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integra
l(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$=\int -(B(m+n+1)\sin(fx+e)-B(m-n))(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] -integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m*
(-c*sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8947 vs. $2(36) = 72$.

Time = 53.15 (sec) , antiderivative size = 8947, normalized size of antiderivative = 248.53

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] (B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*ta
n(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*
e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*
tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5
/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*
e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*
log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1
/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(
1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x
 + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*ta
n(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*ta
n(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 +
 abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x
```

+ 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f* x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*sin(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))*tan(1/2*f*x + 1/2*e)^2 - B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1

/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e
) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan
(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e
)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x +
e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan
(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c
)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1
/2*e)^2 + 1)))*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f
*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e
)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn
(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x
+ 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2
 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi
*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8
*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*ta
n(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e
)))^2 - B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2
 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x
 + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*
e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) +
 2) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^
2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x
 + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*
tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)
*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*
f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 +
 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2
 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/
2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2
 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2
*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*
f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x
+ e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x +
e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan
(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f
*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x
+ e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*t
an(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(

1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(1/2*f*x + 1/2*e)^2 - 2*B*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*sin(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1

/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e))) + B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))))/(f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e))

```
 - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4
*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2
*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi
+ 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x
 + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*
pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(
1/2*f*x + 1/2*e)))^2 + f*tan(1/2*f*x + 1/2*e)^2 + f)
```

Mupad [B] (verification not implemented)

Time = 12.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\ \cos\left(e+f\,x\right)\ {\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n}{f}$$

```
[In] int((B*(m - n) - B*sin(e + f*x)*(m + n + 1))*(a + a*sin(e + f*x))^m*(c - c*
sin(e + f*x))^n,x)
```

```
[Out] (B*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/f
```

3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 46, antiderivative size = 37

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c+c\sin(e+fx))^n}{f}$$

```
[Out] -B*cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {3049}

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

Rule 3049

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
```

```
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c+c\sin(e+fx))^n}{f}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$$
$$=-\frac{B\cos(e+fx)(c(1+\sin(e+fx)))^n(a-a\sin(e+fx))^m}{f}$$

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 +
 m + n)*Sin[e + f*x]),x]
[Out] -((B*Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^n*(a - a*Sin[e + f*x])^m)/f)
```

Maple [A] (verified)

Time = 9.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$-\frac{B(-a(\sin(fx+e)-1))^m(c(1+\sin(fx+e)))^n\cos(fx+e)}{f}$	38

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x,
method=_RETURNVERBOSE)
[Out] -B/f*(-a*(sin(f*x+e)-1))^m*(c*(1+sin(f*x+e)))^n*cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$$
$$=-\frac{(-a\sin{(fx+e)}+a)^m(c\sin{(fx+e)}+c)^nB\cos{(fx+e)}}{f}$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] -(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F]

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$$
$$=B\left(\int m(-a\sin{(e+fx)}+a)^m\,(c\sin{(e+fx)}+c)^n\,dx+\int\left(-n(-a\sin{(e+fx)}+a)^m\,(c\sin{(e+fx)}+c)^n\right)dx+\int(-a\sin{(e+fx)}+a)^m\,(c\sin{(e+fx)}+c)^n\sin{(e+fx)}\,dx+\int m(-a\sin{(e+fx)}+a)^m\,(c\sin{(e+fx)}+c)^n\sin{(e+fx)}\,dx+\int n(-a\sin{(e+fx)}+a)^m\,(c\sin{(e+fx)}+c)^n\sin{(e+fx)}\,dx\right)$$

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**n*(B*(m-n)+B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] B*(Integral(m*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integr
al(-n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integral((-a*s
in(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral(m*(
-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral
(n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

Maxima [F]

$$\int (a - a\sin(e + fx))^m (c + c\sin(e + fx))^n (B(m - n) + B(1 + m + n)\sin(e + fx))\, dx$$
$$= \int (B(m + n + 1)\sin(fx + e) + B(m - n))(-a\sin(fx + e) + a)^m (c\sin(fx + e) + c)^n\, dx$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m*
(c*sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8948 vs. $2(37) = 74$.

Time = 51.92 (sec) , antiderivative size = 8948, normalized size of antiderivative = 241.84

$$\int (a - a\sin(e + fx))^m (c + c\sin(e + fx))^n (B(m - n) + B(1 + m + n)\sin(e + fx))\, dx$$
$$= \text{Too large to display}$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] -(B*cos(2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2
*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4
*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) +
5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*
tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m
*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan
(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*t
an(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*t
an(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2
+ abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*
```

x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f

*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*sin(2*pi*n*floor(-1/8*sgn(4*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) +
4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) +
5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2
*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x
+ e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*
sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/
2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)
^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*t
an(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*
e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3
- 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(
1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/
pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*
f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1
/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*t
an(1/2*f*x + 1/2*e)))*tan(1/2*f*x + 1/2*e)^2 - B*cos(2*pi*n*floor(-1/8*sgn(
4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/
2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e)
+ 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^
2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*
x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(f*x
+ e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*t
an(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/
4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f
*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*log(2) - n*log(2) + m*log(sqr
t(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/
2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x +
e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*ta
n(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f
*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(
1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2
+ 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x +
e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)
) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(
f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)
^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ab
s(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x +

1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2 - B*cos(2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan

(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^
2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(1/2*f*x + 1/2*e)^2
 - 2*B*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x +
e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/
2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*
tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x
+ e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x
 + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*ta
n(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(
f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) +
 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan
(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*ta
n(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
 + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2
)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/
2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*sin(2*pi*n*floor(
-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2
*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x
 + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*t
an(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*n*sgn(4
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e)
+ 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*
tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*tan(-1/4*pi*m*sgn(2*a*tan(
1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e)
- 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*ta
n(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2
*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2
*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*f
loor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*
sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f
*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x +

1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e))) + B*cos(2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))))/(f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(-1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)

```
) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 -
4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/
2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi
 + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*
x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) - 1/4
*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan
(1/2*f*x + 1/2*e)))^2 + f*tan(1/2*f*x + 1/2*e)^2 + f)
```

Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\,\cos\left(e+f\,x\right)\,\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)^n}{f}$$

```
[In] int((B*(m - n) + B*sin(e + f*x)*(m + n + 1))*(a - a*sin(e + f*x))^m*(c + c*
sin(e + f*x))^n,x)
```

```
[Out] -(B*cos(e + f*x)*(-a*(sin(e + f*x) - 1))^m*(c*(sin(e + f*x) + 1))^n)/f
```

3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 140

$$\begin{aligned}\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= \frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}\\&-\frac{a^3A\cos^7(c+dx)}{7d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{8d}\\&-\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{12d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d}\end{aligned}$$

```
[Out] 1/8*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+3/5*a^3*A*cos(d*x+c)^5/d-1/7*a^3*A*cos
(d*x+c)^7/d-1/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/12*a^3*A*cos(d*x+c)*sin(d*x
+c)^3/d+1/3*a^3*A*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 2713, 2715, 8}

$$\begin{aligned}\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= -\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}\\&+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{12d}\\&-\frac{a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^3Ax\end{aligned}$$

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5*
d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[c +
 d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(a^3 A \sin^3(c+dx) + 2a^3 A \sin^4(c+dx) - 2a^3 A \sin^6(c+dx) - a^3 A \sin^7(c+dx)\right) dx$$

$$= \left(a^3 A\right) \int \sin^3(c+dx)\, dx - \left(a^3 A\right) \int \sin^7(c+dx)\, dx + \left(2a^3 A\right) \int \sin^4(c+dx)\, dx - \left(2a^3 A\right) \int \sin^6(c+dx)\, dx$$

$$= -\frac{a^3 A \cos(c+dx) \sin^3(c+dx)}{2d} + \frac{a^3 A \cos(c+dx) \sin^5(c+dx)}{3d} + \frac{1}{2}\left(3a^3 A\right) \int \sin^2(c+dx)\, dx - \frac{1}{3}\left(5a^3 A\right) \int \sin^4(c+dx)\, dx - \frac{\left(a^3 A\right) \text{Subst}\left(\int \left(1-x^2\right) dx, x, \cos(c+dx)\right)}{d} + \frac{\left(a^3 A\right) \text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right) dx, x, \cos(c+dx)\right)}{d}$$

$$=-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos^7(c+dx)}{7d}-\frac{3a^3A\cos(c+dx)\sin(c+dx)}{4d}-\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{12d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d}+\frac{1}{4}(3a^3A)\int 1\,dx-\frac{1}{4}(5a^3A)\int \sin^2(c+dx)\,dx$$

$$=\frac{3}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos^7(c+dx)}{7d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{12d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d}-\frac{1}{8}(5a^3A)\int 1\,dx$$

$$=\frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos^7(c+dx)}{7d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{12d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$=\frac{a^3A(840c+840dx-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c+dx))-15\cos(7(c+dx))-21}{6720d}$$

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Co
s[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c
+ d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

method	result
parallelrisch	$-\frac{A\,a^{3}(-840dx+1365\cos(dx+c)+15\cos(7dx+7c)-70\sin(6dx+6c)-147\cos(5dx+5c)+210\sin(4dx+4c)+175\cos(3dx+}{6720d}$
risch	$\frac{a^{3}Ax}{8}-\frac{13a^{3}A\cos(dx+c)}{64d}-\frac{A\,a^{3}\cos(7dx+7c)}{448d}+\frac{A\,a^{3}\sin(6dx+6c)}{96d}+\frac{7A\,a^{3}\cos(5dx+5c)}{320d}-\frac{A\,a^{3}\sin(4dx+4c)}{32d}$
derivativedivides	$\frac{\frac{A\,a^{3}\left(\frac{16}{5}+\sin^{6}(dx+c)+\frac{6\left(\sin^{4}(dx+c)\right)}{5}+\frac{8\left(\sin^{2}(dx+c)\right)}{5}\right)\cos(dx+c)}{7}-2A\,a^{3}\left(-\frac{\left(\sin^{5}(dx+c)+\frac{5\left(\sin^{3}(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos}{6}\right.}{d}$
default	$\frac{\frac{A\,a^{3}\left(\frac{16}{5}+\sin^{6}(dx+c)+\frac{6\left(\sin^{4}(dx+c)\right)}{5}+\frac{8\left(\sin^{2}(dx+c)\right)}{5}\right)\cos(dx+c)}{7}-2A\,a^{3}\left(-\frac{\left(\sin^{5}(dx+c)+\frac{5\left(\sin^{3}(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos}{6}\right.}{d}$
parts	$-\frac{A\,a^{3}(2+\sin^{2}(dx+c))\cos(dx+c)}{3d}+\frac{2A\,a^{3}\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}-\frac{2A\,a^{3}\left(-\frac{\left(\sin^{5}(dx\right.}{}\right.}{}$
norman	$-\frac{44A\,a^{3}}{105d}+\frac{a^{3}Ax}{8}-\frac{4A\,a^{3}\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24A\,a^{3}\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8A\,a^{3}\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{44A\,a^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{52A\,a^{3}(}{}$

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/6720*A*a^3*(-840*d*x+1365*cos(d*x+c)+15*cos(7*d*x+7*c)-70*sin(6*d*x+6*c)
-147*cos(5*d*x+5*c)+210*sin(4*d*x+4*c)+175*cos(3*d*x+3*c)+210*sin(2*d*x+2*c
)+1408)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{120\,Aa^3\cos(dx+c)^7-504\,Aa^3\cos(dx+c)^5+560\,Aa^3\cos(dx+c)^3-105\,Aa^3dx-35\,(8\,Aa^3\cos(dx}{840\,d}$$

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*cos
(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d*x
 + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(131) = 262$.

Time = 0.48 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.14

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{8} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\,\mathrm{co}}{2} \\ x(-A\sin(c)+A)\,(a\sin(c)+a)^3\sin^3(c) \end{cases}$$

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/8 - 15*A*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/8 + 3*A*a**3*x*sin(c + d*x)**4/4 - 15*A*a**3*x*sin(c + d*x)**2*
cos(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 - 5*A*a**3
*x*cos(c + d*x)**6/8 + 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*sin(c + d*x)**
6*cos(c + d*x)/d + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 2*A*a**3*
sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**
3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin(c + d*
x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 5*A*a
**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)
**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d*x)**3/(3*
d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{96\left(5\cos(dx+c)^7 - 21\cos(dx+c)^5 + 35\cos(dx+c)^3 - 35\cos(dx+c)\right)Aa^3 - 1120\left(\cos(dx+c)^3 - \right.}{}$$

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*
cos(d*x + c))*A*a^3 - 1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 35*(4*
sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*
c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a
^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{1}{8}Aa^3x-\frac{Aa^3\cos(7\,dx+7\,c)}{448\,d}+\frac{7\,Aa^3\cos(5\,dx+5\,c)}{320\,d}-\frac{5\,Aa^3\cos(3\,dx+3\,c)}{192\,d}$$
$$-\frac{13\,Aa^3\cos(dx+c)}{64\,d}+\frac{Aa^3\sin(6\,dx+6\,c)}{96\,d}-\frac{Aa^3\sin(4\,dx+4\,c)}{32\,d}-\frac{Aa^3\sin(2\,dx+2\,c)}{32\,d}$$

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c)
/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A*a
^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2*d*
x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.14

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{A\,a^3\left(105\,c-210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2464\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1400\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4032\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6790\,\mathrm{ta}\right.}{}$$

```
[In] int(sin(c + d*x)^3*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*(105*c - 210*tan(c/2 + (d*x)/2) - 2464*tan(c/2 + (d*x)/2)^2 - 1400*t
an(c/2 + (d*x)/2)^3 - 4032*tan(c/2 + (d*x)/2)^4 + 6790*tan(c/2 + (d*x)/2)^5
 + 2240*tan(c/2 + (d*x)/2)^6 - 14560*tan(c/2 + (d*x)/2)^8 - 6790*tan(c/2 +
(d*x)/2)^9 - 3360*tan(c/2 + (d*x)/2)^10 + 1400*tan(c/2 + (d*x)/2)^11 + 210*
tan(c/2 + (d*x)/2)^13 + 105*d*x + 735*tan(c/2 + (d*x)/2)^2*(c + d*x) + 2205
*tan(c/2 + (d*x)/2)^4*(c + d*x) + 3675*tan(c/2 + (d*x)/2)^6*(c + d*x) + 367
5*tan(c/2 + (d*x)/2)^8*(c + d*x) + 2205*tan(c/2 + (d*x)/2)^10*(c + d*x) + 7
35*tan(c/2 + (d*x)/2)^12*(c + d*x) + 105*tan(c/2 + (d*x)/2)^14*(c + d*x) -
352))/(840*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3}{16}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} - \frac{3a^3A\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d}$$

```
[Out] 3/16*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+2/5*a^3*A*cos(d*x+c)^5/d-3/16*a^3*A*c
os(d*x+c)*sin(d*x+c)/d+5/24*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^3*A*cos(d
*x+c)*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3045, 2715, 8, 2713}

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{2a^3A\cos^5(c+dx)}{5d} - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d} + \frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d} - \frac{3a^3A\sin(c+dx)\cos(c+dx)}{16d} + \frac{3}{16}a^3Ax$$

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)/
(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(a^3 A \sin^2(c+dx) + 2a^3 A \sin^3(c+dx) - 2a^3 A \sin^5(c+dx) - a^3 A \sin^6(c+dx)\right) dx$$

$$= \left(a^3 A\right) \int \sin^2(c+dx)\,dx - \left(a^3 A\right) \int \sin^6(c+dx)\,dx + \left(2a^3 A\right) \int \sin^3(c+dx)\,dx - \left(2a^3 A\right) \int \sin^5(c+dx)\,dx$$

$$= -\frac{a^3 A \cos(c+dx)\sin(c+dx)}{2d} + \frac{a^3 A \cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{2}\left(a^3 A\right) \int 1\,dx - \frac{1}{6}\left(5a^3 A\right) \int \sin^4(c+dx)\,dx - \frac{\left(2a^3 A\right) \operatorname{Subst}\left(\int \left(1-x^2\right) dx, x, \cos(c+dx)\right)}{d} + \frac{\left(2a^3 A\right) \operatorname{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{1}{2}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos(c+dx)\sin(c+dx)}{2d} + \frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{1}{8}(5a^3A)\int \sin^2(c+dx)\,dx$$

$$= \frac{1}{2}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} - \frac{3a^3A\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{1}{16}(5a^3A)\int 1\,dx$$

$$= \frac{3}{16}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} - \frac{3a^3A\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.64

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^3A(180c+180dx-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx))-15\sin(2(c+dx))-45\sin(}{960d}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5
*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{A\,a^{3}(-180dx+240\cos(dx+c)-5\sin(6dx+6c)-24\cos(5dx+5c)+45\sin(4dx+4c)+40\cos(3dx+3c)+15\sin(2dx+2c)+2}{960d}$
risch	$\frac{3a^{3}Ax}{16}-\frac{a^{3}A\cos(dx+c)}{4d}+\frac{A\,a^{3}\sin(6dx+6c)}{192d}+\frac{A\,a^{3}\cos(5dx+5c)}{40d}-\frac{3A\,a^{3}\sin(4dx+4c)}{64d}-\frac{A\,a^{3}\cos(3dx+3c)}{24d}-$
derivativedivides	$\frac{-A\,a^{3}\left(-\frac{\left(\sin^{5}(dx+c)+\frac{5\left(\sin^{3}(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2A\,a^{3}\left(\frac{8}{3}+\sin^{4}(dx+c)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos(d}{5}}{d}$
default	$\frac{-A\,a^{3}\left(-\frac{\left(\sin^{5}(dx+c)+\frac{5\left(\sin^{3}(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2A\,a^{3}\left(\frac{8}{3}+\sin^{4}(dx+c)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos(d}{5}}{d}$
parts	$\frac{A\,a^{3}\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2A\,a^{3}\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3d}+\frac{2A\,a^{3}\left(\frac{8}{3}+\sin^{4}(dx+c)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)}{5d}$
norman	$\frac{-\frac{8A\,a^{3}}{15d}+\frac{3a^{3}Ax}{16}-\frac{16A\,a^{3}\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16A\,a^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{8A\,a^{3}\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3A\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{13A\,a^{3}\left(\tan^{3}\right.}{24d}}{}$

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/960*A*a^3*(-180*d*x+240*cos(d*x+c)-5*sin(6*d*x+6*c)-24*cos(5*d*x+5*c)+45
*sin(4*d*x+4*c)+40*cos(3*d*x+3*c)+15*sin(2*d*x+2*c)+256)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{96\,Aa^3\cos{(dx+c)}^5-160\,Aa^3\cos{(dx+c)}^3+45\,Aa^3dx+5\left(8\,Aa^3\cos{(dx+c)}^5-26\,Aa^3\cos{(dx+c)}^3+\right.}{240\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x +
5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c))
*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(119) = 238.

Time = 0.35 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.97

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{16}-\frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16}-\frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{Aa^3x\sin^2(c+dx)}{2}-\frac{5Aa^3x\cos^6(c+dx)}{16}+ \\ x(-A\sin(c)+A)\left(a\sin(c)+a\right)^3\sin^2(c) \end{cases}$$

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/16 - 15*A*a**3*x*sin(c + d*x)**4*cos
(c + d*x)**2/16 - 15*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + A*a**3*x
*sin(c + d*x)**2/2 - 5*A*a**3*x*cos(c + d*x)**6/16 + A*a**3*x*cos(c + d*x)*
*2/2 + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 2*A*a**3*sin(c + d*x
)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*A*
a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*A*a**3*sin(c + d*x)**2*cos(c
 + d*x)/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a**3*sin(c + d
*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**3*cos(c
+ d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**2,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{128\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)Aa^3+640\left(\cos(dx+c)^3-3\cos(dx+c)\right)Aa^3-}{}$$

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 +
 640*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 + 60
*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x
+ 2*c - sin(2*d*x + 2*c))*A*a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3}{16}\,Aa^3x + \frac{Aa^3\cos(5\,dx+5\,c)}{40\,d} - \frac{Aa^3\cos(3\,dx+3\,c)}{24\,d} - \frac{Aa^3\cos(dx+c)}{4\,d} + \frac{Aa^3\sin(6\,dx+6\,c)}{192\,d} - \frac{3\,Aa^3\sin(4\,dx+4\,c)}{64\,d} - \frac{Aa^3\sin(2\,dx+2\,c)}{64\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)/
d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^3*
sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.12

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{A\,a^3\,\left(45\,c-90\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-768\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+130\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+1500\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-1280\tan\left(\frac{c}{2}+\right.\right.}{}$$

```
[In] int(sin(c + d*x)^2*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*(45*c - 90*tan(c/2 + (d*x)/2) - 768*tan(c/2 + (d*x)/2)^2 + 130*tan(c
/2 + (d*x)/2)^3 + 1500*tan(c/2 + (d*x)/2)^5 - 1280*tan(c/2 + (d*x)/2)^6 - 1
500*tan(c/2 + (d*x)/2)^7 - 1920*tan(c/2 + (d*x)/2)^8 - 130*tan(c/2 + (d*x)/
2)^9 + 90*tan(c/2 + (d*x)/2)^11 + 45*d*x + 270*tan(c/2 + (d*x)/2)^2*(c + d*
x) + 675*tan(c/2 + (d*x)/2)^4*(c + d*x) + 900*tan(c/2 + (d*x)/2)^6*(c + d*x
) + 675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 270*tan(c/2 + (d*x)/2)^10*(c + d*x
) + 45*tan(c/2 + (d*x)/2)^12*(c + d*x) - 128))/(240*d*(tan(c/2 + (d*x)/2)^2
 + 1)^6)
```

3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{1}{4}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}$$

```
[Out] 1/4*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+1/5*a^3*A*cos(d*x+c)^5/d-1/4*a^3*A*cos
(d*x+c)*sin(d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3045, 2718, 2715, 8, 2713}

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A\cos^5(c+dx)}{5d} - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d} - \frac{a^3A\sin(c+dx)\cos(c+dx)}{4d} + \frac{1}{4}a^3Ax$$

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*
x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(a^3 A \sin(c+dx) + 2a^3 A \sin^2(c+dx) - 2a^3 A \sin^4(c+dx) - a^3 A \sin^5(c+dx)\right) dx$$

$$= \left(a^3 A\right) \int \sin(c+dx)\, dx - \left(a^3 A\right) \int \sin^5(c+dx)\, dx + \left(2a^3 A\right) \int \sin^2(c+dx)\, dx - \left(2a^3 A\right) \int \sin^4(c+dx)\, dx$$

$$
\begin{aligned}
&= -\frac{a^3 A\cos(c+dx)}{d} - \frac{a^3 A\cos(c+dx)\sin(c+dx)}{d} \\
&\quad + \frac{a^3 A\cos(c+dx)\sin^3(c+dx)}{2d} + \left(a^3 A\right)\int 1\,dx - \frac{1}{2}\left(3a^3 A\right)\int \sin^2(c+dx)\,dx \\
&\quad + \frac{\left(a^3 A\right)\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(c+dx)\right)}{d} \\
&= a^3 Ax - \frac{2a^3 A\cos^3(c+dx)}{3d} + \frac{a^3 A\cos^5(c+dx)}{5d} - \frac{a^3 A\cos(c+dx)\sin(c+dx)}{4d} \\
&\quad + \frac{a^3 A\cos(c+dx)\sin^3(c+dx)}{2d} - \frac{1}{4}\left(3a^3 A\right)\int 1\,dx \\
&= \frac{1}{4}a^3 Ax - \frac{2a^3 A\cos^3(c+dx)}{3d} + \frac{a^3 A\cos^5(c+dx)}{5d} \\
&\quad - \frac{a^3 A\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^3 A\cos(c+dx)\sin^3(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$
\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx
$$

$$
= \frac{a^3 A\cos(c+dx)\left(-30\arcsin\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right) + \sqrt{\cos^2(c+dx)}\left(-28-15\sin(c+dx)+16\sin^2(c+dx)+30\right.\right.}{60d\sqrt{\cos^2(c+dx)}}
$$

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
[Out] (a^3*A*Cos[c + d*x]*(-30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]] + Sqrt[Cos[
c + d*x]^2]*(-28 - 15*Sin[c + d*x] + 16*Sin[c + d*x]^2 + 30*Sin[c + d*x]^3
+ 12*Sin[c + d*x]^4)))/(60*d*Sqrt[Cos[c + d*x]^2])
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

method	result
parallelrisch	$-\frac{A\,a^3(-60dx+90\cos(dx+c)-3\cos(5dx+5c)+15\sin(4dx+4c)+25\cos(3dx+3c)+112)}{240d}$
risch	$\frac{a^3Ax}{4}-\frac{3a^3A\cos(dx+c)}{8d}+\frac{A\,a^3\cos(5dx+5c)}{80d}-\frac{A\,a^3\sin(4dx+4c)}{16d}-\frac{5A\,a^3\cos(3dx+3c)}{48d}$
derivativedivides	$\frac{\frac{A\,a^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2A\,a^3\left(-\frac{\cos}{}\right.}{d}$
default	$\frac{\frac{A\,a^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2A\,a^3\left(-\frac{\cos}{}\right.}{d}$
parts	$-\frac{a^3A\cos(dx+c)}{d}+\frac{2A\,a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.}{d}$
norman	$\frac{-\frac{14A\,a^3}{15d}+\frac{a^3Ax}{4}-\frac{4A\,a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2A\,a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8A\,a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8A\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{A\,a^3\tan\left(\frac{d}{2}\right.}{2d}}{}$

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*A*a^3*(-60*d*x+90*cos(d*x+c)-3*cos(5*d*x+5*c)+15*sin(4*d*x+4*c)+25*c
os(3*d*x+3*c)+112)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{12\,Aa^3\cos{(dx+c)}^5-40\,Aa^3\cos{(dx+c)}^3+15\,Aa^3dx-15\,\left(2\,Aa^3\cos{(dx+c)}^3-Aa^3\cos{(dx+c)}\right)\sin{(}}{60\,d}$$

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 15
*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(88) = 176.

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.78

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{3Aa^3x\sin^4{(c+dx)}}{4} - \frac{3Aa^3x\sin^2{(c+dx)}\cos^2{(c+dx)}}{2} + Aa^3x\sin^2{(c+dx)} - \frac{3Aa^3x\cos^4{(c+dx)}}{4} + Aa^3x\cos^2{(c+dx)} + \\ x(-A\sin{(c)}+A)\left(a\sin{(c)}+a\right)^3\sin{(c)} \end{cases}$$

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 + A*
a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**3*s
in(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*x)**
3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c + d*x)
*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*x)/d,
Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{16\left(3\cos\left(dx+c\right)^5 - 10\cos\left(dx+c\right)^3 + 15\cos\left(dx+c\right)\right)Aa^3 - 15\left(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) - 8\sin\left(\right.\right.}{240\,d}$$

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 -
15*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d
*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{1}{4}\,Aa^3x + \frac{Aa^3\cos\left(5\,dx+5\,c\right)}{80\,d} - \frac{5\,Aa^3\cos\left(3\,dx+3\,c\right)}{48\,d} - \frac{3\,Aa^3\cos\left(dx+c\right)}{8\,d} - \frac{Aa^3\sin\left(4\,dx+4\,c\right)}{16\,d}$$

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/d
 - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.04

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{A\,a^3\,x}{4} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-120)}{60}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-160)}{60}\right.}{}$$

```
[In] int(sin(c + d*x)*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*x)/4 - (tan(c/2 + (d*x)/2)^8*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(7
5*c + 75*d*x - 120))/60) + tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/12
 - (A*a^3*(75*c + 75*d*x - 160))/60) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c +
 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 80))/60) + tan(c/2 + (d*x)/2)^6*((A
*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 480))/60) + (A*a^3*tan(
c/2 + (d*x)/2))/2 - 3*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c/2 + (d*x)/
2)^7 - (A*a^3*tan(c/2 + (d*x)/2)^9)/2 + (A*a^3*(15*c + 15*d*x))/60 - (A*a^3
*(15*c + 15*d*x - 56))/60)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.227 $\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5}{8}a^3Ax-\frac{5a^3A\cos^3(c+dx)}{12d} + \frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d} - \frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}$$

```
[Out] 5/8*a^3*A*x-5/12*a^3*A*cos(d*x+c)^3/d+5/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/4
*A*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2815, 2757, 2748, 2715, 8}

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{5a^3A\cos^3(c+dx)}{12d} - \frac{A\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{4d} + \frac{5a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3Ax$$

```
[In] Int[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*x)/8 - (5*a^3*A*Cos[c + d*x]^3)/(12*d) + (5*a^3*A*Cos[c + d*x]*Sin
[c + d*x])/(8*d) - (A*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (aA)\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx\\
&= -\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2A\right)\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx\\
&= -\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^3A\right)\int \cos^2(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5a^3A\cos^3(c+dx)}{12d} + \frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d} + \frac{1}{8}(5a^3A)\int 1\,dx \\
&= \frac{5}{8}a^3Ax - \frac{5a^3A\cos^3(c+dx)}{12d} + \frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$
\begin{aligned}
&\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx \\
&= \frac{a^3A(60dx-48\cos(c+dx)-16\cos(3(c+dx))+24\sin(2(c+dx))-3\sin(4(c+dx)))}{96d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
[Out] (a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x)
] - 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

method	result
parallelrisch	$-\frac{A\,a^3(-60dx+48\cos(dx+c)+3\sin(4dx+4c)+16\cos(3dx+3c)-24\sin(2dx+2c)+64)}{96d}$
risch	$\frac{5a^3Ax}{8}-\frac{a^3A\cos(dx+c)}{2d}-\frac{A\,a^3\sin(4dx+4c)}{32d}-\frac{A\,a^3\cos(3dx+3c)}{6d}+\frac{A\,a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{-A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2A\,a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2A\,a^3\cos(dx+c)+A\,a^3(dx+c}{d}$
default	$\frac{-A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2A\,a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2A\,a^3\cos(dx+c)+A\,a^3(dx+c}{d}$
parts	$a^3Ax-\frac{2a^3A\cos(dx+c)}{d}+\frac{2A\,a^3(2+\sin^2(dx+c))\cos(dx+c)}{3d}-\frac{A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
norman	$-\frac{4A\,a^3}{3d}+\frac{5a^3Ax}{8}-\frac{4A\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4A\,a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4A\,a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3A\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{11A\,a^3\left(\tan^3\left(\frac{dx}{2}\right.\right.}{4d}$

```
[In] int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/96*A*a^3*(-60*d*x+48*cos(d*x+c)+3*sin(4*d*x+4*c)+16*cos(3*d*x+3*c)-24*si
n(2*d*x+2*c)+64)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{16\,Aa^3\cos\left(dx+c\right)^3-15\,Aa^3dx+3\left(2\,Aa^3\cos\left(dx+c\right)^3-5\,Aa^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3 -
 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(78) = 156.

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.39

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{8}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}-\frac{3Aa^3x\cos^4(c+dx)}{8}+Aa^3x+\frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{2Aa^3\sin^2(c}{} \\ x(-A\sin(c)+A)\left(a\sin(c)+a\right)^3 \end{cases}$$

```
[In] integrate((a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c + d
*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*A*a
**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d) - 2
*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{64\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)Aa^3+3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\sin\left(2\,dx+2\,c\right)\right)Aa^3-96}{96\,d}$$

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3*c
os(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5}{8}Aa^3x-\frac{Aa^3\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{Aa^3\cos\left(dx+c\right)}{2\,d}-\frac{Aa^3\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{Aa^3\sin\left(2\,dx+2\,c\right)}{4\,d}$$

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/3
2*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.05

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5\,A\,a^3\,x}{8}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-32)}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-96)}{24}\right)+}{}$$

```
[In] int((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*A*a^3*x)/8 - (tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(
60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/6
- (A*a^3*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c + 1
5*d*x))/4 - (A*a^3*(90*c + 90*d*x - 96))/24) - (3*A*a^3*tan(c/2 + (d*x)/2))
/4 - (11*A*a^3*tan(c/2 + (d*x)/2)^3)/4 + (11*A*a^3*tan(c/2 + (d*x)/2)^5)/4
+ (3*A*a^3*tan(c/2 + (d*x)/2)^7)/4 + (A*a^3*(15*c + 15*d*x))/24 - (A*a^3*(1
5*c + 15*d*x - 32))/24)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = a^3Ax - \frac{a^3A\text{arctanh}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos(c+dx)\sin(c+dx)}{d}$$

```
[Out] a^3*A*x-a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-1/3*a^3*A*cos(d*x+c)
^3/d+a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3045, 3855, 2715, 8, 2713}

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A\text{arctanh}(\cos(c+dx))}{d} - \frac{a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos(c+dx)}{d} + \frac{a^3A\sin(c+dx)\cos(c+dx)}{d} + a^3Ax$$

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*A
*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(2a^3A + a^3A\csc(c+dx) - 2a^3A\sin^2(c+dx) - a^3A\sin^3(c+dx)\right)\,dx$$

$$= 2a^3Ax + (a^3A)\int \csc(c+dx)\,dx - (a^3A)\int \sin^3(c+dx)\,dx - (2a^3A)\int \sin^2(c+dx)\,dx$$

$$= 2a^3Ax - \frac{a^3A\text{arctanh}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx)\sin(c+dx)}{d} - (a^3A)\int 1\,dx + \frac{(a^3A)\,\text{Subst}\left(\int (1-x^2)\,dx, x, \cos(c+dx)\right)}{d}$$

$$= a^3Ax - \frac{a^3A\text{arctanh}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos(c+dx)\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^3A\left(9\cos(c+dx)-\cos(3(c+dx))+6\left(-2c+2dx-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\mathrm{s}\right.\right.}{12d}$$

[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] (a^3*A*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(-2*c + 2*d*x - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12*d)

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{A\,a^3\left(12dx+9\cos(dx+c)-\cos(3dx+3c)+6\sin(2dx+2c)+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)}{12d}$
derivativedivides	$\frac{\frac{A\,a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2A\,a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A\,a^3(dx+c)+A\,a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}$
default	$\frac{\frac{A\,a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2A\,a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A\,a^3(dx+c)+A\,a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}$
risch	$a^3Ax+\frac{3A\,a^3e^{i(dx+c)}}{8d}+\frac{3A\,a^3e^{-i(dx+c)}}{8d}-\frac{A\,a^3\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{A\,a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{A\,a^3\cos(3dx+3c)}{12d}+$
norman	$\frac{a^3Ax+a^3Ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4A\,a^3}{3d}+\frac{4A\,a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16A\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2A\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2A\,a^3\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\right.}$

[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/12*A*a^3*(12*d*x+9*cos(d*x+c)-cos(3*d*x+3*c)+6*sin(2*d*x+2*c)+12*ln(tan(1/2*d*x+1/2*c))+8)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{2\,Aa^3\cos\left(dx+c\right)^3-6\,Aa^3dx-6\,Aa^3\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,Aa^3\cos\left(dx+c\right)+3\,Aa^3\log\left(\frac{1}{2}\cos\right.}{6\,d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
[Out] -1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x +
 c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^3*
log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int\left(-2\sin\left(c+dx\right)\csc\left(c+dx\right)\right)\,dx+\int 2\sin^3\left(c+dx\right)\csc\left(c+dx\right)dx\right.$$
$$\left.+\int\sin^4\left(c+dx\right)\csc\left(c+dx\right)dx+\int\left(-\csc\left(c+dx\right)\right)\,dx\right)$$

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*x
)**3*csc(c + d*x), x) + Integral(sin(c + d*x)**4*csc(c + d*x), x) + Integra
l(-csc(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{2\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)Aa^3+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)Aa^3-12\left(dx+c\right)Aa^3+6\,Aa^3\,\mathrm{l}}{6\,d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

[Out] -1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*log(cot(d*x + c) + csc(d*x + c)))/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3\,(dx+c)Aa^3 + 3\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,Aa\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1/2*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d

Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.79

$$\begin{aligned}\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= \frac{-2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,A\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}\\ &\quad+\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{4\,A^2\,a^6}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}\\ &\quad+\frac{A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}\end{aligned}$$

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)

[Out] ((4*A*a^3)/3 + 2*A*a^3*tan(c/2 + (d*x)/2) + 4*A*a^3*tan(c/2 + (d*x)/2)^2 - 2*A*a^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*A*a^3*atan((4*A^2*a^6)/(4*A^2*a^6 - 4*A^2*a^6*tan(c/2 + (d*x)/2)) + (4*A^2*a^6*tan(c/2 + (d*x)/2))/(4*A^2*a^6 - 4*A^2*a^6*tan(c/2 + (d*x)/2))))/d + (A*a^3*log(tan(c/2 + (d*x)/2)))/d

3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 79

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{1}{2}a^3Ax-\frac{2a^3A\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -1/2*a^3*A*x-2*a^3*A*arctanh(cos(d*x+c))/d+2*a^3*A*cos(d*x+c)/d-a^3*A*cot(d
*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3029, 2788, 3855, 3852, 8, 2718, 2715}

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{2a^3A\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{2d}-\frac{1}{2}a^3Ax$$

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a^3*A*x) - (2*a^3*A*ArcTanh[Cos[c + d*x]])/d + (2*a^3*A*Cos[c + d*x])
/d - (a^3*A*Cot[c + d*x])/d + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (aA)\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx \\
&= \frac{A\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c+dx)\right)\,dx}{a} \\
&= \left(a^3A\right)\int \csc^2(c+dx)\,dx-\left(a^3A\right)\int \sin^2(c+dx)\,dx \\
&\quad +\left(2a^3A\right)\int \csc(c+dx)\,dx-\left(2a^3A\right)\int \sin(c+dx)\,dx \\
&= -\frac{2a^3A\text{arctanh}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d} \\
&\quad -\frac{1}{2}\left(a^3A\right)\int 1\,dx-\frac{\left(a^3A\right)\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d} \\
&= -\frac{1}{2}a^3Ax-\frac{2a^3A\text{arctanh}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d} \\
&\quad -\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$
\begin{aligned}
&\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx \\
&= \frac{a^3A\left(-2c-2dx+8\cos(c)\cos(dx)-4\cot(c+dx)-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\right.}{4d}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-2*c - 2*d*x + 8*Cos[c]*Cos[d*x] - 4*Cot[c + d*x] - 8*Log[Cos[(c +
d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] - 8*Sin[c]*Sin[d*x] + Sin[2*(c + d*x)]))
/(4*d)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-A\,a^{3}\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A\,a^{3}\cos(dx+c)+2A\,a^{3}\ln(\csc(dx+c)-\cot(dx+c))-A\,a^{3}\cot(dx+c)}{d}$
default	$\frac{-A\,a^{3}\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A\,a^{3}\cos(dx+c)+2A\,a^{3}\ln(\csc(dx+c)-\cot(dx+c))-A\,a^{3}\cot(dx+c)}{d}$
parallelrisch	$-\frac{a^{3}\left(-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\cos(dx+c)-\frac{\cos(2dx+2c)}{2}+4\sin(dx+c)-\frac{5}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)+dx\right)}{2d}$
risch	$-\frac{a^{3}Ax}{2}-\frac{iA\,a^{3}e^{2i(dx+c)}}{8d}+\frac{A\,a^{3}e^{i(dx+c)}}{d}+\frac{A\,a^{3}e^{-i(dx+c)}}{d}+\frac{iA\,a^{3}e^{-2i(dx+c)}}{8d}-\frac{2iA\,a^{3}}{d\left(e^{2i(dx+c)}-1\right)}-\frac{2A\,a^{3}\ln(e^{i}}{d}$
norman	$\frac{\frac{4A\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{A\,a^{3}}{2d}+\frac{4A\,a^{3}\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12A\,a^{3}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12A\,a^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{A\,a^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{A\,a^{3}}{}}{}$

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a^3*cos(d*x+c)+2
*A*a^3*ln(csc(d*x+c)-cot(d*x+c))-A*a^3*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{Aa^3\cos(dx+c)^3+2\,Aa^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,Aa^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{2\,d\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x + c)
 + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -Aa^3\left(\int(-2\sin(c+dx)\csc^2(c+dx))\,dx + \int 2\sin^3(c+dx)\csc^2(c+dx)\,dx + \int \sin^4(c+dx)\csc^2(c+dx)\,dx + \int(-\csc^2(c+dx))\,dx\right)$$

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c +
d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**2, x)
+ Integral(-csc(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))Aa^3+4\,Aa^3(\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1))-8\,Aa^3\cos(}{4\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
[Out] -1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) - 8*A*a^3*cos(d*x + c) + 4*A*a^3/tan(d*x + c))/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{(dx+c)Aa^3-4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(Aa^3\tan\left(\right.\right.}{}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*((d*x + c)*A*a^3 - 4*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - A*a^3*tan(
1/2*d*x + 1/2*c) + (4*A*a^3*tan(1/2*d*x + 1/2*c) + A*a^3)/tan(1/2*d*x + 1/2
*c) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - A*
a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.86

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{A\,a^3\,\mathrm{atan}\left(\frac{A^2\,a^6}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$-\frac{3\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+A\,a^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+\frac{2\,A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (A*a^3*atan((A^2*a^6)/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2)) - (4*A^2*a^6
*tan(c/2 + (d*x)/2))/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2))))/d - (A*a^3
- 8*A*a^3*tan(c/2 + (d*x)/2) - 8*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c
/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(
c/2 + (d*x)/2)^5)) + (2*A*a^3*log(tan(c/2 + (d*x)/2)))/d + (A*a^3*tan(c/2 +
 (d*x)/2))/(2*d)
```

3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -2a^3Ax - \frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{2d} + \frac{a^3A\cos(c+dx)}{d} - \frac{2a^3A\cot(c+dx)}{d} - \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -2*a^3*A*x-1/2*a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-2*a^3*A*cot(d
*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3045, 3852, 8, 3853, 3855, 2718}

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{2d} + \frac{a^3A\cos(c+dx)}{d} - \frac{2a^3A\cot(c+dx)}{d} - \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d} - 2a^3Ax$$

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d -
 (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-2a^3A + 2a^3A\csc^2(c+dx) + a^3A\csc^3(c+dx) - a^3A\sin(c+dx)\right)\,dx \\
&= -2a^3Ax + \left(a^3A\right)\int \csc^3(c+dx)\,dx - \left(a^3A\right)\int \sin(c+dx)\,dx + \left(2a^3A\right)\int \csc^2(c+dx)\,dx \\
&= -2a^3Ax + \frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d} \\
&\quad + \frac{1}{2}\left(a^3A\right)\int \csc(c+dx)\,dx - \frac{\left(2a^3A\right)\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{d}
\end{aligned}$$

$$= -2a^3Ax - \frac{a^3A\text{arctanh}(\cos(c+dx))}{2d} + \frac{a^3A\cos(c+dx)}{d} - \frac{2a^3A\cot(c+dx)}{d} - \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.82

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -2a^3Ax + \frac{a^3A\cos(c)\cos(dx)}{d} - \frac{2a^3A\cot(c+dx)}{d} - \frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a^3A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^3A\sin(c)\sin(dx)}{d}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A*
Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*Lo
g[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*Sin[
c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{A\,a^3\cos(dx+c)-2A\,a^3(dx+c)-2A\,a^3\cot(dx+c)+A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{A\,a^3\cos(dx+c)-2A\,a^3(dx+c)-2A\,a^3\cot(dx+c)+A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
parallelrisch	$\frac{A\,a^3\left(-16dx+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\sec^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\csc^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)+23\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots\right.}{8d}$
risch	$-2a^3Ax+\frac{A\,a^3e^{i(dx+c)}}{2d}+\frac{A\,a^3e^{-i(dx+c)}}{2d}+\frac{A\,a^3\left(e^{3i(dx+c)}+e^{i(dx+c)}-4ie^{2i(dx+c)}+4i\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{A\,a^3\ln(e^{i(dx+c)}+1\cdots}{2d}$
norman	$\frac{\frac{A\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{A\,a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{A\,a^3}{8d}+\frac{3A\,a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5A\,a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{27A\,a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\cdots}{\cdots}$

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

[Out] 1/d*(A*a^3*cos(d*x+c)-2*A*a^3*(d*x+c)-2*A*a^3*cot(d*x+c)+A*a^3*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.95

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{8\,Aa^3dx\cos{(dx+c)}^2-4\,Aa^3\cos{(dx+c)}^3-8\,Aa^3dx-8\,Aa^3\cos{(dx+c)}\sin{(dx+c)}+2\,Aa^3\cos{(dx}}{4\,(d\cos}$$

[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x - 8*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)

Sympy [F]

$$\begin{aligned}&\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=-Aa^3\left(\int\left(-2\sin{(c+dx)}\csc^3{(c+dx)}\right)\,dx+\int 2\sin^3{(c+dx)}\csc^3{(c+dx)}\,dx\right.\\&\left.\quad+\int\sin^4{(c+dx)}\csc^3{(c+dx)}\,dx+\int\left(-\csc^3{(c+dx)}\right)\,dx\right)\end{aligned}$$

[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)

[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**3, x) + Integral(2*sin(c + d*x)**3*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**3, x) + Integral(-csc(c + d*x)**3, x))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{8\,(dx+c)Aa^3 - Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - 4\,Aa^3\cos\left(dx+c\right.}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*(8*(d*x + c)*A*a^3 - A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 4*A*a^3*cos(d*x + c) + 8*A*a^3
/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 16\,(dx+c)Aa^3 + 4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{16}{\tan\left(\frac{1}{2}\,d\right.}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(ta
n(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x +
1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.82

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{A\,a^3\left(\frac{\cos(c+d\,x)}{2}-4\,\mathrm{atan}\left(\frac{\sqrt{17}\,\left(4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{17\,\cos\left(\frac{c}{2}-\mathrm{atan}(4)+\frac{d\,x}{2}\right)}\right)-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\cos\left(2\,c+2\,d\,x\right)+\frac{\cos(3\,c+3\,d\,x)}{2}+2 \cdots}{2\,d\,\left(\cos\left(c+\cdots\right.\right.}$$

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (A*a^3*(cos(c + d*x)/2 - 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 +
 (d*x)/2)))/(17*cos(c/2 - atan(4) + (d*x)/2))) - log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))/2 + cos(2*c + 2*d*x) + cos(3*c + 3*d*x)/2 + 2*sin(2*c + 2*
d*x) + 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(17*co
s(c/2 - atan(4) + (d*x)/2)))*cos(2*c + 2*d*x) + (log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 - 1))/(2*d*(cos(c + d*x)^2 - 1))
```

3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -a^3Ax+\frac{a^3A\text{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}$$

```
[Out] -a^3*A*x+a^3*A*arctanh(cos(d*x+c))/d-a^3*A*cot(d*x+c)/d-1/3*a^3*A*cot(d*x+c
)^3/d-a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 3855, 3853, 3852}

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A\text{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a^
3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-a^3A - 2a^3A\csc(c+dx) + 2a^3A\csc^3(c+dx) + a^3A\csc^4(c+dx)\right)\,dx \\
&= -a^3Ax + \left(a^3A\right)\int \csc^4(c+dx)\,dx - \left(2a^3A\right)\int \csc(c+dx)\,dx + \left(2a^3A\right)\int \csc^3(c+dx)\,dx \\
&= -a^3Ax + \frac{2a^3A\operatorname{arctanh}(\cos(c+dx))}{d} - \frac{a^3A\cot(c+dx)\csc(c+dx)}{d} \\
&\quad + \left(a^3A\right)\int \csc(c+dx)\,dx - \frac{\left(a^3A\right)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(c+dx)\right)}{d} \\
&= -a^3Ax + \frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{d} - \frac{a^3A\cot(c+dx)}{d} \\
&\quad - \frac{a^3A\cot^3(c+dx)}{3d} - \frac{a^3A\cot(c+dx)\csc(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.81

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A\left(24c+24dx+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
[Out] -1/24*(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 2
4*Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 -
 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2
- 8*Tan[(c + d*x)/2]))/d
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{A a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24dx+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-9\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-24}{24d}$
derivativedivides	$\frac{-A a^3(dx+c)-2A a^3\ln(\csc(dx+c)-\cot(dx+c))+2A a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+A a^3\left(-\frac{2}{3}-\right.}{d}$
default	$\frac{-A a^3(dx+c)-2A a^3\ln(\csc(dx+c)-\cot(dx+c))+2A a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+A a^3\left(-\frac{2}{3}-\right.}{d}$
risch	$-a^3Ax+\frac{2A a^3\left(3\,e^{5i(dx+c)}+6i e^{2i(dx+c)}-2i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{A a^3\ln\left(e^{i(dx+c)}+1\right)}{d}-\frac{A a^3\ln\left(e^{i(dx+c)}-1\right)}{d}$
norman	$\frac{2A a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{A a^3}{24d}+\frac{6A a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11A a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{21A a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{A a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-$

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
[Out] 1/24*A*a^3*(tan(1/2*d*x+1/2*c)^3-cot(1/2*d*x+1/2*c)^3+6*tan(1/2*d*x+1/2*c)^
2-6*cot(1/2*d*x+1/2*c)^2-24*d*x+9*tan(1/2*d*x+1/2*c)-9*cot(1/2*d*x+1/2*c)-2
4*ln(tan(1/2*d*x+1/2*c)))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(76) = 152$.

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.24

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{4\,Aa^3\cos{(dx+c)}^3-6\,Aa^3\cos{(dx+c)}-3\,\left(Aa^3\cos{(dx+c)}^2-Aa^3\right)\log{\left(\frac{1}{2}\,\cos{(dx+c)}+\frac{1}{2}\right)}\sin{(dx+}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c)
^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x + c
)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*cos(d
*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
2 - d)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int \left(-2\sin{(c+dx)}\csc^4{(c+dx)}\right)\,dx+\int 2\sin^3{(c+dx)}\csc^4{(c+dx)}\,dx + \int \sin^4{(c+dx)}\csc^4{(c+dx)}\,dx+\int \left(-\csc^4{(c+dx)}\right)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**4, x) + Integral(2*sin(c +
d*x)**3*csc(c + d*x)**4, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**4, x)
+ Integral(-csc(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{6\,(dx+c)Aa^3-3\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log{(\cos{(dx+c)}+1)}+\log{(\cos{(dx+c)}-1)}\right)-6\,Aa^3(\log{(\cos{(d}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6*(6*(d*x + c)*A*a^3 - 3*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3
)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.92

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)Aa^3-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,Aa}{24\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(d
*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2*d
*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1/2*
c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.14

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{\frac{A\,a^3\,\sin(2\,c+2\,d\,x)}{2}-\frac{A\,a^3\,\cos(3\,c+3\,d\,x)}{6}+\frac{A\,a^3\,\cos(c+d\,x)}{2}-\frac{A\,a^3\,\sin(3\,c+3\,d\,x)\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,A\,a^3\,\sin}{}}{\frac{3\,d\,\sin(c+d\,x)}{4}-\frac{d\,\sin}{}}$$

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] -((A*a^3*sin(2*c + 2*d*x))/2 - (A*a^3*cos(3*c + 3*d*x))/6 + (A*a^3*cos(c +
d*x))/2 - (A*a^3*sin(3*c + 3*d*x)*atan((2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c
/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x)/2))))/2 + (3*A*a^3*sin(c + d*x)*l
og(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*A*a^3*sin(c + d*x)*atan((
2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x
)/2))))/2 - (A*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/((3*d*sin(c + d*x))/4 - (d*sin(3*c + 3*d*x))/4)
```

3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5a^3A\text{arctanh}(\cos(c+dx))}{8d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] 5/8*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-3/8*a^3*A*cot(d*x+
c)*csc(d*x+c)/d-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3045, 3855, 3852, 8, 3853}

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5a^3A\text{arctanh}(\cos(c+dx))}{8d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}$$

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (3
*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x]^
3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-a^3 A\csc(c+dx)-2a^3 A\csc^2(c+dx)+2a^3 A\csc^4(c+dx)+a^3 A\csc^5(c+dx)\right)\,dx \\
&= -\left(\left(a^3A\right)\int \csc(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^5(c+dx)\,dx \\
&\quad -\left(2a^3A\right)\int \csc^2(c+dx)\,dx+\left(2a^3A\right)\int \csc^4(c+dx)\,dx \\
&= \frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^3A\right)\int \csc^3(c+dx)\,dx \\
&\quad +\frac{\left(2a^3A\right)\operatorname{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}-\frac{\left(2a^3A\right)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(c+dx)\right)}{d}
\end{aligned}
$$

$$= \frac{a^3A\text{arctanh}(\cos(c+dx))}{d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{8}(3a^3A)\int\csc(c+dx)\,dx$$

$$= \frac{5a^3A\text{arctanh}(\cos(c+dx))}{8d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. $2(86) = 172$.

Time = 0.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.44

$$\int\csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d} - \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12d}\right)$$

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d
*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[
(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2
]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-A a^3 \ln(\csc(dx+c)-\cot(dx+c))+2A a^3 \cot(dx+c)+2A a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+A a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\right.\right.}{d}$
default	$\frac{-A a^3 \ln(\csc(dx+c)-\cot(dx+c))+2A a^3 \cot(dx+c)+2A a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+A a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\right.\right.}{d}$
parallelrisch	$-\frac{a^3\left(\cot^4\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{16\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}{64d}$
risch	$\frac{A a^3\left(9\,\mathrm{e}^{7i(dx+c)}-33\,\mathrm{e}^{5i(dx+c)}+48i\mathrm{e}^{6i(dx+c)}-33\,\mathrm{e}^{3i(dx+c)}-48i\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+16i\mathrm{e}^{2i(dx+c)}-16i\right)}{12d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}-\frac{5A a^3 \ln(\mathrm{e}}{}$
norman	$-\frac{A a^3}{64d}-\frac{57A a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{19A a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{27A a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{49A a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{A a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{12d}$

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^3*ln(csc(d*x+c)-cot(d*x+c))+2*A*a^3*cot(d*x+c)+2*A*a^3*(-2/3-1/3*
csc(d*x+c)^2)*cot(d*x+c)+A*a^3*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+
c)+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.93

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{32\,Aa^3\cos(dx+c)^3\sin(dx+c)-18\,Aa^3\cos(dx+c)^3+30\,Aa^3\cos(dx+c)-15\left(Aa^3\cos(dx+c)^4-\right.}{48\,(d\,c}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30*
A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*
a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d
*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*co
s(d*x + c)^2 + d)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.69

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{3\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+24\,Aa^3(\log\left(\cos\left(dx\right.\right.}{48\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
[Out] 1/48*(3*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 24
*A*a^3*(log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 96*A*a^3/tan(d*x +
 c) - 32*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(78) = 156.

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.02

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
[Out] 1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2
4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 48*A*a^3*tan(1/2*d*x + 1/2*c) + (250*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 48*A*a
^3*tan(1/2*d*x + 1/2*c)^3 - 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.84

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-16\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d}{2}\right.\right.}{}$$

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^8 - 3*sin(c/2 + (d*x)/2)^8 - 16*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^7 + 16*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 2
4*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 48*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^5 - 48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2
 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4))/(192*d*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^4)
```

3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 105

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A\text{arctanh}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot^5(c+dx)}{5d}+\frac{a^3A\cot(c+dx)\csc(c+dx)}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}$$

```
[Out] 1/4*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-1/5*a^3*A*cot(d*x+
c)^5/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^3/
d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3029, 2788, 3852, 8, 3853, 3855}

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A\text{arctanh}(\cos(c+dx))}{4d}-\frac{a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3A\cot(c+dx)\csc(c+dx)}{4d}$$

```
[In] Int[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (a^3
*A*Cot[c + d*x]^5)/(5*d) + (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^3*A
*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = (a^3A^3)\int \frac{\cot^6(c+dx)}{(A-A\sin(c+dx))^2}\,dx$$

$$
\begin{aligned}
&= \frac{a^3 \int (-A^4 \csc^2(c+dx) - 2A^4 \csc^3(c+dx) + 2A^4 \csc^5(c+dx) + A^4 \csc^6(c+dx)) \, dx}{A^3} \\
&= -\left((a^3 A) \int \csc^2(c+dx) \, dx\right) + (a^3 A) \int \csc^6(c+dx) \, dx \\
&\quad - (2a^3 A) \int \csc^3(c+dx) \, dx + (2a^3 A) \int \csc^5(c+dx) \, dx \\
&= \frac{a^3 A \cot(c+dx) \csc(c+dx)}{d} - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{2d} \\
&\quad - (a^3 A) \int \csc(c+dx) \, dx + \frac{1}{2}(3a^3 A) \int \csc^3(c+dx) \, dx \\
&\quad + \frac{(a^3 A) \operatorname{Subst}(\int 1 \, dx, x, \cot(c+dx))}{d} \\
&\quad - \frac{(a^3 A) \operatorname{Subst}(\int (1+2x^2+x^4) \, dx, x, \cot(c+dx))}{d} \\
&= \frac{a^3 A \operatorname{arctanh}(\cos(c+dx))}{d} - \frac{2a^3 A \cot^3(c+dx)}{3d} \\
&\quad - \frac{a^3 A \cot^5(c+dx)}{5d} + \frac{a^3 A \cot(c+dx) \csc(c+dx)}{4d} \\
&\quad - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{2d} + \frac{1}{4}(3a^3 A) \int \csc(c+dx) \, dx \\
&= \frac{a^3 A \operatorname{arctanh}(\cos(c+dx))}{4d} - \frac{2a^3 A \cot^3(c+dx)}{3d} - \frac{a^3 A \cot^5(c+dx)}{5d} \\
&\quad + \frac{a^3 A \cot(c+dx) \csc(c+dx)}{4d} - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.55

$$
\begin{aligned}
&\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx)) \, dx \\
&= a^3 A \left(\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{30d} + \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{16d} - \frac{19\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{480d} \right. \\
&\quad - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{160d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d} \\
&\quad - \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{16d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{30d} \\
&\quad \left. + \frac{19\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{480d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{160d} \right)
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((7*Cot[(c + d*x)/2])/(30*d) + Csc[(c + d*x)/2]^2/(16*d) - (19*Cot[(c
 + d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - Csc[(c + d*x)/2]^4/(32*d) - (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + Log[Cos[(c + d*x)/2]]/(4*d) - Log
[Sin[(c + d*x)/2]]/(4*d) - Sec[(c + d*x)/2]^2/(16*d) + Sec[(c + d*x)/2]^4/(
32*d) - (7*Tan[(c + d*x)/2])/(30*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d))
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result
parallelrisch	$-\frac{a^3\left(\cot^5\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\left(\cot^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{25\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3}{160d}$
derivativedivides	$\frac{A\,a^3\cot(dx+c)-2A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2A\,a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx}{d}$
default	$\frac{A\,a^3\cot(dx+c)-2A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2A\,a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx}{d}$
risch	$-\frac{A\,a^3\left(-60i e^{8i(dx+c)}+15\,e^{9i(dx+c)}+240i e^{6i(dx+c)}+90\,e^{7i(dx+c)}-40i e^{4i(dx+c)}+80i e^{2i(dx+c)}-90\,e^{3i(dx+c)}-28i-15\,e\right.}{30d\left(e^{2i(dx+c)}-1\right)^5}$

```
[In] int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/160*a^3*(cot(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^5+5*cot(1/2*d*x+1/2*c)^
4-5*tan(1/2*d*x+1/2*c)^4+25/3*cot(1/2*d*x+1/2*c)^3-25/3*tan(1/2*d*x+1/2*c)^
3-30*cot(1/2*d*x+1/2*c)+30*tan(1/2*d*x+1/2*c)+40*ln(tan(1/2*d*x+1/2*c)))*A/
d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.91

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{56\,Aa^3\cos\left(dx+c\right)^5-80\,Aa^3\cos\left(dx+c\right)^3+15\left(Aa^3\cos\left(dx+c\right)^4-2\,Aa^3\cos\left(dx+c\right)^2+Aa^3\right)\log\left(\frac{1}{2}\,\mathrm{c}\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(56*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 + 15*(A*a^3*cos(d*
x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(A*a^3*cos(d*x + c)^3 + A*a^3*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{15\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} - 3\,\log\left(\cos\left(dx+c\right)+1\right) + 3\,\log\left(\cos\left(dx+c\right)-1\right)\right) - 60\,Aa^3\left(\frac{2\,\cos(dx+c}{\cos(dx+c)^2}\right.}{120\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 10*ta
n(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.66

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 2
5*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a
^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\right.\right.}{}$$

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) -
 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 90*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^6 - 90*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 25*cos(c
/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(480*d*cos(c/2 + (d*
x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 130

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3a^3A\text{arctanh}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{2a^3A\cot^5(c+dx)}{5d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}-\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}$$

```
[Out] 3/16*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-2/5*a^3*A*cot(d*x
+c)^5/d+3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-5/24*a^3*A*cot(d*x+c)*csc(d*x+c)
^3/d-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 3853, 3855, 3852}

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3a^3A\text{arctanh}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}$$

```
[In] Int[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d)
- (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc[c
+ d*x]^5)/(6*d)
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(-a^3A\csc^3(c+dx)-2a^3A\csc^4(c+dx)+2a^3A\csc^6(c+dx)+a^3A\csc^7(c+dx)\right)\,dx$$

$$= -\left(\left(a^3A\right)\int \csc^3(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^7(c+dx)\,dx - \left(2a^3A\right)\int \csc^4(c+dx)\,dx+\left(2a^3A\right)\int \csc^6(c+dx)\,dx$$

$$= \frac{a^3 A \cot(c+dx) \csc(c+dx)}{2d} - \frac{a^3 A \cot(c+dx) \csc^5(c+dx)}{6d} - \frac{1}{2}\left(a^3 A\right) \int \csc(c+dx)\,dx + \frac{1}{6}\left(5a^3 A\right) \int \csc^5(c+dx)\,dx + \frac{(2a^3 A)\,\text{Subst}\left(\int (1+x^2)\,dx, x, \cot(c+dx)\right)}{d} - \frac{(2a^3 A)\,\text{Subst}\left(\int (1+2x^2+x^4)\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{a^3 A \text{arctanh}(\cos(c+dx))}{2d} - \frac{2a^3 A \cot^3(c+dx)}{3d} - \frac{2a^3 A \cot^5(c+dx)}{5d} + \frac{a^3 A \cot(c+dx) \csc(c+dx)}{2d} - \frac{5a^3 A \cot(c+dx) \csc^3(c+dx)}{24d} - \frac{a^3 A \cot(c+dx) \csc^5(c+dx)}{6d} + \frac{1}{8}\left(5a^3 A\right) \int \csc^3(c+dx)\,dx$$

$$= \frac{a^3 A \text{arctanh}(\cos(c+dx))}{2d} - \frac{2a^3 A \cot^3(c+dx)}{3d} - \frac{2a^3 A \cot^5(c+dx)}{5d} + \frac{3a^3 A \cot(c+dx) \csc(c+dx)}{16d} - \frac{5a^3 A \cot(c+dx) \csc^3(c+dx)}{24d} - \frac{a^3 A \cot(c+dx) \csc^5(c+dx)}{6d} + \frac{1}{16}\left(5a^3 A\right) \int \csc(c+dx)\,dx$$

$$= \frac{3a^3 A \text{arctanh}(\cos(c+dx))}{16d} - \frac{2a^3 A \cot^3(c+dx)}{3d} - \frac{2a^3 A \cot^5(c+dx)}{5d} + \frac{3a^3 A \cot(c+dx) \csc(c+dx)}{16d} - \frac{5a^3 A \cot(c+dx) \csc^3(c+dx)}{24d} - \frac{a^3 A \cot(c+dx) \csc^5(c+dx)}{6d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 306 vs. $2(130) = 260$.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.35

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d}+\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{240d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{80d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{240d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{80d}\right)$$

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (3*L
og[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Sec[(c
 + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/(384*
d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2
40*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{25a^3\left(\frac{1536\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}+\left(\sec^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)-\frac{13\cos(3dx+3c)}{150}-\frac{3\cos(5dx+5c)}{50}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.\right.}{8192d}$
derivativedivides	$\frac{-A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2A\,a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+2A\,a^3\left(-\frac{8}{15}-\frac{\left(\csc^4(d\right.}{5}\right.}{}$
default	$\frac{-A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2A\,a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+2A\,a^3\left(-\frac{8}{15}-\frac{\left(\csc^4(d\right.}{5}\right.}{}$
risch	$-\frac{A\,a^3\left(45\,e^{11i(dx+c)}+65\,e^{9i(dx+c)}-750\,e^{7i(dx+c)}+960i e^{8i(dx+c)}-750\,e^{5i(dx+c)}-640i e^{6i(dx+c)}+65\,e^{3i(dx+c)}+45\,e^{i(d\right.}{120d\left(e^{2i(dx+c)}-1\right)^6}$

```
[In] int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] -25/8192*a^3*(1536/25*ln(tan(1/2*d*x+1/2*c))+sec(1/2*d*x+1/2*c)^5*(sec(1/2*
d*x+1/2*c)*(cos(d*x+c)-13/150*cos(3*d*x+3*c)-3/50*cos(5*d*x+5*c))*csc(1/2*d
*x+1/2*c)+256/75*cos(d*x+c)+64/75*cos(3*d*x+3*c)-64/375*cos(5*d*x+5*c))*csc
(1/2*d*x+1/2*c)^5)*A/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{90\,Aa^3\cos(dx+c)^5 - 80\,Aa^3\cos(dx+c)^3 - 90\,Aa^3\cos(dx+c) - 45\,\left(Aa^3\cos(dx+c)^6 - 3\,Aa^3\cos(dx\right.}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d*
x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*
x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6 -
3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x +
 c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(d*x +
 c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.59

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{5\,Aa^3\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-1}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(5*A*a^3*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))
/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*
x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 120*A*a^3*(2*cos(d*x + c)/(cos(d*
x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 320*(3*tan
(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3 - 64*(15*tan(d*x + c)^4 + 10*tan(d*x
+ c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(118) = 236.

Time = 0.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.86

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1920*(5*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
45*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^
3*tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 240*A
*a^3*tan(1/2*d*x + 1/2*c) + (882*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 240*A*a^3*t
an(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*tan(1/2*
d*x + 1/2*c)^3 - 45*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*tan(1/2*d*x + 1
/2*c) - 5*A*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.62

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\right.\right.}{}$$

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(A*a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
 - 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2
 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 +
 (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)
/2)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6
*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 129

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{41A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-\frac{199A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

```
[Out] -19/2*A*x/a^3-4*A*cos(d*x+c)/a^3/d+1/2*A*cos(d*x+c)*sin(d*x+c)/a^3/d-2/5*A*
cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+41/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2
-199/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used

$= \{3045, 2718, 2715, 8, 2729, 2727\}$

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d} - \frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)} + \frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2} - \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} - \frac{19Ax}{2a^3}$$

```
[In] Int[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A*C
os[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*a^3
*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{9A}{a^3} + \frac{4A\sin(c+dx)}{a^3} - \frac{A\sin^2(c+dx)}{a^3} + \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{9A}{a^3(1+\sin(c+dx))^2} + \frac{16A}{a^3(1+\sin(c+dx))}\right) dx \\
&= -\frac{9Ax}{a^3} - \frac{A\int \sin^2(c+dx)\,dx}{a^3} + \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} + \frac{(4A)\int \sin(c+dx)\,dx}{a^3} - \frac{(9A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} + \frac{(16A)\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} \\
&= -\frac{9Ax}{a^3} - \frac{4A\cos(c+dx)}{a^3d} + \frac{A\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx)}{a^3d(1+\sin(c+dx))^2} - \frac{16A\cos(c+dx)}{a^3d(1+\sin(c+dx))} - \frac{A\int 1\,dx}{2a^3} + \frac{(4A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3} - \frac{(3A)\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} \\
&= -\frac{19Ax}{2a^3} - \frac{4A\cos(c+dx)}{a^3d} + \frac{A\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{41A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} - \frac{13A\cos(c+dx)}{a^3d(1+\sin(c+dx))} + \frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3} \\
&= -\frac{19Ax}{2a^3} - \frac{4A\cos(c+dx)}{a^3d} + \frac{A\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{41A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} - \frac{199A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.80 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.64

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\sec(c+dx)\sqrt{1-\sin(c+dx)}\Big(140\sqrt{2}\sqrt{a}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1+\sin(}{}$$

```
[In] Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*Sec[c + d*x]*Sqrt[1 - Sin[c + d*x]]*(140*Sqrt[2]*Sqrt[a]*Hypergeometric2
F1[-3/2, -3/2, -1/2, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]) - 360*ArcSin[
Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^4*Sqrt[a*(1 + Sin[c + d*x])] + Sqrt[a]*Sqrt[1 - Sin[c + d*x]]*(-30
8 - 639*Sin[c + d*x] - 433*Sin[c + d*x]^2 - 75*Sin[c + d*x]^3 + 15*Sin[c +
d*x]^4)))/(30*a^(7/2)*d*(1 + Sin[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{32A\left(-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{(\tan^3}{}\right.}{d\,a^3}$
default	$\frac{32A\left(-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{(\tan^3}{}\right.}{d\,a^3}$
risch	$-\frac{19Ax}{2a^3}-\frac{iA\,e^{2i(dx+c)}}{8a^3d}-\frac{2A\,e^{i(dx+c)}}{a^3d}-\frac{2A\,e^{-i(dx+c)}}{a^3d}+\frac{iA\,e^{-2i(dx+c)}}{8a^3d}-\frac{2\left(825iA\,e^{3i(dx+c)}+240A\,e^{4i(dx+c)}-75\right.}{15d\,a^3\left(e^{i(\right.}}$
parallelrisch	$\frac{\left((76dx-120)\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+(380dx-128)\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)+\left(76dx+\frac{3484}{15}\right)\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)+(-760dx-304)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right.}{8d\,a^3\left(-\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+5\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)\right.}$
norman	$\frac{-\frac{19Ax\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{95A\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{19A\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{391A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{1919Ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{448A}{15ad}-\frac{19}{}}{}$

```
[In] int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 32/d*A/a^3*(-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/2
4/(tan(1/2*d*x+1/2*c)+1)^3-5/16/(tan(1/2*d*x+1/2*c)+1)^2-9/16/(tan(1/2*d*x+
```

```
1/2*c)+1)-1/16*(1/2*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)^2-1/2*tan(1/2
*d*x+1/2*c)+4)/(1+tan(1/2*d*x+1/2*c)^2)^2-19/32*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(119) = 238$.

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.92

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{15\,A\cos{(dx+c)}^5+90\,A\cos{(dx+c)}^4+(285\,Adx+683\,A)\cos{(dx+c)}^3-1140\,Adx+(855\,Adx-52}{30\,\left(a^3d\cos{(dx}$$

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos(
d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*x
+ 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 1140*A
*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d*x +
 c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos
(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3614 vs. $2(126) = 252$.

Time = 22.20 (sec) , antiderivative size = 3614, normalized size of antiderivative = 28.02

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-285*A*d*x*tan(c/2 + d*x/2)**9/(30*a**3*d*tan(c/2 + d*x/2)**9 +
150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*
d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2
 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)
**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2
)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*
a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*ta
n(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d
*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 3
```

```
0*a**3*d) - 3420*A*d*x*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3
*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2
)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/
2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360
*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*t
an(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**9
+ 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c
/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/
2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x
/2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 36
0*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*
tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
 d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
 30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/2)**3/(30*a**3*d*tan(c/2 + d*x/2)**9
 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a*
*3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(
c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x
/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 3420*A*d*x*tan(c/2 + d*
x/2)**2/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 3
60*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d
*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2
+ d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2)
+ 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3
*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2
)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 285*A*d*x/(30*a**3*d*tan(
c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x
/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 +
780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*
d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 570*A*ta
n(c/2 + d*x/2)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x
/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 +
780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*
d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2
 + d*x/2) + 30*a**3*d) - 2850*A*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*
x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 +
 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3
*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/
2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 6650*A*tan(c/2 +
```

```
 d*x/2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8
+ 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**
3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c
/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/
2) + 30*a**3*d) - 10450*A*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**
9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a
**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan
(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*
x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 12846*A*tan(c/2 + d*x/
2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360
*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*t
an(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
30*a**3*d) - 11270*A*tan(c/2 + d*x/2)**3/(30*a**3*d*tan(c/2 + d*x/2)**9 + 1
50*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d
*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2
+ d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)*
*2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7902*A*tan(c/2 + d*x/2)**2/
(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*
d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2
 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)
**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**
3*d) - 3910*A*tan(c/2 + d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*
tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 +
 d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**
4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a
**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 896*A/(30*a**3*d*tan(c/2 + d*x/2)**9
+ 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c
/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/
2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d), Ne(d, 0)), (x*(-A*sin(c)
+ A)*sin(c)**4/(a*sin(c) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(119) = 238$.

Time = 0.35 (sec) , antiderivative size = 715, normalized size of antiderivative = 5.54

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

[Out] -1/15*(A*((1325*sin(d*x + c)/(cos(d*x + c) + 1) + 2673*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4329*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 3575*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 227
5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 975*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + 195*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 304)/(a^3 + 5*a^3*sin(d*x
+ c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 20*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 26*a^3*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 26*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 12*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) +
1)^9) + 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 6*A*((105*sin(d*
x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 200*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
))/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{285\,(dx+c)A}{a^3}+\frac{30\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+615\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\cdots\right.}{a^3\left(\tan\left(\frac{1}{2}\cdots\right.\right.}}{30\,d}$$

[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")

[Out] -1/30*(285*(d*x + c)*A/a^3 + 30*(A*tan(1/2*d*x + 1/2*c)^3 + 8*A*tan(1/2*d*x
 + 1/2*c)^2 - A*tan(1/2*d*x + 1/2*c) + 8*A)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2
a^3) + 4(135*A*tan(1/2*d*x + 1/2*c)^4 + 615*A*tan(1/2*d*x + 1/2*c)^3 + 10
25*A*tan(1/2*d*x + 1/2*c)^2 + 685*A*tan(1/2*d*x + 1/2*c) + 164*A)/(a^3*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.53

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\left(\frac{95\,A\,(c+d\,x)}{2}-\frac{A\,(1425\,c+1425\,d\,x+570)}{30}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(114\,A\,(c+d\,x)-\frac{A\,(3420\,c+3420\,d\,x+2850)}{30}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$
$$-\frac{19\,A\,x}{2\,a^3}$$

```
[In] int((sin(c + d*x)^4*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 3910))/30
) + tan(c/2 + (d*x)/2)^8*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 570)
)/30) + tan(c/2 + (d*x)/2)^7*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 285
0))/30) + tan(c/2 + (d*x)/2)^2*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 7
902))/30) + tan(c/2 + (d*x)/2)^6*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x +
 6650))/30) + tan(c/2 + (d*x)/2)^3*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x
 + 11270))/30) + tan(c/2 + (d*x)/2)^5*(247*A*(c + d*x) - (A*(7410*c + 7410*
d*x + 10450))/30) + tan(c/2 + (d*x)/2)^4*(247*A*(c + d*x) - (A*(7410*c + 74
10*d*x + 12846))/30) + (19*A*(c + d*x))/2 - (A*(285*c + 285*d*x + 896))/30)
/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (19*A*x)
/(2*a^3)
```

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{104A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

```
[Out] 4*A*x/a^3+A*cos(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-31/15*
A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+104/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c)
)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 2718, 2729, 2727}

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

```
[In] Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + S
in[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (10
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{4A}{a^3} - \frac{A\sin(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} + \frac{7A}{a^3(1+\sin(c+dx))^2} - \frac{9A}{a^3(1+\sin(c+dx))}\right) dx$$

$$= \frac{4Ax}{a^3} - \frac{A\int \sin(c+dx)\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} + \frac{(7A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{(9A)\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}$$

$$= \frac{4Ax}{a^3} + \frac{A\cos(c+dx)}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{7A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} + \frac{9A\cos(c+dx)}{a^3d(1+\sin(c+dx))} - \frac{(4A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3} + \frac{(7A)\int \frac{1}{1+\sin(c+dx)}\,dx}{3a^3}$$

$$= \frac{4Ax}{a^3} + \frac{A\cos(c+dx)}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{20A\cos(c+dx)}{3a^3d(1+\sin(c+dx))} - \frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3}$$

$$= \frac{4Ax}{a^3} + \frac{A\cos(c+dx)}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{104A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\sec(c+dx)\left(-120\arcsin\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\sqrt{1-\sin(c+dx)}\sqrt{a(}\right.}{15a^{7/2}d(1+\text{s}}$$

```
[In] Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] -1/15*(A*Sec[c + d*x]*(-120*ArcSin[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt
[a])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sqrt[1 - Sin[c + d*x]]*Sqrt[a
*(1 + Sin[c + d*x])] + Sqrt[a]*(-94 - 128*Sin[c + d*x] + 73*Sin[c + d*x]^2
+ 134*Sin[c + d*x]^3 + 15*Sin[c + d*x]^4)))/(a^(7/2)*d*(1 + Sin[c + d*x])^2
)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{16A\left(\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+\frac{1}{8+8\left(\tan^2\left(\frac{dx}{2}\right.\right.}\right.}{d\,a^3}$
default	$\frac{16A\left(\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+\frac{1}{8+8\left(\tan^2\left(\frac{dx}{2}\right.\right.}\right.}{d\,a^3}$
risch	$\frac{4Ax}{a^3}+\frac{A\,e^{i(dx+c)}}{2a^3d}+\frac{A\,e^{-i(dx+c)}}{2a^3d}+\frac{2A\left(435i e^{3i(dx+c)}+135\,e^{4i(dx+c)}-385i e^{i(dx+c)}-605\,e^{2i(dx+c)}+104\right)}{15d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
parallelrisch	$\frac{A\left(120dx\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)-600dx\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)+600dx\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)+120dx\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)-1200dx\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-1200d\right.}{30d\,a^3\left(\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)-5\sin\right.}$
norman	$\frac{164A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{284Ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{188A}{15ad}+\frac{40A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8A\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{204Ax\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{20Ax(}{}$

```
[In] int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/d*A/a^3*(1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan(1/2*d*x+1/2*c)+1)^4+1/12/
(tan(1/2*d*x+1/2*c)+1)^3+3/8/(tan(1/2*d*x+1/2*c)+1)^2+1/2/(tan(1/2*d*x+1/2*
c)+1)+1/8/(1+tan(1/2*d*x+1/2*c)^2)+1/2*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(97) = 194$.

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.18

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{15\,A\cos{(dx+c)}^4 + (60\,Adx + 149\,A)\cos{(dx+c)}^3 - 240\,Adx + (180\,Adx - 103\,A)\cos{(dx+c)}^2 - 3\,(40}{15\,\left(a^3d\cos{(dx+c)}^3 + 3\,a^3d\cos{(dx+c)}^2 -}$$

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x +
 (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) + (1
5*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*(40
*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(100) = 200$.

Time = 12.94 (sec) , antiderivative size = 2290, normalized size of antiderivative = 22.23

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**7/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75
*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*t
an(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 +
d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d
*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 1
65*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d
*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 +
 d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d
*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 +
```

```
 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3
*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*
x*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 +
d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4
 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**
3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/2)**3/(15*a**3*
d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2
+ d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)*
*3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*
d) + 660*A*d*x*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3
*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/
2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2
)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d*x/2)
/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*
d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2
 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
+ 15*a**3*d) + 60*A*d*x/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2
+ d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)*
*4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 120*A*tan(c/2 + d*x/2)**6/(15*a**3*d*
tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 +
d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3
 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 + 600*A*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan
(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*
x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 +
 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1280*A*tan(c/2 + d*x/2)**4/(15*a
**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(
c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x
/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a
**3*d) + 1540*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**
3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c
/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/
2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1468*A*tan(c/2 + d*x/2)**
2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3
*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/
2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
 + 15*a**3*d) + 820*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*
a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*ta
n(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d
*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 188*A/(15*a**3*d*tan(c
/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2
)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 16
5*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(
d, 0)), (x*(-A*sin(c) + A)*sin(c)**3/(a*sin(c) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(97) = 194.

Time = 0.32 (sec) , antiderivative size = 543, normalized size of antiderivative = 5.27

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2\left(3\,A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{189\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{160\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{75\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+24}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{11\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{15\,\mathrm{arc}}{}\right.\right.}{}$$

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c
) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos
(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2\left(\frac{30\,(dx+c)A}{a^3}+\frac{15\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+285\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+505\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+335\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}\right.}{15\,d}$$

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

[Out] 2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A*
tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x +
1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c) +
1)^5))/d

Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.53

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4\,A\,x}{a^3} - \frac{\left(20\,A\,(c+d\,x)-\frac{4\,A\,(75\,c+75\,d\,x+30)}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(44\,A\,(c+d\,x)-\frac{4\,A\,(165\,c+165\,d\,x+150)}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{}$$

[In] int((sin(c + d*x)^3*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)

[Out] (4*A*x)/a^3 - (tan(c/2 + (d*x)/2)*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 2
05))/15) + tan(c/2 + (d*x)/2)^6*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 30)
)/15) + tan(c/2 + (d*x)/2)^5*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 150)
)/15) + tan(c/2 + (d*x)/2)^2*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 367)
)/15) + tan(c/2 + (d*x)/2)^4*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 320)
)/15) + tan(c/2 + (d*x)/2)^3*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 385)
)/15) + 4*A*(c + d*x) - (4*A*(15*c + 15*d*x + 47))/15)/(a^3*d*(tan(c/2 + (d
x)/2) + 1)^5(tan(c/2 + (d*x)/2)^2 + 1))

3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{Ax}{a^3} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

```
[Out] -A*x/a^3-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+7/5*A*cos(d*x+c)/a^3/d/(1+
sin(d*x+c))^2-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3045, 2729, 2727}

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)} + \frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2} - \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} - \frac{Ax}{a^3}$$

```
[In] Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{A}{a^3}+\frac{2A}{a^3(1+\sin(c+dx))^3}-\frac{5A}{a^3(1+\sin(c+dx))^2}+\frac{4A}{a^3(1+\sin(c+dx))}\right) dx\\
&= -\frac{Ax}{a^3}+\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(5A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}\\
&= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{5A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}\\
&\quad -\frac{4A\cos(c+dx)}{a^3d(1+\sin(c+dx))}+\frac{(4A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3}-\frac{(5A)\int \frac{1}{1+\sin(c+dx)}\,dx}{3a^3}\\
&= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}\\
&\quad -\frac{7A\cos(c+dx)}{3a^3d(1+\sin(c+dx))}+\frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3}\\
&= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\sec(c+dx)\sqrt{1-\sin(c+dx)}\left(20\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1+\sin(c+\right.}{15a^3d(1+\sin(c+dx))^2}$$

```
[In] Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*Sec[c + d*x]*Sqrt[1 - Sin[c + d*x]]*(20*Sqrt[2]*Hypergeometric2F1[-3/2,
-3/2, -1/2, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x
]]*(-4 + 3*Sin[c + d*x] + Sin[c + d*x]^2)))/(15*a^3*d*(1 + Sin[c + d*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{Ax}{a^3}-\frac{2\left(-75A\,e^{2i(dx+c)}+55iA\,e^{3i(dx+c)}+20A\,e^{4i(dx+c)}-45iA\,e^{i(dx+c)}+13A\right)}{5d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
derivativedivides	$\frac{8A\left(-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{}{4\left(\tan\left(\right.\right.}\right.}{d\,a^3}$
default	$\frac{8A\left(-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{}{4\left(\tan\left(\right.\right.}\right.}{d\,a^3}$
parallelrisch	$-\frac{\left(\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)dx+(5dx+2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10dx+10)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10dx+22)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(5dx+14\right.}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
norman	$\frac{-\frac{Ax}{a}-\frac{16A}{5ad}-\frac{5Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{13Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{25Ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{38Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{46Ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{}$

```
[In] int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -A*x/a^3-2/5*(-75*A*exp(2*I*(d*x+c))+55*I*A*exp(3*I*(d*x+c))+20*A*exp(4*I*(
d*x+c))-45*I*A*exp(I*(d*x+c))+13*A)/d/a^3/(exp(I*(d*x+c))+I)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{(5\,Adx+13\,A)\cos{(dx+c)}^3-20\,Adx+3\,(5\,Adx-2\,A)\cos{(dx+c)}^2-(10\,Adx+21\,A)\cos{(dx+c)}-}{5\,\left(a^3d\cos{(dx+c)}^3+3\,a^3d\cos{(dx+c)}^2-2\,a^3d\cos{(dx+c)}-4\,a^3d\right.}$$

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d*
x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*co
s(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3
*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(85) = 170.

Time = 7.41 (sec) , antiderivative size = 1268, normalized size of antiderivative = 14.25

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-5*A*d*x*tan(c/2 + d*x/2)**5/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*
a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(
c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2
 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4
+ 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d
*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(
c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2
)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*
d) - 50*A*d*x*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d
*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 +
d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2 + d*x
/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3
*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2
+ d*x/2) + 5*a**3*d) - 5*A*d*x/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*ta
n(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x
```

```
/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 10*A*tan(c/2 + d*x/2)**4/
(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*t
an(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*
x/2) + 5*a**3*d) - 50*A*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(c/2 + d*x/2)**5 +
 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*
tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 110*A*tan(c/
2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4
 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*
d*tan(c/2 + d*x/2) + 5*a**3*d) - 70*A*tan(c/2 + d*x/2)/(5*a**3*d*tan(c/2 +
d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 +
 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 1
6*A/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3
*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2
+ d*x/2) + 5*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)**2/(a*sin(c) + a
)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(83) = 166$.

Time = 0.34 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.40

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(A\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{\cdots}{a^3+\frac{5a^3\cdots}{\cos\cdots}}\cdots}{15\,d}$$

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -2/15*(A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{5\,(dx+c)A}{a^3}+\frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*tan(1/2*d*x + 1/2*c)^4 + 25*A*tan(1/2*d*x
+ 1/2*c)^3 + 55*A*tan(1/2*d*x + 1/2*c)^2 + 35*A*tan(1/2*d*x + 1/2*c) + 8*A)
/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{\left(5\,A\,(c+d\,x)-\frac{A\,(25\,c+25\,d\,x+10)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(10\,A\,(c+d\,x)-\frac{A\,(50\,c+50\,d\,x+50)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(10\,A\right.}{a^3\,d} - \frac{A\,x}{a^3}$$

```
[In] int((sin(c + d*x)^2*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 70))/5) + tan(c/2
+ (d*x)/2)^4*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 10))/5) + tan(c/2 + (d*x)
/2)^3*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 50))/5) + tan(c/2 + (d*x)/2)^2*
(10*A*(c + d*x) - (A*(50*c + 50*d*x + 110))/5) + A*(c + d*x) - (A*(5*c + 5*
d*x + 16))/5)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5) - (A*x)/a^3
```

3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

```
[Out] 2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-11/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x
+c))^2+4/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3045, 2729, 2727}

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)} - \frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2} + \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

```
[In] Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(15
*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*
x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{3A}{a^3(1+\sin(c+dx))^2}-\frac{A}{a^3(1+\sin(c+dx))}\right)dx \\
&= -\frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(3A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} \\
&= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))^2} \\
&\quad +\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{(4A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3}+\frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} \\
&= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-\frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3} \\
&= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$
\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(15\cos\left(c+\frac{dx}{2}\right)-5\cos\left(c+\frac{3dx}{2}\right)+25\sin\left(\frac{dx}{2}\right)+15\sin\left(2c+\frac{3dx}{2}\right)-4\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}
$$

```
[In] Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] -1/30*(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 15
*Sin[2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{2A\left(15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{15d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
derivativedivides	$\frac{4A\left(\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}\right)}{d\,a^3}$
default	$\frac{4A\left(\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}\right)}{d\,a^3}$
risch	$\frac{2A\left(15i\,e^{3i(dx+c)}+15\,e^{4i(dx+c)}-5i\,e^{i(dx+c)}-25\,e^{2i(dx+c)}+4\right)}{15d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
norman	$\frac{-\frac{2A}{15ad}-\frac{14A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{10A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{2A\left(\tan^6\left(\frac{dx}{2}\right.\right.}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

```
[In] int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -2/15*A*(15*tan(1/2*d*x+1/2*c)^3-5*tan(1/2*d*x+1/2*c)^2+5*tan(1/2*d*x+1/2*c
)+1)/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(76) = 152.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4\,A\cos{(dx+c)}^3+7\,A\cos{(dx+c)}^2-3\,A\cos{(dx+c)}-\left(4\,A\cos{(dx+c)}^2-3\,A\cos{(dx+c)}-6\right.}{15\left(a^3d\cos{(dx+c)}^3+3\,a^3d\cos{(dx+c)}^2-2\,a^3d\cos{(dx+c)}-4\,a^3d+\left(a^3d\cos{(dx+c)}^2-2\,a^3d\cos{(d\right.\right.}$$

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
[Out] 1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*cos
(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos
(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(78) = 156$.

Time = 4.17 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.62

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \begin{cases} -\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} + \frac{}{15a^3d\tan^5\left(\frac{c}{2}+\frac{d}{}\right.} \\ \frac{x(-A\sin(c)+A)\sin(c)}{(a\sin(c)+a)^3} \end{cases}$$

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2 +
d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150
*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*ta
n(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)/(a*sin(c)
 + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(76) = 152$.

Time = 0.25 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.24

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2\left(\frac{2A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c}{(\cos(dx+c)+1)}}\right.}{15d}$$

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 2/15*(2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d
```

```
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3 + 5*a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(1
/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\,A\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-5\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+15\,\mathrm{si}\right.}{15\,a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^5}$$

```
[In] int((sin(c + d*x)*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(2*A*cos(c/2 + (d*x)/2)^2*(cos(c/2 + (d*x)/2)^3 + 15*sin(c/2 + (d*x)/2)^3
- 5*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 5*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)))/(15*a^3*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx=-\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}$$

```
[Out] -1/5*a*A*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^4-1/15*A*cos(d*x+c)^3/d/(a+a*sin(d
*x+c))^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2815, 2751, 2750}

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx=-\frac{A\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^3}-\frac{aA\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}$$

```
[In] Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*(a*A*Cos[c + d*x]^3)/(d*(a + a*Sin[c + d*x])^4) - (A*Cos[c + d*x]^3)/(
15*d*(a + a*Sin[c + d*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= (aA)\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx\\ &= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}+\frac{1}{5}A\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx\\ &= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(-15\cos\left(c+\frac{dx}{2}\right)+5\cos\left(c+\frac{3dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

```
[In] Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c
+ (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^5)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

method	result
risch	$\frac{2iA\left(-5i\mathrm{e}^{2i(dx+c)}+15\,\mathrm{e}^{3i(dx+c)}-i-5\,\mathrm{e}^{i(dx+c)}\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}}$
parallelrisch	$-\frac{2A\left(15\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+25\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}{15d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{2A\left(\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}\right)}{d\,a^{3}}$
default	$\frac{2A\left(\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}\right)}{d\,a^{3}}$
norman	$\frac{-\frac{8A}{15ad}-\frac{58A\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{8A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16A\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{2A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2A\left(\tan^{6}\left(\frac{dx}{2}+\right.\right.}{ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

```
[In] int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*I*A*(-5*I*exp(2*I*(d*x+c))+15*exp(3*I*(d*x+c))-I-5*exp(I*(d*x+c)))/d/a
^3/(exp(I*(d*x+c))+I)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(54) = 108.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.66

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$=\frac{A\cos\left(dx+c\right)^{3}-2\,A\cos\left(dx+c\right)^{2}+3\,A\cos\left(dx+c\right)-\left(A\cos\left(dx+c\right)^{2}+3\,A\cos\left(dx+c\right)+6\,A\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx\right.\right.}$$

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*x
 + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x
 + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(53) = 106$.

Time = 2.32 (sec) , antiderivative size = 573, normalized size of antiderivative = 9.88

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \begin{cases} -\frac{30A\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} - \frac{}{15a^3d\tan^5\left(\frac{c}{2}+\right.} \\ \frac{x(-A\sin(c)+A)}{(a\sin(c)+a)^3} \end{cases}$$

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 30*A*tan(c/2 +
d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 50*A*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/
2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)
**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3
*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(
c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x
/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 8*A/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
, Ne(d, 0)), (x*(-A*sin(c) + A)/(a*sin(c) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(54) = 108$.

Time = 0.23 (sec) , antiderivative size = 387, normalized size of antiderivative = 6.67

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(\frac{A\left(\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+}{(\cos(dx+c)}\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin}{(\cos(dx+}}\right.}{15d}$$

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
```

```
s(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,A\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*tan
(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x +
 1/2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 12.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\,A\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+25\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\right.}{15\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

```
[In] int((A - A*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(2*A*cos(c/2 + (d*x)/2)*(4*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
+ 15*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 5*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2) + 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*a^3*d*(c
os(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} + \frac{8A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

```
[Out] -A*arctanh(cos(d*x+c))/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+3/5*A*
cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+8/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3045, 3855, 2729, 2727}

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)} + \frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2} + \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

```
[In] Int[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin
[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{A\csc(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{A}{a^3(1+\sin(c+dx))^2} - \frac{A}{a^3(1+\sin(c+dx))}\right) dx \\
&= \frac{A\int \csc(c+dx)\,dx}{a^3} - \frac{A\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} \\
&= -\frac{A\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} \\
&\quad + \frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))} - \frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{3a^3} - \frac{(4A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3} \\
&= -\frac{A\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} \\
&\quad + \frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} + \frac{4A\cos(c+dx)}{3a^3d(1+\sin(c+dx))} - \frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3} \\
&= -\frac{A\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} \\
&\quad + \frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} + \frac{8A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(98) = 196.

Time = 6.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.19

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(2\cos\left(\frac{c}{2}\right)-2\sin\left(\frac{c}{2}\right)+3\cos\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

```
[In] Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2*Cos[c/2] - 2*Sin[c/2] + 3*Cos[c/
2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c/2]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2 - 5*Log[Cos[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + 5*Log[Sin[(c + d*x)/2]]*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 2*Sin[(d*x)/2]*(-17 + 4*C
os[2*(c + d*x)] - 19*Sin[c + d*x]))*(A - A*Sin[c + d*x]))/(5*a^3*d*(Cos[c/2
] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^5)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{A\left(\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{d\,a^3}$
default	$\frac{A\left(\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{d\,a^3}$
risch	$\frac{2A\left(25i\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{4i(dx+c)}-35i\mathrm{e}^{i(dx+c)}-55\,\mathrm{e}^{2i(dx+c)}+8\right)}{5d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^5}+\frac{A\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d\,a^3}-\frac{A\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d\,a^3}$
norman	$\frac{\frac{8A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{18A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{38A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{26A}{5ad}+\frac{22A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{40A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{176A\left(\tan^2\left(\frac{d}{}\right.\right.}{5ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
parallelrisch	$\frac{5\left(\left(\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)-\frac{\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)}{5}+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)-\frac{\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)}{5}+2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{d\,a^3\left(-\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+5\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)-5\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)-\cos\left(\right.\right.}$

```
[In] int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*A/a^3*(16/5/(tan(1/2*d*x+1/2*c)+1)^5-8/(tan(1/2*d*x+1/2*c)+1)^4+12/(tan
(1/2*d*x+1/2*c)+1)^3-10/(tan(1/2*d*x+1/2*c)+1)^2+8/(tan(1/2*d*x+1/2*c)+1)+l
```

```
n(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(92) = 184.

Time = 0.26 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.16

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{16\,A\cos{(dx+c)}^3 - 22\,A\cos{(dx+c)}^2 - 42\,A\cos{(dx+c)} - 5\left(A\cos{(dx+c)}^3 + 3\,A\cos{(dx+c)}^2 - 2\,A\mathrm{c}\right.}{}$$

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A*
cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2
- 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) +
 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x +
 c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c) - 4*A
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4
*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x +
c))
```

Sympy [F]

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc{(c+dx)}}{\sin^3{(c+dx)}+3\sin^2{(c+dx)}+3\sin{(c+dx)}+1}\right)dx + \int\frac{\sin{(c+dx)}\csc{(c+dx)}}{\sin^3{(c+dx)}+3\sin^2{(c+dx)}+3\sin{(c+dx)}+1}\,dx\right)}{a^3}$$

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
 d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*si
n(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(92) = 184$.

Time = 0.22 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.42

$$\int \frac{\csc(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(\frac{2\left(\frac{115\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{135\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+32\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2A\left(\frac{20\sin(d}{\cos(dx+}\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+}}{15\,d}$$

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{\csc(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{5A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{2\left(20A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+55A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+75A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+45A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13A\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{5\,d}$$

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)^
4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(1/
2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.03

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+90\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+150\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+110\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+25\right.}{}$$

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(5*log(tan(c/2 + (d*x)/2)) + 90*tan(c/2 + (d*x)/2) + 150*tan(c/2 + (d*x)
/2)^2 + 110*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^4 + 25*log(tan(c/2
 + (d*x)/2))*tan(c/2 + (d*x)/2) + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x
)/2)^2 + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 25*log(tan(c/2 +
 (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 5*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)
/2)^5 + 26))/(5*a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 113

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2} - \frac{104A\cot(c+dx)}{15a^3d(1+\csc(c+dx))}$$

```
[Out] 4*A*arctanh(cos(d*x+c))/a^3/d-94/15*A*cot(d*x+c)/a^3/d+2/5*A*cot(d*x+c)/a^3
/d/(1+sin(d*x+c))^3+13/15*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))^2+4*A*cot(d*x+c
)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {3029, 2788, 3855, 3852, 8, 3862, 4007, 4004, 3879}

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{104A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)} + \frac{31A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)^2} - \frac{2A\cot(c+dx)}{5a^3d(\csc(c+dx)+1)^3}$$

```
[In] Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (A*Cot[c + d*x])/(a^3*d) - (2*A*Cot[c
 + d*x])/(5*a^3*d*(1 + Csc[c + d*x])^3) + (31*A*Cot[c + d*x])/(15*a^3*d*(1
+ Csc[c + d*x])^2) - (104*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (aA)\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx \\
&= \frac{A\int\left(\frac{9}{a^2}-\frac{4\csc(c+dx)}{a^2}+\frac{\csc^2(c+dx)}{a^2}-\frac{2}{a^2(1+\csc(c+dx))^3}+\frac{9}{a^2(1+\csc(c+dx))^2}-\frac{16}{a^2(1+\csc(c+dx))}\right)dx}{a} \\
&= \frac{9Ax}{a^3}+\frac{A\int\csc^2(c+dx)\,dx}{a^3}-\frac{(2A)\int\frac{1}{(1+\csc(c+dx))^3}\,dx}{a^3} \\
&\quad -\frac{(4A)\int\csc(c+dx)\,dx}{a^3}+\frac{(9A)\int\frac{1}{(1+\csc(c+dx))^2}\,dx}{a^3}-\frac{(16A)\int\frac{1}{1+\csc(c+dx)}\,dx}{a^3} \\
&= \frac{9Ax}{a^3}+\frac{4A\operatorname{arctanh}(\cos(c+dx))}{a^3d}-\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} \\
&\quad +\frac{3A\cot(c+dx)}{a^3d(1+\csc(c+dx))^2}-\frac{16A\cot(c+dx)}{a^3d(1+\csc(c+dx))}+\frac{(2A)\int\frac{-5+2\csc(c+dx)}{(1+\csc(c+dx))^2}\,dx}{5a^3} \\
&\quad -\frac{(3A)\int\frac{-3+\csc(c+dx)}{1+\csc(c+dx)}\,dx}{a^3}+\frac{(16A)\int -1\,dx}{a^3}-\frac{A\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^3d} \\
&= \frac{2Ax}{a^3}+\frac{4A\operatorname{arctanh}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)}{a^3d} \\
&\quad -\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3}+\frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2} \\
&\quad -\frac{16A\cot(c+dx)}{a^3d(1+\csc(c+dx))}-\frac{(2A)\int\frac{15-7\csc(c+dx)}{1+\csc(c+dx)}\,dx}{15a^3}-\frac{(12A)\int\frac{\csc(c+dx)}{1+\csc(c+dx)}\,dx}{a^3}
\end{aligned}$$

$$= \frac{4A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2} - \frac{4A\cot(c+dx)}{a^3d(1+\csc(c+dx))} + \frac{(44A)\int \frac{\csc(c+dx)}{1+\csc(c+dx)}\,dx}{15a^3}$$

$$= \frac{4A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2} - \frac{104A\cot(c+dx)}{15a^3d(1+\csc(c+dx))}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\Big(15\cot\left(\frac{1}{2}(c+dx)\right) - 120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{12}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))}}{30a^3d}$$

```
[In] Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c
+ d*x)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*x)]
 - 354*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[(c +
 d*x)/2]))/(a^3*d)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{}{(\tan}\right.}{2d\,a^{3}}$
default	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{}{(\tan}\right.}{2d\,a^{3}}$
parallelrisch	$-\frac{\left(8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+52\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+161\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{649\left(\tan^{2}\left(\frac{dx}{2}\right.\right.}{3}\right.}{2a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
risch	$-\frac{4\left(-320A\,e^{4i(dx+c)}+150iA\,e^{5i(dx+c)}+367A\,e^{2i(dx+c)}-385iA\,e^{3i(dx+c)}-47A+205iA\,e^{i(dx+c)}+30A\,e^{6i(dx+c)}\right)}{15\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^{5}a^{3}d}-4$
norman	$\frac{-\frac{A}{2ad}+\frac{A\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{3811A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}-\frac{893A\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{413A\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{161A\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-805}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

```
[In] int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d*A/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c
))-32/5/(tan(1/2*d*x+1/2*c)+1)^5+16/(tan(1/2*d*x+1/2*c)+1)^4-88/3/(tan(1/2*
d*x+1/2*c)+1)^3+28/(tan(1/2*d*x+1/2*c)+1)^2-36/(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(107) = 214$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.59

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$=\frac{94\,A\cos\left(dx+c\right)^{4}+222\,A\cos\left(dx+c\right)^{3}-115\,A\cos\left(dx+c\right)^{2}-237\,A\cos\left(dx+c\right)+30\,\left(A\cos\left(dx+c\right)^{4}\right.}{}$$

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 - 2
37*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x
 + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*c
os(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30*(A*
cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c)
```

```
 - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x
 + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 128*A*co
s(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x
 + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

```
[In] integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(107) = 214$.

Time = 0.25 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.59

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{3A\left(\frac{\frac{121\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\sin\cdots}{a^3(\cos\cdots}\cdots\right)}{30\cdots}$$

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/30*(3*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
```

```
 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + 2*A*(2*(115*sin(d*x + c)/(cos
(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{120\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{15\left(8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\cdots\right.}{a^3\left(\tan\left(\frac{1}{2}\,c\cdots\right.\right.}}{30\,d}$$

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c)
/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(13
5*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/2*d
*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 15.92 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.86

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^3\,d}-\frac{4\,A\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d} - \frac{37\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+121\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{514\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{338\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{491\,A\tan\cdots}{\cdots}}{d\left(2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+10\,a^3\tan\left(\frac{c}{2}\cdots\right.\right.}$$

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*tan(c/2 + (d*x)/2))/(2*a^3*d) - (4*A*log(tan(c/2 + (d*x)/2)))/(a^3*d) -
(A + (491*A*tan(c/2 + (d*x)/2))/15 + (338*A*tan(c/2 + (d*x)/2)^2)/3 + (514*
A*tan(c/2 + (d*x)/2)^3)/3 + 121*A*tan(c/2 + (d*x)/2)^4 + 37*A*tan(c/2 + (d*
x)/2)^5)/(d*(10*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3 + 20
*a^3*tan(c/2 + (d*x)/2)^4 + 10*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (
d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)))
```

3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19A\operatorname{arctanh}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{29A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{164A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

```
[Out] -19/2*A*arctanh(cos(d*x+c))/a^3/d+4*A*cot(d*x+c)/a^3/d-1/2*A*cot(d*x+c)*csc
(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+29/15*A*cos(d*x+c)/a^
3/d/(1+sin(d*x+c))^2+164/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used

$= \{3045, 3855, 3852, 8, 3853, 2729, 2727\}$

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19A\text{arctanh}(\cos(c+dx))}{2a^3d} + \frac{4A\cot(c+dx)}{a^3d} + \frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)} + \frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2} + \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[In] Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*C
ot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[
c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (164*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{9A\csc(c+dx)}{a^3} - \frac{4A\csc^2(c+dx)}{a^3} + \frac{A\csc^3(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{5A}{a^3(1+\sin(c+dx))^2} - \frac{9A}{a^3(1+\sin(c+dx))}\right) dx \\
&= \frac{A\int \csc^3(c+dx)\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} - \frac{(4A)\int \csc^2(c+dx)\,dx}{a^3} - \frac{(5A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} + \frac{(9A)\int \csc(c+dx)\,dx}{a^3} - \frac{(9A)\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} \\
&= -\frac{9A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{5A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} + \frac{9A\cos(c+dx)}{a^3d(1+\sin(c+dx))} + \frac{A\int \csc(c+dx)\,dx}{2a^3} - \frac{(4A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3} - \frac{(5A)\int \frac{1}{1+\sin(c+dx)}\,dx}{3a^3} + \frac{(4A)\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{a^3d} \\
&= -\frac{19A\text{arctanh}(\cos(c+dx))}{2a^3d} + \frac{4A\cot(c+dx)}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{29A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{32A\cos(c+dx)}{3a^3d(1+\sin(c+dx))} - \frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3}
\end{aligned}$$

$$= -\frac{19A\operatorname{arctanh}(\cos(c+dx))}{2a^3d} + \frac{4A\cot(c+dx)}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{29A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{164A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

Mathematica [A] (verified)

Time = 2.90 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.78

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(240\cot\left(\frac{1}{2}(c+dx)\right) - 15\csc^2\left(\frac{1}{2}(c+dx)\right) - 1140\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 1140\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \right.}{}$$

```
[In] Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] (A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/2
]] + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^4 - (464*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*x)/2]
)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120*a^3*d
)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{A\left(\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} - 8\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} - \frac{32}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} + \frac{208}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{72}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} + \right.}{4d\,a^3}$
default	$\frac{A\left(\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} - 8\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} - \frac{32}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} + \frac{208}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{72}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} + \right.}{4d\,a^3}$
parallelrisch	$-\frac{\left(-76\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 11\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 472\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 1504\left(\tan^3\right.\right.}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
risch	$\frac{A\left(1425i e^{7i(dx+c)} + 285\,e^{8i(dx+c)} - 5225i e^{5i(dx+c)} - 3325\,e^{6i(dx+c)} + 5635i e^{3i(dx+c)} + 6423\,e^{4i(dx+c)} - 1955i e^{i(dx+c)} - 395\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^5 a^3d}$
norman	$\frac{\frac{57A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} - \frac{A}{8ad} + \frac{11A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} - \frac{11A\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad} + \frac{A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad} + \frac{1943A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad} + \frac{2627A\left(\tan\right.}{6}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}$

```
[In] int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d*A/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-8*tan(1/2*d*x+1/2*c)+64/5/(tan(1/2*d*
x+1/2*c)+1)^5-32/(tan(1/2*d*x+1/2*c)+1)^4+208/3/(tan(1/2*d*x+1/2*c)+1)^3-72
/(tan(1/2*d*x+1/2*c)+1)^2+128/(tan(1/2*d*x+1/2*c)+1)-1/2/tan(1/2*d*x+1/2*c)
^2+8/tan(1/2*d*x+1/2*c)+38*ln(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(128) = 256.

Time = 0.28 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.61

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{896\,A\cos{(dx+c)}^5 - 1222\,A\cos{(dx+c)}^4 - 3218\,A\cos{(dx+c)}^3 + 1168\,A\cos{(dx+c)}^2 + 2292\,A\cos{(dx+}$$

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
+ 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c)^5 + 3*A
*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c
) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d
*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) + 285*(A*cos
(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2
+ 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x +
 c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*cos(d*x + c
)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos(d*x + c)^
5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*
x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x
 + c))
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

```
[In] integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(128) = 256$.

Time = 0.27 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.51

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{12A\left(\frac{\frac{121\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\sin}{a^3(\cos}\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a
^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)/(cos(d*
x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9285*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*
x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{1140\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{15\left(114\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} + \frac{15\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6} + }{120\,d}$$

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x +
1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 15
*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(2
40*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan(1/2
*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*x + 1
/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(165\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4234\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+14090\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+19780\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+12060\tan\left(\frac{c}{2}+\frac{d}{}\right.\right.}{}$$

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(165*tan(c/2 + (d*x)/2) + 4234*tan(c/2 + (d*x)/2)^2 + 14090*tan(c/2 + (d
*x)/2)^3 + 19780*tan(c/2 + (d*x)/2)^4 + 12060*tan(c/2 + (d*x)/2)^5 + 1830*t
an(c/2 + (d*x)/2)^6 - 1050*tan(c/2 + (d*x)/2)^7 - 165*tan(c/2 + (d*x)/2)^8
+ 15*tan(c/2 + (d*x)/2)^9 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)
^2 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 11400*log(tan(c/2
+ (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 11400*log(tan(c/2 + (d*x)/2))*tan(c/2 +
(d*x)/2)^5 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 1140*log(t
an(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 - 15))/(120*a^3*d*tan(c/2 + (d*x)/2
)^2*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{18A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{93A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

```
[Out] 18*A*arctanh(cos(d*x+c))/a^3/d-10*A*cot(d*x+c)/a^3/d-1/3*A*cot(d*x+c)^3/a^3
/d+2*A*cot(d*x+c)*csc(d*x+c)/a^3/d-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-
13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2-93/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x
+c))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used

$= \{3045, 3855, 3852, 8, 3853, 2729, 2727\}$

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{18A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)} - \frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2} - \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}$$

```
[In] Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Cot
[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{16A\csc(c+dx)}{a^3}+\frac{9A\csc^2(c+dx)}{a^3}-\frac{4A\csc^3(c+dx)}{a^3}+\frac{A\csc^4(c+dx)}{a^3}\right.\\
&\qquad\left.+\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{7A}{a^3(1+\sin(c+dx))^2}+\frac{16A}{a^3(1+\sin(c+dx))}\right)dx\\
&= \frac{A\int\csc^4(c+dx)\,dx}{a^3}+\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}-\frac{(4A)\int\csc^3(c+dx)\,dx}{a^3}\\
&\quad+\frac{(7A)\int\frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}+\frac{(9A)\int\csc^2(c+dx)\,dx}{a^3}\\
&\quad-\frac{(16A)\int\csc(c+dx)\,dx}{a^3}+\frac{(16A)\int\frac{1}{1+\sin(c+dx)}\,dx}{a^3}\\
&= \frac{16A\operatorname{arctanh}(\cos(c+dx))}{a^3d}+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}\\
&\quad-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{7A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{16A\cos(c+dx)}{a^3d(1+\sin(c+dx))}\\
&\quad+\frac{(4A)\int\frac{1}{(1+\sin(c+dx))^2}\,dx}{5a^3}-\frac{(2A)\int\csc(c+dx)\,dx}{a^3}+\frac{(7A)\int\frac{1}{1+\sin(c+dx)}\,dx}{3a^3}\\
&\quad-\frac{A\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,\cot(c+dx)\right)}{a^3d}-\frac{(9A)\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{a^3d}\\
&= \frac{18A\operatorname{arctanh}(\cos(c+dx))}{a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}\\
&\quad+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}\\
&\quad-\frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{55A\cos(c+dx)}{3a^3d(1+\sin(c+dx))}+\frac{(4A)\int\frac{1}{1+\sin(c+dx)}\,dx}{15a^3}
\end{aligned}$$

$$= \frac{18A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{93A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 348 vs. $2(153) = 306$.

Time = 6.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.27

$$\int \frac{\csc^4(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(-\frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{6d} + \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d} + \frac{18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{\sec^2\left(\frac{1}{2}(c+\right.}{2d}\right.}{}$$

```
[In] Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[Si
n[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*Sin[(c
+ d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c + d*x)/
2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)))/a^3
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{A\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{160}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\right.}{8d\,a^3}$
default	$\frac{A\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{160}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\right.}{8d\,a^3}$
parallelrisch	$-\frac{\left(432\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-67\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2637\left(\tan^4\left(\right.\right.\right.}{24}$
risch	$-\frac{4\left(675iA\,e^{9i(dx+c)}+135A\,e^{10i(dx+c)}-3150iA\,e^{7i(dx+c)}-1710A\,e^{8i(dx+c)}+5180iA\,e^{5i(dx+c)}+4572A\,e^{6i(dx+c)}-3590i\right.}{15\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)^5a^3d}$
norman	$\frac{-\frac{A}{24ad}+\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{17A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{17A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{7A\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{A\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{66469A\left(\tan\right.}{12}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$

```
[In] int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d*A/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+39*tan(1/2*d*x
+1/2*c)-128/5/(tan(1/2*d*x+1/2*c)+1)^5+64/(tan(1/2*d*x+1/2*c)+1)^4-160/(tan
(1/2*d*x+1/2*c)+1)^3+176/(tan(1/2*d*x+1/2*c)+1)^2-400/(tan(1/2*d*x+1/2*c)+1
)-1/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2-39/tan(1/2*d*x+1/2*c)-144
*ln(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.88

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$=\frac{424\,A\cos{(dx+c)}^6+1002\,A\cos{(dx+c)}^5-944\,A\cos{(dx+c)}^4-2074\,A\cos{(dx+c)}^3+531\,A\cos{(dx+}}{}$$

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4 -
 2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135*(
A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x +
c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(
```

```
d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4
*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x + c)^6
 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A
*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c)
 - 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 578*A*cos(d*
x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A*cos(d*x +
c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*cos(d*x + c)^
5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*cos(d*x + c)^
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*
x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*
x + c) + 4*a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

```
[In] integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(145) = 290$.

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.61

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 25
80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c
```

```
)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a
^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((20*si
n(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)/(a^3*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 9
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.39

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{2160\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{5\left(792\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-117\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} + \frac{48\left(125\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4\right.}{}}{}$$

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/(a
^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*tan(1
/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x + 1/2*
c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x + 1/2
*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2*c))/a
^9)/d
```

Mupad [B] (verification not implemented)

Time = 15.98 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{A\left(335\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7559\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+24610\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+33170\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{}$$

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(A*(335*tan(c/2 + (d*x)/2)^2 - 35*tan(c/2 + (d*x)/2) + 7559*tan(c/2 + (d*x
)/2)^3 + 24610*tan(c/2 + (d*x)/2)^4 + 33170*tan(c/2 + (d*x)/2)^5 + 18670*ta
n(c/2 + (d*x)/2)^6 + 1310*tan(c/2 + (d*x)/2)^7 - 2375*tan(c/2 + (d*x)/2)^8
- 335*tan(c/2 + (d*x)/2)^9 + 35*tan(c/2 + (d*x)/2)^10 - 5*tan(c/2 + (d*x)/2
)^11 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 10800*log(tan(c/
2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2
+ (d*x)/2)^5 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 10800*l
og(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 + 2160*log(tan(c/2 + (d*x)/2))*
tan(c/2 + (d*x)/2)^8 + 5))/(120*a^3*d*tan(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)
/2) + 1)^5)
```

3.244 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 33, antiderivative size = 327

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{8}a\big(B\big(4c^3+12c^2d+9cd^2+3d^3\big)+A\big(8c^3+12c^2d+12cd^2+3d^3\big)\big)\,x \\
&\quad -\frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-B(3c^4-15c^3d-52c^2d^2-60cd^3-16d^4))\cos(e+fx)}{30df} \\
&\quad -\frac{a(5Ad(6c^2+20cd+9d^2)-B(6c^3-30c^2d-71cd^2-45d^3))\cos(e+fx)\sin(e+fx)}{120f} \\
&\quad -\frac{a(4(5A+4B)d^2-3c(Bc-5(A+B)d))\cos(e+fx)(c+d\sin(e+fx))^2}{60df} \\
&\quad +\frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} \\
&\quad -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}
\end{aligned}$$

```
[Out] 1/8*a*(B*(4*c^3+12*c^2*d+9*c*d^2+3*d^3)+A*(8*c^3+12*c^2*d+12*c*d^2+3*d^3))*
x-1/30*a*(5*A*d*(3*c^3+16*c^2*d+12*c*d^2+4*d^3)-B*(3*c^4-15*c^3*d-52*c^2*d^
2-60*c*d^3-16*d^4))*cos(f*x+e)/d/f-1/120*a*(5*A*d*(6*c^2+20*c*d+9*d^2)-B*(6
*c^3-30*c^2*d-71*c*d^2-45*d^3))*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(4*(5*A+4*B)
*d^2-3*c*(B*c-5*(A+B)*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*a*(B*c-5*(
A+B)*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a*B*cos(f*x+e)*(c+d*sin(f*x+e
))^4/d/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3047, 3102, 2832, 2813}

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx\\
&=-\frac{a(5Ad(6c^2+20cd+9d^2)-B(6c^3-30c^2d-71cd^2-45d^3))\sin(e+fx)\cos(e+fx)}{120f}\\
&\quad+\frac{1}{8}ax\left(A\left(8c^3+12c^2d+12cd^2+3d^3\right)+B\left(4c^3+12c^2d+9cd^2+3d^3\right)\right)\\
&\quad-\frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-B(3c^4-15c^3d-52c^2d^2-60cd^3-16d^4))\cos(e+fx)}{30df}\\
&\quad-\frac{a(4d^2(5A+4B)-3c(Bc-5d(A+B)))\cos(e+fx)(c+d\sin(e+fx))^2}{60df}\\
&\quad+\frac{a(Bc-5d(A+B))\cos(e+fx)(c+d\sin(e+fx))^3}{20df}\\
&\quad-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}
\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c*d^2
 + 3*d^3))*x)/8 - (a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) - B*(3*c^
4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/(30*d*f) - (a
*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45*d^3)
)*Cos[e + f*x]*Sin[e + f*x])/(120*f) - (a*(4*(5*A + 4*B)*d^2 - 3*c*(B*c - 5
*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d*f) + (a*(B*c - 5*(A
 + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a*B*Cos[e + f*x]*
(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int (c+d\sin(e+fx))^3\left(aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)\right)\,dx$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int (c+d\sin(e+fx))^3(a(5A+4B)d-a(Bc-5(A+B)d)\sin(e+fx))\,dx}{5d}$$

$$= \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int (c+d\sin(e+fx))^2\left(ad(20Ac+13Bc+15Ad+15Bd)+a(4(5A+4B)d^2-3c(Bc-5(A+\right.}{20d}$$

$$= -\frac{a(4(5A+4B)d^2-3c(Bc-5(A+B)d))\cos(e+fx)(c+d\sin(e+fx))^2}{60df} + \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int (c+d\sin(e+fx))\left(ad(60Ac^2+33Bc^2+75Acd+75Bcd+40Ad^2+32Bd^2)+a(5Ad(6c^2+\right.}{60d}$$

$$
\begin{aligned}
&= \frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A\left(8c^3+12c^2d+12cd^2+3d^3\right)\right)x \\
&\quad - \frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-B(3c^4-15c^3d-52c^2d^2-60cd^3-16d^4))\cos(e+fx)}{30df} \\
&\quad - \frac{a(5Ad(6c^2+20cd+9d^2)-B(6c^3-30c^2d-71cd^2-45d^3))\cos(e+fx)\sin(e+fx)}{120f} \\
&\quad - \frac{a(4(5A+4B)d^2-3c(Bc-5(A+B)d))\cos(e+fx)(c+d\sin(e+fx))^2}{60df} \\
&\quad + \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} \\
&\quad - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.82

$$
\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx
= \frac{a(1+\sin(e+fx))\left(-60(2A(4c^3+12c^2d+9cd^2+3d^3)+B(8c^3+18c^2d+18cd^2+5d^3))\cos(e+fx)+10\right.}{}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(8
*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d) +
 B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)] +
 15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c
*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2*(e +
 f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

method	result
parts	$\frac{(Aa\,d^3+3Bac\,d^2+d^3Ba)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(A\,c^3a+3Aa\,c^2d+Ba\,c^3)\cos(fx+e)}{f}$
parallelrisch	$\frac{\left(((-A-B)d^3-3d^2c(A+B)-3c^2d(A+B)-B\,c^3)\sin(2fx+2e)+\left(\frac{\left(A+\frac{5B}{4}\right)d^2}{3}+dc(A+B)+B\,c^2\right)d\cos(3fx+3e)+\frac{d^2(3B}{}\right.}{}$
derivativedivides	$\frac{-A\cos(fx+e)a\,c^3+3Aa\,c^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aac\,d^2(2+\sin^2(fx+e))\cos(fx+e)+Aa\,d^3\left(-\frac{\left(\sin^3(fx+e\right.}{}\right.}{}$
default	$\frac{-A\cos(fx+e)a\,c^3+3Aa\,c^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aac\,d^2(2+\sin^2(fx+e))\cos(fx+e)+Aa\,d^3\left(-\frac{\left(\sin^3(fx+e\right.}{}\right.}{}$
risch	$-\frac{3\sin(2fx+2e)Aa\,c^2d}{4f}-\frac{3\sin(2fx+2e)Aac\,d^2}{4f}-\frac{3\sin(2fx+2e)Ba\,c^2d}{4f}-\frac{3\sin(2fx+2e)Bac\,d^2}{4f}+Aa\,c^3x+$
norman	Expression too large to display

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] (A*a*d^3+3*B*a*c*d^2+B*a*d^3)/f*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x
+e)+3/8*f*x+3/8*e)-(A*a*c^3+3*A*a*c^2*d+B*a*c^3)/f*cos(f*x+e)-1/3*(3*A*a*c*
d^2+A*a*d^3+3*B*a*c^2*d+3*B*a*c*d^2)/f*(2+sin(f*x+e)^2)*cos(f*x+e)+(3*A*a*c
^2*d+3*A*a*c*d^2+B*a*c^3+3*B*a*c^2*d)/f*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x
+1/2*e)+A*a*c^3*x-1/5*d^3*B*a/f*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{24\,Bad^3\cos{(fx+e)}^5-40\,(3\,Bac^2d+3\,(A+B)acd^2+(A+2\,B)ad^3)\cos{(fx+e)}^3-15\,(4\,(2\,A+B)}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 + (
A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3*(A
```

```
 + B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2*(3*
B*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.05

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f*
x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*x)*
*2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*cos(
e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d**2*co
s(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*
A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f*x)**3/(3
*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2 - B*a*c**
3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*B*a*c**2*d*
x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*c**2*d*sin(e
 + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) -
2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)**4/8 + 9*B*a*c
*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*cos(e + f*x)**4/
8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a*c*d**2*sin(e +
 f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) -
 2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*x)**4/8 + 3*B*a*d*
*3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*cos(e + f*x)**4/8 - B
*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*B*a*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*d**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a*d**3*cos(e + f*x)**5/(15*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.24

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{480\,(fx+e)Aac^3+120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac^3+360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aac^2d+}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^3
 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*d +
 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d^2 +
45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2 + 160*
(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e
)^3 + 15*cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*cos(f*x +
 e) - 1440*A*a*c^2*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.94

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx &= -\frac{Bad^3\cos(5\,fx+5\,e)}{80\,f}\\ &+\frac{1}{8}\left(8\,Aac^3+4\,Bac^3+12\,Aac^2d+12\,Bac^2d+12\,Aacd^2+9\,Bacd^2+3\,Aad^3+3\,Bad^3\right)x\\ &+\frac{\left(12\,Bac^2d+12\,Aacd^2+12\,Bacd^2+4\,Aad^3+5\,Bad^3\right)\cos(3\,fx+3\,e)}{48\,f}\\ &-\frac{\left(8\,Aac^3+8\,Bac^3+24\,Aac^2d+18\,Bac^2d+18\,Aacd^2+18\,Bacd^2+6\,Aad^3+5\,Bad^3\right)\cos(fx+e)}{8\,f}\\ &+\frac{\left(3\,Bacd^2+Aad^3+Bad^3\right)\sin(4\,fx+4\,e)}{32\,f}\\ &-\frac{\left(Bac^3+3\,Aac^2d+3\,Bac^2d+3\,Aacd^2+3\,Bacd^2+Aad^3+Bad^3\right)\sin(2\,fx+2\,e)}{4\,f}\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2*
d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x +
1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3)*c
os(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*c^2*
d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/f + 1
/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c^3 + 3
*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*s
in(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.54

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$= \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(8Ac^3+3Ad^3+4Bc^3+3Bd^3+12Acd^2+12Ac^2d+9Bcd^2+12Bc^2d\right)}{4\left(2Aac^3+\frac{3Aad^3}{4}+Bac^3+\frac{3Bad^3}{4}+3Aacd^2+3Aac^2d+\frac{9Bacd^2}{4}+3Bac^2d\right)}\right)\left(8Ac^3+3Ad^3+4Bc^3+3B\ldots\right.}{4f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aad^3}{4}+Bac^3+\frac{3Bad^3}{4}+3Aacd^2+3Aac^2d+\frac{9Bacd^2}{4}+3Bac^2d\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(\ldots\right.}{\ldots}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A
*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*(2*A*a*c^3 + (3*A*a*d^3)/
4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 +
 3*B*a*c^2*d)))*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 12*A*
c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*f) - (tan(e/2 + (f*x)/2)*((3*A*a*d^3)/4
 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 +
3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^3 + 2*B*a*c^3 + 6*A*a*c^2*d) +
 tan(e/2 + (f*x)/2)^2*(8*A*a*c^3 + (20*A*a*d^3)/3 + 8*B*a*c^3 + (16*B*a*d^3
)/3 + 20*A*a*c*d^2 + 24*A*a*c^2*d + 20*B*a*c*d^2 + 20*B*a*c^2*d) + tan(e/2
+ (f*x)/2)^4*(12*A*a*c^3 + (28*A*a*d^3)/3 + 12*B*a*c^3 + (32*B*a*d^3)/3 + 2
8*A*a*c*d^2 + 36*A*a*c^2*d + 28*B*a*c*d^2 + 28*B*a*c^2*d) + tan(e/2 + (f*x)
/2)^6*(8*A*a*c^3 + 4*A*a*d^3 + 8*B*a*c^3 + 12*A*a*c*d^2 + 24*A*a*c^2*d + 12
*B*a*c*d^2 + 12*B*a*c^2*d) - tan(e/2 + (f*x)/2)^9*((3*A*a*d^3)/4 + B*a*c^3
+ (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 + 3*B*a*c^2*d
) + tan(e/2 + (f*x)/2)^3*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a
*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) - tan(e/2 + (f*x)/2)
^7*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d +
 (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) + 2*A*a*c^3 + (4*A*a*d^3)/3 + 2*B*a*c^3 +
(16*B*a*d^3)/15 + 4*A*a*c*d^2 + 6*A*a*c^2*d + 4*B*a*c*d^2 + 4*B*a*c^2*d)/(f
*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^
6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.245 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= \frac{1}{8}a\big(4A\big(2c^2+2cd+d^2\big)+B\big(4c^2+8cd+3d^2\big)\big)x \\ &\quad -\frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e+fx)}{6df} \\ &\quad -\frac{a(3(4A+3B)d^2-2c(Bc-4(A+B)d))\cos(e+fx)\sin(e+fx)}{24f} \\ &\quad +\frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^2}{12df} \\ &\quad -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}\end{aligned}$$

```
[Out] 1/8*a*(4*A*(2*c^2+2*c*d+d^2)+B*(4*c^2+8*c*d+3*d^2))*x-1/6*a*(4*A*d*(c^2+3*c
*d+d^2)-B*(c^3-4*c^2*d-8*c*d^2-4*d^3))*cos(f*x+e)/d/f-1/24*a*(3*(4*A+3*B)*d
^2-2*c*(B*c-4*(A+B)*d))*cos(f*x+e)*sin(f*x+e)/f+1/12*a*(B*c-4*(A+B)*d)*cos(
f*x+e)*(c+d*sin(f*x+e))^2/d/f-1/4*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2832, 2813}

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{a(-8cd(A+B)-3d^2(4A+3B)+2Bc^2)\sin(e+fx)\cos(e+fx)}{24f} \\
&\quad+\frac{1}{8}ax\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right) \\
&\quad-\frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e+fx)}{6df} \\
&\quad+\frac{a(Bc-4d(A+B))\cos(e+fx)(c+d\sin(e+fx))^2}{12df} \\
&\quad-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}
\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/8 - (a*(4*A*d
*(c^2 + 3*c*d + d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/(
6*d*f) + (a*(2*B*c^2 - 8*(A + B)*c*d - 3*(4*A + 3*B)*d^2)*Cos[e + f*x]*Sin[
e + f*x])/(24*f) + (a*(B*c - 4*(A + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*d*f) - (a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int (c+d\sin(e+fx))^2 \left(aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)\right)\,dx$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df} + \frac{\int (c+d\sin(e+fx))^2(a(4A+3B)d-a(Bc-4(A+B)d)\sin(e+fx))\,dx}{4d}$$

$$= \frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^2}{12df} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df} + \frac{\int (c+d\sin(e+fx))\left(ad(12Ac+7Bc+8Ad+8Bd)-a(2Bc^2-8(A+B)cd-3(4A+3B)d^2\right.}{12d}$$

$$= \frac{1}{8}a\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)x - \frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e+fx)}{6df} + \frac{a(2Bc^2-8(A+B)cd-3(4A+3B)d^2)\cos(e+fx)\sin(e+fx)}{24f} + \frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^2}{12df} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{a(1+\sin(e+fx))\left(-24(B(4c^2+6cd+3d^2)+A(4c^2+8cd+3d^2))\cos(e+fx)+8d(Ad+B(2c+d))\cos\right.}{96f\left(\mathrm{co}\right.}$$

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2, x]

[Out] (a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d + 3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4*A *(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2 + A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)])))/(96*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81

method	result
parts	$-\frac{(Aa\,d^2+2Bacd+d^2Ba)\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3f}-\frac{(a\,c^2A+2Aacd+Ba\,c^2)\cos(fx+e)}{f}+\frac{(2Aacd+Aa\,d^2+Ba\,c^2\cdots}{}$
parallelrisch	$\frac{\left(((-3B-3A)d^2-6dc(A+B)-3B\,c^2)\sin(2fx+2e)+(2Bc+(A+B)d)d\cos(3fx+3e)+\frac{3d^2B\sin(4fx+4e)}{8}+\left((-9A-9B)d^2-\cdots\right.\right.}{}$
derivativedivides	$\frac{-a\,c^2A\cos(fx+e)+2Aacd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aa\,d^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Ba\,c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}\cdots\right.}{}$
default	$\frac{-a\,c^2A\cos(fx+e)+2Aacd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aa\,d^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Ba\,c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}\cdots\right.}{}$
risch	$Aa\,c^2x+Aacdx+\frac{Aa\,d^2x}{2}+\frac{Ba\,c^2x}{2}+Bacdx+\frac{3Ba\,d^2x}{8}-\frac{a\cos(fx+e)A\,c^2}{f}-\frac{2a\cos(fx+e)Acd}{f}-\frac{3\cdots}{}$
norman	$\frac{(a\,c^2A+Aacd+\frac{1}{2}Aa\,d^2+\frac{1}{2}Ba\,c^2+Bacd+\frac{3}{8}d^2Ba)x+(a\,c^2A+Aacd+\frac{1}{2}Aa\,d^2+\frac{1}{2}Ba\,c^2+Bacd+\frac{3}{8}d^2Ba)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\cdots}{}$

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] -1/3*(A*a*d^2+2*B*a*c*d+B*a*d^2)/f*(2+sin(f*x+e)^2)*cos(f*x+e)-(A*a*c^2+2*A*a*c*d+B*a*c^2)/f*cos(f*x+e)+(2*A*a*c*d+A*a*d^2+B*a*c^2+2*B*a*c*d)/f*(-1/2*

```
cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+A*a*c^2*x+d^2*B*a/f*(-1/4*(sin(f*x+e)^
3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.75

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{8\,(2\,Bacd+(A+B)ad^2)\cos{(fx+e)}^3+3\,(4\,(2\,A+B)ac^2+8\,(A+B)acd+(4\,A+3\,B)ad^2)fx-24\,((A}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2 +
 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)*a
*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*a*c
^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(201) = 402.

Time = 0.23 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.68

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\begin{cases} Aac^2x-\frac{Aac^2\cos{(e+fx)}}{f}+Aacdx\sin^2{(e+fx)}+Aacdx\cos^2{(e+fx)}-\frac{Aacd\sin{(e+fx)}\cos{(e+fx)}}{f}-\frac{2Aacd\cos{(e}}{f} \\ x(A+B\sin{(e)})\,(c+d\sin{(e)})^2\,(a\sin{(e)}+a) \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**2
 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a*
c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f*x)
**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)*cos
(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e + f*x
)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e + f*x)/
(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*d*x*cos
(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*sin(e + f
*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*x*sin(e +
 f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**2*x*
```

```
cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - B*a*d**
2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
 d*sin(e))**2*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.24

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$$
$$= \frac{96\,(fx + e)Aac^2 + 24\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Bac^2 + 48\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aacd + 64\,(\mathrm{co}}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a
*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e) - 96
*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

$$\begin{aligned}\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx &= \frac{Bad^2\sin(4\,fx + 4\,e)}{32\,f} + \frac{1}{8}\left(8\,Aac^2 + 4\,Bac^2 + 8\,Aacd + 8\,Bacd + 4\,Aad^2 + 3\,Bad^2\right)x \\ &\quad + \frac{(2\,Bacd + Aad^2 + Bad^2)\cos(3\,fx + 3\,e)}{12\,f} \\ &\quad - \frac{(4\,Aac^2 + 4\,Bac^2 + 8\,Aacd + 6\,Bacd + 3\,Aad^2 + 3\,Bad^2)\cos(fx + e)}{4\,f} \\ &\quad - \frac{(Bac^2 + 2\,Aacd + 2\,Bacd + Aad^2 + Bad^2)\sin(2\,fx + 2\,e)}{4\,f}\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

[Out] 1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^2) *cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d + 3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*a*c* d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f

Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.57

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8\,A\,c^2+4\,A\,d^2+4\,B\,c^2+3\,B\,d^2+8\,A\,c\,d+8\,B\,c\,d)}{4\left(2\,A\,a\,c^2+A\,a\,d^2+B\,a\,c^2+\frac{3\,B\,a\,d^2}{4}+2\,A\,a\,c\,d+2\,B\,a\,c\,d\right)}\right)(8\,A\,c^2+4\,A\,d^2+4\,B\,c^2+3\,B\,d^2+8\,A\,c\,d+8\,B}{4\,f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(2\,A\,a\,c^2+2\,B\,a\,c^2+4\,A\,a\,c\,d)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(A\,a\,d^2+B\,a\,c^2+\frac{3\,B\,a\,d^2}{4}+2\,A\,a\,c\,d+2\,B\right.}{}$$

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)

[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A* c*d + 8*B*c*d))/(4*(2*A*a*c^2 + A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c *d + 2*B*a*c*d)))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*c*d + 8*B*c* d))/(4*f) - (tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 + 2*B*a*c^2 + 4*A*a*c*d) + tan (e/2 + (f*x)/2)*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) + tan(e/2 + (f*x)/2)^4*(6*A*a*c^2 + 4*A*a*d^2 + 6*B*a*c^2 + 4*B*a*d^2 + 12 *A*a*c*d + 8*B*a*c*d) + tan(e/2 + (f*x)/2)^2*(6*A*a*c^2 + (16*A*a*d^2)/3 + 6*B*a*c^2 + (16*B*a*d^2)/3 + 12*A*a*c*d + (32*B*a*c*d)/3) - tan(e/2 + (f*x) /2)^7*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) + tan(e/2 + (f*x)/2)^3*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) - tan(e/2 + (f*x)/2)^5*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2* B*a*c*d) + 2*A*a*c^2 + (4*A*a*d^2)/3 + 2*B*a*c^2 + (4*B*a*d^2)/3 + 4*A*a*c* d + (8*B*a*c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4* tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))

3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 111

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{1}{2}a(B(c+d)+A(2c+d))x-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f} - \frac{a(3Bc+3Ad-Bd)\cos(e+fx)\sin(e+fx)}{6f}-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af}$$

```
[Out] 1/2*a*(B*(c+d)+A*(2*c+d))*x-1/3*a*(3*A*(c+d)+B*(3*c+d))*cos(f*x+e)/f-1/6*a*
(3*A*d+3*B*c-B*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e
))^2/a/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3047, 3102, 2813}

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{6f} +\frac{1}{2}ax(A(2c+d)+B(c+d))-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(B*(c + d) + A*(2*c + d))*x)/2 - (a*(3*A*(c + d) + B*(3*c + d))*Cos[e +
f*x])/(3*f) - (a*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (
B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(3*a*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int (a + a\sin(e+fx))\left(Ac + (Bc+Ad)\sin(e+fx) + Bd\sin^2(e+fx)\right)\,dx$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af} + \frac{\int (a+a\sin(e+fx))(a(3Ac+2Bd) + a(3Bc+3Ad-Bd)\sin(e+fx))\,dx}{3a}$$

$$= \frac{1}{2}a(B(c+d)+A(2c+d))x - \frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f} - \frac{a(3Bc+3Ad-Bd)\cos(e+fx)\sin(e+fx)}{6f} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{a(12Acfx + 6Bcfx + 6Adfx + 6Bdfx - 3(4A(c + d) + B(4c + 3d))\cos(e + fx) + Bd\cos(3(e + fx)) - 3}{12f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4*
c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] - 3*
A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)]))/(12*f)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

method	result
parts	$\frac{(Aad+Bac+dBa)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(Aac+Aad+Bac)\cos(fx+e)}{f}+aAcx-\frac{dBa\left(2+\sin^2(fx+e)\right)}{3f}$
parallelrisch	$-\frac{\left((B(c+d)+dA)\sin(2fx+2e)-\frac{Bd\cos(3fx+3e)}{3}+((4c+3d)B+4A(c+d))\cos(fx+e)+\left(-2cfx-2dfx+4c+\frac{8}{3}d\right)B-4\left(\left(\frac{fx}{2}\right.\right.\right.}{4f}$
derivativedivides	$\frac{-\frac{dBa\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aad\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+dBa\left(-\frac{\cos(}{}\right.}{f}$
default	$\frac{-\frac{dBa\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aad\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+dBa\left(-\frac{\cos(}{}\right.}{f}$
risch	$aAcx+\frac{Aadx}{2}+\frac{Bacx}{2}+\frac{Badx}{2}-\frac{a\cos(fx+e)Ac}{f}-\frac{a\cos(fx+e)dA}{f}-\frac{a\cos(fx+e)Bc}{f}-\frac{3a\cos(fx+e)dB}{4f}+$
norman	$\frac{\left(Aac+\frac{1}{2}Aad+\frac{1}{2}Bac+\frac{1}{2}dBa\right)x+\left(Aac+\frac{1}{2}Aad+\frac{1}{2}Bac+\frac{1}{2}dBa\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3Aac+\frac{3}{2}Aad+\frac{3}{2}Bac+\frac{3}{2}dBa\right)x\left(\tan^2\left(\right.\right.}{}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (A*a*d+B*a*c+B*a*d)/f*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-(A*a*c+A*a
*d+B*a*c)/f*cos(f*x+e)+a*A*c*x-1/3*d*B*a/f*(2+sin(f*x+e)^2)*cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{2\,Bad\cos{(fx+e)}^3+3\,((2\,A+B)ac+(A+B)ad)fx-3\,(Bac+(A+B)ad)\cos{(fx+e)}\sin{(fx+e)}}{6\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a*
c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d)*
cos(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(100) = 200.

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.50

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\begin{cases} Aacx-\frac{Aac\cos{(e+fx)}}{f}+\frac{Aadx\sin^2{(e+fx)}}{2}+\frac{Aadx\cos^2{(e+fx)}}{2}-\frac{Aad\sin{(e+fx)}\cos{(e+fx)}}{2f}-\frac{Aad\cos{(e+fx)}}{f}+\frac{Bacx\sin^2{(e+fx)}}{2} \\ x(A+B\sin{(e)})\,(c+d\sin{(e)})\,(a\sin{(e)}+a) \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*a
*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*cos(
e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a*c*
sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e + f*
x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*x)/f
- B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{12\,(fx+e)Aac + 3\,(2\,fx+2\,e-\sin{(2\,fx+2\,e)})Bac + 3\,(2\,fx+2\,e-\sin{(2\,fx+2\,e)})Aad + 4\,(\cos{(fx}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(f*x + e)*A*a*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*A*a*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*B
*a*d + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*d - 12*A*a*c*cos(f*x + e) - 1
2*B*a*c*cos(f*x + e) - 12*A*a*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{Bad\cos{(3\,fx+3\,e)}}{12\,f} + \frac{1}{2}\,(2\,Aac+Bac+Aad+Bad)x - \frac{(4\,Aac+4\,Bac+4\,Aad+3\,Bad)\cos{(fx+e)}}{4\,f} - \frac{(Bac+Aad+Bad)\sin{(2\,fx+2\,e)}}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x - 1
/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c + A*
a*d + B*a*d)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 12.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{\frac{3\,A\,a\,d\,\sin(2\,e+2\,f\,x)}{2}-\frac{B\,a\,d\,\cos(3\,e+3\,f\,x)}{2}+\frac{3\,B\,a\,c\,\sin(2\,e+2\,f\,x)}{2}+\frac{3\,B\,a\,d\,\sin(2\,e+2\,f\,x)}{2}+6\,A\,a\,c\,\cos\left(e+f\,x\right)+6\,A\,a$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*A*a*d*sin(2*e + 2*f*x))/2 - (B*a*d*cos(3*e + 3*f*x))/2 + (3*B*a*c*sin(
2*e + 2*f*x))/2 + (3*B*a*d*sin(2*e + 2*f*x))/2 + 6*A*a*c*cos(e + f*x) + 6*A
*a*d*cos(e + f*x) + 6*B*a*c*cos(e + f*x) + (9*B*a*d*cos(e + f*x))/2 - 6*A*a
*c*f*x - 3*A*a*d*f*x - 3*B*a*c*f*x - 3*B*a*d*f*x)/(6*f)
```

3.247 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{1}{2}a(2A+B)x - \frac{a(A+B)\cos(e+fx)}{f} - \frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*A+B)*x-a*(A+B)*cos(f*x+e)/f-1/2*a*B*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2813}

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = -\frac{a(A+B)\cos(e+fx)}{f} + \frac{1}{2}ax(2A+B) - \frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}a(2A+B)x - \frac{a(A+B)\cos(e+fx)}{f} - \frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{a(2Be+4Afx+2Bfx-4(A+B)\cos(e+fx)-B\sin(2(e+fx)))}{4f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
[Out] (a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*Cos[e + f*x] - B*Sin[2*(e + f*x)]
))/(4*f)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{\left(-\frac{B\sin(2fx+2e)}{4}+(-A-B)\cos(fx+e)+fxA+\frac{fxB}{2}+A+B\right)a}{f}$
parts	$axA-\frac{(aA+Ba)\cos(fx+e)}{f}+\frac{Ba\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
risch	$axA+\frac{aBx}{2}-\frac{a\cos(fx+e)A}{f}-\frac{a\cos(fx+e)B}{f}-\frac{Ba\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{Ba\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-aA\cos(fx+e)-Ba\cos(fx+e)+aA(fx+e)}{f}$
default	$\frac{Ba\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-aA\cos(fx+e)-Ba\cos(fx+e)+aA(fx+e)}{f}$
norman	$\frac{(aA+\frac{1}{2}Ba)x+(aA+\frac{1}{2}Ba)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2aA+Ba)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(2aA+2Ba)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{Ba\left(\tan^3\left(\frac{fx}{2}+\right.\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] (-1/4*B*sin(2*f*x+2*e)+(-A-B)*cos(f*x+e)+f*x*A+1/2*f*x*B+A+B)*a/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \frac{(2\,A + B)afx - Ba\cos\left(fx + e\right)\sin\left(fx + e\right) - 2\,(A + B)a\cos\left(fx + e\right)}{2\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*A + B)*a*f*x - B*a*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*a*cos(f*x
+ e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \begin{cases} Aax - \frac{Aa\cos\left(e+fx\right)}{f} + \frac{Bax\sin^2\left(e+fx\right)}{2} + \frac{Bax\cos^2\left(e+fx\right)}{2} - \frac{Ba\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{Ba\cos\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x(A + B\sin\left(e\right))\left(a\sin\left(e\right) + a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*cos
(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f,
Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \frac{4\,(fx + e)Aa + (2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right))Ba - 4\,Aa\cos\left(fx + e\right) - 4\,Ba\cos\left(fx + e\right)}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*x
 + e) - 4*B*a*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{1}{2}\,(2\,Aa+Ba)x - \frac{Ba\sin\left(2\,fx+2\,e\right)}{4\,f} - \frac{(Aa+Ba)\cos\left(fx+e\right)}{f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e)
/f
```

Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.08

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = A\,a\,x - \frac{-B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(2\,A\,a+2\,B\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,a+2\,B\,a}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)} + \frac{B\,a\,x}{2}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

```
[Out] A*a*x - (2*A*a + 2*B*a + tan(e/2 + (f*x)/2)^2*(2*A*a + 2*B*a) - B*a*tan(e/2
 + (f*x)/2)^3 + B*a*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/
2 + (f*x)/2)^4 + 1)) + (B*a*x)/2
```

3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{a(Bc-(A+B)d)x}{d^2}+\frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{aB\cos(e+fx)}{df}$$

```
[Out] -a*(B*c-(A+B)*d)*x/d^2-a*B*cos(f*x+e)/d/f+2*a*(c-d)*(-A*d+B*c)*arctan((d+c*
tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3047, 3102, 2814, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d^2}-\frac{aB\cos(e+fx)}{df}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a*(B*c - (A + B)*d)*x)/d^2) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a*B*Cos[e + f*x
])/(d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{aA + (aA + aB)\sin(e + fx) + aB\sin^2(e + fx)}{c + d\sin(e + fx)}\,dx$$

$$= -\frac{aB\cos(e + fx)}{df} + \frac{\int \frac{aAd - a(Bc - (A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$=-\frac{a(Bc-(A+B)d)x}{d^2}-\frac{aB\cos(e+fx)}{df}+\frac{(a(c-d)(Bc-Ad))\int\frac{1}{c+d\sin(e+fx)}\,dx}{d^2}$$

$$=-\frac{a(Bc-(A+B)d)x}{d^2}-\frac{aB\cos(e+fx)}{df}+\frac{(2a(c-d)(Bc-Ad))\text{Subst}\left(\int\frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2f}$$

$$=-\frac{a(Bc-(A+B)d)x}{d^2}-\frac{aB\cos(e+fx)}{df}-\frac{(4a(c-d)(Bc-Ad))\text{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2f}$$

$$=-\frac{a(Bc-(A+B)d)x}{d^2}+\frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}f}-\frac{aB\cos(e+fx)}{df}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.00

$$\int\frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$=\frac{a\left(Adx+B(-c+d)x-\frac{Bd\cos(e)\cos(fx)}{f}+\frac{2(c-d)(Bc-Ad)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin}{\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x])/f + (2*(c - d)*(B*c - A*d)
*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)
/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(
Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/f)*(
1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{2a\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-Bc+dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\left(-Acd+A\,d^2+B\,c^2-cdB\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}\right)}{f}$
default	$\frac{2a\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-Bc+dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\left(-Acd+A\,d^2+B\,c^2-cdB\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}\right)}{f}$
risch	$\frac{xaA}{d}-\frac{xaBc}{d^2}+\frac{xaB}{d}-\frac{Ba\,e^{i(fx+e)}}{2df}-\frac{Ba\,e^{-i(fx+e)}}{2df}+\frac{\sqrt{-(c-d)(c+d)}\,a\ln\left(e^{i(fx+e)}+\frac{ic-\sqrt{-(c-d)(c+d)}}{d}\right)A}{(c+d)fd}-$

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 2/f*a*(1/d^2*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-B*c+B*d)*arctan(tan(1/2*f*x+1/2*e)))+(-A*c*d+A*d^2+B*c^2-B*c*d)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$=\left[-\frac{2\,Bad\cos(fx+e)+2\,(Bac-(A+B)ad)fx-(Bac-Aad)\sqrt{-\frac{c-d}{c+d}}\log\left(-\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(f}{}\right.}{2\,d^2f}\right.$$
$$\left.-\frac{Bad\cos(fx+e)+(Bac-(A+B)ad)fx+(Bac-Aad)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)}{d^2f}\right]$$

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x - (B*a*c - A*a*d)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x

```
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d)*f*x
 + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt(
(c - d)/(c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5508 vs. 2(82) = 164.

Time = 113.71 (sec) , antiderivative size = 5508, normalized size of antiderivative = 56.20

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*sin(e))*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0)
& Eq(f, 0)), ((A*a*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + A*
a*f*x/(f*tan(e/2 + f*x/2)**2 + f) + A*a*log(tan(e/2 + f*x/2))*tan(e/2 + f*x
/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + A*a*log(tan(e/2 + f*x/2))/(f*tan(e/2 +
 f*x/2)**2 + f) + B*a*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) +
 B*a*f*x/(f*tan(e/2 + f*x/2)**2 + f) - 2*B*a/(f*tan(e/2 + f*x/2)**2 + f))/d
, Eq(c, 0)), (A*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3
+ d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)*
*(3/2)) + A*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*
f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2))
 + 2*A*a*d**2*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(
e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A
*a*d**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(
3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x*sqrt(d**2)*tan(e/2
+ f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2
)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x*sqrt(d**2)/(d**
3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2
 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d
**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e
/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*
x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f
*(d**2)**(3/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)
**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d*
*2)**(3/2)) - B*a*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3
+ d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)*
*(3/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*
f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2))
 - B*a*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(
d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*B*a*d**2*tan(e/2 +
```

f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)*
*(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - 2*B*a*d**2*tan(e/2 + f*x/2)
/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*ta
n(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*B*a*d**2/(d**3*f*tan(e/2 + f*x/2)*
*3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**
2)**(3/2)) + B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x
/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*
(d**2)**(3/2)) - B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 +
f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2
- f*(d**2)**(3/2)) + B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)/(d**3*f*tan(e/2
+ f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2
- f*(d**2)**(3/2)) - B*a*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d*
*3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/
2)) + 2*B*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(
3/2)) + 4*B*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f
x/2) - f(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)), Eq(c, -sqrt
(d**2))), (A*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d
3*f*tan(e/2 + f*x/2) + f*(d2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3
/2)) + A*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*t
an(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) +
2*A*a*d**2*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2
+ f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*A*a*
d**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2
)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + A*a*d*f*x*sqrt(d**2)*tan(e/2 + f
*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**
(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + A*a*d*f*x*sqrt(d**2)/(d**3*f
*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 +
f*x/2)**2 + f*(d**2)**(3/2)) - 2*A*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3
*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2
+ f*x/2)**2 + f*(d**2)**(3/2)) - 2*A*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2
)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d
2)(3/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3
+ d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)
(3/2)) - B*a*d2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d
3*f*tan(e/2 + f*x/2) + f*(d2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3
/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*t
an(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) -
B*a*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**
2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*B*a*d**2*tan(e/2 + f*x
/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3
/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - 2*B*a*d**2*tan(e/2 + f*x/2)/(d
3*f*tan(e/2 + f*x/2)3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e
/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*B*a*d**2/(d**3*f*tan(e/2 + f*x/2)**3
+ d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)*

```
*(3/2)) - B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)
**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d*
*2)**(3/2)) + B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*
x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f
*(d**2)**(3/2)) - B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f
*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 +
f*(d**2)**(3/2)) + B*a*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*
f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2))
 - 2*B*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**
3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2
)) - 4*B*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/
2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)), Eq(c, sqrt(d**
2))), ((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*cos(e
+ f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f)/c,
Eq(d, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(c + d*sin(e)), Eq(f, 0)), (-A*
a*c*d*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**
2/(d**2*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2
)) - A*a*c*d*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d**2*f*sqr
t(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + A*a*c*d*
log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**
2*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + A
*a*c*d*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**
2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + A*a*d**2*log(t
an(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*s
qrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + A*a*d*
*2*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 +
d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - A*a*d**2*log(tan(e
/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(
-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - A*a*d**2*l
og(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 + d**2
)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + A*a*d*f*x*sqrt(-c**2 +
 d**2)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 +
 d**2*f*sqrt(-c**2 + d**2)) + A*a*d*f*x*sqrt(-c**2 + d**2)/(d**2*f*sqrt(-c*
*2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + B*a*c**2*log(
tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*
sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + B*a*c
**2*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 +
 d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - B*a*c**2*log(tan(
e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt
(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - B*a*c**2*
log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 + d**
2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - B*a*c*d*log(tan(e/2 +
 f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(-c**
2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - B*a*c*d*log(ta
n(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 + d**2)*tan
```

```
(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + B*a*c*d*log(tan(e/2 + f*x/2
) + d/c + sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(-c**2 + d*
*2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + B*a*c*d*log(tan(e/2
+ f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 + d**2)*tan(e/2 +
 f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - B*a*c*f*x*sqrt(-c**2 + d**2)*tan(
e/2 + f*x/2)**2/(d**2*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqr
t(-c**2 + d**2)) - B*a*c*f*x*sqrt(-c**2 + d**2)/(d**2*f*sqrt(-c**2 + d**2)*
tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + B*a*d*f*x*sqrt(-c**2 + d
**2)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d
**2*f*sqrt(-c**2 + d**2)) + B*a*d*f*x*sqrt(-c**2 + d**2)/(d**2*f*sqrt(-c**2
 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - 2*B*a*d*sqrt(-c
**2 + d**2)/(d**2*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c
**2 + d**2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{\frac{(Bac-Aad-Bad)(fx+e)}{d^2} + \frac{2\,Ba}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d} - \frac{2\,\left(Bac^2-Aacd-Bacd+Aad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 +
1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 3074, normalized size of antiderivative = 31.37

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*A*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*B*a*at
an(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (B*a*cos(e + f*x))
/(f*(c + d)) + (2*A*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(
c + d)) - (A*a*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i + A^2*
d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2)*(d
^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*
d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2)*(d
^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + A^
2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (f*x)
/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1
i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e/2 +
 (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^3*d
^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x)/2)
*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*3i
 - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*sin(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c
^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^2*c^
4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*x)/2
)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*6i
+ A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2 + (
f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/
2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d^4*c
os(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)*(d^
2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A
*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2) + 2
*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*A^2*
c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5*d^2
*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*sin(
```

e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f*x)/2) + 2*B^2*c^4*d^3*sin(e/2 + (f*x)/2) + 4*A*B*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c*d^6*cos(e/2 + (f*x)/2) + B^2*c*d^6*cos(e/2 + (f*x)/2) + 4*A^2*c*d^6*sin(e/2 + (f*x)/2) - 4*A*B*c^3*d^4*cos(e/2 + (f*x)/2) + 2*A*B*c^5*d^2*cos(e/2 + (f*x)/2) - 8*A*B*c^2*d^5*sin(e/2 + (f*x)/2) + 4*A*B*c^4*d^3*sin(e/2 + (f*x)/2) + 2*A*B*c*d^6*cos(e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(d*f*(c + d)) - (2*B*a*c^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d^2*f*(c + d)) - (B*a*c*cos(e + f*x))/(d*f*(c + d)) + (B*a*c*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i + A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*3i - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*6i + A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*A^2*c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5*d^2*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*sin(e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f*x)/2) + 2*B^2*c^4*d^3*sin(e/2 + (f*x)/2) + 4*A*B*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c*d^6*cos(e/2 + (f*x)/2) + B^2*c*d^6*cos(e/2 + (f*x)/2) + 4*A^2*c*d^6*sin(e/2 + (f*x)/2) - 4*A*B*c^3*d^4*cos(e/2 + (f*x)/2) + 2*A*B*c^5*d^2*cos(e/2 + (f*x)/2) - 8*A*B*c^2*d^5*sin(e/2 + (f*x)/2) + 4*A*B*c^4*d^3*sin(e/2 + (f*x)/2) + 2*A*B*c*d^6*cos(e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(d^2*f*(c + d))

3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{aBx}{d^2}+\frac{2a((A+B)(c-d)d^2-Bc(c^2-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f}+\frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] a*B*x/d^2+2*a*((A+B)*(c-d)*d^2-B*c*(c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e
))/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f
/(c+d*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3047, 3100, 2814, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{2a(d^2(A+B)(c-d)-Bc(c^2-d^2))\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{aBx}{d^2}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/d^2 + (2*a*((A + B)*(c - d)*d^2 - B*c*(c^2 - d^2))*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d
)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{aA + (aA + aB)\sin(e+fx) + aB\sin^2(e+fx)}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\int \frac{-a(A+B)(c-d)d-aB(c^2-d^2)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d(c^2-d^2)}$$
$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{(a(Ad^2-B(c^2+cd-d^2)))\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2(c+d)}$$
$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{(2a(Ad^2-B(c^2+cd-d^2)))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2(c+d)f}$$
$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{(4a(Ad^2-B(c^2+cd-d^2)))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2(c+d)f}$$
$$= \frac{aBx}{d^2} + \frac{2a(Ad^2-B(c^2+cd-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{a(1+\sin(e+fx))\left(Bx + \frac{2(Ad^2-B(c^2+cd-d^2))\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{(c+d)\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{(-B}{}\right.}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[(
f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c
^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-(B*c) + A*d)*Csc[e]*(c*Cos[e]
 + d*Sin[f*x]))/((c + d)*f*(c + d*Sin[e + f*x]))))/(d^2*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{2a\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\frac{-\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-cdB+d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}\right.}{f}$
default	$\frac{2a\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\frac{-\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-cdB+d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}\right.}{f}$
risch	$\frac{aBx}{d^2}-\frac{2ia(-dA+Bc)\left(id+c\,e^{i(fx+e)}\right)}{d^2(c+d)f\left(-ie^{2i(fx+e)}d+id+2c\,e^{i(fx+e)}\right)}-\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)A}{\sqrt{-c^2+d^2}\,(c+d)f}+\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-}}{}\right.}{\sqrt{-c^2+d^2}}$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a*(B/d^2*arctan(tan(1/2*f*x+1/2*e))+1/d^2*((-d^2*(A*d-B*c)/(c+d)/c*tan(
1/2*f*x+1/2*e)-d*(A*d-B*c)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1
/2*e)+c)+(A*d^2-B*c^2-B*c*d+B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(119) = 238.

Time = 0.29 (sec) , antiderivative size = 655, normalized size of antiderivative = 5.28

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \left[\frac{2\left(Bac^3d+Bac^2d^2-Bacd^3-Bad^4\right)fx\sin\left(fx+e\right)+2\left(Bac^4+Bac^3d-Bac^2d^2-Bacd^3\right)fx+(Bac\right.}{}\right.$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) +
2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*
d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))
```

```
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c^3*d -
 A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c*d
^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f), ((B*a*
c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a*c^4 + B*
a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (A + B)*a*c
*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqrt(c^2 - d^2
)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (B*a*c^3*d
 - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c
*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{(fx+e)Ba}{d^2} - \frac{2\,(Bac^2+Bacd-Aad^2-Bad^2)\left(\pi\left\lfloor \frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^2+d^3)\sqrt{c^2-d^2}} + \frac{2\,(Bacd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Aad^2\tan\left(\frac{1}{2}\,fx\right.}{(c^2d+cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\,\mathrm{ta}\right.}}{f}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] ((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2*e)
 - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 5102, normalized size of antiderivative = 41.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] (2*B*a*atan(((B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d
))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 +
2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B
^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2
*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a
*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c
^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d
^3 + d^4 + c^2*d^2) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 +
 d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4
*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2))/d^2 +
(B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 +
 d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3
*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d
+ 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5
+ c^2*d^3) + (B*a*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5
```

))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*
c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^
2*d^3) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^
2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*
c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2))/d^2)/((64*(B^3*a^3*
c^3 + A*B^2*a^3*c^3 - B^3*a^3*c*d^2 + B^3*a^3*c^2*d - 2*A*B^2*a^3*c*d^2 + A
*B^2*a^3*c^2*d - A^2*B*a^3*c*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (64*tan(e/2
+ (f*x)/2)*(2*B^3*a^3*c*d^3 - 2*B^3*a^3*c^4 - 4*B^3*a^3*c^3*d + 2*A*B^2*a^3
*c*d^3 + 2*A*B^2*a^3*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (B*a*((32*(B^2*a
^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2)
+ (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2
*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*
d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (B*
a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*
a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*
a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (B*a*((
32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2
+ (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d
^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2)*1i)/d^2 + (B*a*((32*(B^2*a^2*c^2*d^3
+ 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(
e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 -
A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*
B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(B*
a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^
2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*
B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(c^2*d^
7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2
)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 +
c^2*d^3))*1i)/d^2)*1i)/d^2)*1i)/d^2)))/(d^2*f) - ((2*(A*a*d - B*a*c))/(d*(
c + d)) + (2*a*tan(e/2 + (f*x)/2)*(A*d - B*c))/(c*(c + d)))/(f*(c + 2*d*tan
(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (a*atan(((a*(-(c + d)^3*(c - d
))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d
^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2
*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^
5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 +
d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A
*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*
c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^
5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8
+ c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3
(c - d))^(1/2)(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3
- c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 -
c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*1i)/(2*c*d^5 + d^6 - 2*c^3*d^3 -
c^4*d^2) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*

c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^ 5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1 /2)*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a* c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d ^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B* c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*1 i)/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))/((64*(B^3*a^3*c^3 + A*B^2*a^3*c^3 - B^3*a^3*c*d^2 + B^3*a^3*c^2*d - 2*A*B^2*a^3*c*d^2 + A*B^2*a^3*c^2*d - A^ 2*B*a^3*c*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (64*tan(e/2 + (f*x)/2)*(2*B^3*a ^3*c*d^3 - 2*B^3*a^3*c^4 - 4*B^3*a^3*c^3*d + 2*A*B^2*a^3*c*d^3 + 2*A*B^2*a^ 3*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (a*(-(c + d)^3*(c - d))^(1/2)*((32* (B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2 *d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B ^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^ 2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B *a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c *d^3 + d^4 + c^2*d^2) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4 *c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d ^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2 *c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2* d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^ 5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*(B*a*c*d ^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c ^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a*((32*(c^2*d^7 + 2*c ^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c* d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^ 3))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2)))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*2i)/(f*(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))

3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\left(c^2-d^2\right)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))}$$

```
[Out] a*(2*A*c-A*d+B*c-2*B*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c
+d)/(c^2-d^2)^(3/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e)
)^2-1/2*a*(A*(c-2*d)*d+B*(c^2+2*c*d-2*d^2))*cos(f*x+e)/(c-d)/d/(c+d)^2/f/(c
+d*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3047, 3100, 2833, 12, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(2Ac-Ad+Bc-2Bd)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}} - \frac{a(Ad(c-2d)+B(c^2+2cd-2d^2))\cos(e+fx)}{2df(c-d)(c+d)^2(c+d\sin(e+fx))} + \frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c + d)*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2
))*Cos[e + f*x])/(2*(c - d)*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^3}\,dx \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a(A+B)(c-d)d-a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2d\left(c^2-d^2\right)} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{a(c-d)d(2Ac+Bc-Ad-2Bd)}{c+d\sin(e+fx)}\,dx}{2d\left(c^2-d^2\right)^2} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))} \\
&\quad + \frac{(a(2Ac+Bc-Ad-2Bd))\int \frac{1}{c+d\sin(e+fx)}\,dx}{2(c-d)(c+d)^2} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))} \\
&\quad + \frac{(a(2Ac+Bc-Ad-2Bd))\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)(c+d)^2f} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))} \\
&\quad - \frac{(2a(2Ac+Bc-Ad-2Bd))\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)(c+d)^2f} \\
&= \frac{a(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}f} \\
&\quad + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.96

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= \frac{a(1+\sin(e+fx))\left(\frac{4(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{(2c^2+d^2)(A\cdots}{\cdots}\right.}{4(c-d)\cdots}$$

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]*
(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]
*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(
Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2
*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2))*Cos
[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d^2))*S
in[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e + f*x]))
/(d^2*(c + d*Sin[e + f*x])^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(167) = 334.

Time = 1.10 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.41

method	result
derivativedivides	$2a\left(\frac{-\frac{\left(3c^2dA-2d^2cA-2A\,d^3-B\,c^3+2c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^3+c^2d-d^2c-d^3\right)}-\frac{\left(2A\,c^4-2A\,c^3d+3A\,c^2d^2-4Ac\,d^3-2A\,d^4+2B\,c^4-B\,c^3d+4B\,c^2d^2\cdots\right.}{2\left(c^3+c^2d-d^2c-d^3\right)c^2}\cdots}{\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2c\cdots\right.}\right.$
default	$2a\left(\frac{-\frac{\left(3c^2dA-2d^2cA-2A\,d^3-B\,c^3+2c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^3+c^2d-d^2c-d^3\right)}-\frac{\left(2A\,c^4-2A\,c^3d+3A\,c^2d^2-4Ac\,d^3-2A\,d^4+2B\,c^4-B\,c^3d+4B\,c^2d^2\cdots\right.}{2\left(c^3+c^2d-d^2c-d^3\right)c^2}\cdots}{\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2c\cdots\right.}\right.$
risch	Expression too large to display

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a*((-1/2*(3*A*c^2*d-2*A*c*d^2-2*A*d^3-B*c^3+2*B*c^2*d)/c/(c^3+c^2*d-c*d
^2-d^3)*tan(1/2*f*x+1/2*e)^3-1/2*(2*A*c^4-2*A*c^3*d+3*A*c^2*d^2-4*A*c*d^3-2
*A*d^4+2*B*c^4-B*c^3*d+4*B*c^2*d^2-2*B*c*d^3)/(c^3+c^2*d-c*d^2-d^3)/c^2*tan
(1/2*f*x+1/2*e)^2-1/2*(5*A*c^2*d-6*A*c*d^2-2*A*d^3+B*c^3+6*B*c^2*d-4*B*c*d^
2)/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)-1/2*(2*A*c^2-2*A*c*d-A*d^2+2*
B*c^2-B*c*d)/(c^3+c^2*d-c*d^2-d^3))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x
+1/2*e)+c)^2+1/2*(2*A*c-A*d+B*c-2*B*d)/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(167) = 334$.

Time = 0.30 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.49

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*
c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A
+ 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 -
 (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2
)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))
*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)
*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d
^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3
*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 -
 c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)*a*c^3*d - (
2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*si
n(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A
+ 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2
*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-
(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*(A + B)*a*c^4 - (
2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*d^3 + A*a*d^4)*cos
(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos
(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)
*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*
d^6 + d^7)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(167) = 334$.

Time = 0.33 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.24

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx$$
$$= \frac{\frac{(2\,Aac+Bac-Aad-2\,Bad)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^3+c^2d-cd^2-d^3)\sqrt{c^2-d^2}} + \frac{Bac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,Aac^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Bac^3d}{}}{}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d -
c*d^2 - d^3)*sqrt(c^2 - d^2)) + (B*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 3*A*a*c^3
*d*tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c^2*
d^2*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a*c^4
```

```
*tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^3*d*ta
n(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*d^2*tan
(1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*A*a*c*d^3*ta
n(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*d^4*tan(1
/2*f*x + 1/2*e)^2 - B*a*c^4*tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*tan(1/2*f*x
+ 1/2*e) - 6*B*a*c^3*d*tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*tan(1/2*f*x + 1
/2*e) + 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*
e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2*d^2)/((c^5 +
 c^4*d - c^3*d^2 - c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1
/2*e) + c)^2))/f
```

Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{A\,a\,d^2-2\,A\,a\,c^2-2\,B\,a\,c^2+2\,A\,a\,c\,d+B\,a\,c\,d}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,d^3-B\,c^3+6\,A\,c\,d^2-5\,A\,c^2\,d+4\,B\,c\,d^2-6\,B\,c^2\,d\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(2\,A\,d^3\cdots\right.}{c\,(-c\cdots}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2+4\,d^2\right)+c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+c^2+4\,c\,d\,t\cdots\right.}$$

$$-\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,(2\,A\,c-A\,d+B\,c-2\,B\,d)\,\left(-2\,c^3\,d-2\,c^2\,d^2+2\,c\,d^3+2\,d^4\right)}{2\,(c+d)^{5/2}\,(c-d)^{3/2}\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A\,c-A\,d+B\,c-2\,B\,d)}{(c+d)^{5/2}\,(c-d)^{3/2}}\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}{2\,A\,a\,c-A\,a\,d+B\,a\,c-2\,B\,a\,d}\right)\,(2\,A\,c-\cdots}{f\,(c+d)^{5/2}\,(c-d)^{3/2}}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((A*a*d^2 - 2*A*a*c^2 - 2*B*a*c^2 + 2*A*a*c*d + B*a*c*d)/(c*d^2 - c^2*d -
 c^3 + d^3) + (a*tan(e/2 + (f*x)/2)*(2*A*d^3 - B*c^3 + 6*A*c*d^2 - 5*A*c^2*
d + 4*B*c*d^2 - 6*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 +
(f*x)/2)^3*(2*A*d^3 + B*c^3 + 2*A*c*d^2 - 3*A*c^2*d - 2*B*c^2*d))/(c*(c*d^2
 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*d^2 - 2*A
*c^2 - 2*B*c^2 + 2*A*c*d + B*c*d))/(c^2*(c*d^2 - c^2*d - c^3 + d^3)))/(f*(t
an(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*
d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (a*atan((((a*(2*A*c -
 A*d + B*c - 2*B*d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2*(c + d)^(5/
2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 + (f*x)/2)*(2*
A*c - A*d + B*c - 2*B*d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2 - c^2*d - c
^3 + d^3))/(2*A*a*c - A*a*d + B*a*c - 2*B*a*d))*(2*A*c - A*d + B*c - 2*B*d)
)/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 35, antiderivative size = 464

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{16}a^2\big(6A\big(4c^3+8c^2d+7cd^2+2d^3\big)+B\big(16c^3+42c^2d+36cd^2+11d^3\big)\big)\,x \\
&\quad + \frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)-B(2c^5-12c^4d+47c^3d^2+208c^2d^3+216cd^4+64d^5))\,c}{60d^2f} \\
&\quad + \frac{a^2(6Ad(2c^3-20c^2d-57cd^2-30d^3)-B(4c^4-24c^3d+96c^2d^2+284cd^3+165d^4))\cos(e+fx)\sin(e+}{240df} \\
&\quad + \frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f} \\
&\quad + \frac{a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f} \\
&\quad + \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} \\
&\quad - \frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))\,(c+d\sin(e+fx))^4}{6df}
\end{aligned}
$$

```
[Out] 1/16*a^2*(6*A*(4*c^3+8*c^2*d+7*c*d^2+2*d^3)+B*(16*c^3+42*c^2*d+36*c*d^2+11*
d^3))*x+1/60*a^2*(6*A*d*(c^4-10*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)-B*(2*c^5-
12*c^4*d+47*c^3*d^2+208*c^2*d^3+216*c*d^4+64*d^5))*cos(f*x+e)/d^2/f+1/240*a
^2*(6*A*d*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)-B*(4*c^4-24*c^3*d+96*c^2*d^2+284
*c*d^3+165*d^4))*cos(f*x+e)*sin(f*x+e)/d/f+1/120*a^2*(6*A*d*(c^2-10*c*d-12*
d^2)-B*(2*c^3-12*c^2*d+51*c*d^2+64*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/
f+1/120*a^2*(6*A*(c-10*d)*d-B*(2*c^2-12*c*d+55*d^2))*cos(f*x+e)*(c+d*sin(f*
x+e))^3/d^2/f+1/30*a^2*(-6*A*d+2*B*c-7*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d
^2/f-1/6*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^4/d/f
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \frac{a^2(6Ad(c-10d)-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}$$
$$+\frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f}$$
$$+\frac{1}{16}a^2x\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c^2d+36cd^2+11d^3\right)\right)$$
$$+\frac{a^2(6Ad(2c^3-20c^2d-57cd^2-30d^3)-B(4c^4-24c^3d+96c^2d^2+284cd^3+165d^4))\sin(e+fx)\cos(e+}{240df}$$
$$+\frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)-B(2c^5-12c^4d+47c^3d^2+208c^2d^3+216cd^4+64d^5))\text{co}}{60d^2f}$$
$$+\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}$$
$$-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c
*d^2 + 11*d^3))*x)/16 + (a^2*(6*A*d*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3
 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 + 208*c^2*d^3 + 216*c*d^4 + 6
4*d^5))*Cos[e + f*x])/(60*d^2*f) + (a^2*(6*A*d*(2*c^3 - 20*c^2*d - 57*c*d^2
 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c*d^3 + 165*d^4))*Cos[e
 + f*x]*Sin[e + f*x])/(240*d*f) + (a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2) - B*(
2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)
/(120*d^2*f) + (a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^2*(2*B*c - 6*A*d - 7*B*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df} + \frac{\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3(a(6Ad+B(c+4d))-a(2Bc-6Ad-7Bd)\sin(e+fx))\,dx}{6d}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df} + \frac{\int (c+d\sin(e+fx))^3\left(a^2(6Ad+B(c+4d))+\left(-a^2(2Bc-6Ad-7Bd)+a^2(6Ad+B(c+4d\right.\right.}{6d}$$

$$= \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}$$
$$+ \frac{\int (c+d\sin(e+fx))^3\left(-3a^2d(Bc-18Ad-16Bd)-a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\right.}{30d^2}$$

$$= \frac{a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}$$
$$+ \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}$$
$$+ \frac{\int (c+d\sin(e+fx))^2\left(3a^2d(6Ad(11c+10d)-B(2c^2-52cd-55d^2))-3a^2(6Ad(c^2-10cd-1\right.}{120d^2}$$

$$= \frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f}$$
$$+ \frac{a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}$$
$$+ \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}$$
$$+ \frac{\int (c+d\sin(e+fx))\left(3a^2d(6Ad(31c^2+50cd+24d^2)-B(2c^3-132c^2d-267cd^2-128d^3))-3a\right.}{3}$$

$$= \frac{1}{16}a^2\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c^2d+36cd^2+11d^3\right)\right)x$$
$$+ \frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)-B(2c^5-12c^4d+47c^3d^2+208c^2d^3+216cd^4+6}{60d^2f}$$
$$+ \frac{a^2(6Ad(2c^3-20c^2d-57cd^2-30d^3)-B(4c^4-24c^3d+96c^2d^2+284cd^3+165d^4))\cos(e+fx)}{240df}$$
$$+ \frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f}$$
$$+ \frac{a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}$$
$$+ \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}$$

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.94

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx =$$

$$-\frac{a^2\cos(e + fx)\left(60(6A(4c^3 + 8c^2d + 7cd^2 + 2d^3) + B(16c^3 + 42c^2d + 36cd^2 + 11d^3))\arcsin\left(\frac{\sqrt{1-\sin(e+f}}{\sqrt{2}}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/480*(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(
16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*B*c
^2*d + 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*(5*c
^2 + 10*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2*(e +
 f*x)] + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Sin[e +
f*x] + 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c^2*d*Si
n[e + f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x] + 540*A*
d^3*Sin[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e + f*x)] - 9
0*A*c*d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*d^3*Sin[3*
(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)])))/(f*Sqr
t[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a^2\left(\left(\left(-2A-\frac{31B}{16}\right)d^3-6d^2c(A+B)-6c^2d(A+B)-c^3(A+2B)\right)\sin(2fx+2e)+\left(\left(\frac{3A}{4}+\frac{5B}{6}\right)d^3+2\left(\frac{9B}{8}+A\right)c\,d^2+c^2(A+2B\right.\right.}{}$
parts	$-\frac{(A\,a^2d^3+3B\,a^2d^2c+2B\,a^2d^3)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}-\frac{(2A\,a^2c^3+3A\,a^2c^2d+B\,a^2c^3)\cos(fx-}{f}$
derivativedivides	$\frac{A\,a^2c^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-A\,a^2c^2d(2+\sin^2(fx+e))\cos(fx+e)+3A\,a^2d^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(}{4}\right.}{}$
default	$\frac{A\,a^2c^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-A\,a^2c^2d(2+\sin^2(fx+e))\cos(fx+e)+3A\,a^2d^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(}{4}\right.}{}$
risch	$-\frac{3a^2d^2\cos(5fx+5e)Bc}{80f}+\frac{3\sin(4fx+4e)A\,a^2d^2c}{32f}+\frac{3\sin(4fx+4e)B\,a^2c^2d}{32f}+\frac{3\sin(4fx+4e)B\,a^2d^2c}{16f}+\frac{a^2\cos(3fx}{4}$
norman	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/4*a^2*(((-2*A-31/16*B)*d^3-6*d^2*c*(A+B)-6*c^2*d*(A+B)-c^3*(A+2*B))*sin(2
*f*x+2*e)+((3/4*A+5/6*B)*d^3+2*(9/8*B+A)*c*d^2+c^2*(A+2*B)*d+1/3*B*c^3)*cos
(3*f*x+3*e)+3/8*((2/3*A+5/6*B)*d^2+c*(A+2*B)*d+B*c^2)*d*sin(4*f*x+4*e)-1/20
*((A+2*B)*d+3*B*c)*d^2*cos(5*f*x+5*e)-1/48*B*d^3*sin(6*f*x+6*e)+((-5*B-11/2
*A)*d^3-18*(11/12*B+A)*c*d^2-21*(A+6/7*B)*c^2*d-8*c^3*(A+7/8*B))*cos(f*x+e)
+(-64/15*B+3*f*x*A+11/4*f*x*B-24/5*A)*d^3+21/2*c*(f*x*A+6/7*f*x*B-32/21*A-4
8/35*B)*d^2+12*c^2*(f*x*A+7/8*f*x*B-5/3*A-4/3*B)*d+6*c^3*(f*x*A+2/3*f*x*B-4
/3*A-10/9*B))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.78

$$\int(a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$

$$-\frac{48\left(3\,Ba^2cd^2+(A+2\,B)a^2d^3\right)\cos\left(fx+e\right)^5-80\left(Ba^2c^3+3\,(A+2\,B)a^2c^2d+3\,(2\,A+3\,B)a^2cd^2+(3\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*c
^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^3)
*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d + 6*(
7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^3 +
3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x + e) +
 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2*c*d^2
 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 + 6*(8*A
+ 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3)*cos(f*
x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1865 vs. $2(450) = 900$.

Time = 0.52 (sec) , antiderivative size = 1865, normalized size of antiderivative = 4.02

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a**2*c**3*x*sin(e + f*x)**2/2 + A*a**2*c**3*x*cos(e + f*x)**2/
2 + A*a**2*c**3*x - A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**3*cos(e + f*x)/f + 3*A*a**2*c**2*d*x*sin(e + f*x)**2 + 3*A*a**2*c**2*d*x
*cos(e + f*x)**2 - 3*A*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**
2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a**2*c**2*d*cos(e + f*x)**3/f -
3*A*a**2*c**2*d*cos(e + f*x)/f + 9*A*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*A*
a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**2*c*d**2*x*sin(e +
 f*x)**2/2 + 9*A*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**2*c*d**2*x*cos(e
+ f*x)**2/2 - 15*A*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*A*a**
2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**2*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 3*A*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a
**2*c*d**2*cos(e + f*x)**3/f + 3*A*a**2*d**3*x*sin(e + f*x)**4/4 + 3*A*a**2
*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**2*d**3*x*cos(e + f*x)**4
/4 - A*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*d**3*sin(e + f*x
)**3*cos(e + f*x)/(4*f) - 4*A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*
f) - A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*d**3*sin(e + f*x
)*cos(e + f*x)**3/(4*f) - 8*A*a**2*d**3*cos(e + f*x)**5/(15*f) - 2*A*a**2*d
**3*cos(e + f*x)**3/(3*f) + B*a**2*c**3*x*sin(e + f*x)**2 + B*a**2*c**3*x*c
os(e + f*x)**2 - B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c**3*s
in(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c**3*cos(e + f*x)**3/(3*f) - B*a**2*c
**3*cos(e + f*x)/f + 9*B*a**2*c**2*d*x*sin(e + f*x)**4/8 + 9*B*a**2*c**2*d*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**2*c**2*d*x*sin(e + f*x)**2/2 +
 9*B*a**2*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**2*c**2*d*x*cos(e + f*x)**2/2
- 15*B*a**2*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*B*a**2*c**2*d*sin
(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)**3/
```

```
(8*f) - 3*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*c**2*d*c
os(e + f*x)**3/f + 9*B*a**2*c*d**2*x*sin(e + f*x)**4/4 + 9*B*a**2*c*d**2*x*
sin(e + f*x)**2*cos(e + f*x)**2/2 + 9*B*a**2*c*d**2*x*cos(e + f*x)**4/4 - 3
*B*a**2*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**2*c*d**2*sin(e + f*
x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 3*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(4*f) - 8*B*a**2*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*
a**2*c*d**2*cos(e + f*x)**3/f + 5*B*a**2*d**3*x*sin(e + f*x)**6/16 + 15*B*a
**2*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**2*d**3*x*sin(e + f*x
)**4/8 + 15*B*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*d**
3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*B*a**2*d**3*x*cos(e + f*x)**6/16
+ 3*B*a**2*d**3*x*cos(e + f*x)**4/8 - 11*B*a**2*d**3*sin(e + f*x)**5*cos(e
+ f*x)/(16*f) - 2*B*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**
3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*B*a**2*d**3*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 8*B*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*B
*a**2*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*d**3*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 16*B*a**2*d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.56

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx$$
$$= \frac{240\,(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right))Aa^2c^3 + 960\,(fx + e)Aa^2c^3 + 320\,\left(\cos\left(fx + e\right)^3 - 3\,\cos\left(fx + e\right)\right)Ba^2c^3}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2
*c^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c^2*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(cos(f*x +
 e)^3 - 3*cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*
a^2*c^2*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d^2 + 90*(12*f*x
 + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*(2*f*x +
 2*e - sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x +
 e)^3 + 15*cos(f*x + e))*B*a^2*c*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e)
)*B*a^2*c*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*B*a^2*c*d^2 - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*
```

```
A*a^2*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^3 + 60*(12*f*x +
12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^3 - 128*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^3 + 5*(4*sin(2*f*x + 2*
e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*d^3
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^3 - 19
20*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e) - 2880*A*a^2*c^2*d*c
os(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{Ba^2d^3\sin(6\,fx+6\,e)}{192\,f} + \frac{1}{16}\left(24\,Aa^2c^3+16\,Ba^2c^3+48\,Aa^2c^2d+42\,Ba^2c^2d+42\,Aa^2cd^2+36\,Ba^2cd^2+12\,Aa^2d^3+11\,Ba^2d^3\right)x - \frac{(3\,Ba^2cd^2+Aa^2d^3+2\,Ba^2d^3)\cos(5\,fx+5\,e)}{80\,f} + \frac{(4\,Ba^2c^3+12\,Aa^2c^2d+24\,Ba^2c^2d+24\,Aa^2cd^2+27\,Ba^2cd^2+9\,Aa^2d^3+10\,Ba^2d^3)\cos(3\,fx+3\,e)}{48\,f} - \frac{(16\,Aa^2c^3+14\,Ba^2c^3+42\,Aa^2c^2d+36\,Ba^2c^2d+36\,Aa^2cd^2+33\,Ba^2cd^2+11\,Aa^2d^3+10\,Ba^2d^3)\cos(}{8\,f} + \frac{(6\,Ba^2c^2d+6\,Aa^2cd^2+12\,Ba^2cd^2+4\,Aa^2d^3+5\,Ba^2d^3)\sin(4\,fx+4\,e)}{64\,f} - \frac{(16\,Aa^2c^3+32\,Ba^2c^3+96\,Aa^2c^2d+96\,Ba^2c^2d+96\,Aa^2cd^2+96\,Ba^2cd^2+32\,Aa^2d^3+31\,Ba^2d^3)\sin(}{64\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 + 4
8*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a^2
*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*cos
(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d + 24*
A*a^2*c*d^2 + 27*B*a^2*c*d^2 + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x + 3*e)
/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2*d + 3
6*A*a^2*c*d^2 + 33*B*a^2*c*d^2 + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*x + e)/
f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c*d^2 + 12*B*a^2*c*d^2 + 4*A*a^2*d^3 + 5*
B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3 + 96*A*a^
2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c*d^2 + 96*B*a^2*c*d^2 + 32*A*a^2*d^3 +
 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 1291, normalized size of antiderivative = 2.78

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(24*A*c^3 + 12*A*d^3 + 16*B*c^3 + 11*B*d^
3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))/(8*(3*A*a^2*c^3 + (
3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*
a^2*c^2*d + (9*B*a^2*c*d^2)/2 + (21*B*a^2*c^2*d)/4)))*(24*A*c^3 + 12*A*d^3
+ 16*B*c^3 + 11*B*d^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))
/(8*f) - (tan(e/2 + (f*x)/2)*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (
11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 +
(21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*(20*A*a^2*c^3 + 4*A*a^2*d^3 + 14
*B*a^2*c^3 + 24*A*a^2*c*d^2 + 42*A*a^2*c^2*d + 12*B*a^2*c*d^2 + 24*B*a^2*c^
2*d) - tan(e/2 + (f*x)/2)^11*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (
11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 +
(21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^5*(2*A*a^2*c^3 + 7*A*a^2*d^3 + 4*B
*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^2*d + 21*B*a^
2*c*d^2 + (33*B*a^2*c^2*d)/2) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^3 + 7*A*a^2
*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^2*d
 + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) + tan(e/2 + (f*x)/2)^3*(3*A*a^2*c^3
 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24 + (87*A*a^2*c*d^2)/4
 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2*d)/4) - tan(e/2 + (f
*x)/2)^9*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24
 + (87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2
*d)/4) + tan(e/2 + (f*x)/2)^4*(40*A*a^2*c^3 + 32*A*a^2*d^3 + 36*B*a^2*c^3 +
 32*B*a^2*d^3 + 96*A*a^2*c*d^2 + 108*A*a^2*c^2*d + 96*B*a^2*c*d^2 + 96*B*a^
2*c^2*d) + tan(e/2 + (f*x)/2)^2*(20*A*a^2*c^3 + (72*A*a^2*d^3)/5 + 18*B*a^2
*c^3 + (64*B*a^2*d^3)/5 + 48*A*a^2*c*d^2 + 54*A*a^2*c^2*d + (216*B*a^2*c*d^
2)/5 + 48*B*a^2*c^2*d) + tan(e/2 + (f*x)/2)^6*(40*A*a^2*c^3 + 24*A*a^2*d^3
+ (100*B*a^2*c^3)/3 + (64*B*a^2*d^3)/3 + 80*A*a^2*c*d^2 + 100*A*a^2*c^2*d +
 72*B*a^2*c*d^2 + 80*B*a^2*c^2*d) + tan(e/2 + (f*x)/2)^10*(4*A*a^2*c^3 + 2*
B*a^2*c^3 + 6*A*a^2*c^2*d) + 4*A*a^2*c^3 + (12*A*a^2*d^3)/5 + (10*B*a^2*c^3
)/3 + (32*B*a^2*d^3)/15 + 8*A*a^2*c*d^2 + 10*A*a^2*c^2*d + (36*B*a^2*c*d^2)
/5 + 8*B*a^2*c^2*d)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 +
20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10
 + tan(e/2 + (f*x)/2)^12 + 1))
```

3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 336

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad^2+6Bd^2\right)x \\
&+ \frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d+16c^2d^2+40cd^3+18d^4))\cos(e+fx)}{30d^2f} \\
&+ \frac{a^2(5Ad(2c^2-16cd-21d^2)-B(4c^3-20c^2d+66cd^2+90d^3))\cos(e+fx)\sin(e+fx)}{120df} \\
&+ \frac{a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f} \\
&+ \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f} \\
&- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df}
\end{aligned}$$

```
[Out] 1/8*a^2*(12*A*c^2+16*A*c*d+7*A*d^2+8*B*c^2+14*B*c*d+6*B*d^2)*x+1/30*a^2*(5*
A*d*(c^3-8*c^2*d-20*c*d^2-8*d^3)-2*B*(c^4-5*c^3*d+16*c^2*d^2+40*c*d^3+18*d^
4))*cos(f*x+e)/d^2/f+1/120*a^2*(5*A*d*(2*c^2-16*c*d-21*d^2)-B*(4*c^3-20*c^2
*d+66*c*d^2+90*d^3))*cos(f*x+e)*sin(f*x+e)/d/f+1/60*a^2*(5*A*(c-8*d)*d-2*B*
(c^2-5*c*d+18*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/20*a^2*(2*B*(c-3*
d)-5*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f-1/5*B*cos(f*x+e)*(a^2+a^2*sin
(f*x+e))*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&=\frac{a^2(5Ad(c-8d)-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f} \\
&\quad+\frac{1}{8}a^2x\left(12Ac^2+16Acd+7Ad^2+8Bc^2+14Bcd+6Bd^2\right) \\
&\quad+\frac{a^2(5Ad(2c^2-16cd-21d^2)-B(4c^3-20c^2d+66cd^2+90d^3))\sin(e+fx)\cos(e+fx)}{120df} \\
&\quad+\frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d+16c^2d^2+40cd^3+18d^4))\cos(e+fx)}{30d^2f} \\
&\quad+\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f} \\
&\quad-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}
\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*A*c^2 + 8*B*c^2 + 16*A*c*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/8 +
(a^2*(5*A*d*(c^3 - 8*c^2*d - 20*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^
2*d^2 + 40*c*d^3 + 18*d^4))*Cos[e + f*x])/(30*d^2*f) + (a^2*(5*A*d*(2*c^2 -
 16*c*d - 21*d^2) - B*(4*c^3 - 20*c^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*
Sin[e + f*x])/(120*d*f) + (a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2
))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d^2*f) + (a^2*(2*B*(c - 3*d) -
5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d^2*f) - (B*Cos[e + f*x]*(a
^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(5*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df} + \frac{\int (a+a\sin(e+fx))(c+d\sin(e+fx))^2(a(5Ad+B(c+3d))-a(2Bc-5Ad-6Bd)\sin(e+fx))\,dx}{5d}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df} + \frac{\int (c+d\sin(e+fx))^2\left(a^2(5Ad+B(c+3d))+(-a^2(2Bc-5Ad-6Bd)+a^2(5Ad+B(c+3d)\right.}{5d}$$

$$= \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df}$$
$$+ \frac{\int(c+d\sin(e+fx))^2\left(-a^2d(2Bc-35Ad-30Bd)-a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))\sin\right.}{20d^2}$$
$$= \frac{a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f}$$
$$+ \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df}$$
$$+ \frac{\int(c+d\sin(e+fx))\left(a^2d(5Ad(19c+16d)-B(2c^2-70cd-72d^2))-a^2(5Ad(2c^2-16cd-21d\right.}{60d^2}$$
$$= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad^2+6Bd^2\right)x$$
$$+ \frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d+16c^2d^2+40cd^3+18d^4))\cos(e+fx)}{30d^2f}$$
$$+ \frac{a^2(5Ad(2c^2-16cd-21d^2)-B(4c^3-20c^2d+66cd^2+90d^3))\cos(e+fx)\sin(e+fx)}{120df}$$
$$+ \frac{a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f}$$
$$+ \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f}$$
$$- \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.88

$$\int(a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{a^2\cos(e+fx)\left(60(2B(4c^2+7cd+3d^2)+A(12c^2+16cd+7d^2))\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -1/240*(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16*
c*d + 7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]
*(480*A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2 - 8
```

```
*(10*A*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*d^2*
Cos[4*(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 480*A*
c*d*Sin[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] + 270*B*
d^2*Sin[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f*x)] -
30*B*d^2*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{\left(\left((-3B-3A)d^2-6dc(A+B)-\frac{3c^2(A+2B)}{2}\right)\sin(2fx+2e)+\left(\left(\frac{9B}{8}+A\right)d^2+c(A+2B)d+\frac{B\,c^2}{2}\right)\cos(3fx+3e)+\frac{3((A+2B)d}{}\right.}{}$
parts	$\frac{(A\,a^2d^2+2B\,a^2cd+2B\,a^2d^2)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(2A\,a^2c^2+2A\,a^2cd+B\,a^2c^2)\cos(}{f}$
risch	$-\frac{7a^2\cos(fx+e)Acd}{2f}-\frac{3a^2\cos(fx+e)cdB}{f}+\frac{\sin(4fx+4e)B\,a^2cd}{16f}+\frac{a^2\cos(3fx+3e)Acd}{6f}+\frac{a^2\cos(3fx+3e)cdB}{3f}$
derivativedivides	$\frac{A\,a^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2A\,a^2cd\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+A\,a^2d^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+}{4}\right.}{}$
default	$\frac{A\,a^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2A\,a^2cd\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+A\,a^2d^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+}{4}\right.}{}$
norman	$\frac{\left(\frac{3}{2}A\,a^2c^2+2A\,a^2cd+\frac{7}{8}A\,a^2d^2+B\,a^2c^2+\frac{7}{4}B\,a^2cd+\frac{3}{4}B\,a^2d^2\right)x+\left(15A\,a^2c^2+20A\,a^2cd+\frac{35}{4}A\,a^2d^2+10B\,a^2c^2+\frac{35}{2}B\,a^2cd\right.}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/6*(((-3*B-3*A)*d^2-6*d*c*(A+B)-3/2*c^2*(A+2*B))*sin(2*f*x+2*e)+((9/8*B+A)
*d^2+c*(A+2*B)*d+1/2*B*c^2)*cos(3*f*x+3*e)+3/16*((A+2*B)*d+2*B*c)*d*sin(4*f
*x+4*e)-3/40*B*d^2*cos(5*f*x+5*e)+((-9*A-33/4*B)*d^2-21*(A+6/7*B)*c*d-12*c^
2*(A+7/8*B))*cos(f*x+e)+(-36/5*B+21/4*f*x*A+9/2*f*x*B-8*A)*d^2+12*c*(f*x*A+
7/8*f*x*B-5/3*A-4/3*B)*d+9*c^2*(f*x*A+2/3*f*x*B-4/3*A-10/9*B))*a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{24\,Ba^2d^2\cos\left(fx+e\right)^5-40\,(Ba^2c^2+2\,(A+2\,B)a^2cd+(2\,A+3\,B)a^2d^2)\cos\left(fx+e\right)^3-15\,(4\,(3\,A+2}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/120*(24*B*a^2*d^2*cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d +
 (2*A + 3*B)*a^2*d^2)*cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A +
 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A + B)
*a^2*c*d + (A + B)*a^2*d^2)*cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*B)*a
^2*d^2)*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d + (9*
A + 10*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(330) = 660.

Time = 0.35 (sec) , antiderivative size = 1129, normalized size of antiderivative = 3.36

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a**2*c**2*x*sin(e + f*x)**2/2 + A*a**2*c**2*x*cos(e + f*x)**2/
2 + A*a**2*c**2*x - A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**2*cos(e + f*x)/f + 2*A*a**2*c*d*x*sin(e + f*x)**2 + 2*A*a**2*c*d*x*cos(e
 + f*x)**2 - 2*A*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A*a**2*c*d*sin
(e + f*x)*cos(e + f*x)/f - 4*A*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*A*a**2*c*
d*cos(e + f*x)/f + 3*A*a**2*d**2*x*sin(e + f*x)**4/8 + 3*A*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*d**2*x*sin(e + f*x)**2/2 + 3*A*a**2*
d**2*x*cos(e + f*x)**4/8 + A*a**2*d**2*x*cos(e + f*x)**2/2 - 5*A*a**2*d**2*
sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a**2*d**2*sin(e + f*x)**2*cos(e +
f*x)/f - 3*A*a**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*d**2*sin
(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a**2*d**2*cos(e + f*x)**3/(3*f) + B*a**2
*c**2*x*sin(e + f*x)**2 + B*a**2*c**2*x*cos(e + f*x)**2 - B*a**2*c**2*sin(e
 + f*x)**2*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a
**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e + f*x)/f + 3*B*a**2*c*d*
```

```
x*sin(e + f*x)**4/4 + 3*B*a**2*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*
a**2*c*d*x*sin(e + f*x)**2 + 3*B*a**2*c*d*x*cos(e + f*x)**4/4 + B*a**2*c*d*
x*cos(e + f*x)**2 - 5*B*a**2*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a
**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*c*d*sin(e + f*x)*cos(e +
f*x)**3/(4*f) - B*a**2*c*d*sin(e + f*x)*cos(e + f*x)/f - 8*B*a**2*c*d*cos(e
 + f*x)**3/(3*f) + 3*B*a**2*d**2*x*sin(e + f*x)**4/4 + 3*B*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/2 + 3*B*a**2*d**2*x*cos(e + f*x)**4/4 - B*a**2*
d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - 4*B*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*d**2*sin(e + f*x)*cos(e + f*
x)**3/(4*f) - 8*B*a**2*d**2*cos(e + f*x)**5/(15*f) - 2*B*a**2*d**2*cos(e +
f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a
)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.42

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2c^2+480\,(fx+e)Aa^2c^2+160\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)Ba^2c}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2
*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e)^3
- 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(
2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c*d +
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
 e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2 + 30*(12*f
*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 960*A*a^2*c^
2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= -\frac{Ba^2d^2\cos(5\,fx+5\,e)}{80\,f} + \frac{1}{8}\left(12\,Aa^2c^2+8\,Ba^2c^2+16\,Aa^2cd+14\,Ba^2cd+7\,Aa^2d^2+6\,Ba^2d^2\right)x + \frac{(4\,Ba^2c^2+8\,Aa^2cd+16\,Ba^2cd+8\,Aa^2d^2+9\,Ba^2d^2)\cos(3\,fx+3\,e)}{48\,f} - \frac{(16\,Aa^2c^2+14\,Ba^2c^2+28\,Aa^2cd+24\,Ba^2cd+12\,Aa^2d^2+11\,Ba^2d^2)\cos(fx+e)}{8\,f} + \frac{(2\,Ba^2cd+Aa^2d^2+2\,Ba^2d^2)\sin(4\,fx+4\,e)}{32\,f} - \frac{(Aa^2c^2+2\,Ba^2c^2+4\,Aa^2cd+4\,Ba^2cd+2\,Aa^2d^2+2\,Ba^2d^2)\sin(2\,fx+2\,e)}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/80*B*a^2*d^2*cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*A
*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^2
+ 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*cos(3*f*x + 3*e)/
f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12*A*a
^2*d^2 + 11*B*a^2*d^2)*cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2 + 2*B
*a^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d +
 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.28

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(12\,A\,c^2+7\,A\,d^2+8\,B\,c^2+6\,B\,d^2+16\,A\,c\,d+14\,B\,c\,d)}{4\,\left(3\,A\,a^2\,c^2+\frac{7\,A\,a^2\,d^2}{4}+2\,B\,a^2\,c^2+\frac{3\,B\,a^2\,d^2}{2}+4\,A\,a^2\,c\,d+\frac{7\,B\,a^2\,c\,d}{2}\right)}\right)\,(12\,A\,c^2+7\,A\,d^2+8\,B\,c^2+6\,B\,d^2+16\,A\,c\,d}{4\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,(4\,A\,a^2\,c^2+2\,B\,a^2\,c^2+4\,A\,a^2\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a^2\,c^2+\frac{7\,A\,a^2\,d^2}{4}+2\,B\,a^2\,c^2+\frac{3\,B\,a^2\,d^2}{2}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)

[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2 +
 16*A*c*d + 14*B*c*d))/(4*(3*A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3
*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)))*(12*A*c^2 + 7*A*d^2 + 8*B*
c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^
2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d) + tan(e/2 + (f*x)/2)*(A*a^2*c^2 + (7*A*a
^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)
- tan(e/2 + (f*x)/2)^9*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^
2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) + tan(e/2 + (f*x)/2)^3*(2*A*a^2*c
^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*
c*d) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 +
 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(16*A*a^2
*c^2 + 8*A*a^2*d^2 + 12*B*a^2*c^2 + 4*B*a^2*d^2 + 24*A*a^2*c*d + 16*B*a^2*c
*d) + tan(e/2 + (f*x)/2)^2*(16*A*a^2*c^2 + (40*A*a^2*d^2)/3 + (44*B*a^2*c^2
)/3 + 12*B*a^2*d^2 + (88*A*a^2*c*d)/3 + (80*B*a^2*c*d)/3) + tan(e/2 + (f*x)
/2)^4*(24*A*a^2*c^2 + (56*A*a^2*d^2)/3 + (64*B*a^2*c^2)/3 + 20*B*a^2*d^2 +
(128*A*a^2*c*d)/3 + (112*B*a^2*c*d)/3) + 4*A*a^2*c^2 + (8*A*a^2*d^2)/3 + (1
0*B*a^2*c^2)/3 + (12*B*a^2*d^2)/5 + (20*A*a^2*c*d)/3 + (16*B*a^2*c*d)/3)/(f
*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^
6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 166

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx\\
&=\frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)}{6f}\\
&\quad-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)\sin(e+fx)}{24f}\\
&\quad-\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^2}{12f}\\
&\quad-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}
\end{aligned}$$

```
[Out] 1/8*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*x-1/6*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos
(f*x+e)/f-1/24*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/12*
(4*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*d*cos(f*x+e)*(a+a*s
in(f*x+e))^3/a/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2830, 2723}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= -\frac{a^2(12Ac+8Ad+8Bc+7Bd)\cos(e+fx)}{6f} - \frac{a^2(12Ac+8Ad+8Bc+7Bd)\sin(e+fx)\cos(e+fx)}{24f} + \frac{1}{8}a^2x(12Ac+8Ad+8Bc+7Bd) - \frac{(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{12f} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*x)/8 - (a^2*(12*A*c + 8*B*c + 8*A*d +
 7*B*d)*Cos[e + f*x])/(6*f) - (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*Cos[e +
 f*x]*Sin[e + f*x])/(24*f) - ((4*B*c + 4*A*d - B*d)*Cos[e + f*x]*(a + a*Sin
[e + f*x])^2)/(12*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a+a\sin(e+fx))^2\left(Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)\right)\,dx\\
&= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}\\
&\quad+\frac{\int(a+a\sin(e+fx))^2(a(4Ac+3Bd)+a(4Bc+4Ad-Bd)\sin(e+fx))\,dx}{4a}\\
&= -\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^2}{12f}\\
&\quad-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}\\
&\quad+\frac{1}{12}(12Ac+8Bc+8Ad+7Bd)\int(a+a\sin(e+fx))^2\,dx\\
&= \frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)}{6f}\\
&\quad-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)\sin(e+fx)}{24f}\\
&\quad-\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^2}{12f}\\
&\quad-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int(a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$=\frac{\cos(e+fx)\left(-\frac{1}{3}a^3(4Bc+4Ad-Bd)(1+\sin(e+fx))^2-Bd(a+a\sin(e+fx))^3-\frac{a^3(12Ac+8Bc+8Ad+7Bd)}{}\right.}{4af}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] (Cos[e + f*x]*(-1/3*(a^3*(4*B*c + 4*A*d - B*d)*(1 + Sin[e + f*x])^2) - B*d*
(a + a*Sin[e + f*x])^3 - (a^3*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*(6*ArcSin[Sq
rt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(4 + Sin[e + f*x])))/(
6*Sqrt[Cos[e + f*x]^2])))/(4*a*f)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{\left(\left((-6c-6d)B-3A(c+2d)\right)\sin(2fx+2e)+(dA+B(c+2d))\cos(3fx+3e)+\frac{3dB\sin(4fx+4e)}{8}+\left((-21c-18d)B-24\left(c+\frac{7d}{8}\right)\right.\right.}{12f}$
parts	$-\frac{(A\,a^2d+B\,a^2c+2a^2dB)\,(2+\sin^2(fx+e))\cos(fx+e)}{3f}-\frac{(2a^2Ac+A\,a^2d+B\,a^2c)\cos(fx+e)}{f}+\frac{(a^2Ac+2A\,a^2d+2B}{}$
risch	$\frac{3A\,a^2cx}{2}+A\,a^2dx+B\,a^2cx+\frac{7B\,a^2dx}{8}-\frac{2a^2\cos(fx+e)Ac}{f}-\frac{7a^2\cos(fx+e)dA}{4f}-\frac{7a^2\cos(fx+e)Bc}{4f}-$
derivativedivides	$\frac{a^2Ac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-\frac{B\,a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2dB\left(-\frac{\left(\sin^3\right.}{}\right.}{}$
default	$\frac{a^2Ac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-\frac{B\,a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2dB\left(-\frac{\left(\sin^3\right.}{}\right.}{}$
norman	$\frac{(\frac{3}{2}a^2Ac+A\,a^2d+B\,a^2c+\frac{7}{8}a^2dB)x+(6a^2Ac+4A\,a^2d+4B\,a^2c+\frac{7}{2}a^2dB)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6a^2Ac+4A\,a^2d+4B\,a^2c+\frac{7}{2}}{}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/12*(((-6*c-6*d)*B-3*A*(c+2*d))*sin(2*f*x+2*e)+(d*A+B*(c+2*d))*cos(3*f*x+3
*e)+3/8*d*B*sin(4*f*x+4*e)+((-21*c-18*d)*B-24*(c+7/8*d)*A)*cos(f*x+e)+(12*c
*f*x+21/2*d*f*x-20*c-16*d)*B+18*((2/3*f*x-10/9)*d+c*(f*x-4/3))*A)*a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{8\,(Ba^2c+(A+2\,B)a^2d)\cos{(fx+e)}^3+3\,(4\,(3\,A+2\,B)a^2c+(8\,A+7\,B)a^2d)fx-48\,((A+B)a^2c+(A}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*c
 + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x + e)
 + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)*co
s(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(163) = 326$.

Time = 0.23 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.44

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \begin{cases} \frac{Aa^2cx\sin^2(e+fx)}{2} + \frac{Aa^2cx\cos^2(e+fx)}{2} + Aa^2cx - \frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^2c\cos(e+fx)}{f} + Aa^2dx\sin^2(e+fx) + \\ x(A + B\sin(e))(c + d\sin(e))(a\sin(e) + a)^2 \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A*
a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e + f*
x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*d*s
in(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*
a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*sin(e +
 f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(e + f*
x)/f - B*a**2*c*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x)**3/(3
*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*B*a**2*d
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/2 + 3*B*a
**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a**2*d*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e + f*x)/f -
 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin
(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{24(2fx + 2e - \sin(2fx + 2e))Aa^2c + 96(fx + e)Aa^2c + 32(\cos(fx + e)^3 - 3\cos(fx + e))Ba^2c + 48(}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c +
32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*
a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d +
 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x + e) - 9
6*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{Ba^2d\sin(4fx+4e)}{32f} + \frac{1}{8}\left(12Aa^2c+8Ba^2c+8Aa^2d+7Ba^2d\right)x + \frac{(Ba^2c+Aa^2d+2Ba^2d)\cos(3fx+3e)}{12f} - \frac{(8Aa^2c+7Ba^2c+7Aa^2d+6Ba^2d)\cos(fx+e)}{4f} - \frac{(Aa^2c+2Ba^2c+2Aa^2d+2Ba^2d)\sin(2fx+2e)}{4f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d +
 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f - 1
/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*(A*
a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.99 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.96

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d)}{4\left(3\,A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B\,a^2\,d}{4}\right)}\right)(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d)}{4\,f}$$
$$-\frac{a^2\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d)}{4\,f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{15\,B\,a^2\,d}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B}{}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*(3*A
*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4)))*(12*A*c + 8*A*d + 8*B*c +
 7*B*d))/(4*f) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*A*c + 8*A*d
+ 8*B*c + 7*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(A*a^2*c + 2*A*a^2*d + 2*B*
a^2*c + (15*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^7*(A*a^2*c + 2*A*a^2*d + 2*B*a
^2*c + (7*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^5*(A*a^2*c + 2*A*a^2*d + 2*B*a^2
*c + (15*B*a^2*d)/4) + tan(e/2 + (f*x)/2)^4*(12*A*a^2*c + 10*A*a^2*d + 10*B
*a^2*c + 8*B*a^2*d) + tan(e/2 + (f*x)/2)^2*(12*A*a^2*c + (34*A*a^2*d)/3 + (
34*B*a^2*c)/3 + (32*B*a^2*d)/3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c + 2*A*a^2
*d + 2*B*a^2*c) + tan(e/2 + (f*x)/2)*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*
B*a^2*d)/4) + 4*A*a^2*c + (10*A*a^2*d)/3 + (10*B*a^2*c)/3 + (8*B*a^2*d)/3)/
(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^
6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.254 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \frac{1}{2}a^2(3A+2B)x - \frac{2a^2(3A+2B)\cos(e+fx)}{3f} - \frac{a^2(3A+2B)\cos(e+fx)\sin(e+fx)}{6f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}$$

```
[Out] 1/2*a^2*(3*A+2*B)*x-2/3*a^2*(3*A+2*B)*cos(f*x+e)/f-1/6*a^2*(3*A+2*B)*cos(f*
x+e)*sin(f*x+e)/f-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2830, 2723}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{2a^2(3A+2B)\cos(e+fx)}{3f} - \frac{a^2(3A+2B)\sin(e+fx)\cos(e+fx)}{6f} + \frac{1}{2}a^2x(3A+2B) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 - (2*a^2*(3*A + 2*B)*Cos[e + f*x])/(3*f) - (a^2*(3*A
+ 2*B)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}+\frac{1}{3}(3A+2B)\int(a+a\sin(e+fx))^2\,dx\\ &= \frac{1}{2}a^2(3A+2B)x-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}\\ &\quad-\frac{a^2(3A+2B)\cos(e+fx)\sin(e+fx)}{6f}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx=$$

$$-\frac{a^2\cos(e+fx)\left(6(3A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(2(6A+5B)+3(A+2B)\sin(e+f}{6f\sqrt{\cos^2(e+fx)}}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/6*(a^2*Cos[e + f*x]*(6*(3*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*Sin[e + f*x] + 2*B*Si
n[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a^2(3(-A-2B)\sin(2fx+2e)+\cos(3fx+3e)B+3(-8A-7B)\cos(fx+e)+18fxA+12fxB-24A-20B)}{12f}$
parts	$a^2xA+\frac{(A\,a^2+2B\,a^2)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(2A\,a^2+B\,a^2)\cos(fx+e)}{f}-\frac{B\,a^2(2+\sin^2(fx+e))\cos(}{3f}$
risch	$\frac{3a^2xA}{2}+a^2xB-\frac{2a^2\cos(fx+e)A}{f}-\frac{7a^2\cos(fx+e)B}{4f}+\frac{B\,a^2\cos(3fx+3e)}{12f}-\frac{\sin(2fx+2e)A\,a^2}{4f}-\frac{\sin(2fx+}{2}$
derivativedivides	$\frac{A\,a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{B\,a^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2A\,a^2\cos(fx+e)+2B\,a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.}{f}$
default	$\frac{A\,a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{B\,a^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2A\,a^2\cos(fx+e)+2B\,a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.}{f}$
norman	$\frac{(\frac{3}{2}A\,a^2+B\,a^2)x+(\frac{3}{2}A\,a^2+B\,a^2)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{2}A\,a^2+3B\,a^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{2}A\,a^2+3B\,a^2)x\left(\tan^4\left(\frac{fx}{2}+\right.\right.}{(1}$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a^2*(3*(-A-2*B)*sin(2*f*x+2*e)+cos(3*f*x+3*e)*B+3*(-8*A-7*B)*cos(f*x+e
)+18*f*x*A+12*f*x*B-24*A-20*B)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$=\frac{2\,Ba^2\cos{(fx+e)}^3+3\,(3\,A+2\,B)a^2fx-3\,(A+2\,B)a^2\cos{(fx+e)}\sin{(fx+e)}-12\,(A+B)a^2\cos{(f}}{6\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*a^2*cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*cos(f
*x + e)*sin(f*x + e) - 12*(A + B)*a^2*cos(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$=\begin{cases}\frac{Aa^2x\sin^2(e+fx)}{2}+\frac{Aa^2x\cos^2(e+fx)}{2}+Aa^2x-\frac{Aa^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{2Aa^2\cos(e+fx)}{f}+Ba^2x\sin^2(e+fx)+Ba^2\\ x(A+B\sin(e))(a\sin(e)+a)^2\end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**2
*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B*a
**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**2*c
os(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f*x)*
*3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$=\frac{3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2+12\,(fx+e)Aa^2+4\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)Ba^2+6\,(2\,fx+}{12\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*B
*a^2 - 24*A*a^2*cos(f*x + e) - 12*B*a^2*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \frac{Ba^2\cos\left(3\,fx+3\,e\right)}{12\,f} + \frac{1}{2}\left(3\,Aa^2+2\,Ba^2\right)x - \frac{\left(8\,Aa^2+7\,Ba^2\right)\cos\left(fx+e\right)}{4\,f} - \frac{\left(Aa^2+2\,Ba^2\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 +
7*B*a^2)*cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{\frac{3\,A\,a^2\,\sin(2\,e+2\,f\,x)}{2} - \frac{B\,a^2\,\cos(3\,e+3\,f\,x)}{2} + 3\,B\,a^2\,\sin\left(2\,e+2\,f\,x\right) + 12\,A\,a^2\,\cos\left(e+f\,x\right) + \frac{21\,B\,a^2\,\cos(e+f\,x)}{2} - 9\ldots}{6\,f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] -((3*A*a^2*sin(2*e + 2*f*x))/2 - (B*a^2*cos(3*e + 3*f*x))/2 + 3*B*a^2*sin(2
*e + 2*f*x) + 12*A*a^2*cos(e + f*x) + (21*B*a^2*cos(e + f*x))/2 - 9*A*a^2*f
*x - 6*B*a^2*f*x)/(6*f)
```

3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 171

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx &= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} \\ &\quad -\frac{2a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} \\ &\quad +\frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f}-\frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2df}\end{aligned}$$

```
[Out] -1/2*a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x/d^3+1/2*a^2*(-2*A*d+2*B*c-
3*B*d)*cos(f*x+e)/d^2/f-1/2*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/f-2*a^2*(c-
d)^2*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2
-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{3055, 3047, 3102, 2814, 2739, 632, 210\}$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= -\frac{2a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}} - \frac{a^2x(2Ad(c-2d)-B(2c^2-4cd+3d^2))}{2d^3} + \frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/2*(a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/d^3 - (2*a^2*(c
- d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*S
qrt[c^2 - d^2]*f) + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(2*d^2*f) -
(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} \\
&\quad+\frac{\int \frac{(a+a\sin(e+fx))(a(Bc+2Ad)-a(2Bc-2Ad-3Bd)\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{2d} \\
&= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} \\
&\quad+\frac{\int \frac{a^2(Bc+2Ad)+\left(a^2(Bc+2Ad)-a^2(2Bc-2Ad-3Bd)\right)\sin(e+fx)-a^2(2Bc-2Ad-3Bd)\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx}{2d} \\
&= \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f}-\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} \\
&\quad+\frac{\int \frac{a^2d(Bc+2Ad)-a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{2d^2} \\
&= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3}+\frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} \\
&\quad-\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df}-\frac{\left(a^2(c-d)^2(Bc-Ad)\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^3}
\end{aligned}
$$

$$= -\frac{a^2(2A(c-2d)d - B(2c^2-4cd+3d^2))\,x}{2d^3} + \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2df} - \frac{(2a^2(c-d)^2(Bc-Ad))\,\mathrm{Subst}\left(\int\frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^3f}$$

$$= -\frac{a^2(2A(c-2d)d - B(2c^2-4cd+3d^2))\,x}{2d^3} + \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2df} + \frac{(4a^2(c-d)^2(Bc-Ad))\,\mathrm{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^3f}$$

$$= -\frac{a^2(2A(c-2d)d - B(2c^2-4cd+3d^2))\,x}{2d^3} - \frac{2a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} + \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2df}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{a^2(1+\sin(e+fx))^2\left(2(2Ad(-c+2d)+B(2c^2-4cd+3d^2))\,(e+fx) - \frac{8(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+f\right.}{\sqrt{c^2-d^2}}\right.}{\sqrt{c^2-d^2}}\right.}{4d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2))
*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] - B*
d^2*Sin[2*(e + f*x)]))/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{2a^2\left(-\frac{\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-cdB+2d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-cdB+2d^2B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{(2Acd-4A\,d^2-2B\,c^2-}{}}{d^3}\right.}{f}$
default	$\frac{2a^2\left(-\frac{\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-cdB+2d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-cdB+2d^2B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{(2Acd-4A\,d^2-2B\,c^2-}{}}{d^3}\right.}{f}$
risch	$-\frac{x\,a^2Ac}{d^2}+\frac{2x\,a^2A}{d}+\frac{x\,a^2B\,c^2}{d^3}-\frac{2x\,a^2cB}{d^2}+\frac{3x\,a^2B}{2d}-\frac{a^2e^{i(fx+e)}A}{2df}+\frac{a^2e^{i(fx+e)}Bc}{2d^2f}-\frac{a^2e^{i(fx+e)}B}{df}-a^2e$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a^2*(-1/d^3*((-1/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(A*d^2-B*c*d+2*B*d^2)*tan
(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)*d^2+A*d^2-c*d*B+2*d^2*B)/(1+tan(
1/2*f*x+1/2*e)^2)^2+1/2*(2*A*c*d-4*A*d^2-2*B*c^2+4*B*c*d-3*B*d^2)*arctan(ta
n(1/2*f*x+1/2*e)))+(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2*d-B*c*d^2)/d^3/(c
^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.64

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$=\left[-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e)-(2\,Ba^2c^2-2\,(A+2\,B)a^2cd+(4\,A+3\,B)a^2d^2)fx+(Ba^2c^2-(A+}{}\right.$$

$$-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e)-(2\,Ba^2c^2-2\,(A+2\,B)a^2cd+(4\,A+3\,B)a^2d^2)fx-2\,(Ba^2c^2-(A+}{2\,d^3f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x + (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f),
 -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^
2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))
/((c - d)*cos(f*x + e))) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/
(d^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2 (A + B\sin(e + fx))}{c + d\sin(e + fx)}\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^2 (A + B\sin(e + fx))}{c + d\sin(e + fx)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.78

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= \frac{\frac{(2\,Ba^2c^2-2\,Aa^2cd-4\,Ba^2cd+4\,Aa^2d^2+3\,Ba^2d^2)(fx+e)}{d^3}-\frac{4\,(Ba^2c^3-Aa^2c^2d-2\,Ba^2c^2d+2\,Aa^2cd^2+Ba^2cd^2-Aa^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\mathrm{ar}\right.}{\sqrt{c^2-d^2}d^3}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)*
(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^2
+ B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3) + 2*
(B*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^
2*d*tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*tan(1/2*f*x + 1/2*e)^2 - B*a^2*d*tan
(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((tan(1/2*f*x + 1/2*
e)^2 + 1)^2*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.39 (sec) , antiderivative size = 7371, normalized size of antiderivative = 43.11

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x)),x)
```

```
[Out] (atan(((((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 +
9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^
5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*
B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*
x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i
)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^
8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*tan(e/2
+ (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^
2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*
(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x)
/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a
^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 -
```

44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*1i)/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*1i)/d^3)/((16*(2*B^3*a^6*c^7 + 20*A^3*a^6*c^2*d^5 - 16*A^3*a^6*c^3*d^4 + 4*A^3*a^6*c^4*d^3 + 3*B^3*a^6*c^3*d^4 - 10*B^3*a^6*c^4*d^3 + 13*B^3*a^6*c^5*d^2 - 8*A^3*a^6*c*d^6 - 8*B^3*a^6*c^6*d - 6*A^2*B*a^6*c*d^6 + 3*A*B^2*a^6*c^2*d^5 - 6*A*B^2*a^6*c^3*d^4 + 3*A*B^2*a^6*c^4*d^3 + 24*A^2*B*a^6*c^2*d^5 - 36*A^2*B*a^6*c^3*d^4 + 24*A^2*B*a^6*c^4*d^3 - 6*A^2*B*a^6*c^5*d^2))/d^5 - (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8*(8*A*a^2*c*d^8 + 6*B*a^2

*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + ((32*c
^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i
+ (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*tan(
e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^
2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*
d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (
f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A
^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^
5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^
4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*
B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8)
)/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*
1i)/2))/d^3 - (16*tan(e/2 + (f*x)/2)*(104*A^3*a^6*c^3*d^5 - 96*A^3*a^6*c^2*
d^6 - 8*B^3*a^6*c^8 - 48*A^3*a^6*c^4*d^4 + 8*A^3*a^6*c^5*d^3 - 18*B^3*a^6*c
^2*d^6 + 84*B^3*a^6*c^3*d^5 - 170*B^3*a^6*c^4*d^4 + 192*B^3*a^6*c^5*d^3 - 1
28*B^3*a^6*c^6*d^2 + 32*A^3*a^6*c*d^7 + 48*B^3*a^6*c^7*d + 18*A*B^2*a^6*c*d
^7 + 24*A*B^2*a^6*c^7*d + 48*A^2*B*a^6*c*d^7 - 132*A*B^2*a^6*c^2*d^6 + 354*
A*B^2*a^6*c^3*d^5 - 480*A*B^2*a^6*c^4*d^4 + 360*A*B^2*a^6*c^5*d^3 - 144*A*B
^2*a^6*c^6*d^2 - 216*A^2*B*a^6*c^2*d^6 + 384*A^2*B*a^6*c^3*d^5 - 336*A^2*B*
a^6*c^4*d^4 + 144*A^2*B*a^6*c^5*d^3 - 24*A^2*B*a^6*c^6*d^2))/d^6))*(B*a^2*c
^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*2i)/(d^3
f) - ((2(A*a^2*d - B*a^2*c + 2*B*a^2*d))/d^2 + (2*tan(e/2 + (f*x)/2)^2*(A
*a^2*d - B*a^2*c + 2*B*a^2*d))/d^2 - (B*a^2*tan(e/2 + (f*x)/2)^3)/d + (B*a^
2*tan(e/2 + (f*x)/2))/d)/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4
+ 1)) + (a^2*atan(((a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*(16*A^2*
a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 -
24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^
6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*
B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^
4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 4
3*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6
*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4
*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16
*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c + d)*(c -
d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A
*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 -
(8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*
B*a^2*c^3*d^6))/d^5 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 -
8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 + d^4)))/(
c*d^3 + d^4))*1i)/(c*d^3 + d^4) + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/
2)*((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2
*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3
+ 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4
*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*
d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*

a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 + d^4)))/(c*d^3 + d^4))*1i)/(c*d^3 + d^4))/((16*(2*B^3*a^6*c^7 + 20*A^3*a^6*c^2*d^5 - 16*A^3*a^6*c^3*d^4 + 4*A^3*a^6*c^4*d^3 + 3*B^3*a^6*c^3*d^4 - 10*B^3*a^6*c^4*d^3 + 13*B^3*a^6*c^5*d^2 - 8*A^3*a^6*c*d^6 - 8*B^3*a^6*c^6*d - 6*A^2*B*a^6*c*d^6 + 3*A*B^2*a^6*c^2*d^5 - 6*A*B^2*a^6*c^3*d^4 + 3*A*B^2*a^6*c^4*d^3 + 24*A^2*B*a^6*c^2*d^5 - 36*A^2*B*a^6*c^3*d^4 + 24*A^2*B*a^6*c^4*d^3 - 6*A^2*B*a^6*c^5*d^2))/d^5 - (16*tan(e/2 + (f*x)/2)*(104*A^3*a^6*c^3*d^5 - 96*A^3*a^6*c^2*d^6 - 8*B^3*a^6*c^8 - 48*A^3*a^6*c^4*d^4 + 8*A^3*a^6*c^5*d^3 - 18*B^3*a^6*c^2*d^6 + 84*B^3*a^6*c^3*d^5 - 170*B^3*a^6*c^4*d^4 + 192*B^3*a^6*c^5*d^3 - 128*B^3*a^6*c^6*d^2 + 32*A^3*a^6*c*d^7 + 48*B^3*a^6*c^7*d + 18*A*B^2*a^6*c*d^7 + 24*A*B^2*a^6*c^7*d + 48*A^2*B*a^6*c*d^7 - 132*A*B^2*a^6*c^2*d^6 + 354*A*B^2*a^6*c^3*d^5 - 480*A*B^2*a^6*c^4*d^4 + 360*A*B^2*a^6*c^5*d^3 - 144*A*B^2*a^6*c^6*d^2 - 216*A^2*B*a^6*c^2*d^6 + 384*A^2*B*a^6*c^3*d^5 - 336*A^2*B*a^6*c^4*d^4 + 144*A^2*B*a^6*c^5*d^3 - 24*A^2*B*a^6*c^6*d^2))/d^6 - (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 + d^4)))/(c*d^3 + d^4)))/(c*d^3 + d^4) + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 8

```
0*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5
*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c
+ d)*(c - d)^3)^(1/2)*((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7
- 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*A*a^2*
c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3
*d^7 - 8*B*a^2*c^4*d^6))/d^6 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12
*c*d^10 - 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 +
 d^4)))/(c*d^3 + d^4)))/(c*d^3 + d^4)))*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1
/2)*2i)/(f*(c*d^3 + d^4))
```

3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 198

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} - \frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}\,f} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] -a^2*(-A*d+2*B*c-2*B*d)*x/d^3+a^2*(A*d-B*(2*c+d))*cos(f*x+e)/d^2/(c+d)/f+(-
A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))-2*a^2*(
c-d)*(A*d*(c+2*d)-B*(2*c^2+2*c*d-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2
-d^2)^(1/2))/d^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{3054, 3047, 3102, 2814, 2739, 632, 210\}$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= -\frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}$$
$$-\frac{a^2x(-Ad+2Bc-2Bd)}{d^3}+\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2f(c+d)}$$
$$+\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*(2*B*c - A*d - 2*B*d)*x)/d^3) - (2*a^2*(c - d)*(A*d*(c + 2*d) - B*(2
*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3
*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(A*d - B*(2*c + d))*Cos[e + f*x])/(d^2*(
c + d)*f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(d*(c + d)*
f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} \\
&\quad+\frac{\int \frac{(a+a\sin(e+fx))(-a(B(c-d)-2Ad)-a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{d(c+d)} \\
&= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} \\
&\quad+\frac{\int \frac{-a^2(B(c-d)-2Ad)+\left(-a^2(B(c-d)-2Ad)-a^2(Ad-B(2c+d))\right)\sin(e+fx)-a^2(Ad-B(2c+d))\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx}{d(c+d)} \\
&= \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} \\
&\quad+\frac{\int \frac{-a^2d(B(c-d)-2Ad)-a^2(c+d)(2B(c-d)-Ad)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d^2(c+d)}
\end{aligned}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\right)\int\frac{1}{c+d\sin(e+fx)}\,dx}{d^3(c+d)}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\right)\text{Subst}\left(\int\frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^3(c+d)f}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(4a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\right)\text{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx\right.\right.}{d^3(c+d)f}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} - \frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}f} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int\frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(1+\sin(e+fx))^2\left((-2Bc+Ad+2Bd)(e+fx)+\frac{2(c-d)\left(-Ad(c+2d)+B\left(2c^2+2cd-d^2\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right.}{d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

[Out] (a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(-(A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{2a^2\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-2Bc+2dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}-\frac{-\frac{d^2\left(Acd-A\,d^2-B\,c^2+cdB\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-A\,d^2-B\,c\right.}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}\right.}{f}$
default	$\frac{2a^2\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-2Bc+2dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}-\frac{-\frac{d^2\left(Acd-A\,d^2-B\,c^2+cdB\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-A\,d^2-B\,c\right.}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}\right.}{f}$
risch	$\frac{x\,a^2A}{d^2}-\frac{2x\,a^2Bc}{d^3}+\frac{2x\,a^2B}{d^2}-\frac{B\,a^2\mathrm{e}^{i(fx+e)}}{2d^2f}-\frac{B\,a^2\mathrm{e}^{-i(fx+e)}}{2d^2f}+\frac{2ia^2\left(-Acd+A\,d^2+B\,c^2-cdB\right)\left(id+c\,\mathrm{e}^{i(fx+e)}\right)}{d^3(c+d)f\left(-i\mathrm{e}^{2i(fx+e)}d+id+2c\,\mathrm{e}^{i(fx+e)}\right)}$

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 2/f*a^2*(1/d^3*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-2*B*c+2*B*d)*arctan(tan(1/2*f*x+1/2*e)))-1/d^3*((-d^2*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(A*c^2*d+A*c*d^2-2*A*d^3-2*B*c^3+3*B*c*d^2-B*d^3)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.69

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \left[-\frac{2\,(2\,Ba^2c^3 - Aa^2c^2d - (A+2\,B)a^2cd^2)fx + (2\,Ba^2c^3 - (A-2\,B)a^2c^2d - (2\,A+B)a^2cd^2 + (2\,Ba^2c^2d}{}\right.$$

$$-\frac{(2\,Ba^2c^3 - Aa^2c^2d - (A+2\,B)a^2cd^2)fx + (2\,Ba^2c^3 - (A-2\,B)a^2c^2d - (2\,A+B)a^2cd^2 + (2\,Ba^2c^2d -}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c
^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)
*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((
2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*
d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c +
 d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B*a^2*c
^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A*a^2*c*
d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(
f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*B*a^2*
c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A - 2*B)*a^2
*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*d^2 - (2*A
+ B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
 d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c^2*d - A*a^2*
c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d^2 - (A + 2*B)
*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/((c*d
^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(194) = 388$.

Time = 0.30 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.42

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx$$
$$= \frac{\frac{2\,(2\,Ba^2c^3 - Aa^2c^2d - Aa^2cd^2 - 3\,Ba^2cd^2 + 2\,Aa^2d^3 + Ba^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + d}{\sqrt{c^2 - d^2}}\right)\right)}{(cd^3 + d^4)\sqrt{c^2 - d^2}} - \frac{2\left(Ba^2c^2d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right.}{}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3 +
 B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*
x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B*a^
2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B*a^2
```

```
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a^2*c*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a^2*c*d^
2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d^3*tan(1
/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan(1/2*f*x
+ 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*
tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^2*d - 2*B*a
^2*d)*(f*x + e)/d^3)/f
```

Mupad [B] (verification not implemented)

Time = 22.06 (sec) , antiderivative size = 8706, normalized size of antiderivative = 43.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (
f*x)/2)^2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2
 + (f*x)/2)*(A*a^2*d^2 + 3*B*a^2*c^2 - A*a^2*c*d + B*a^2*c*d))/(c*d*(c + d)
) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2*d^2 + B*a^2*c^2 - A*a^2*c*d - B*a^2*c*d)
)/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2
+ c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) - (atan((((B*a^2*c*2i
 - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*
c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B
*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/
(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((((32*(c^2*
d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x
)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 +
 d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9
+ 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c
^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10
 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4
*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c
*d^7 + d^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4
*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^
7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4
*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*
a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 +
4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3 + ((B*a^2*c*2i - a^2*d
*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4
+ 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2
*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6
 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 +
```

2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4 *d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/ (2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a ^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c ^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d ^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4 *c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B ^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3* d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^ 4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3)/((64*(4*B^3*a^6*c^6 - 2*A^3*a ^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*d^3 - 2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5*d + 9 *A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2*a^6*c ^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c^2*d^4 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d ^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((((32*(c^2*d^10 + 2*c ^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c* d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2 *d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9 + 2*B*a^2* c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2 *c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2 *d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c ^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2 *a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^ 6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4* c*d^8))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 - ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c ^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B* a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2* d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d ^6 + d^7 + c^2*d^5) + (((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^ 7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4 *c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2 *A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*

a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6)
))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*
A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^
6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*
c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*
a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*
c*d^7 + d^8 + c^2*d^6)))/d^3 + (64*tan(e/2 + (f*x)/2)*(16*B^3*a^6*c^7 + 2*A
^3*a^6*c^2*d^5 - 4*A^3*a^6*c^3*d^4 - 2*A^3*a^6*c^4*d^3 - 32*B^3*a^6*c^2*d^5
 + 16*B^3*a^6*c^3*d^4 + 48*B^3*a^6*c^4*d^3 - 40*B^3*a^6*c^5*d^2 + 4*A^3*a^6
*c*d^6 + 8*B^3*a^6*c*d^6 - 16*B^3*a^6*c^6*d + 24*A*B^2*a^6*c*d^6 - 24*A*B^2
*a^6*c^6*d + 18*A^2*B*a^6*c*d^6 - 48*A*B^2*a^6*c^2*d^5 - 24*A*B^2*a^6*c^3*d
^4 + 72*A*B^2*a^6*c^4*d^3 - 12*A^2*B*a^6*c^2*d^5 - 30*A^2*B*a^6*c^3*d^4 + 1
2*A^2*B*a^6*c^4*d^3 + 12*A^2*B*a^6*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6)))*(B
*a^2*c*2i - a^2*d*(A + 2*B)*1i)*2i)/(d^3*f) - (a^2*atan(((a^2*(-(c + d)^3*(
c - d))^(1/2)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 +
 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*
d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6
+ d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^
3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*
a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 -
2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6
 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c
*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*tan
(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^
2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c
^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (32*(A*a^2*c*d^9 + 2
*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*
d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8
))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^1
1 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)
^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 +
 d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))
/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d
- 2*B*c*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) + (a^2*(-(c + d)^3*(c
- d))^(1/2)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4
*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^
6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 +
d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*
d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^
4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*
A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 -
 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d
^8))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A*a^
2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 -
 B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^

2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4
*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^
6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))
/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11
+ c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3
(c - d))^(1/2)(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d
^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(
3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d -
2*B*c*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((64*(4*B^3*a^6*c^6 - 2
*A^3*a^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*
d^3 - 2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5
*d + 9*A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2
*a^6*c^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c
^2*d^4 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (64*tan(e/2 + (f
x)/2)(16*B^3*a^6*c^7 + 2*A^3*a^6*c^2*d^5 - 4*A^3*a^6*c^3*d^4 - 2*A^3*a^6*
c^4*d^3 - 32*B^3*a^6*c^2*d^5 + 16*B^3*a^6*c^3*d^4 + 48*B^3*a^6*c^4*d^3 - 40
*B^3*a^6*c^5*d^2 + 4*A^3*a^6*c*d^6 + 8*B^3*a^6*c*d^6 - 16*B^3*a^6*c^6*d + 2
4*A*B^2*a^6*c*d^6 - 24*A*B^2*a^6*c^6*d + 18*A^2*B*a^6*c*d^6 - 48*A*B^2*a^6*
c^2*d^5 - 24*A*B^2*a^6*c^3*d^4 + 72*A*B^2*a^6*c^4*d^3 - 12*A^2*B*a^6*c^2*d^
5 - 30*A^2*B*a^6*c^3*d^4 + 12*A^2*B*a^6*c^4*d^3 + 12*A^2*B*a^6*c^5*d^2))/(2
*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A^2*a^4*c^2
*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*
c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B
*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2
+ (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A
^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5
+ 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d
^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a
^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6
) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10
+ 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4
*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c
*d^7 + d^8 + c^2*d^6) - (32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 +
B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5)
+ (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) +
(32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^
5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*
B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2
*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c
^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*
c^2*d^4 + c^3*d^3) - (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A^2*a^4*c^2*d^6
+ 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d
^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*
c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*
x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^

```
4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28
*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 +
22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^
5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (
a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^
3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 +
c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^
2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c
^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a
^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*t
an(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8
))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2
 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^
2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^
3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d
^4 + c^3*d^3)))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c
*d - 2*B*c*d)*2i)/(f*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
```

3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 215

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2Bx}{d^3}+\frac{a^2(3Ad^3-B(2c^3+4c^2d+cd^2-4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}\,f}+\frac{(Bc-Ad)\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))}$$

```
[Out] a^2*B*x/d^3+1/2*(-A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*s
in(f*x+e))^2-1/2*a^2*(3*A*d^2-B*(2*c^2+3*c*d-2*d^2))*cos(f*x+e)/d^2/(c+d)^2
/f/(c+d*sin(f*x+e))+a^2*(3*A*d^3-B*(2*c^3+4*c^2*d+c*d^2-4*d^3))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

= {3054, 3047, 3100, 2814, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= \frac{a^2(3Ad^3-B(2c^3+4c^2d+cd^2-4d^3))\arctan\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}$$
$$-\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}$$
$$+\frac{(Bc-Ad)\cos(e+fx)\,(a^2\sin(e+fx)+a^2)}{2df(c+d)(c+d\sin(e+fx))^2}+\frac{a^2Bx}{d^3}$$

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*B*x)/d^3 + (a^2*(3*A*d^3 - B*(2*c^3 + 4*c^2*d + c*d^2 - 4*d^3))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*
f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - (a^2*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f
*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad+\frac{\int \frac{(a+a\sin(e+fx))(-a(Bc-3Ad-2Bd)+2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)} \\
&= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad+\frac{\int \frac{-a^2(Bc-3Ad-2Bd)+\left(2a^2B(c+d)-a^2(Bc-3Ad-2Bd)\right)\sin(e+fx)+2a^2B(c+d)\sin^2(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}
\end{aligned}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\int \frac{-a^2(c-d)d(3Ad+B(c+4d))-2a^2B(c-d)(c+d)^2\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{2(c-d)d^2(c+d)^2}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(a^2(2Bc(c+d)^2-d^2(3Ad+B(c+4d)))\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{2d^3(c+d)^2}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(a^2(2Bc(c+d)^2-d^2(3Ad+B(c+4d)))\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^3(c+d)^2f}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{\left(2a^2(2Bc(c+d)^2-d^2(3Ad+B(c+4d)))\right)\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^3(c+d)^2f}$$

$$= \frac{a^2Bx}{d^3} - \frac{a^2(2Bc(c+d)^2-d^2(3Ad+B(c+4d)))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= \frac{a^2(1+\sin(e+fx))^2\left(2B(e+fx)-\frac{2(-3Ad^3+B(2c^3+4c^2d+cd^2-4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}}-\frac{d(-c+d)(-Bc+Ad)\cos}{(c+d)(c+d\sin(e+}\right.}{2d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2*
d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c +
 d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*
(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2))*C
os[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(206) = 412.

Time = 1.02 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{2a^2\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{d^2\left(c^2dA-4d^2cA-2A\,d^3+B\,c^3+4c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(4A\,c^3d^2+A\,c^2d^3+8Ac\,d^4+2A\,d^5-2B\right.}{}}{}\right.}{}$
default	$\frac{2a^2\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{d^2\left(c^2dA-4d^2cA-2A\,d^3+B\,c^3+4c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(4A\,c^3d^2+A\,c^2d^3+8Ac\,d^4+2A\,d^5-2B\right.}{}}{}\right.}{}$
risch	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2*(B/d^3*arctan(tan(1/2*f*x+1/2*e))+1/d^3*((1/2*d^2*(A*c^2*d-4*A*c*d^
2-2*A*d^3+B*c^3+4*B*c^2*d)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(4*
A*c^3*d^2+A*c^2*d^3+8*A*c*d^4+2*A*d^5-2*B*c^5-4*B*c^4*d-3*B*c^3*d^2-8*B*c^2
*d^3+2*B*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2-1/2*d^2*(A*c^2*d+1
2*A*c*d^2+2*A*d^3-7*B*c^3-12*B*c^2*d+4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f
*x+1/2*e)-1/2*d*(4*A*c*d^2+A*d^3-2*B*c^3-4*B*c^2*d+B*c*d^2)/(c^2+2*c*d+d^2)
)/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(3*A*d^3-2*B*c^3-
4*B*c^2*d-B*c*d^2+4*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(206) = 412$.

Time = 0.33 (sec) , antiderivative size = 1483, normalized size of antiderivative = 6.90

$$\int \frac{(a + a\sin(e + fx))^2 (A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*c
os(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*d^
4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*a^2
*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a^2*c
^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x + e)^
2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*a^2*c*
d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c
*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)
) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3 - (A + 4*B
)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) - 2*(4*(B*a^2
*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x + (3*B*a^2*c^
4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A - 4*B)*a^2*c*d
^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^5 + 2*c^3*d^6
 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5 - 2*c^2*d^7 - c
*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2*d^7 - 2*c*d^8 -
 d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d
^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a
^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d
+ 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (
2*B*a^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(
f*x + e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*
B)*a^2*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(s
qrt(c^2 - d^2)*cos(f*x + e))) - (2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3
```

```
*B)*a^2*c^3*d^3 - (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*
cos(f*x + e) - (4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*
c*d^5)*f*x + (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2
*d^4 + (A - 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^4*d^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2
*c^4*d^5 - 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d
^5 - c^2*d^7 - 2*c*d^8 - d^9)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(206) = 412$.

Time = 0.33 (sec) , antiderivative size = 678, normalized size of antiderivative = 3.15

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\frac{(fx+e)Ba^2}{d^3} - \frac{(2Ba^2c^3+4Ba^2c^2d+Ba^2cd^2-3Aa^2d^3-4Ba^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^3+2cd^4+d^5)\sqrt{c^2-d^2}} + \frac{Ba^2c^4d\tan\left(\frac{1}{2}fx+\right.}{}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^2
*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*ta
n(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c
^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/2*f*
x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^3*tan
(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^2*c^3*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 8
*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2
- 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e) -
 A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)
 - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan(1/2*f*x + 1/
2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B*a^2*c^4*d - 4
*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2
*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 8632, normalized size of antiderivative = 40.15

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (2*B*a^2*atan(((B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*
a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c
^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 +
 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*
c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4
*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*
c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2 + (f*x
)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*
d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5
*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6)
 - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^
7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))
/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12
 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*
c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*
d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(
4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3 +
```

(B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3 *d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^ 3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^ 2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24* A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^ 2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3* d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^ 3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d ^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^1 0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^ 3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3* d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3)/((16*(2*B^3*a^ 6*c^5 + 17*B^3*a^6*c^3*d^2 - 16*B^3*a^6*c*d^4 + 12*B^3*a^6*c^4*d - 24*A*B^2 *a^6*c*d^4 + 6*A*B^2*a^6*c^4*d - 9*A^2*B*a^6*c*d^4 + 12*A*B^2*a^6*c^3*d^2)) /(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (16*tan(e/2 + (f*x)/2) *(28*B^3*a^6*c^2*d^4 - 8*B^3*a^6*c^6 - 8*B^3*a^6*c^3*d^3 - 44*B^3*a^6*c^4*d ^2 + 16*B^3*a^6*c*d^5 - 32*B^3*a^6*c^5*d + 12*A*B^2*a^6*c*d^5 + 24*A*B^2*a^ 6*c^2*d^4 + 12*A*B^2*a^6*c^3*d^3))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^ 4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2 *d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 7 5*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^ 6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c ^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c* d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2 + (f*x)/ 2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^ 9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d ^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^ 2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^ 13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4* c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3 + (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^ 3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c ^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B ^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24

*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10
+ 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c
^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3
*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c
^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^1
1 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*
d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^
10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^1
1 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c
^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3
*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^1
0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3)))/(d^3*f)
- ((A*a^2*d^3 - 2*B*a^2*c^3 + 4*A*a^2*c*d^2 + B*a^2*c*d^2 - 4*B*a^2*c^2*d)/
(d^2*(2*c*d + c^2 + d^2)) - (tan(e/2 + (f*x)/2)^3*(B*a^2*c^3 - 2*A*a^2*d^3
- 4*A*a^2*c*d^2 + A*a^2*c^2*d + 4*B*a^2*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) +
(tan(e/2 + (f*x)/2)*(2*A*a^2*d^3 - 7*B*a^2*c^3 + 12*A*a^2*c*d^2 + A*a^2*c^
2*d + 4*B*a^2*c*d^2 - 12*B*a^2*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) + (tan(e/2
+ (f*x)/2)^2*(c^2 + 2*d^2)*(A*a^2*d^3 - 2*B*a^2*c^3 + 4*A*a^2*c*d^2 + B*a^
2*c*d^2 - 4*B*a^2*c^2*d))/(c^2*d^2*(2*c*d + c^2 + d^2)))/(f*(tan(e/2 + (f*x
)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (
f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) + (a^2*atan(((a^2*(-(c + d)^5*(c - d
))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 +
16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^
3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c
^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B
^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24
*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10
+ 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*ta
n(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 1
2*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 -
32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^
7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*
A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a
^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (a^2*(-(c
+ d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*
d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*ta
n(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*
c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7
+ c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d
^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^
3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d
^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^
2*d)*1i)/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3))
+ (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d
^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8

+ d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a ^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4 *c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*ta n(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 1 2*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^ 7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^1 1 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c ^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3 *d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^1 0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^ 2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6 *d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4 *B*d^3 + B*c*d^2 + 4*B*c^2*d)*1i)/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))/((16*(2*B^3*a^6*c^5 + 17*B^3*a^6*c^3*d^2 - 16*B^3 *a^6*c*d^4 + 12*B^3*a^6*c^4*d - 24*A*B^2*a^6*c*d^4 + 6*A*B^2*a^6*c^4*d - 9* A^2*B*a^6*c*d^4 + 12*A*B^2*a^6*c^3*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3 *d^6 + c^4*d^5) - (16*tan(e/2 + (f*x)/2)*(28*B^3*a^6*c^2*d^4 - 8*B^3*a^6*c^ 6 - 8*B^3*a^6*c^3*d^3 - 44*B^3*a^6*c^4*d^2 + 16*B^3*a^6*c*d^5 - 32*B^3*a^6* c^5*d + 12*A*B^2*a^6*c*d^5 + 24*A*B^2*a^6*c^2*d^4 + 12*A*B^2*a^6*c^3*d^3))/ (4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c ^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4* c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8* B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 2 4*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*t an(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d ^7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6 *A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B* a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5 *d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*t an(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36 *c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^ 7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*

```
d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d
^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*
d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c
^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)) +
(a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5
 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 +
 d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4
*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 3
2*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 +
 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c
*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^
5*(c - d))^(1/2)*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 -
 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*
B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(
e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*
A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32
*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7
+ c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11
+ 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3
*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d
^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10
+ 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2
+ 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d
^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^
9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B
*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*
c^5*d^4 - c^6*d^3))))*(-(c + d)^5*(c - d))^(1/2)*(2*B*c^3 - 3*A*d^3 - 4*B*d
^3 + B*c*d^2 + 4*B*c^2*d)*1i)/(f*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4
*c^5*d^4 - c^6*d^3))
```

3.258 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 35, antiderivative size = 604

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A\left(40c^3+90c^2d+78cd^2+23d^3\right)\right)x$$
$$-\frac{a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189c^3d^3-9\cdots}{420d^3f}$$
$$-\frac{a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-1263cd^4-\cdots}{1680d^2f}$$
$$-\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))\cos(e+fx)(c-\cdots}{840d^3f}$$
$$-\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{840d^3f}$$
$$-\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f}$$
$$-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df}$$
$$+\frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f}$$

```
[Out] 1/16*a^3*(3*B*(10*c^3+26*c^2*d+23*c*d^2+7*d^3)+A*(40*c^3+90*c^2*d+78*c*d^2+
23*d^3))*x-1/420*a^3*(7*A*d*(2*c^5-18*c^4*d+107*c^3*d^2+472*c^2*d^3+456*c*d
^4+136*d^5)-3*B*(2*c^6-14*c^5*d+51*c^4*d^2-189*c^3*d^3-920*c^2*d^4-952*c*d^
5-288*d^6))*cos(f*x+e)/d^3/f-1/1680*a^3*(7*A*d*(4*c^4-36*c^3*d+216*c^2*d^2+
626*c*d^3+345*d^4)-3*B*(4*c^5-28*c^4*d+104*c^3*d^2-392*c^2*d^3-1263*c*d^4-7
35*d^5))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/840*a^3*(7*A*d*(2*c^3-18*c^2*d+111*c
```

```
*d^2+136*d^3)-B*(6*c^4-42*c^3*d+165*c^2*d^2-651*c*d^3-864*d^4))*cos(f*x+e)*
(c+d*sin(f*x+e))^2/d^3/f-1/840*a^3*(7*A*d*(2*c^2-18*c*d+115*d^2)-B*(6*c^3-4
2*c^2*d+177*c*d^2-735*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f-1/210*a^3*(
-14*A*c*d+91*A*d^2+6*B*c^2-27*B*c*d+87*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4
/d^3/f-1/7*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4/d/f+1/42*(3
*B*(c-3*d)-7*A*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^4/d^2/f
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= -\frac{a^3(-14Acd+91Ad^2+6Bc^2-27Bcd+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f}$$
$$-\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{840d^3f}$$
$$+\frac{1}{16}a^3x\left(A\left(40c^3+90c^2d+78cd^2+23d^3\right)+3B\left(10c^3+26c^2d+23cd^2+7d^3\right)\right)$$
$$-\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))\cos(e+fx)(c+}{840d^3f}$$
$$-\frac{a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-1263cd^4-7}{1680d^2f}$$
$$-\frac{a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189c^3d^3-92}{420d^3f}$$
$$+\frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^4}{42d^2f}$$
$$-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 7
8*c*d^2 + 23*d^3))*x)/16 - (a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 189
*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/(420*d^3*f) -
(a^3*(7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4
*c^5 - 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e
+ f*x]*Sin[e + f*x])/(1680*d^2*f) - (a^3*(7*A*d*(2*c^3 - 18*c^2*d + 111*c*d
^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*d^3 - 864*d^4))*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(840*d^3*f) - (a^3*(7*A*d*(2*c^2 - 18*c
*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3))*Cos[e + f*x]*(c
```

```
 + d*Sin[e + f*x])^3)/(840*d^3*f) - (a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 9
1*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(210*d^3*f) - (a*B
*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(7*d*f) + ((3*
B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f
*x])^4)/(42*d^2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df} + \frac{\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^3(a(7Ad+2B(c+2d))-a(3Bc-7Ad-9Bd)\sin(e+fx))\,dx}{7d}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df} + \frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f} + \frac{\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\left(a^2(7Ad(c+10d)-B(3c^2-9cd-60d^2))+a^2(6Bc^2-\right.}{42d^2}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df} + \frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f} + \frac{\int (c+d\sin(e+fx))^3\left(a^3(7Ad(c+10d)-B(3c^2-9cd-60d^2))+(a^3(6Bc^2-14Acd-27Bcd+\right.}{}$$

$$= -\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df} + \frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f} + \frac{\int (c+d\sin(e+fx))^3\left(-3a^3d(7A(c-34d)d-3B(c^2-7cd+72d^2))+a^3(7Ad(2c^2-18cd+115\right.}{210d^3}$$

$$= -\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx)}{840d^3f} - \frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df} + \frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f} + \frac{\int (c+d\sin(e+fx))^2\left(-3a^3d(7Ad(2c^2-118cd-115d^2)-B(6c^3-42c^2d+687cd^2+735d^3))+\right.}{}$$

$$= -\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))\cos(e+}{840d^3f}$$
$$-\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx}{840d^3f}$$
$$-\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f}$$
$$-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df}$$
$$+\frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f}$$
$$+\frac{\int(c+d\sin(e+fx))\left(-3a^3d(7Ad(2c^3-318c^2d-567cd^2-272d^3)-3B(2c^4-14c^3d+577c^2d}{}$$

$$= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A\left(40c^3+90c^2d+78cd^2+23d^3\right)\right)x$$
$$-\frac{a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189}{420d^3f}$$
$$-\frac{a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-12}{1680d^2f}$$
$$-\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))\cos(e+}{840d^3f}$$
$$-\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx}{840d^3f}$$
$$-\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f}$$
$$-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df}$$
$$+\frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f}$$

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.87

$$\int(a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{a^3\cos(e+fx)\left(420(3B(10c^3+26c^2d+23cd^2+7d^3)+A(40c^3+90c^2d+78cd^2+23d^3))\arcsin\left(\frac{\sqrt{1-\mathrm{si}}}{}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/3360*(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3)
+ A*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/
Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2*d
+ 32676*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*d^3
- (112*A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*c^2*d
 + 2912*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) + B*(14*
c^2 + 42*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)] + 5040
*A*c^3*Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e + f*x]
+ 22050*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*B*c*d^2*S
in[e + f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] - 210*B*c^3
*Sin[3*(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*Sin[3*(e +
f*x)] - 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e + f*x)] - 980
*A*d^3*Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*c*d^2*Sin[5*(
e + f*x)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e + f*x)])))/(f*Sq
rt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{\left(\left(\left(\frac{\left(19A+\frac{81B}{4}\right)d^3}{4}+\frac{51c\left(A+\frac{19B}{17}\right)d^2}{4}+9\left(\frac{17B}{12}+A\right)c^2d+c^3(A+3B)\right)\cos(3fx+3e)+3\left(\frac{(-63A-61B)d^3}{16}-12c\left(A+\frac{63B}{64}\right)d^2-\right.\right.\right.}{}$
parts	$\frac{\left(A\,a^3d^3+3B\,a^3d^2c+3B\,a^3d^3\right)\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}-\frac{\left(3A\,a^3c^3+3A\,a^3\right.}{}$
risch	$-\frac{189\sin(2fx+2e)B\,a^3d^2c}{64f}-\frac{15a^3\cos(fx+e)A\,c^3}{4f}-\frac{21a^3\cos(fx+e)A\,d^3}{8f}-\frac{13a^3\cos(fx+e)B\,c^3}{4f}-\frac{155a^3\cos(fx+}{64f}$
derivativedivides	Expression too large to display
default	Expression too large to display
norman	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/12*((1/4*(19*A+81/4*B)*d^3+51/4*c*(A+19/17*B)*d^2+9*(17/12*B+A)*c^2*d+c^3
*(A+3*B))*cos(3*f*x+3*e)+3*(1/16*(-63*A-61*B)*d^3-12*c*(A+63/64*B)*d^2-12*c
^2*d*(A+B)-3*(A+4/3*B)*c^3)*sin(2*f*x+2*e)+3/8*(1/2*(11*B+9*A)*d^3+9*(3/2*B
+A)*c*d^2+3*c^2*(A+3*B)*d+B*c^3)*sin(4*f*x+4*e)-9/20*d*((A+19/12*B)*d^2+c*(
A+3*B)*d+B*c^2)*cos(5*f*x+5*e)-1/16*((A+3*B)*d+3*B*c)*d^2*sin(6*f*x+6*e)+3/
112*B*d^3*cos(7*f*x+7*e)+3*(1/2*(-155/8*B-21*A)*d^3-69/2*(21/23*B+A)*c*d^2-
39*c^2*(23/26*B+A)*d-15*c^3*(A+13/15*B))*cos(f*x+e)+(-864/35*B+69/4*f*x*A+6
3/4*f*x*B-136/5*A)*d^3+117/2*c*(f*x*A+23/26*f*x*B-304/195*A-272/195*B)*d^2+
```

```
135/2*c^2*(f*x*A+13/15*f*x*B-8/5*A-304/225*B)*d+30*c^3*(f*x*A+3/4*f*x*B-22/
15*A-6/5*B))*a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{240\,Ba^3d^3\cos\left(fx+e\right)^7-1008\left(Ba^3c^2d+(A+3\,B)a^3cd^2+(A+2\,B)a^3d^3\right)\cos\left(fx+e\right)^5+560\left((A+3\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c*
d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A +
 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x +
e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*A +
23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 + 3*(A
 + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e) - 35*
(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c^3 + 18
*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^3*d^3)*c
os(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c^2*d + 3*
(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*sin(f*x + e)
)/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. 2(598) = 1196.

Time = 0.72 (sec) , antiderivative size = 2878, normalized size of antiderivative = 4.76

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x)
**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9
*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x*sin(
e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d*x*cos
```

(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*A*
a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*
A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f + 27*A*a**
3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d**2*x*cos(e +
f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*d**2*sin(e +
f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*a**3*c*d**2*sin
(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*c*d**2*
cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f + 5*A*a**3*d**3*x
*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 +
9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*x*sin(e + f*x)**2*cos(e
+ f*x)**4/16 + 9*A*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*A*a**
3*d**3*x*cos(e + f*x)**6/16 + 9*A*a**3*d**3*x*cos(e + f*x)**4/8 - 11*A*a**3
*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*A*a**3*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 5*A*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*
a**3*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**3*sin(e + f*x)**
2*cos(e + f*x)**3/f - A*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**3
*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*A*a**3*d**3*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 8*A*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*A*a**3*d**3*cos
(e + f*x)**3/(3*f) + 3*B*a**3*c**3*x*sin(e + f*x)**4/8 + 3*B*a**3*c**3*x*si
n(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**3*x*sin(e + f*x)**2/2 + 3*B*a
3*c3*x*cos(e + f*x)**4/8 + 3*B*a**3*c**3*x*cos(e + f*x)**2/2 - 5*B*a**3
*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3*c**3*sin(e + f*x)**2*co
s(e + f*x)/f - 3*B*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*
c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c**3*cos(e + f*x)**3/f - B*
a**3*c**3*cos(e + f*x)/f + 27*B*a**3*c**2*d*x*sin(e + f*x)**4/8 + 27*B*a**3
*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**2*d*x*sin(e + f*x
)**2/2 + 27*B*a**3*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**3*c**2*d*x*cos(e + f
*x)**2/2 - 3*B*a**3*c**2*d*sin(e + f*x)**4*cos(e + f*x)/f - 45*B*a**3*c**2*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a**3*c**2*d*sin(e + f*x)**2*cos(
e + f*x)**3/f - 9*B*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 27*B*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*d*sin(e + f*x)*co
s(e + f*x)/(2*f) - 8*B*a**3*c**2*d*cos(e + f*x)**5/(5*f) - 6*B*a**3*c**2*d*
cos(e + f*x)**3/f + 15*B*a**3*c*d**2*x*sin(e + f*x)**6/16 + 45*B*a**3*c*d**
2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 27*B*a**3*c*d**2*x*sin(e + f*x)**4
/8 + 45*B*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 27*B*a**3*c*d*
*2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*c*d**2*x*cos(e + f*x)**6
/16 + 27*B*a**3*c*d**2*x*cos(e + f*x)**4/8 - 33*B*a**3*c*d**2*sin(e + f*x)*
*5*cos(e + f*x)/(16*f) - 9*B*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5
*B*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 45*B*a**3*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 12*B*a**3*c*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/f - 3*B*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 15*B*a**3*c*d*

```
*2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 27*B*a**3*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 24*B*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*a**3*c*d**
2*cos(e + f*x)**3/f + 15*B*a**3*d**3*x*sin(e + f*x)**6/16 + 45*B*a**3*d**3*
x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**3*d**3*x*sin(e + f*x)**4/8 +
45*B*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**3*d**3*x*sin(e
 + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*d**3*x*cos(e + f*x)**6/16 + 3*B*a*
*3*d**3*x*cos(e + f*x)**4/8 - B*a**3*d**3*sin(e + f*x)**6*cos(e + f*x)/f -
33*B*a**3*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*B*a**3*d**3*sin(e +
f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*B*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 5*B*a**3*d**3*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) - 8*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) - 4*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 15*B*a**3*d**3*s
in(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**3*d**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 16*B*a**3*d**3*cos(e + f*x)**7/(35*f) - 8*B*a**3*d**3*cos(e +
f*x)**5/(5*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a
)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 1056, normalized size of antiderivative = 1.75

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^3 + 5040*(2*f*x + 2*
e - sin(2*f*x + 2*e))*A*a^3*c^3 + 6720*(f*x + e)*A*a^3*c^3 + 6720*(cos(f*x
+ e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 210*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a
^3*c^3 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2*d + 630*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2*d + 15120*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^3*c^2*d - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a^3*c^2*d + 20160*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x +
2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2*d - 134
4*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c*d^2 + 20
160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*B*a^3*c*d^2 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*
c*d^2 + 105*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
```

```
*sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
 + 15*cos(f*x + e))*A*a^3*d^3 + 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^
3*d^3 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*d^3 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin
(2*f*x + 2*e))*A*a^3*d^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*c
os(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*d^3 - 1344*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^3 + 105*(4*sin(2*f*x + 2*e)^3 + 60*
f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^3 + 210*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^3 - 20160*A*a^3
*c^3*cos(f*x + e) - 6720*B*a^3*c^3*cos(f*x + e) - 20160*A*a^3*c^2*d*cos(f*x
 + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.93

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{Ba^3d^3\cos(7\,fx+7\,e)}{448\,f}$$
$$+\frac{1}{16}\left(40\,Aa^3c^3+30\,Ba^3c^3+90\,Aa^3c^2d+78\,Ba^3c^2d+78\,Aa^3cd^2+69\,Ba^3cd^2+23\,Aa^3d^3+21\,Ba^3d^3\right)x$$
$$-\frac{(12\,Ba^3c^2d+12\,Aa^3cd^2+36\,Ba^3cd^2+12\,Aa^3d^3+19\,Ba^3d^3)\cos(5\,fx+5\,e)}{320\,f}$$
$$+\frac{(16\,Aa^3c^3+48\,Ba^3c^3+144\,Aa^3c^2d+204\,Ba^3c^2d+204\,Aa^3cd^2+228\,Ba^3cd^2+76\,Aa^3d^3+81\,Ba^3d^3)\,\mathrm{co}}{192\,f}$$
$$-\frac{(240\,Aa^3c^3+208\,Ba^3c^3+624\,Aa^3c^2d+552\,Ba^3c^2d+552\,Aa^3cd^2+504\,Ba^3cd^2+168\,Aa^3d^3+155\,Ba^3}{64\,f}$$
$$-\frac{(3\,Ba^3cd^2+Aa^3d^3+3\,Ba^3d^3)\sin(6\,fx+6\,e)}{192\,f}$$
$$+\frac{(2\,Ba^3c^3+6\,Aa^3c^2d+18\,Ba^3c^2d+18\,Aa^3cd^2+27\,Ba^3cd^2+9\,Aa^3d^3+11\,Ba^3d^3)\sin(4\,fx+4\,e)}{64\,f}$$
$$-\frac{(48\,Aa^3c^3+64\,Ba^3c^3+192\,Aa^3c^2d+192\,Ba^3c^2d+192\,Aa^3cd^2+189\,Ba^3cd^2+63\,Aa^3d^3+61\,Ba^3d^3)\,\mathrm{si}}{64\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 90
*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^3*
d^3 + 21*B*a^3*d^3)*x - 1/320*(12*B*a^3*c^2*d + 12*A*a^3*c*d^2 + 36*B*a^3*c
*d^2 + 12*A*a^3*d^3 + 19*B*a^3*d^3)*cos(5*f*x + 5*e)/f + 1/192*(16*A*a^3*c^
3 + 48*B*a^3*c^3 + 144*A*a^3*c^2*d + 204*B*a^3*c^2*d + 204*A*a^3*c*d^2 + 22
8*B*a^3*c*d^2 + 76*A*a^3*d^3 + 81*B*a^3*d^3)*cos(3*f*x + 3*e)/f - 1/64*(240
*A*a^3*c^3 + 208*B*a^3*c^3 + 624*A*a^3*c^2*d + 552*B*a^3*c^2*d + 552*A*a^3*
```

```
c*d^2 + 504*B*a^3*c*d^2 + 168*A*a^3*d^3 + 155*B*a^3*d^3)*cos(f*x + e)/f - 1
/192*(3*B*a^3*c*d^2 + A*a^3*d^3 + 3*B*a^3*d^3)*sin(6*f*x + 6*e)/f + 1/64*(2
*B*a^3*c^3 + 6*A*a^3*c^2*d + 18*B*a^3*c^2*d + 18*A*a^3*c*d^2 + 27*B*a^3*c*d
^2 + 9*A*a^3*d^3 + 11*B*a^3*d^3)*sin(4*f*x + 4*e)/f - 1/64*(48*A*a^3*c^3 +
64*B*a^3*c^3 + 192*A*a^3*c^2*d + 192*B*a^3*c^2*d + 192*A*a^3*c*d^2 + 189*B*
a^3*c*d^2 + 63*A*a^3*d^3 + 61*B*a^3*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 1395, normalized size of antiderivative = 2.31

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d^
3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 + 78*B*c^2*d))/(8*(5*A*a^3*c^3 + (
23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4
+ (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4)))*(40*A*c^3
 + 23*A*d^3 + 30*B*c^3 + 21*B*d^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 +
78*B*c^2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3*c^3 + (23*A*a^3*d^3)/8 +
(15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (45*A*a^3*c^2*d)
/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^10*(40*A
*a^3*c^3 + 4*A*a^3*d^3 + 24*B*a^3*c^3 + 36*A*a^3*c*d^2 + 72*A*a^3*c^2*d + 1
2*B*a^3*c*d^2 + 36*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^13*(3*A*a^3*c^3 + (23*
A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (
45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f
*x)/2)^3*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 + (35*B*a^3*d^3)/
2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 + 57*B*a^3*c^2*d)
 - tan(e/2 + (f*x)/2)^11*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 +
 (35*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 +
 57*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^8*((322*A*a^3*c^3)/3 + (148*A*a^3*d^3
)/3 + 82*B*a^3*c^3 + 32*B*a^3*d^3 + 188*A*a^3*c*d^2 + 246*A*a^3*c^2*d + 148
*B*a^3*c*d^2 + 188*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*((448*A*a^3*c^3)/3 +
 (328*A*a^3*d^3)/3 + 128*B*a^3*c^3 + 112*B*a^3*d^3 + 344*A*a^3*c*d^2 + 384*
A*a^3*c^2*d + 328*B*a^3*c*d^2 + 344*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^2*((1
36*A*a^3*c^3)/3 + (476*A*a^3*d^3)/15 + 40*B*a^3*c^3 + (144*B*a^3*d^3)/5 + (
532*A*a^3*c*d^2)/5 + 120*A*a^3*c^2*d + (476*B*a^3*c*d^2)/5 + (532*B*a^3*c^2
*d)/5) + tan(e/2 + (f*x)/2)^5*(15*A*a^3*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^
3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 +
(841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*(15*A*a^3
*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a
^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 + (841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)
/4) + tan(e/2 + (f*x)/2)^4*(114*A*a^3*c^3 + (456*A*a^3*d^3)/5 + 102*B*a^3*c
```

```
^3 + (432*B*a^3*d^3)/5 + (1416*A*a^3*c*d^2)/5 + 306*A*a^3*c^2*d + (1368*B*a
^3*c*d^2)/5 + (1416*B*a^3*c^2*d)/5) + tan(e/2 + (f*x)/2)^12*(6*A*a^3*c^3 +
2*B*a^3*c^3 + 6*A*a^3*c^2*d) + (22*A*a^3*c^3)/3 + (68*A*a^3*d^3)/15 + 6*B*a
^3*c^3 + (144*B*a^3*d^3)/35 + (76*A*a^3*c*d^2)/5 + 18*A*a^3*c^2*d + (68*B*a
^3*c*d^2)/5 + (76*B*a^3*c^2*d)/5)/(f*(7*tan(e/2 + (f*x)/2)^2 + 21*tan(e/2 +
 (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/
2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 + 1))
```

3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 463

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60cd+26d^2\right)\right)x$$
$$- \frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)-B(2c^5-12c^4d+37c^3d^2-112c^2d^3-304cd^4-136d^5}{60d^3f}$$
$$- \frac{a^3(2Ad(4c^3-30c^2d+146cd^2+195d^3)-B(4c^4-24c^3d+76c^2d^2-236cd^3-345d^4))\cos(e+fx)\sin(}{240d^2f}$$
$$- \frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^3f}$$
$$+ \frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f}$$
$$- \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df}$$
$$+ \frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f}$$

```
[Out] 1/16*a^3*(B*(30*c^2+52*c*d+23*d^2)+A*(40*c^2+60*c*d+26*d^2))*x-1/60*a^3*(2*
A*d*(2*c^4-15*c^3*d+72*c^2*d^2+180*c*d^3+76*d^4)-B*(2*c^5-12*c^4*d+37*c^3*d
^2-112*c^2*d^3-304*c*d^4-136*d^5))*cos(f*x+e)/d^3/f-1/240*a^3*(2*A*d*(4*c^3
-30*c^2*d+146*c*d^2+195*d^3)-B*(4*c^4-24*c^3*d+76*c^2*d^2-236*c*d^3-345*d^4
))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/120*a^3*(2*A*d*(2*c^2-15*c*d+76*d^2)-B*(2*
c^3-12*c^2*d+41*c*d^2-136*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^3/f+1/40*a^
3*(2*A*(2*c-11*d)*d-B*(2*c^2-8*c*d+21*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d
^3/f-1/6*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3/d/f+1/30*(-6*
A*d+3*B*c-8*B*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^3/d^2/f
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{1}{16}a^3x\big(A\big(40c^2+60cd+26d^2\big)+B\big(30c^2+52cd+23d^2\big)\big)$$
$$+\frac{a^3(2Ad(2c-11d)-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f}$$
$$-\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^3f}$$
$$-\frac{a^3(2Ad(4c^3-30c^2d+146cd^2+195d^3)-B(4c^4-24c^3d+76c^2d^2-236cd^3-345d^4))\sin(e+fx)\cos(e}{240d^2f}$$
$$-\frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)-B(2c^5-12c^4d+37c^3d^2-112c^2d^3-304cd^4-136d^5)}{60d^3f}$$
$$+\frac{(-6Ad+3Bc-8Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^3}{30d^2f}$$
$$-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/16 -
(a^3*(2*A*d*(2*c^4 - 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5
 - 12*c^4*d + 37*c^3*d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x]
)/(60*d^3*f) - (a^3*(2*A*d*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*
c^4 - 24*c^3*d + 76*c^2*d^2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*
x])/(240*d^2*f) - (a^3*(2*A*d*(2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2
*d + 41*c*d^2 - 136*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^3*f)
+ (a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*d + 21*d^2))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^3)/(40*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c
 + d*Sin[e + f*x])^3)/(6*d*f) + ((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3
+ a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(30*d^2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} + \frac{\int(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2(a(2Bc+6Ad+3Bd)-a(3Bc-6Ad-8Bd)\sin(e+fx))\,d}{6d}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} + \frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f} + \frac{\int(a+a\sin(e+fx))(c+d\sin(e+fx))^2\left(3a^2(2Ad(c+8d)-B(c^2-3cd-13d^2))-3a^2(2A(2c\right.}{30d^2}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} + \frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f} + \frac{\int(c+d\sin(e+fx))^2\left(3a^3(2Ad(c+8d)-B(c^2-3cd-13d^2))+(3a^3(2Ad(c+8d)-B(c^2-3c\right.}{}$$

$$= \frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} + \frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f} + \frac{\int(c+d\sin(e+fx))^2\left(-3a^3d(2A(2c-65d)d-B(2c^2-12cd+115d^2))+3a^3(2Ad(2c^2-15cd\right.}{120d^3}$$

$$= -\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^3f} + \frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} + \frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f} + \frac{\int(c+d\sin(e+fx))\left(-3a^3d(2Ad(2c^2-165cd-152d^2)-B(2c^3-12c^2d+263cd^2+272d^3))+\right.}{}$$

$$= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60cd+26d^2\right)\right)x$$
$$-\frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)-B(2c^5-12c^4d+37c^3d^2-112c^2d^3-304cd^4}{60d^3f}$$
$$-\frac{a^3(2Ad(4c^3-30c^2d+146cd^2+195d^3)-B(4c^4-24c^3d+76c^2d^2-236cd^3-345d^4))\cos(e+}{240d^2f}$$
$$-\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))}{120d^3f}$$
$$+\frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f}$$
$$-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df}$$
$$+\frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{a^3\cos(e+fx)\left(60(B(30c^2+52cd+23d^2)+A(40c^2+60cd+26d^2))\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e}\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -1/480*(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60
*c*d + 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^
2]*(1840*A*c^2 + 1680*B*c^2 + 3360*A*c*d + 3112*B*c*d + 1556*A*d^2 + 1468*B
*d^2 - 16*(A*(5*c^2 + 30*c*d + 22*d^2) + B*(15*c^2 + 44*c*d + 26*d^2))*Cos[
2*(e + f*x)] + 12*d*(2*B*c + A*d + 3*B*d)*Cos[4*(e + f*x)] + 720*A*c^2*Sin[
e + f*x] + 990*B*c^2*Sin[e + f*x] + 1980*A*c*d*Sin[e + f*x] + 2100*B*c*d*Si
n[e + f*x] + 1050*A*d^2*Sin[e + f*x] + 1085*B*d^2*Sin[e + f*x] - 30*B*c^2*S
in[3*(e + f*x)] - 60*A*c*d*Sin[3*(e + f*x)] - 180*B*c*d*Sin[3*(e + f*x)] -
90*A*d^2*Sin[3*(e + f*x)] - 140*B*d^2*Sin[3*(e + f*x)] + 5*B*d^2*Sin[5*(e +
 f*x)]))))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{a^3\left(\left(\frac{(17A+19B)d^2}{4}+6\left(\frac{17B}{12}+A\right)cd+c^2(A+3B)\right)\cos(3fx+3e)+3\left(\left(-\frac{63B}{16}-4A\right)d^2-8dc(A+B)-3\left(A+\frac{4B}{3}\right)c^2\right)\sin(2fx\right.}{}$
parts	$-\frac{(A a^3d^2+2B a^3cd+3B a^3d^2)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}-\frac{(3A a^3c^2+2A a^3cd+B a^3c^2)\cos(fx+e}{f}$
risch	$\frac{13B a^3cdx}{4}-\frac{15a^3\cos(fx+e)A c^2}{4f}-\frac{23a^3\cos(fx+e)A d^2}{8f}-\frac{13a^3\cos(fx+e)B c^2}{4f}-\frac{21a^3\cos(fx+e)d^2B}{8f}-\frac{B a^3d}{}$
derivativedivides	$\frac{-\frac{A a^3c^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A a^3cd\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{A a^3d^2\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}}{}$
default	$\frac{-\frac{A a^3c^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A a^3cd\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{A a^3d^2\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}}{}$
norman	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/12*a^3*((1/4*(17*A+19*B)*d^2+6*(17/12*B+A)*c*d+c^2*(A+3*B))*cos(3*f*x+3*e
)+3*((-63/16*B-4*A)*d^2-8*d*c*(A+B)-3*(A+4/3*B)*c^2)*sin(2*f*x+2*e)+3/4*(3/
2*(3/2*B+A)*d^2+c*(A+3*B)*d+1/2*B*c^2)*sin(4*f*x+4*e)-3/20*((A+3*B)*d+2*B*c
)*d*cos(5*f*x+5*e)-1/16*B*d^2*sin(6*f*x+6*e)+3*(1/2*(-23*A-21*B)*d^2-26*c*(
23/26*B+A)*d-15*c^2*(A+13/15*B))*cos(f*x+e)+(-136/5*B+39/2*f*x*A+69/4*f*x*B
-152/5*A)*d^2+45*c*(f*x*A+13/15*f*x*B-8/5*A-304/225*B)*d+30*c^2*(f*x*A+3/4*
f*x*B-22/15*A-6/5*B))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.65

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{48\,(2\,Ba^3cd+(A+3\,B)a^3d^2)\cos{(fx+e)}^5-80\,((A+3\,B)a^3c^2+2\,(3\,A+5\,B)a^3cd+(5\,A+7\,B)a^3d^2)}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B)
*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 - 15
*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d^2)
*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(f*x
+ e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a^3*c*
d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^2 + 4*
(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/
f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(449) = 898$.

Time = 0.50 (sec) , antiderivative size = 1804, normalized size of antiderivative = 3.90

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^2\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x)
**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 + 3*A*
a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*sin(e + f*x)*
*2 + 3*A*a**3*c*d*x*cos(e + f*x)**4/4 + 3*A*a**3*c*d*x*cos(e + f*x)**2 - 5*
A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e + f*x)**
2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*A*a*
*3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/f - 2*A*a
**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*sin(e + f*x)**2/2 + 9*A
*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/2 - A*a**3*d
**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*A*a**
3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*d**2
*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f + 3*B*a**3*c**2*x
*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*
B*a**3*c**2*x*sin(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e + f*x)**4/8 + 3*B*a
**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 3*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f)
 - 2*B*a**3*c**2*cos(e + f*x)**3/f - B*a**3*c**2*cos(e + f*x)/f + 9*B*a**3*
```

```
c*d*x*sin(e + f*x)**4/4 + 9*B*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2
+ B*a**3*c*d*x*sin(e + f*x)**2 + 9*B*a**3*c*d*x*cos(e + f*x)**4/4 + B*a**3*
c*d*x*cos(e + f*x)**2 - 2*B*a**3*c*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*
a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 8*B*a**3*c*d*sin(e + f*x)**2*
cos(e + f*x)**3/(3*f) - 6*B*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a
**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*d*sin(e + f*x)*cos(e
+ f*x)/f - 16*B*a**3*c*d*cos(e + f*x)**5/(15*f) - 4*B*a**3*c*d*cos(e + f*x)
**3/f + 5*B*a**3*d**2*x*sin(e + f*x)**6/16 + 15*B*a**3*d**2*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*B*a**3*d**2*x*sin(e + f*x)**4/8 + 15*B*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*d**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 5*B*a**3*d**2*x*cos(e + f*x)**6/16 + 9*B*a**3*d**2*x*cos(e
+ f*x)**4/8 - 11*B*a**3*d**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*B*a**3
*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)**3*cos(e
+ f*x)**3/(6*f) - 15*B*a**3*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a
**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - B*a**3*d**2*sin(e + f*x)**2*co
s(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*B*a**3
*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a**3*d**2*cos(e + f*x)**5/(5
*f) - 2*B*a**3*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c
+ d*sin(e))**2*(a*sin(e) + a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.52

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{320\left(\cos(fx+e)^3-3\cos(fx+e)\right)Aa^3c^2+720\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^3c^2+960\,(fx+e)Aa^3c^2}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e)^
3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2
+ 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(
f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c*d + 18
0*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d + 480*(
2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5 - 10*cos(f
*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x +
```

```
e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 192*(3*cos(f*x
 + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 320*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e +
 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*a^3*c^2*cos(f*x +
 e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{Ba^3d^2\sin(6fx+6e)}{192f} + \frac{1}{16}\left(40Aa^3c^2+30Ba^3c^2+60Aa^3cd+52Ba^3cd+26Aa^3d^2+23Ba^3d^2\right)x - \frac{(2Ba^3cd+Aa^3d^2+3Ba^3d^2)\cos(5fx+5e)}{80f} + \frac{(4Aa^3c^2+12Ba^3c^2+24Aa^3cd+34Ba^3cd+17Aa^3d^2+19Ba^3d^2)\cos(3fx+3e)}{48f} - \frac{(30Aa^3c^2+26Ba^3c^2+52Aa^3cd+46Ba^3cd+23Aa^3d^2+21Ba^3d^2)\cos(fx+e)}{8f} + \frac{(2Ba^3c^2+4Aa^3cd+12Ba^3cd+6Aa^3d^2+9Ba^3d^2)\sin(4fx+4e)}{64f} - \frac{(48Aa^3c^2+64Ba^3c^2+128Aa^3cd+128Ba^3cd+64Aa^3d^2+63Ba^3d^2)\sin(2fx+2e)}{64f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 + 6
0*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a^3
*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 + 12
*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)*cos
(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46*B*a^
3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c^2 + 4
*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4*e)/f -
 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d + 64*A*a
^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.11

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^
2 + 60*A*c*d + 52*B*c*d))/(8*(5*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^
2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2)))*(40*A*c^2
+ 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*f) - (tan(e/2 +
 (f*x)/2)^10*(6*A*a^3*c^2 + 2*B*a^3*c^2 + 4*A*a^3*c*d) + tan(e/2 + (f*x)/2)
*(3*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (1
5*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^11*(3*A*a^3*c^2 + (
13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 +
(13*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^8*(34*A*a^3*c^2 + 12*A*a^3*d^2 + 22*
B*a^3*c^2 + 4*B*a^3*d^2 + 44*A*a^3*c*d + 24*B*a^3*c*d) + tan(e/2 + (f*x)/2)
^5*(6*A*a^3*c^2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 +
19*A*a^3*c*d + 25*B*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*A*a^3*c^2 + (25*A*a^
3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 + 19*A*a^3*c*d + 25*B*a^3*c*
d) + tan(e/2 + (f*x)/2)^4*(76*A*a^3*c^2 + 64*A*a^3*d^2 + 68*B*a^3*c^2 + 64*
B*a^3*d^2 + 136*A*a^3*c*d + 128*B*a^3*c*d) + tan(e/2 + (f*x)/2)^3*(9*A*a^3*
c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^3*
c*d)/2 + (63*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^9*(9*A*a^3*c^2 + (63*A*a^3*
d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^3*c*d)/2 + (63*B*a
^3*c*d)/2) + tan(e/2 + (f*x)/2)^2*(38*A*a^3*c^2 + (152*A*a^3*d^2)/5 + 34*B*
a^3*c^2 + (136*B*a^3*d^2)/5 + 68*A*a^3*c*d + (304*B*a^3*c*d)/5) + tan(e/2 +
 (f*x)/2)^6*((220*A*a^3*c^2)/3 + (152*A*a^3*d^2)/3 + 60*B*a^3*c^2 + (136*B*
a^3*d^2)/3 + 120*A*a^3*c*d + (304*B*a^3*c*d)/3) + (22*A*a^3*c^2)/3 + (76*A*
a^3*d^2)/15 + 6*B*a^3*c^2 + (68*B*a^3*d^2)/15 + 12*A*a^3*c*d + (152*B*a^3*c
*d)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 +
 (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
 (f*x)/2)^12 + 1))
```

3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 201

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx &= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x-\frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{5f}\\ &\quad+\frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos^3(e+fx)}{60f}\\ &\quad-\frac{3a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sin(e+fx)}{40f}\\ &\quad-\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f}\\ &\quad-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af}\end{aligned}$$

```
[Out] 1/8*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*x-1/5*a^3*(20*A*c+15*A*d+15*B*c+13*B*
d)*cos(f*x+e)/f+1/60*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)^3/f-3/40*
a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/20*(5*A*d+5*B*c
-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^3/f-1/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^4
/a/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3047, 3102, 2830, 2724, 2718, 2715, 8, 2713}

$$\begin{aligned}&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx\\ &=\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos^3(e+fx)}{60f}\\ &\quad-\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{5f}\\ &\quad-\frac{3a^3(20Ac+15Ad+15Bc+13Bd)\sin(e+fx)\cos(e+fx)}{40f}\\ &\quad+\frac{1}{8}a^3x(20Ac+15Ad+15Bc+13Bd)\\ &\quad-\frac{(5Ad+5Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^3}{20f}\\ &\quad-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*x)/8 - (a^3*(20*A*c + 15*B*c + 15*
A*d + 13*B*d)*Cos[e + f*x])/(5*f) + (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d
)*Cos[e + f*x]^3)/(60*f) - (3*a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e
 + f*x]*Sin[e + f*x])/(40*f) - ((5*B*c + 5*A*d - B*d)*Cos[e + f*x]*(a + a*S
in[e + f*x])^3)/(20*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (a+a\sin(e+fx))^3 \left(Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)\right)\,dx \\
&= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} \\
&\quad +\frac{\int (a+a\sin(e+fx))^3(a(5Ac+4Bd)+a(5Bc+5Ad-Bd)\sin(e+fx))\,dx}{5a}
\end{aligned}
$$

$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{1}{20}(20Ac+15Bc+15Ad+13Bd)\int (a+a\sin(e+fx))^3\,dx$$

$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{1}{20}(20Ac+15Bc+15Ad+13Bd)\int \left(a^3 + 3a^3\sin(e+fx) + 3a^3\sin^2(e+fx) + a^3\sin^3(e+fx)\right)\,dx$$

$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{1}{20}\left(a^3(20Ac+15Bc+15Ad+13Bd)\right)\int \sin^3(e+fx)\,dx + \frac{1}{20}\left(3a^3(20Ac+15Bc+15Ad+13Bd)\right)\int \sin(e+fx)\,dx + \frac{1}{20}\left(3a^3(20Ac+15Bc+15Ad+13Bd)\right)\int \sin^2(e+fx)\,dx$$

$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{3a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{20f} - \frac{3a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sin(e+fx)}{40f} - \frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{1}{40}\left(3a^3(20Ac+15Bc+15Ad+13Bd)\right)\int 1\,dx - \frac{\left(a^3(20Ac+15Bc+15Ad+13Bd)\right)\text{Subst}\left(\int (1-x^2)\,dx, x, \cos(e+fx)\right)}{20f}$$

$$= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{5f}$$
$$+ \frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos^3(e+fx)}{60f}$$
$$- \frac{3a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sin(e+fx)}{40f}$$
$$- \frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f}$$
$$- \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{\cos(e+fx)\left(-\frac{1}{4}a^4(5Bc+5Ad-Bd)(1+\sin(e+fx))^3 - Bd(a+a\sin(e+fx))^4 - \frac{a^4(20Ac+15Bc+15Ad+1}{}\right.}{5af}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] (Cos[e + f*x]*(-1/4*(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3) - B*d*
(a + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*ArcSi
n[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(22 + 9*Sin[e + f*
x] + 2*Sin[e + f*x]^2)))/(24*Sqrt[Cos[e + f*x]^2])))/(5*a*f)
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(\left(\left(3c+\frac{17d}{4}\right)B+A(c+3d)\right)\cos(3fx+3e)+3\left(4(-c-d)B-3\left(c+\frac{4d}{3}\right)A\right)\sin(2fx+2e)+\frac{3(B(c+3d)+dA)\sin(4fx+4e)}{8}-\frac{3Bd\cos}{}\right.}{}$
parts	$\frac{(A a^3 d+B a^3 c+3a^3 dB)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(3A a^3 c+A a^3 d+B a^3 c)\cos(fx+e)}{f}-$
risch	$\frac{5A a^3 cx}{2}+\frac{15A a^3 dx}{8}+\frac{15B a^3 cx}{8}+\frac{13B a^3 dx}{8}-\frac{15a^3\cos(fx+e)Ac}{4f}-\frac{13a^3\cos(fx+e)dA}{4f}-\frac{13a^3\cos(fx+e)Bc}{4f}$
derivativedivides	$\frac{-\frac{A a^3 c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+A a^3 d\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B a^3 c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin}{}\right.}{}\right.}{}$
default	$\frac{-\frac{A a^3 c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+A a^3 d\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B a^3 c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin}{}\right.}{}\right.}{}$
norman	$\frac{\left(\frac{5}{2}A a^3 c+\frac{15}{8}A a^3 d+\frac{15}{8}B a^3 c+\frac{13}{8}a^3 dB\right)x+\left(25A a^3 c+\frac{75}{4}A a^3 d+\frac{75}{4}B a^3 c+\frac{65}{4}a^3 dB\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(25A a^3 c+\frac{75}{4}A a^3\right.}{}$

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVE RBOSE)

[Out] 1/12*(((3*c+17/4*d)*B+A*(c+3*d))*cos(3*f*x+3*e)+3*(4*(-c-d)*B-3*(c+4/3*d)*A)*sin(2*f*x+2*e)+3/8*(B*(c+3*d)+d*A)*sin(4*f*x+4*e)-3/20*B*d*cos(5*f*x+5*e) +3*((-13*c-23/2*d)*B-15*(c+13/15*d)*A)*cos(f*x+e)+(45/2*c*f*x+39/2*d*f*x-36 *c-152/5*d)*B+30*(3*(-2/5+1/4*f*x)*d+c*(f*x-22/15))*A)*a^3/f

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{24\,Ba^3d\cos(fx+e)^5-40\left((A+3\,B)a^3c+(3\,A+5\,B)a^3d\right)\cos(fx+e)^3-15\left(5\,(4\,A+3\,B)a^3c+(15\,A\right.}{}$$

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm ="fricas")

[Out] -1/120*(24*B*a^3*d*cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*d)*cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 480

```
*((A + B)*a^3*c + (A + B)*a^3*d)*cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*B)*
a^3*d)*cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d)*cos(f*x
 + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(201) = 402$.

Time = 0.33 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*c*x*sin(e + f*x)**2/2 + 3*A*a**3*c*x*cos(e + f*x)**2/2
+ A*a**3*c*x - A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*sin(e +
 f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c*cos(e + f*x)**3/(3*f) - 3*A*a**3*c*co
s(e + f*x)/f + 3*A*a**3*d*x*sin(e + f*x)**4/8 + 3*A*a**3*d*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*A*a**3*d*x*sin(e + f*x)**2/2 + 3*A*a**3*d*x*cos(e +
 f*x)**4/8 + 3*A*a**3*d*x*cos(e + f*x)**2/2 - 5*A*a**3*d*sin(e + f*x)**3*co
s(e + f*x)/(8*f) - 3*A*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*d*sin(e + f*x)*cos(e + f*x)/(2
*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B*a**3*c*x
*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a
**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B*a**3*c*x*c
os(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c*cos(e + f*
x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x)**4/8 + 9*B*a*
*3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e + f*x)**2/2 + 9
*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2/2 - B*a**3*d*sin
(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a**3*d*sin(e + f
*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a
**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*d*cos(e + f*x)**5/(15*f) -
 2*B*a**3*d*cos(e + f*x)**3/f, Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*
(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(189) = 378.

Time = 0.22 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.98

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{160\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^3c+360\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^3c+480\left(fx+e\right)Aa^3c+\cdots}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*A*a^3*c + 480*(f*x + e)*A*a^3*c + 480*(cos(f*x + e)^3 - 3*
cos(f*x + e))*B*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*B*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 480*(cos(f
*x + e)^3 - 3*cos(f*x + e))*A*a^3*d + 15*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*A*a^3*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*
d - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d + 4
80*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d + 120*(2*f*x + 2*e - sin(2*f*x + 2*e
))*B*a^3*d - 1440*A*a^3*c*cos(f*x + e) - 480*B*a^3*c*cos(f*x + e) - 480*A*a
^3*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\begin{aligned}&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx\\&=-\frac{Ba^3d\cos\left(5fx+5e\right)}{80f}+\frac{1}{8}\left(20Aa^3c+15Ba^3c+15Aa^3d+13Ba^3d\right)x\\&\quad+\frac{\left(4Aa^3c+12Ba^3c+12Aa^3d+17Ba^3d\right)\cos\left(3fx+3e\right)}{48f}\\&\quad-\frac{\left(30Aa^3c+26Ba^3c+26Aa^3d+23Ba^3d\right)\cos\left(fx+e\right)}{8f}\\&\quad+\frac{\left(Ba^3c+Aa^3d+3Ba^3d\right)\sin\left(4fx+4e\right)}{32f}\\&\quad-\frac{\left(3Aa^3c+4Ba^3c+4Aa^3d+4Ba^3d\right)\sin\left(2fx+2e\right)}{4f}\end{aligned}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/80*B*a^3*d*cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3*
d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*d)
*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a^3*
d)*cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d)*sin(4*f*x + 4*e)/f
 - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.74

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{a^3\,\mathrm{atan}\left(\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(20\,A\,c+15\,A\,d+15\,B\,c+13\,B\,d)}{4\left(5\,A\,a^3\,c+\frac{15\,A\,a^3\,d}{4}+\frac{15\,B\,a^3\,c}{4}+\frac{13\,B\,a^3\,d}{4}\right)}\right)(20\,A\,c+15\,A\,d+15\,B\,c+13\,B\,d)}{4\,f}$$
$$- \frac{a^3\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)(20\,A\,c+15\,A\,d+15\,B\,c+13\,B\,d)}{4\,f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(6\,A\,a^3\,c+\frac{19\,A\,a^3\,d}{2}+\frac{19\,B\,a^3\,c}{2}+\frac{25\,B\,a^3\,d}{2}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\left(3\,A\,a^3\,c+\frac{15\,A\,a^3\,d}{4}+\frac{15\,B\,a^3\,c}{4}+\ldots\right.}{}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*(
5*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4)))*(20*A*c + 1
5*A*d + 15*B*c + 13*B*d))/(4*f) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(6*A*a^
3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/2) - tan(e/2 + (f*x)/2
)^9*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4) - tan(e/
2 + (f*x)/2)^7*(6*A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/
2) + tan(e/2 + (f*x)/2)^6*(28*A*a^3*c + 20*A*a^3*d + 20*B*a^3*c + 12*B*a^3*
d) + tan(e/2 + (f*x)/2)^2*((92*A*a^3*c)/3 + 28*A*a^3*d + 28*B*a^3*c + (76*B
*a^3*d)/3) + tan(e/2 + (f*x)/2)^4*((136*A*a^3*c)/3 + 40*A*a^3*d + 40*B*a^3*
c + (116*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^8*(6*A*a^3*c + 2*A*a^3*d + 2*B*a^
3*c) + tan(e/2 + (f*x)/2)*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (1
3*B*a^3*d)/4) + (22*A*a^3*c)/3 + 6*A*a^3*d + 6*B*a^3*c + (76*B*a^3*d)/15)/(
f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)
^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx &= \frac{5}{8}a^3(4A+3B)x-\frac{5a^3(4A+3B)\cos(e+fx)}{6f}-\frac{5a^3(4A+3B)\cos(e+fx)\sin(e+fx)}{24f}\\&\quad-\frac{a(4A+3B)\cos(e+fx)(a+a\sin(e+fx))^2}{12f}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}\end{aligned}$$

```
[Out] 5/8*a^3*(4*A+3*B)*x-5/6*a^3*(4*A+3*B)*cos(f*x+e)/f-5/24*a^3*(4*A+3*B)*cos(f
*x+e)*sin(f*x+e)/f-1/12*a*(4*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*c
os(f*x+e)*(a+a*sin(f*x+e))^3/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2830, 2724, 2718, 2715, 8, 2713}

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx &= \frac{a^3(4A+3B)\cos^3(e+fx)}{12f}-\frac{a^3(4A+3B)\cos(e+fx)}{f}\\&\quad-\frac{3a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{8f}\\&\quad+\frac{5}{8}a^3x(4A+3B)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}\end{aligned}$$

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 - (a^3*(4*A + 3*B)*Cos[e + f*x])/f + (a^3*(4*A + 3*
B)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*A + 3*B)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int(a+a\sin(e+fx))^3\,dx\\ &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}\\ &\quad+\frac{1}{4}(4A+3B)\int\left(a^3+3a^3\sin(e+fx)+3a^3\sin^2(e+fx)+a^3\sin^3(e+fx)\right)\,dx\end{aligned}$$

$$= \frac{1}{4}a^3(4A+3B)x - \frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f} + \frac{1}{4}\left(a^3(4A+3B)\right)\int \sin^3(e+fx)\,dx + \frac{1}{4}\left(3a^3(4A+3B)\right)\int \sin(e+fx)\,dx + \frac{1}{4}\left(3a^3(4A+3B)\right)\int \sin^2(e+fx)\,dx$$

$$= \frac{1}{4}a^3(4A+3B)x - \frac{3a^3(4A+3B)\cos(e+fx)}{4f} - \frac{3a^3(4A+3B)\cos(e+fx)\sin(e+fx)}{8f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f} + \frac{1}{8}\left(3a^3(4A+3B)\right)\int 1\,dx - \frac{\left(a^3(4A+3B)\right)\operatorname{Subst}\left(\int (1-x^2)\,dx, x, \cos(e+fx)\right)}{4f}$$

$$= \frac{5}{8}a^3(4A+3B)x - \frac{a^3(4A+3B)\cos(e+fx)}{f} + \frac{a^3(4A+3B)\cos^3(e+fx)}{12f} - \frac{3a^3(4A+3B)\cos(e+fx)\sin(e+fx)}{8f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = -\frac{a^3\cos(e+fx)\left(30(4A+3B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right) + \sqrt{\cos^2(e+fx)}(88A+72B+9(4A+5B)\sin(e+ \ldots\right)}{24f\sqrt{\cos^2(e+fx)}}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
[Out] -1/24*(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*(A
 + 3*B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{a^3(8(A+3B)\cos(3fx+3e)+24(-3A-4B)\sin(2fx+2e)+3B\sin(4fx+4e)+24(-15A-13B)\cos(fx+e)+240fxA+180f}{96f}$
risch	$\frac{5a^3Ax}{2}+\frac{15a^3Bx}{8}-\frac{15a^3\cos(fx+e)A}{4f}-\frac{13a^3\cos(fx+e)B}{4f}+\frac{B\,a^3\sin(4fx+4e)}{32f}+\frac{a^3\cos(3fx+3e)A}{12f}+\frac{a^3\cos}{}$
parts	$a^3Ax-\frac{(A\,a^3+3B\,a^3)(2+\sin^2(fx+e))\cos(fx+e)}{3f}-\frac{(3A\,a^3+B\,a^3)\cos(fx+e)}{f}+\frac{(3A\,a^3+3B\,a^3)\left(-\frac{\cos(fx+e)\sin}{2}\right.}{f}$
derivativedivides	$\frac{-\frac{A\,a^3(2+\sin^2(fx+e))\cos(fx+e)}{3}+B\,a^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3A\,a^3\left(-\frac{\cos(fx+e)\sin(fx+e}{2}\right.}{}$
default	$\frac{-\frac{A\,a^3(2+\sin^2(fx+e))\cos(fx+e)}{3}+B\,a^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3A\,a^3\left(-\frac{\cos(fx+e)\sin(fx+e}{2}\right.}{}$
norman	$\frac{(\frac{5}{2}A\,a^3+\frac{15}{8}B\,a^3)x+(10A\,a^3+\frac{15}{2}B\,a^3)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10A\,a^3+\frac{15}{2}B\,a^3)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(15A\,a^3+\frac{45}{4}B\,a^3)x(t}{}$

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*a^3*(8*(A+3*B)*cos(3*f*x+3*e)+24*(-3*A-4*B)*sin(2*f*x+2*e)+3*B*sin(4*f
*x+4*e)+24*(-15*A-13*B)*cos(f*x+e)+240*f*x*A+180*f*x*B-352*A-288*B)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\frac{8\,(A+3\,B)a^3\cos{(fx+e)}^3+15\,(4\,A+3\,B)a^3fx-96\,(A+B)a^3\cos{(fx+e)}+3\,\big(2\,Ba^3\cos{(fx+e)}^3-}{24\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)*
a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e)^3 - (12*A + 17*B)*a^3*cos(f*x +
e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(119) = 238.

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.92

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\begin{cases}\frac{3Aa^3x\sin^2(e+fx)}{2}+\frac{3Aa^3x\cos^2(e+fx)}{2}+Aa^3x-\frac{Aa^3\sin^2(e+fx)\cos(e+fx)}{f}-\frac{3Aa^3\sin(e+fx)\cos(e+fx)}{2f}-\frac{2Aa^3\cos^3(e+fx)}{3f} \\ x(A+B\sin(e))(a\sin(e)+a)^3\end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A*
a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/f +
 3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*a**3*
x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**
3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f*x)**3/f
 - B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\frac{32\left(\cos(fx+e)^3-3\cos(fx+e)\right)Aa^3+72\left(2fx+2e-\sin(2fx+2e)\right)Aa^3+96(fx+e)Aa^3+96\left(\cos\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3 + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3 + 96*(f*x + e)*A*a^3 + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*B*a^3 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3 - 288*A*a^3*cos(f*x + e) - 96*B
*a^3*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \frac{Ba^3\sin(4fx+4e)}{32f} + \frac{5}{8}(4Aa^3+3Ba^3)x + \frac{(Aa^3+3Ba^3)\cos(3fx+3e)}{12f} - \frac{(15Aa^3+13Ba^3)\cos(fx+e)}{4f} - \frac{(3Aa^3+4Ba^3)\sin(2fx+2e)}{4f}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1/4
*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.60

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \frac{5a^3\,\mathrm{atan}\left(\frac{5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4A+3B)}{4\left(5Aa^3+\frac{15Ba^3}{4}\right)}\right)(4A+3B)}{4f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3Aa^3+\frac{15Ba^3}{4}\right)+\frac{22Aa^3}{3}+6Ba^3+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(6Aa^3+2Ba^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(3\ldots\right.}{f\left(\tan\left(\frac{e}{2}+\ldots\right.\right.} - \frac{5a^3(4A+3B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*A + 3*B))/(4*(5*A*a^3 + (15*B*a^3)
/4)))*(4*A + 3*B))/(4*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3 + (15*B*a^3)/4) + (
22*A*a^3)/3 + 6*B*a^3 + tan(e/2 + (f*x)/2)^6*(6*A*a^3 + 2*B*a^3) - tan(e/2
+ (f*x)/2)^7*(3*A*a^3 + (15*B*a^3)/4) + tan(e/2 + (f*x)/2)^3*(3*A*a^3 + (23
*B*a^3)/4) - tan(e/2 + (f*x)/2)^5*(3*A*a^3 + (23*B*a^3)/4) + tan(e/2 + (f*x
)/2)^4*(22*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^2*((70*A*a^3)/3 + 22*B*a^
3))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)
/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (5*a^3*(4*A + 3*B)*(atan(tan(e/2 + (f*
x)/2)) - (f*x)/2))/(4*f)
```

3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 246

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx &= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} \\ &\quad + \frac{2a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}\,f} \\ &\quad + \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} \\ &\quad - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} \\ &\quad + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{6d^2f}\end{aligned}$$

```
[Out] 1/2*a^3*(A*d*(2*c^2-6*c*d+7*d^2)-B*(2*c^3-6*c^2*d+7*c*d^2-5*d^3))*x/d^4+1/2
*a^3*(A*(2*c-5*d)*d-B*(2*c^2-5*c*d+5*d^2))*cos(f*x+e)/d^3/f-1/3*a*B*cos(f*x
+e)*(a+a*sin(f*x+e))^2/d/f+1/6*(-3*A*d+3*B*c-5*B*d)*cos(f*x+e)*(a^3+a^3*sin
(f*x+e))/d^2/f+2*a^3*(c-d)^3*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^
2-d^2)^(1/2))/d^4/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3055, 3047, 3102, 2814, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f\sqrt{c^2-d^2}} + \frac{a^3(Ad(2c-5d)-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} + \frac{a^3x(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))}{2d^4} + \frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{6d^2f} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(A*d*(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*
x)/(2*d^4) + (2*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(d^4*Sqrt[c^2 - d^2]*f) + (a^3*(A*(2*c - 5*d)*d - B*(2*c^2
 - 5*c*d + 5*d^2))*Cos[e + f*x])/(2*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^2)/(3*d*f) + ((3*B*c - 3*A*d - 5*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[
e + f*x]))/(6*d^2*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{\int \frac{(a+a\sin(e+fx))^2(a(2Bc+3Ad)-a(3Bc-3Ad-5Bd)\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{3d}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6d^2f} + \frac{\int \frac{(a+a\sin(e+fx))\left(-3a^2(Bc(c-3d)-Ad(c+2d))-3a^2\left(A(2c-5d)d-B\left(2c^2-5cd+5d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{6d^2}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6d^2f} + \frac{\int \frac{-3a^3(Bc(c-3d)-Ad(c+2d))+\left(-3a^3(Bc(c-3d)-Ad(c+2d))-3a^3\left(A(2c-5d)d-B\left(2c^2-5cd+5d^2\right)\right)\right)\sin(e+fx)-3a^3\left(A(2c-5d)\right.}{c+d\sin(e+fx)}}{6d^2}$$

$$= \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6d^2f} + \frac{\int \frac{-3a^3d(Bc(c-3d)-Ad(c+2d))+3a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{6d^3}$$

$$= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} + \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6d^2f} + \frac{\left(a^3(c-d)^3(Bc-Ad)\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^4}$$

$$= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} + \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6d^2f} + \frac{\left(2a^3(c-d)^3(Bc-Ad)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^4f}$$

$$
\begin{aligned}
&= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} \\
&\quad + \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} \\
&\quad - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} \\
&\quad + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{6d^2f} \\
&\quad - \frac{(4a^3(c-d)^3(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^4f} \\
&= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} \\
&\quad + \frac{2a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}\,f} \\
&\quad + \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} \\
&\quad - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} \\
&\quad + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{6d^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx
$$

$$
= \frac{a^3(1+\sin(e+fx))^3\left(6(Ad(2c^2-6cd+7d^2)+B(-2c^3+6c^2d-7cd^2+5d^3))\,(e+fx)+\frac{24(c-d)^3(Bc-Ad)\,\mathrm{a}\ldots}{\sqrt{\ldots}}\ldots\right.}{12d\ldots}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*c
^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d +
c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c + 3*
d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)] - 3
*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)]))/(12*d^4*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.54

method	result
derivativedivides	$2a^3\left(\frac{\left(-A c^3 d+3A c^2 d^2-3Ac d^3+A d^4+B c^4-3B c^3 d+3B c^2 d^2-B c d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A d^3-\frac{1}{2}d^2 cB+\frac{3}{2}d^3 B\right)}{}\right.$
default	$2a^3\left(\frac{\left(-A c^3 d+3A c^2 d^2-3Ac d^3+A d^4+B c^4-3B c^3 d+3B c^2 d^2-B c d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A d^3-\frac{1}{2}d^2 cB+\frac{3}{2}d^3 B\right)}{}\right.$
risch	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a^3*((-A*c^3*d+3*A*c^2*d^2-3*A*c*d^3+A*d^4+B*c^4-3*B*c^3*d+3*B*c^2*d^2-
B*c*d^3)/d^4/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))+1/d^4*(((1/2*A*d^3-1/2*d^2*c*B+3/2*d^3*B)*tan(1/2*f*x+1/2*e)^5+(
A*c*d^2-3*A*d^3-B*c^2*d+3*B*c*d^2-3*B*d^3)*tan(1/2*f*x+1/2*e)^4+(2*A*c*d^2-
6*A*d^3-2*B*c^2*d+6*B*c*d^2-8*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3+1/2*d
^2*c*B-3/2*d^3*B)*tan(1/2*f*x+1/2*e)+d^2*c*A-3*A*d^3-c^2*d*B+3*d^2*c*B-11/3
*d^3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(2*A*c^2*d-6*A*c*d^2+7*A*d^3-2*B*c^3
+6*B*c^2*d-7*B*c*d^2+5*B*d^3)*arctan(tan(1/2*f*x+1/2*e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$=\left[\frac{2\,Ba^3d^3\cos\left(fx+e\right)^3-3\left(2\,Ba^3c^3-2\left(A+3\,B\right)a^3c^2d+\left(6\,A+7\,B\right)a^3cd^2-\left(7\,A+5\,B\right)a^3d^3\right)fx+3\,(B}{}\right.$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d +
 (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3
```

```
*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) + 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d
 + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d
^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*
x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^
2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d - (A +
 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f), 1/6*(2*B*a^3*
d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a
^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*c
os(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a
^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sq
rt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 6*(B*a^3*c^2*d - (A + 3*B)*a^
3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(233) = 466$.

Time = 0.31 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.43

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx =$$

$$-\frac{\frac{3\,(2\,Ba^3c^3 - 2\,Aa^3c^2d - 6\,Ba^3c^2d + 6\,Aa^3cd^2 + 7\,Ba^3cd^2 - 7\,Aa^3d^3 - 5\,Ba^3d^3)(fx+e)}{d^4} - \frac{12\,(Ba^3c^4 - Aa^3c^3d - 3\,Ba^3c^3d + 3\,Aa^3c^2d^2 + 3\,Ba^3c^2 \cdots}{\cdots}}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B*
a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*a^
3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*d^3
 - B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4) + 2
*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 -
 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^4 -
6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 +
18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 +
 12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^2
- 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^2
 + 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*x + 1/2*e) +
 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e) + 6*B*
a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^3*d^2)/((tan(1
/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 10256, normalized size of antiderivative = 41.69

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x)),x)
```

```
[Out] - ((2*(9*A*a^3*d^2 + 3*B*a^3*c^2 + 11*B*a^3*d^2 - 3*A*a^3*c*d - 9*B*a^3*c*d
))/(3*d^3) - (tan(e/2 + (f*x)/2)^5*(A*a^3*d - B*a^3*c + 3*B*a^3*d))/d^2 + (
4*tan(e/2 + (f*x)/2)^2*(3*A*a^3*d^2 + B*a^3*c^2 + 4*B*a^3*d^2 - A*a^3*c*d -
 3*B*a^3*c*d))/d^3 + (2*tan(e/2 + (f*x)/2)^4*(3*A*a^3*d^2 + B*a^3*c^2 + 3*B
*a^3*d^2 - A*a^3*c*d - 3*B*a^3*c*d))/d^3 + (tan(e/2 + (f*x)/2)*(A*a^3*d - B
```

*a^3*c + 3*B*a^3*d))/d^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)
^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*
a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 +
25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^
6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 7
0*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6
*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x
)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116
*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2
*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 13
6*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^
9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*
A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^
6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9
+ ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B*
a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 -
(a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d
^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3
*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A
*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24
*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i +
(a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*
c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2
- (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4 +
(((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2
*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8
+ 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a
^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 +
188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6
*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^
2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8
*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6
*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 +
48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*
d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 25
2*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c
^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (((8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11
- 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^
9 - 2*B*a^3*c^4*d^8))/d^8 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13
- 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*
d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*tan(e/2
+ (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a
^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*
a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^
3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i

+ (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4)/((16*(2*B^3*a^9*c^10 - 47*A^3*a^9*c^2*d^8 + 55*A^3*a^9*c^3*d^7 - 21*A^3*a^9*c^4*d^6 - 7*A^3*a^9*c^5*d^5 + 8*A^3*a^9*c^6*d^4 - 2*A^3*a^9*c^7*d^3 - 15*B^3*a^9*c^3*d^7 + 71*B^3*a^9*c^4*d^6 - 148*B^3*a^9*c^5*d^5 + 180*B^3*a^9*c^6*d^4 - 139*B^3*a^9*c^7*d^3 + 67*B^3*a^9*c^8*d^2 + 14*A^3*a^9*c*d^9 - 18*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B*a^9*c*d^9 + 5*A*B^2*a^9*c^2*d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4*d^6 - 280*A*B^2*a^9*c^5*d^5 + 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3 + 44*A*B^2*a^9*c^8*d^2 - 32*A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45*A^2*B*a^9*c^4*d^6 - 97*A^2*B*a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B*a^9*c^7*d^3 + 6*A^2*B*a^9*c^8*d^2))/d^8 + (((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8 *d^4 + 140*A*B*a^6*c*d^11))/d^9 + (((8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 + (16*tan(e/2 + (f*x)/2)*(8*B^3*a^9*c^11 - 462*A^3*a^9*c^2*d^9 + 926*A^3*a^9*c^3*d^8 - 1034*A^3*a^9*c^4*d^7 + 704*A^3*a^9*c^5*d^6 - 296*A^3*a^9*c^6*d^5 + 72*A^3*a^9*c^7*d^4 - 8*A^3*a^9*c^8*d^3 - 50*B^3*a^9*c^2*d^9 + 290*B^3*a^9*c^3*d^8 - 788*B^3*a^9*c^4*d^7 + 1332*B^3*a^9*c^5*d^6 - 1546*B^3*a^9*c^6*d^5 + 1274*B^3*a^9*c^7*d^4 - 744*B^3*a^9*c^8*d^3 + 296*B^3*a^9*c^9*d^2 + 98*A^3*a^9*c*d^10 - 72*B^3*a^9*c^10*d + 50*A*B^2*a^9*c*d^10 - 24*A*B^2*a^9*c^10*d + 140*A^2*B*a^9*c*d^10 - 430*A*B^2*a^9*c^2*d^9 + 1524*A*B^2*a^9*c^3*d^8 - 3076*A*B^2*a^9*c^4*d^7 + 4018*A*B^2*a^9*c^5*d^6 - 3582*A*B^2*a^9*c^6*d^5 + 2192*A*B^2*a^9*c^7*d^4 - 888*A*B^2*a^9*c^8*d^3 + 216*A*B^2*a^9*c^9*d^2 - 834*A^2*B*a^9*c^2*d^9 + 2206*A^2*B*a^9*c^3*d^8 - 3398*A^2*B*a^9*c^4*d^7 + 3342*A^2*B*a^9*c^5*d^6 - 2152*A^2*B*a^9*c^6*d^5 + 888*A^2*B*a^9*c^7*d^4 - 216*A^2*B*a^9*c^8*d^3 + 24*A^2*B*a^9*c^9*d^2))/d^9))*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)*2i)/(d^4*f) - (a^3*atan(((a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5))*1i)/(c*d^4 + d^5) + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2 *a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a ^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^ 6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f* x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 11 6*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^ 2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 1 36*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c ^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258 *A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c ^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(14*A*a^3*c*d^11 + 10*B* a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B* a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 - (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^ 11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 + (a^3*(3 2*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5))*1i)/(c*d^4 + d ^5))/((16*(2*B^3*a^9*c^10 - 47*A^3*a^9*c^2*d^8 + 55*A^3*a^9*c^3*d^7 - 21*A^ 3*a^9*c^4*d^6 - 7*A^3*a^9*c^5*d^5 + 8*A^3*a^9*c^6*d^4 - 2*A^3*a^9*c^7*d^3 - 15*B^3*a^9*c^3*d^7 + 71*B^3*a^9*c^4*d^6 - 148*B^3*a^9*c^5*d^5 + 180*B^3*a^ 9*c^6*d^4 - 139*B^3*a^9*c^7*d^3 + 67*B^3*a^9*c^8*d^2 + 14*A^3*a^9*c*d^9 - 1 8*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B*a^9*c*d^9 + 5*A*B^2*a^9*c^2* d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4*d^6 - 280*A*B^2*a^9*c^5*d^5 + 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3 + 44*A*B^2*a^9*c^8*d^2 - 32 *A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45*A^2*B*a^9*c^4*d^6 - 97*A^2*B *a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B*a^9*c^7*d^3 + 6*A^2*B*a^9*c^ 8*d^2))/d^8 + (16*tan(e/2 + (f*x)/2)*(8*B^3*a^9*c^11 - 462*A^3*a^9*c^2*d^9 + 926*A^3*a^9*c^3*d^8 - 1034*A^3*a^9*c^4*d^7 + 704*A^3*a^9*c^5*d^6 - 296*A^ 3*a^9*c^6*d^5 + 72*A^3*a^9*c^7*d^4 - 8*A^3*a^9*c^8*d^3 - 50*B^3*a^9*c^2*d^9 + 290*B^3*a^9*c^3*d^8 - 788*B^3*a^9*c^4*d^7 + 1332*B^3*a^9*c^5*d^6 - 1546* B^3*a^9*c^6*d^5 + 1274*B^3*a^9*c^7*d^4 - 744*B^3*a^9*c^8*d^3 + 296*B^3*a^9* c^9*d^2 + 98*A^3*a^9*c*d^10 - 72*B^3*a^9*c^10*d + 50*A*B^2*a^9*c*d^10 - 24* A*B^2*a^9*c^10*d + 140*A^2*B*a^9*c*d^10 - 430*A*B^2*a^9*c^2*d^9 + 1524*A*B^ 2*a^9*c^3*d^8 - 3076*A*B^2*a^9*c^4*d^7 + 4018*A*B^2*a^9*c^5*d^6 - 3582*A*B^ 2*a^9*c^6*d^5 + 2192*A*B^2*a^9*c^7*d^4 - 888*A*B^2*a^9*c^8*d^3 + 216*A*B^2* a^9*c^9*d^2 - 834*A^2*B*a^9*c^2*d^9 + 2206*A^2*B*a^9*c^3*d^8 - 3398*A^2*B*a ^9*c^4*d^7 + 3342*A^2*B*a^9*c^5*d^6 - 2152*A^2*B*a^9*c^6*d^5 + 888*A^2*B*a^ 9*c^7*d^4 - 216*A^2*B*a^9*c^8*d^3 + 24*A^2*B*a^9*c^9*d^2))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3* d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2* a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^ 6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^ 6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6

+ 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19 *A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6 *c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6* c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 - (8*(14 *A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14* B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5)))/(c*d^4 + d^5) - (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6* c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25* B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A* B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5 *d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2) *(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2 *a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B* a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(14*A*a^3*c*d^11 + 10*B*a^3*c *d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c ^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 - (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24 *A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 + (a^3*(32*c^2 *d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5)))/(c*d^4 + d^5)))*(A *d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*2i)/(f*(c*d^4 + d^5))

3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 283

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} + \frac{2a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)\sqrt{c^2-d^2}\,f} - \frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c+d))\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] -1/2*a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/d^4-1/2*a^3*(4*A*c*d-B*
(6*c^2-3*c*d-5*d^2))*cos(f*x+e)/d^3/(c+d)/f+1/2*(2*A*d-B*(3*c+d))*cos(f*x+e
)*(a^3+a^3*sin(f*x+e))/d^2/(c+d)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))
^2/d/(c+d)/f/(c+d*sin(f*x+e))+2*a^3*(c-d)^2*(A*d*(2*c+3*d)-B*(3*c^2+3*c*d-d
^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^4/(c+d)/f/(c^2-d^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3054, 3055, 3047, 3102, 2814, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{2a^3(c-d)^2\left(Ad(2c+3d)-B(3c^2+3cd-d^2)\right)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)\sqrt{c^2-d^2}} - \frac{a^3x(2Ad(2c-3d)-B(6c^2-12cd+7d^2))}{2d^4} - \frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3f(c+d)} + \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2d^2f(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*(a^3*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*d^2))*x)/d^4 + (2*a^3*
(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)*Sqrt[c^2 - d^2]*f) - (a^3*(4*A*c*
d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(2*d^3*(c + d)*f) + ((2*A*d -
B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d^2*(c + d)*f) + (a*
(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Sin[e
+ f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^2(-a(B(2c-d)-3Ad)-a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))\left(-a^2\left(2A(c-3d)d-B\left(3c^2-3cd+2d^2\right)\right)+a^2\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{2d^2(c+d)}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{-a^3\left(2A(c-3d)d-B\left(3c^2-3cd+2d^2\right)\right)+\left(a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)-a^3\left(2A(c-3d)d-B\left(3c^2-3cd+2d^2\right)\right)\right)\sin(e+fx)+a^3\left(4Acd\right.}{c+d\sin(e+fx)}}{2d^2(c+d)}$$

$$= -\frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{-a^3d\left(2A(c-3d)d-B\left(3c^2-3cd+2d^2\right)\right)-a^3(c+d)\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{2d^3(c+d)}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} - \frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c+d))\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{(a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2)))\int\frac{1}{c+d\sin(e+fx)}\,dx}{d^4(c+d)}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} - \frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c+d))\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{(2a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2)))\,\text{Subst}\left(\int\frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^4(c+d)f}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} - \frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c+d))\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} - \frac{(4a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2)))\,\text{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+\right.\right.}{d^4(c+d)f}$$

$$= -\frac{a^3(2A(2c-3d)d - B(6c^2-12cd+7d^2))\,x}{2d^4} + \frac{2a^3(c-d)^2\,(Ad(2c+3d) - B(3c^2+3cd-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)\sqrt{c^2-d^2}\,f} - \frac{a^3(4Acd - B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad - B(3c+d))\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 5.95 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.86

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a^3(1+\sin(e+fx))^3\left(2(2Ad(-2c+3d)+B(6c^2-12cd+7d^2))\,(e+fx) - \frac{8(c-d)^2(-Ad(2c+3d)+B(3c^2+3cd-d^2)}{(c+d)\sqrt{c^2-}}\right.}{4d^4f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d^
2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2))
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]
) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d)*Co
s[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)]))/(4*d^
4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.43

method	result
derivativedivides	$2a^3\left(-\frac{\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-2cdB+3d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-2cdB+3d^2B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{(4Acd-6A\,d^2-6I}{}}{d^4}\right.$
default	$2a^3\left(-\frac{\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-2cdB+3d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-2cdB+3d^2B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{(4Acd-6A\,d^2-6I}{}}{d^4}\right.$
risch	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3*(-1/d^4*((-1/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(A*d^2-2*B*c*d+3*B*d^2)*t
an(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)*d^2+A*d^2-2*c*d*B+3*d^2*B)/(1+
tan(1/2*f*x+1/2*e)^2)^2+1/2*(4*A*c*d-6*A*d^2-6*B*c^2+12*B*c*d-7*B*d^2)*arct
an(tan(1/2*f*x+1/2*e)))+1/d^4*((-d^2*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2
*d-B*c*d^2)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B
*c^2*d-B*c*d^2)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(2
*A*c^3*d-A*c^2*d^2-4*A*c*d^3+3*A*d^4-3*B*c^4+3*B*c^3*d+4*B*c^2*d^2-5*B*c*d^
3+B*d^4)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 1027, normalized size of antiderivative = 3.63

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*
B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*B*
a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3*
B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*si
n(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*
```

```
d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (
2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*c^3*d -
2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3*d^4)*f*
x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)*cos(f*x
 + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f),
1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*B
)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + 2*(3*B
*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3
*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*s
in(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d
)/(c + d))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2
*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*
c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3
*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)
*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d
^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(271) = 542$.

Time = 0.31 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{\frac{4\,(3\,Ba^3c^4-2\,Aa^3c^3d-3\,Ba^3c^3d+Aa^3c^2d^2-4\,Ba^3c^2d^2+4\,Aa^3cd^3+5\,Ba^3cd^3-3\,Aa^3d^4-Ba^3d^4)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{\sqrt{c^2-d^2}}\right.\right.}{(cd^4+d^5)\sqrt{c^2-d^2}}}{}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B*
a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(1/2*
f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(1/2*f*
x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*d - 2*B
*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d^3 + c*d
^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (6*B*a^3*c
^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x + e)/d^4
- 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*
A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)^2 - B*a^3*d
*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((tan(1/2*f*x +
1/2*e)^2 + 1)^2*d^3))/f
```

Mupad [B] (verification not implemented)

Time = 24.89 (sec) , antiderivative size = 11993, normalized size of antiderivative = 42.38

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^
2 + 3*B*a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^4*(A*a^3*d^3 - 3*
B*a^3*c^3 - B*a^3*d^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + B*a^3*c*d^2 + 3*B*a^3
*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^2*(2*A*a^3*d^3 - 6*B*a^3*c^3
 + B*a^3*d^3 - 2*A*a^3*c*d^2 + 4*A*a^3*c^2*d + 5*B*a^3*c*d^2 + 6*B*a^3*c^2*
d))/(d^3*(c + d)) + (4*tan(e/2 + (f*x)/2)^3*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^
```

3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^3 - 3*B*a^3*c^3 - 4*A*a^3*c*d^2 + 2*A*a^3 *c^2*d - 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + (tan(e/2 + (f*x) /2)*(2*A*a^3*d^3 - 9*B*a^3*c^3 + 6*A*a^3*c^2*d + 10*B*a^3*c*d^2 + 9*B*a^3*c ^2*d))/(c*d^2*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 + 3*c*tan(e/2 + (f*x)/2)^4 + c*tan(e/2 + (f*x)/2)^6 + 4*d*tan(e/2 + (f*x)/2)^3 + 2*d*tan(e/2 + (f*x)/2)^5)) - (atan(((((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6* c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 14 4*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^ 8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88* A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3 *d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32* A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6* c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c *d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 5 72*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6 *c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((((8*(4*c^2 *d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^1 2 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d ^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)* (24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^ 3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/ 2)*1i)/d^4 + (((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4 *d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^ 2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d ^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B* a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)* (144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2 *a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^ 10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B ^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9 *d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A* B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6 *d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c *d^10 + d^11 + c^2*d^9) + (((8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3 *c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2)*1i)/d^4)/((16*(132*A^3*a^9*c^3*d^6 - 252*A^3*a^9*c^2*d^7 - 54*B^3*a^9*c^9 + 76*A^3*a^9*c^4*d^5 - 80*A^3*a^9*c^5*d^4 + 16*A^3*a^9*c^6*d^3 - 115*B^3*a^9*c^2*d^7 + 350*B^3*a^9*c^3*d^6 - 537*B^3*a^9*c^4*d^5 + 387*B^3*a^9*c^5*d^4 + 36*B^3*a^9*c^6*d^3 - 297*B^3*a^9*c^7*d^2 + 108*A^3*a^9*c*d^8 + 14*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 96*A*B^2*a^9*c*d^8 + 108*A*B^2*a^9*c^8*d + 198*A^2*B*a^9*c*d^8 - 573*A*B^2*a^9*c^2*d^7 + 1239*A*B^2*a^9*c^3*d^6 - 1125*A*B^2*a^9*c^4*d^5 + 93*A*B^2*a^9*c^5*d^4 + 630*A*B^2*a^9*c^6*d^3 - 468*A*B^2*a^9*c^7*d^2 - 768*A^2*B*a^9*c^2*d^7 + 996*A^2*B*a^9*c^3*d^6 - 288*A^2*B*a^9*c^4*d^5 - 402*A^2*B*a^9*c^5*d^4 + 336*A^2*B*a^9*c^6*d^3 - 72*A^2*B*a^9*c^7*d^2))/(2*c*d^9 + d^10 + c^2*d^8) + (16*tan(e/2 + (f*x)/2)*(520*A^3*a^9*c^4*d^6 - 360*A^3*a^9*c^2*d^8 - 168*A^3*a^9*c^3*d^7 - 216*B^3*a^9*c^10 - 112*A^3*a^9*c^5*d^5 - 160*A^3*a^9*c^6*d^4 + 64*A^3*a^9*c^7*d^3 - 728*B^3*a^9*c^2*d^8 + 1702*B^3*a^9*c^3*d^7 - 1090*B^3*a^9*c^4*d^6 - 1584*B^3*a^9*c^5*d^5 + 2898*B^3*a^9*c^6*d^4 - 1080*B^3*a^9*c^7*d^3 - 864*B^3*a^9*c^8*d^2 + 216*A^3*a^9*c*d^9 + 98*B^3*a^9*c*d^9 + 864*B^3*a^9*c^9*d + 462*A*B^2*a^9*c*d^9 + 432*A*B^2*a^9*c^9*d + 576*A^2*B*a^9*c*d^9 - 2178*A*B^2*a^9*c^2*d^8 + 2982*A*B^2*a^9*c^3*d^7 + 594*A*B^2*a^9*c^4*d^6 - 4668*A*B^2*a^9*c^5*d^5 + 3096*A*B^2*a^9*c^6*d^4 + 792*A*B^2*a^9*c^7*d^3 - 1512*A*B^2*a^9*c^8*d^2 - 1752*A^2*B*a^9*c^2*d^8 + 912*A^2*B*a^9*c^3*d^7 + 2016*A^2*B*a^9*c^4*d^6 - 2352*A^2*B*a^9*c^5*d^5 + 24*A^2*B*a^9*c^6*d^4 + 864*A^2*B*a^9*c^7*d^3 - 288*A^2*B*a^9*c^8*d^2))/(2*c*d^10 + d^11 + c^2*d^9) + (((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d

^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c
^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^
2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (
8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10
 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10
+ c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*
a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32
*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))
/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i -
(a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2
)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (((8*(36*A^2*a^6*c^2*d^9 + 24*
A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*
d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^
2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^
3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*
a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c
^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9
 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*
a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*
d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*
B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^1
1 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A
*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8
*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((8*(12*A*a^3*c*
d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4
*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (
((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8
*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8
*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*
B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*tan(e/2 + (f*x)/2)*(24*
A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*
a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56
*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^
2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B
*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/
d^4))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*
1i)/2)*2i)/(d^4*f) - (a^3*atan(((a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(36*A
^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d
^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*
a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4
 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a
^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))
/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 -
 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a
^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^

9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^
2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^1
1 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B
*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*
d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9)
 + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^1
3 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^1
0 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*
d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9) - (8*(12*A*a^3*c*d^12
 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9
 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (a^3*
((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8
*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8
*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*
d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^
3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3
*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d)*1i)/(3
*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8
*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6
*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 5
9*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c
^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148
*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7
*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*
d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32
*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*
c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 +
156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6
*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 2
08*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^
6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^
2*d^9) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(12*A*a^3*c*d^12 + 14*B*a^3*
c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*
c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) - (8*tan(e/2 + (f*x
)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^
11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*
d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9) +
 (a^3*((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8
) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^
12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)
*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^
5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d
^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d)*
1i)/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))/((16*(132*A^3*a^9*c^3*d^6 - 252*
A^3*a^9*c^2*d^7 - 54*B^3*a^9*c^9 + 76*A^3*a^9*c^4*d^5 - 80*A^3*a^9*c^5*d^4

+ 16*A^3*a^9*c^6*d^3 - 115*B^3*a^9*c^2*d^7 + 350*B^3*a^9*c^3*d^6 - 537*B^3*
a^9*c^4*d^5 + 387*B^3*a^9*c^5*d^4 + 36*B^3*a^9*c^6*d^3 - 297*B^3*a^9*c^7*d^
2 + 108*A^3*a^9*c*d^8 + 14*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 96*A*B^2*a^9
*c*d^8 + 108*A*B^2*a^9*c^8*d + 198*A^2*B*a^9*c*d^8 - 573*A*B^2*a^9*c^2*d^7
+ 1239*A*B^2*a^9*c^3*d^6 - 1125*A*B^2*a^9*c^4*d^5 + 93*A*B^2*a^9*c^5*d^4 +
630*A*B^2*a^9*c^6*d^3 - 468*A*B^2*a^9*c^7*d^2 - 768*A^2*B*a^9*c^2*d^7 + 996
*A^2*B*a^9*c^3*d^6 - 288*A^2*B*a^9*c^4*d^5 - 402*A^2*B*a^9*c^5*d^4 + 336*A^
2*B*a^9*c^6*d^3 - 72*A^2*B*a^9*c^7*d^2))/(2*c*d^9 + d^10 + c^2*d^8) + (16*t
an(e/2 + (f*x)/2)*(520*A^3*a^9*c^4*d^6 - 360*A^3*a^9*c^2*d^8 - 168*A^3*a^9*
c^3*d^7 - 216*B^3*a^9*c^10 - 112*A^3*a^9*c^5*d^5 - 160*A^3*a^9*c^6*d^4 + 64
*A^3*a^9*c^7*d^3 - 728*B^3*a^9*c^2*d^8 + 1702*B^3*a^9*c^3*d^7 - 1090*B^3*a^
9*c^4*d^6 - 1584*B^3*a^9*c^5*d^5 + 2898*B^3*a^9*c^6*d^4 - 1080*B^3*a^9*c^7*
d^3 - 864*B^3*a^9*c^8*d^2 + 216*A^3*a^9*c*d^9 + 98*B^3*a^9*c*d^9 + 864*B^3*
a^9*c^9*d + 462*A*B^2*a^9*c*d^9 + 432*A*B^2*a^9*c^9*d + 576*A^2*B*a^9*c*d^9
 - 2178*A*B^2*a^9*c^2*d^8 + 2982*A*B^2*a^9*c^3*d^7 + 594*A*B^2*a^9*c^4*d^6
- 4668*A*B^2*a^9*c^5*d^5 + 3096*A*B^2*a^9*c^6*d^4 + 792*A*B^2*a^9*c^7*d^3 -
 1512*A*B^2*a^9*c^8*d^2 - 1752*A^2*B*a^9*c^2*d^8 + 912*A^2*B*a^9*c^3*d^7 +
2016*A^2*B*a^9*c^4*d^6 - 2352*A^2*B*a^9*c^5*d^5 + 24*A^2*B*a^9*c^6*d^4 + 86
4*A^2*B*a^9*c^7*d^3 - 288*A^2*B*a^9*c^8*d^2))/(2*c*d^10 + d^11 + c^2*d^9) +
 (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3
*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^
2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d
^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*
a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6
 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (
8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*
a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5
 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^
2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8
*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*
a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*
d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*
A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (a^3*(-(c + d)^3*(c - d)^3)^
(1/2)*((8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^
2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3
*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*
d^10 + d^11 + c^2*d^9) - (8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^
2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*
c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (a^3*((8*(4*c^2*d^13 + 8*c^3*d^12 +
4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15
+ 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 +
c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d
 - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*
d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 -
 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4

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) - (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*
c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49
*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^
5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A
*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*
d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8)
+ (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A
^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*
d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91
*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*
c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A
*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c
^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 1
44*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (a^3*(-(c + d)^3*(c - d)^
3)^(1/2)*((8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a
^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*
d^9 + d^10 + c^2*d^8) - (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*
d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c
^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^
3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9) + (a^3*((8*(4*c^2*d^13 + 8*c^3*d^12
 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^
15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11
 + c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*
c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 +
 B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^
2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*
d^4)))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d -
3*B*c*d)*2i)/(f*(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))
```

3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 305

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx &= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} \\ &\quad -\frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)^2\sqrt{c^2-d^2}\,f} \\ &\quad -\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} \\ &\quad +\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} \\ &\quad -\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))}\end{aligned}$$

```
[Out] -a^3*(-A*d+3*B*c-3*B*d)*x/d^4-1/2*a^3*(3*B*c*(2*c+3*d)-A*d*(2*c+5*d))*cos(f
*x+e)/d^3/(c+d)^2/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/(c+d)/
f/(c+d*sin(f*x+e))^2-1/2*(A*d*(c+4*d)-B*(3*c^2+4*c*d-2*d^2))*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))/d^2/(c+d)^2/f/(c+d*sin(f*x+e))-a^3*(c-d)*(A*d*(2*c^2+6*c*
d+7*d^2)-3*B*(2*c^3+4*c^2*d+c*d^2-2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(
c^2-d^2)^(1/2))/d^4/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3054, 3047, 3102, 2814, 2739, 632, 210}

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= -\frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)^2\sqrt{c^2-d^2}} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2d^2f(c+d)^2(c+d\sin(e+fx))} - \frac{a^3x(-Ad+3Bc-3Bd)}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3f(c+d)^2} + \frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}$$

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(3*B*c - A*d - 3*B*d)*x)/d^4) - (a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7
*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/
2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (a^3*(3*B*c*(2*c
+ 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x])/(2*d^3*(c + d)^2*f) + (a*(B*c - A*d
)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^
2) - ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e + f*x]*(a^3 + a^3*S
in[e + f*x]))/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{a(Bc - Ad)\cos(e + fx)(a + a\sin(e + fx))^2}{2d(c + d)f(c + d\sin(e + fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))^2(-2a(B(c-d)-2Ad)+a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx}{2d(c + d)}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))\left(a^2\left(Ad(c+7d)-3B\left(c^2+cd-2d^2\right)\right)+a^2(3Bc(2c+3d)-Ad(2c+5d))\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{2d^2(c+d)^2}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{a^3\left(Ad(c+7d)-3B\left(c^2+cd-2d^2\right)\right)+\left(a^3(3Bc(2c+3d)-Ad(2c+5d))+a^3\left(Ad(c+7d)-3B\left(c^2+cd-2d^2\right)\right)\right)\sin(e+fx)+a^3(3Bc(2c+3}{c+d\sin(e+fx)}}{2d^2(c+d)^2}$$

$$= -\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{a^3d\left(Ad(c+7d)-3B\left(c^2+cd-2d^2\right)\right)-2a^3(c+d)^2(3B(c-d)-Ad)\sin(e+fx)}{c+d\sin(e+fx)}dx}{2d^3(c+d)^2}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{2d^4(c+d)^2}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}dx,x,\tan\left(\right.\right.}{d^4(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{(2a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right))\,\mathrm{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d\right.}{d^4(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)^2\sqrt{c^2-d^2}f} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 830 vs. $2(305) = 610$.

Time = 7.18 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.72

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a^3(1+\sin(e+fx))^3\left(\frac{4(c-d)\left(-Ad(2c^2+6cd+7d^2)+3B(2c^3+4c^2d+cd^2-2d^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{-12Bc^5e+4Ac^4de-12B}{\ }\right.}{\ }$$

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*B
*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^
2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e + 8*
A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4*e +
 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x - 12*B*
c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B*c^2*d
```

```
^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x - d*(2*A
*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^2*d^2 +
4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B*d)*(e +
f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Cos[3*(e + f
*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*c^3*d^2*e*S
in[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[e + f*x] + 2
4*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c*d^4*e*Sin[e
+ f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e + f*x] - 24*B*
c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 24*B*c^2*d^3*f*x
*Sin[e + f*x] + 8*A*c*d^4*f*x*Sin[e + f*x] + 24*B*c*d^4*f*x*Sin[e + f*x] -
9*B*c^3*d^2*Sin[2*(e + f*x)] + 3*A*c^2*d^3*Sin[2*(e + f*x)] - 9*B*c^2*d^3*S
in[2*(e + f*x)] + 3*A*c*d^4*Sin[2*(e + f*x)] + 4*B*c*d^4*Sin[2*(e + f*x)] -
 6*A*d^5*Sin[2*(e + f*x)] - 2*B*d^5*Sin[2*(e + f*x)])/(c + d*Sin[e + f*x])^
2))/(4*d^4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.92

method	result
derivativedivides	$2a^3\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-3Bc+3dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^4}-\frac{-\frac{d^2\left(A\,c^3d+5A\,c^2d^2-4Ac\,d^3-2A\,d^4-3B\,c^4-3B\,c^3d+6B\,c^2d^2\right.}{2\left(c^2+2cd+d^2\right)c}}{}\right.$
default	$2a^3\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-3Bc+3dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^4}-\frac{-\frac{d^2\left(A\,c^3d+5A\,c^2d^2-4Ac\,d^3-2A\,d^4-3B\,c^4-3B\,c^3d+6B\,c^2d^2\right.}{2\left(c^2+2cd+d^2\right)c}}{}\right.$
risch	Expression too large to display

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3*(1/d^4*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-3*B*c+3*B*d)*arctan(tan(
1/2*f*x+1/2*e)))-1/d^4*((-1/2*d^2*(A*c^3*d+5*A*c^2*d^2-4*A*c*d^3-2*A*d^4-3*
B*c^4-3*B*c^3*d+6*B*c^2*d^2)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(
2*A*c^5*d+4*A*c^4*d^2-A*c^3*d^3+7*A*c^2*d^4-10*A*c*d^5-2*A*d^6-4*B*c^6-2*B*
c^5*d-B*c^4*d^2-5*B*c^3*d^3+14*B*c^2*d^4-2*B*c*d^5)/(c^2+2*c*d+d^2)/c^2*tan
(1/2*f*x+1/2*e)^2-1/2*d^2*(7*A*c^3*d+11*A*c^2*d^2-16*A*c*d^3-2*A*d^4-13*B*c
^4-5*B*c^3*d+22*B*c^2*d^2-4*B*c*d^3)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-1
/2*d*(2*A*c^3*d+4*A*c^2*d^2-5*A*c*d^3-A*d^4-4*B*c^4-2*B*c^3*d+7*B*c^2*d^2-B
```

```
*c*d^3)/(c^2+2*c*d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^
2+1/2*(2*A*c^3*d+4*A*c^2*d^2+A*c*d^3-7*A*d^4-6*B*c^4-6*B*c^3*d+9*B*c^2*d^2+
9*B*c*d^3-6*B*d^4)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(294) = 588$.

Time = 0.34 (sec) , antiderivative size = 1670, normalized size of antiderivative = 5.48

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 -
 (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 +
 B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3
*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x
 - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*
A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a
^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*
a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2
*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) -
 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(
A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*B*a^3*c^4*d -
(A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x
+ (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3
*A + B)*a^3*d^5)*cos(f*x + e))*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*c
os(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 +
 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B*a^3*c^3*d^2 - (A -
 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x
+ e)^2 + 2*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 + B*a^3*d^5)*cos(f*x + e)^3 - 2*(
3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*
A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^
3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A -
B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2
*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B
*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B
)*a^3*c*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (6*B*a^3*c^4*d - 2*(A -
```

```
3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B
)*a^3*d^5)*cos(f*x + e) - (4*(3*B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A
+ 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3
*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e)
)*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 +
2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*
d^7 + d^8)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(294) = 588$.

Time = 0.34 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.12

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*c
^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(
c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((tan(1/
2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A*a^3*
c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 5
*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e
)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/2*f*x +
1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(1/2*f*x +
 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^2*tan(1/2*
f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^3*d^3*tan(1
/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*A*a^3*c^2*d^
4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 10*A*a
^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*
A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e) - 7*
A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) -
 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2
*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2*d^4*tan(1/2*f*x +
 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^6 - 2*A*a^3*c^5*d
+ 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 + 5*A*a^3*c^3*d^3 + B*a
^3*c^3*d^3 + A*a^3*c^2*d^4)/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*(c*tan(1/2*f*x
 + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - (3*B*a^3*c - A*a^3*d - 3*B
*a^3*d)*(f*x + e)/d^4)/f
```

Mupad [B] (verification not implemented)

Time = 25.54 (sec) , antiderivative size = 13891, normalized size of antiderivative = 45.54

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3 +
 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2)/(d^3*(c + d)^2) + (4*ta
n(e/2 + (f*x)/2)^3*(A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d
 + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2))/(c*d^2
*(c + d)^2) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^4 + 3*B*a^3*c^4 + 4*A*a^3*c*
d^3 - A*a^3*c^3*d + 3*B*a^3*c^3*d - 5*A*a^3*c^2*d^2 - 6*B*a^3*c^2*d^2))/(c*
d^2*(c + d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*a^3*d^6 + 6*B*a^3*c^6 + 5*A*a^3
*c*d^5 - 2*A*a^3*c^5*d + B*a^3*c*d^5 + 6*B*a^3*c^5*d - 3*A*a^3*c^2*d^4 + 3*
A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 3*B*a^3*c^2*d^4 + 11*B*a^3*c^3*d^3 + 3*B*
a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)^4*(2*A*a^3*d^6 + 6*
B*a^3*c^6 + 10*A*a^3*c*d^5 - 2*A*a^3*c^5*d + 2*B*a^3*c*d^5 + 6*B*a^3*c^5*d
- 7*A*a^3*c^2*d^4 + A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 14*B*a^3*c^2*d^4 + 5*
```

B*a^3*c^3*d^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)
*(2*A*a^3*d^4 + 21*B*a^3*c^4 + 16*A*a^3*c*d^3 - 7*A*a^3*c^3*d + 4*B*a^3*c*d
^3 + 21*B*a^3*c^3*d - 11*A*a^3*c^2*d^2 - 14*B*a^3*c^2*d^2))/(c*d^2*(c + d)^
2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c^2 + 4*d^2) + tan(e/2 + (f*x)/2)^4*(3*c^2
+ 4*d^2) + c^2*tan(e/2 + (f*x)/2)^6 + c^2 + 8*c*d*tan(e/2 + (f*x)/2)^3 + 4*
c*d*tan(e/2 + (f*x)/2)^5 + 4*c*d*tan(e/2 + (f*x)/2))) - (atan((((B*a^3*c*3i
 - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2
*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9
+ 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^
6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 +
72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c
^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c
^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 +
 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c
^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 -
81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^
6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 28
2*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^
6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 -
 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9)
+ ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^1
4 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d
^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^
3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^
3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A
*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a
^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B
*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c
^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^
13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 +
 c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 +
16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6
*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))
/d^4)*1i)/d^4 + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 +
 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c
^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144
*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8
*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*
B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12
+ 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2
*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A
^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*
d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*
B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d
^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*

A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))/d^4)*1i)/d^4)/((16*(54*B^3*a^9*c^8 - 29*A^3*a^9*c^3*d^5 - 18*A^3*a^9*c^4*d^4 - 2*A^3*a^9*c^5*d^3 - 324*B^3*a^9*c^2*d^6 + 81*B^3*a^9*c^3*d^5 + 405*B^3*a^9*c^4*d^4 - 135*B^3*a^9*c^5*d^3 - 243*B^3*a^9*c^6*d^2 + 49*A^3*a^9*c*d^7 + 108*B^3*a^9*c*d^7 + 54*B^3*a^9*c^7*d + 288*A*B^2*a^9*c*d^7 - 54*A*B^2*a^9*c^7*d + 231*A^2*B*a^9*c*d^7 - 576*A*B^2*a^9*c^2*d^6 - 135*A*B^2*a^9*c^3*d^5 + 540*A*B^2*a^9*c^4*d^4 + 135*A*B^2*a^9*c^5*d^3 - 198*A*B^2*a^9*c^6*d^2 - 231*A^2*B*a^9*c^2*d^6 - 201*A^2*B*a^9*c^3*d^5 + 69*A^2*B*a^9*c^4*d^4 + 114*A^2*B*a^9*c^5*d^3 + 18*A^2*B*a^9*c^6*d^2))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (16*tan(e/2 + (f*x)/2)*(216*B^3*a^9*c^9 + 52*A^3*a^9*c^2*d^7 + 4*A^3*a^9*c^3*d^6 - 44*A^3*a^9*c^4*d^5 - 32*A^3*a^9*c^5*d^4 - 8*A^3*a^9*c^6*d^3 - 324*B^3*a^9*c^2*d^7 - 756*B^3*a^9*c^3*d^6 + 864*B^3*a^9*c^4*d^5 + 1080*B^3*a^9*c^5*d^4 - 756*B^3*a^9*c^6*d^3 - 756*B^3*a^9*c^7*d^2 + 28*A^3*a^9*c*d^8 + 216*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 396*A*B^2*a^9*c*d^8 - 216*A*B^2*a^9*c^8*d + 192*A^2*B*a^9*c*d^8 - 108*A*B^2*a^9*c^2*d^7 - 1224*A*B^2*a^9*c^3*d^6 + 1260*A*B^2*a^9*c^5*d^4 + 324*A*B^2*a^9*c^6*d^3 - 432*A*B^2*a^9*c^7*d^2 + 156*A^2*B*a^9*c^2*d^7 - 372*A^2*B*a^9*c^3*d^6 - 372*A^2*B*a^9*c^4*d^5 + 108*A^2*B*a^9*c^5*d^4 + 216*A^2*B*a^9*c^6*d^3 + 72*A^2*B*a^9*c^7*d^2))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3

*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144
*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^1
3 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)
*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d
^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^
6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^
3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2
*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*
a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B
*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6
*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (((
8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c
*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(1
2*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d
^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(
B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))/d^4))/d^4 - ((B*a^3*c*3i - a^3*d*(A
 + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^
7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^
6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 +
 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*
c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24
*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (
8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6
*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 25
2*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*
c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 -
 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c
^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 +
 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6
*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c
*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c
^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*
c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^
3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e
/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*
a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12
*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10
 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*
c^3*d^10 + c^4*d^9) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5
*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) +
 (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14
 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 +
 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))/d^4))/d^4)
)*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*2i)/(d^4*f) - (a^3*atan(((a^3*(-(c + d)
^5*(c - d))^(1/2)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*

c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*1i)/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B

*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*
(-(c + d)^5*(c - d))^(1/2)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*
c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3
*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a
^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(
e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A
*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 1
2*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^1
0 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4
*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16
*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c
^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*
d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))
/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3
+ 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 +
d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 +
6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d
^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 +
6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*1i)/(2*(5*c*d^8 +
d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))/((16*(54*B^3*a^9*c^
8 - 29*A^3*a^9*c^3*d^5 - 18*A^3*a^9*c^4*d^4 - 2*A^3*a^9*c^5*d^3 - 324*B^3*a
^9*c^2*d^6 + 81*B^3*a^9*c^3*d^5 + 405*B^3*a^9*c^4*d^4 - 135*B^3*a^9*c^5*d^3
- 243*B^3*a^9*c^6*d^2 + 49*A^3*a^9*c*d^7 + 108*B^3*a^9*c*d^7 + 54*B^3*a^9*
c^7*d + 288*A*B^2*a^9*c*d^7 - 54*A*B^2*a^9*c^7*d + 231*A^2*B*a^9*c*d^7 - 57
6*A*B^2*a^9*c^2*d^6 - 135*A*B^2*a^9*c^3*d^5 + 540*A*B^2*a^9*c^4*d^4 + 135*A
*B^2*a^9*c^5*d^3 - 198*A*B^2*a^9*c^6*d^2 - 231*A^2*B*a^9*c^2*d^6 - 201*A^2*
B*a^9*c^3*d^5 + 69*A^2*B*a^9*c^4*d^4 + 114*A^2*B*a^9*c^5*d^3 + 18*A^2*B*a^9
*c^6*d^2))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (16*tan(e
/2 + (f*x)/2)*(216*B^3*a^9*c^9 + 52*A^3*a^9*c^2*d^7 + 4*A^3*a^9*c^3*d^6 - 4
4*A^3*a^9*c^4*d^5 - 32*A^3*a^9*c^5*d^4 - 8*A^3*a^9*c^6*d^3 - 324*B^3*a^9*c^
2*d^7 - 756*B^3*a^9*c^3*d^6 + 864*B^3*a^9*c^4*d^5 + 1080*B^3*a^9*c^5*d^4 -
756*B^3*a^9*c^6*d^3 - 756*B^3*a^9*c^7*d^2 + 28*A^3*a^9*c*d^8 + 216*B^3*a^9*
c*d^8 + 216*B^3*a^9*c^8*d + 396*A*B^2*a^9*c*d^8 - 216*A*B^2*a^9*c^8*d + 192
*A^2*B*a^9*c*d^8 - 108*A*B^2*a^9*c^2*d^7 - 1224*A*B^2*a^9*c^3*d^6 + 1260*A*
B^2*a^9*c^5*d^4 + 324*A*B^2*a^9*c^6*d^3 - 432*A*B^2*a^9*c^7*d^2 + 156*A^2*B
*a^9*c^2*d^7 - 372*A^2*B*a^9*c^3*d^6 - 372*A^2*B*a^9*c^4*d^5 + 108*A^2*B*a^
9*c^5*d^4 + 216*A^2*B*a^9*c^6*d^3 + 72*A^2*B*a^9*c^7*d^2))/(4*c*d^12 + d^13
+ 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8
*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*
c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*
B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7
*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*
B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^
4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*
x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A

^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d
^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B
^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9
*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A
*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6
*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*
d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d)
)^(1/2)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^
3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A
*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 +
36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13
+ 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13
 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9
 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*
d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8
) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*
d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9
 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15
+ 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 +
 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d
^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 +
10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^
2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 1
0*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2
 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10
*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)) - (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*A
^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d
^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a
^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4
+ 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6
*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(
4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)
*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^
6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 -
 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^
6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3
- 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^
6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6
+ 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12
+ d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/
2)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d
^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*
d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 +
 d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3
*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3

```
*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*
a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 2
4*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (
a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13
+ 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^
4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*
c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^
2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 +
2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^
3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2
*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3
*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*
A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*
d^6 + 5*c^4*d^5 + c^5*d^4))))*(-(c + d)^5*(c - d))^(1/2)*(7*A*d^3 - 6*B*c^3
 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*1i)/(f*(5*c*d^
8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4))
```

3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 220

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx &= \frac{(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))\,x}{2a} \\ &+ \frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{3af} \\ &+ \frac{d^2(6Ac-11Bc-9Ad+9Bd)\cos(e+fx)\sin(e+fx)}{6af} \\ &+ \frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} \\ &- \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))}\end{aligned}$$

```
[Out] 1/2*(3*A*d*(2*c^2-2*c*d+d^2)+B*(2*c^3-6*c^2*d+9*c*d^2-3*d^3))*x/a+2/3*d*(3*
A*(c^2-3*c*d+d^2)-B*(7*c^2-9*c*d+4*d^2))*cos(f*x+e)/a/f+1/6*d^2*(6*A*c-9*A*
d-11*B*c+9*B*d)*cos(f*x+e)*sin(f*x+e)/a/f+1/3*(3*A-4*B)*d*cos(f*x+e)*(c+d*s
in(f*x+e))^2/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3056, 2832, 2813}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{3af} + \frac{x(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))}{2a} + \frac{d^2(6Ac-9Ad-11Bc+9Bd)\sin(e+fx)\cos(e+fx)}{6af} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)} + \frac{d(3A-4B)\cos(e+fx)(c+d\sin(e+fx))^2}{3af}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((3*A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/(
2*a) + (2*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f
*x])/(3*a*f) + (d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f
*x])/(6*a*f) + ((3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a*f)
- ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} \\ &\quad+\frac{\int(c+d\sin(e+fx))^2(a(B(c-3d)+3Ad)-a(3A-4B)d\sin(e+fx))\,dx}{a^2} \\ &=\frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} \\ &\quad+\frac{\int(c+d\sin(e+fx))\left(a(3A(3c-2d)d+B(3c^2-9cd+8d^2))-ad(6Ac-11Bc-9Ad+9Bd)\sin\right.}{3a^2} \\ &=\frac{(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))\,x}{2a} \\ &\quad+\frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{3af} \\ &\quad+\frac{d^2(6Ac-11Bc-9Ad+9Bd)\cos(e+fx)\sin(e+fx)}{6af} \\ &\quad+\frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} \\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 788 vs. $2(220) = 440$.

Time = 7.02 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.58

$$\int\frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(4Ad(6c^2(e+fx)-3cd(1+2e+2fx)+d^2(1+3e+3fx))+B(8c\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*(
1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d*(
1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x)))*C
os[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos[(3*
(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f*x))/
2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A*c^3*S
in[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f*x)/2]
+ 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180*B*c*d^2
*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e + f*x)/2] +
 24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72*B*c^2*d*e
*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*e*Sin[(e +
f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x)/2] + 24*B*
c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 72*B*c^2*d*f*x
*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c*d^2*f*x*Sin[(
e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*Sin[(e + f*x)/2]
 - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e + f*x))/2] + 27*B
*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2] - 18*B*d^3*Sin[(
3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^3*Sin[(5*(e + f*x)
)/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e + f*x))/2]))/(24*a*f*
(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{\frac{2\left(\left(\frac{1}{2}A\,d^{3}+\frac{3}{2}d^{2}cB-\frac{1}{2}d^{3}B\right)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-3d^{2}cA+A\,d^{3}-3c^{2}dB+3d^{2}cB-d^{3}B\right)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-6d^{2}cA+2A\,d^{3}-6c^{2}dB+6\right.}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}}{}$
default	$\frac{\frac{2\left(\left(\frac{1}{2}A\,d^{3}+\frac{3}{2}d^{2}cB-\frac{1}{2}d^{3}B\right)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-3d^{2}cA+A\,d^{3}-3c^{2}dB+3d^{2}cB-d^{3}B\right)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-6d^{2}cA+2A\,d^{3}-6c^{2}dB+6\right.}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}}{}$
parallelrisch	$\frac{\left((36fxA-36fxB+84A-109B)d^{3}-72c\left(\frac{(-3fx-7)B}{2}+A\left(fx+\frac{7}{2}\right)\right)d^{2}+72\left(\left(-fx-\frac{7}{2}\right)B+A(fx+2)\right)c^{2}d-48\left(\left(-\frac{fx}{2}-1\right)B\right.\right.}{}$
risch	$-\frac{d^{3}\sin(2fx+2e)A}{4af}+\frac{3xA\,d^{3}}{2a}+\frac{xB\,c^{3}}{a}-\frac{3x\,d^{3}B}{2a}+\frac{3x\,c^{2}dA}{a}-\frac{3x\,d^{2}cA}{a}-\frac{3x\,c^{2}dB}{a}+\frac{9x\,d^{2}cB}{2a}+\frac{B\,d^{3}\cos(}{12a}$
norman	Expression too large to display

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(((1/2*A*d^3+3/2*d^2*c*B-1/2*d^3*B)*tan(1/2*f*x+1/2*e)^5+(-3*A*c*d^2+
A*d^3-3*B*c^2*d+3*B*c*d^2-B*d^3)*tan(1/2*f*x+1/2*e)^4+(-6*A*c*d^2+2*A*d^3-6
```

```
*B*c^2*d+6*B*c*d^2-4*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3-3/2*d^2*c*B+1/
2*d^3*B)*tan(1/2*f*x+1/2*e)-3*d^2*c*A+A*d^3-3*c^2*d*B+3*d^2*c*B-5/3*d^3*B)/
(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(6*A*c^2*d-6*A*c*d^2+3*A*d^3+2*B*c^3-6*B*c^2
*d+9*B*c*d^2-3*B*d^3)*arctan(tan(1/2*f*x+1/2*e))-(A*c^3-3*A*c^2*d+3*A*c*d^2
-A*d^3-B*c^3+3*B*c^2*d-3*B*c*d^2+B*d^3)/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(212) = 424.

Time = 0.28 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.14

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \frac{2\,Bd^3\cos{(fx+e)}^4 - 6\,(A-B)c^3 + 18\,(A-B)c^2d - 18\,(A-B)cd^2 + 6\,(A-B)d^3 + (9\,Bcd^2 + (3\,A-B}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B)
*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(2*
B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(3*B
*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)*c^3
 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^3 + 6
*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x + e) + (
2*B*d^3*cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A - B)*c*d^
2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*
(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^2 - 3*(6*B*c^2*
d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14644 vs. 2(204) = 408.

Time = 3.87 (sec) , antiderivative size = 14644, normalized size of antiderivative = 66.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*
f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2
)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(
```

```
e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**2/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 12*A*c**3/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*tan(e/2 + f*x/2)**7
/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*
f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*t
an(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
 f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) +
 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan
(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
 f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**3/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(
e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18
*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*
x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18
*A*c**2*d*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f) + 18*A*c**2*d*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f
*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f
*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
+ 6*a*f) + 36*A*c**2*d*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/
2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan
(e/2 + f*x/2) + 6*a*f) + 108*A*c**2*d*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 108*A*c**2*d*tan(e/2 + f*x/2)**2/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*A*c**2*d/(6*a*f*t
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an(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*A*c*d**2*f*x*tan(e/2 +
f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*t
an(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**
3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*A*c*d
**2*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*
x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*
tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
 6*a*f) - 54*A*c*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 54*A*c*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c*d**2*f*x*tan(e/2 + f*x
/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c*d**2
*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2
)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan
(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*
a*f) - 18*A*c*d**2*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*
tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)*
*4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/
2 + f*x/2) + 6*a*f) - 18*A*c*d**2*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 36*A*c*d**2*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*A*c*d**2*tan(e/2 + f*x/2)**5/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 144*A*c*d**2*tan(e/2 + f*x/
2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e
/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 72*A*c*d**2*
tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
+ 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
+ f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
- 180*A*c*d**2*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e
/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 +
18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f
*x/2) + 6*a*f) - 36*A*c*d**2*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 +
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6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 72*A*c*d**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*
tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)*
*4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/
2 + f*x/2) + 6*a*f) + 9*A*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 9*A*d**3*f*x*tan(e/2 + f*x/2)**6/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 27*A*d**3*f*x*tan(e/2 +
 f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*
tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)*
*3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 27*A*d*
*3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) + 27*A*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/
2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan
(e/2 + f*x/2) + 6*a*f) + 27*A*d**3*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 +
 f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2
)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 9*A*d**3*f*x*tan(e/2 + f*x/2)/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 9*A*d**3*f*x/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*d**3*tan(e/2 + f*x/2)*
*6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*d**3*tan(e
/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x
/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 48*
A*d**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) + 24*A*d**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 54*A*d**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
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2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6*A*d**3*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 +
 f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2
)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*A*d**3/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6*B*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6*B*c**3*f*x*tan(e/2 + f*x/
2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e
/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*B*c**3*f*
x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**
6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/
2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f
) + 18*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) + 18*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*B*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
5 + 18*a*f*tan(e/2 + f*x/2)4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6*B*c**3*f*x*tan(e/2 +
f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6*B*c**3*f*
x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 12*B*c**3*tan(e/
2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a
*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/
2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*B
*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) + 36*B*c**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) + 12*B*c**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)

 + 6*a*f) - 18*B*c**2*d*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7
+ 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 +
 f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*
f*tan(e/2 + f*x/2) + 6*a*f) - 18*B*c**2*d*f*x*tan(e/2 + f*x/2)**6/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*B*c**2*d*f*x*tan(e/2 + f
*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*ta
n(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3
 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*B*c**2
*d*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) - 54*B*c**2*d*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 54*B*c**2*d*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*B*c**2*d*f*x*tan(e/2 + f*x/
2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*B*c**2*d*f*x
/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*
f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*B*c**2*d*tan(e
/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x
/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*
B*c**2*d*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f
*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f
*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
+ 6*a*f) - 144*B*c**2*d*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*
a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x
/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*ta
n(e/2 + f*x/2) + 6*a*f) - 72*B*c**2*d*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 180*B*c2*d*tan(e/2 + f*x/2)**2/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*B*c**2*d*tan(e/2
+ f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*ta
n(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3
 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 72*B*c**2

```
*d/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 27*B*c*d**2*f*x
*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 + 27*B*c*d**2*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 81*B*c*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f
*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*
tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)*
*2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 81*B*c*d**2*f*x*tan(e/2 + f*x/2)**4/
(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f
*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 81*B*c*d**2*f*x*ta
n(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) +
81*B*c*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(
e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 +
 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
f*x/2) + 6*a*f) + 27*B*c*d**2*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) + 27*B*c*d**2*f*x/(6*a*f*tan(e/2 + f*x/2)**7
 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
+ f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a
*f*tan(e/2 + f*x/2) + 6*a*f) + 54*B*c*d**2*tan(e/2 + f*x/2)**6/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*B*c*d**2*tan(e/2 + f*x/2)**
5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 144*B*c*d**2*tan
(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 7
2*B*c*d**2*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f) + 162*B*c*d**2*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) + 18*B*c*d**2*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f
```

*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*
tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)*
*2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*B*c*d**2/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*B*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*B*d**3*f*x*tan(e/2 + f*x/
2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e
/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 27*B*d**3*f*
x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**
6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/
2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f
) - 27*B*d**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 27*B*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 27*B*d**3*f*x*tan(e/2 + f*x/2)**2/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
5 + 18*a*f*tan(e/2 + f*x/2)4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*B*d**3*f*x*tan(e/2 +
f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*B*d**3*f*
x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*B*d**3*tan(e/
2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a
*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/
2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*B
*d**3*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) - 48*B*d**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 48*B*d**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**
7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*
a*f*tan(e/2 + f*x/2) + 6*a*f) - 78*B*d**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18

```
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 14*B*d**3*tan(e/2 + f*x/2)/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 32*B*d**3/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(c
 + d*sin(e))**3/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(212) = 424$.

Time = 0.32 (sec) , antiderivative size = 1124, normalized size of antiderivative = 5.11

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*d^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 9*B*c*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*d^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a) + 18*B*c^2*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 18*A*c*d^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
```

```
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*B*
c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1))) - 18*A*c^2*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 6*A*c^3/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(212) = 424$.

Time = 0.29 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.09

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx$$

$$= \frac{\frac{3\,(2\,Bc^3+6\,Ac^2d-6\,Bc^2d-6\,Acd^2+9\,Bcd^2+3\,Ad^3-3\,Bd^3)(fx+e)}{a} - \frac{12\,(Ac^3-Bc^3-3\,Ac^2d+3\,Bc^2d+3\,Acd^2-3\,Bcd^2-Ad^3+Bd^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{2\,(9\,Bc}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3 -
 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c*d
^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*c*d
^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*tan(1/
2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(1/2*f
*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x + 1/2*e)^
2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 +
 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c*
d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3*tan(1/2*f
*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 10*B*d^3)/((
tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.81

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(12*A*c^3*cos(e/2 + (f*x)/2) - 18*A*d^3*cos(e/2 + (f*x)/2) - 12*B*c^3*cos(
e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2) + 6*A*d^3*cos(e/2 + (f*x)/2)^3
```

- 12*A*d^3*cos(e/2 + (f*x)/2)^5 - 6*B*d^3*cos(e/2 + (f*x)/2)^3 + 36*B*d^3*cos(e/2 + (f*x)/2)^5 - 16*B*d^3*cos(e/2 + (f*x)/2)^7 - 9*A*d^3*cos(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*cos(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*cos(e/2 + (f*x)/2)*(e + f*x) - 9*A*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*sin(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*A*d^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*B*d^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2) - 16*B*d^3*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2) - 36*A*c^2*d*cos(e/2 + (f*x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^3 + 18*B*c*d^2*cos(e/2 + (f*x)/2)^3 + 36*B*c^2*d*cos(e/2 + (f*x)/2)^3 - 36*B*c*d^2*cos(e/2 + (f*x)/2)^5 + 18*A*c*d^2*cos(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*cos(e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d^2*cos(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*cos(e/2 + (f*x)/2)*(e + f*x) + 18*A*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*sin(e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*sin(e/2 + (f*x)/2)*(e + f*x) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 36*B*c*d^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/(6*a*f*cos(e/2 + (f*x)/2) + 6*a*f*sin(e/2 + (f*x)/2))

3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 143

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx\\&=\frac{(2A(2c-d)d+B(2c^2-4cd+3d^2))\,x}{2a}+\frac{2(A(c-d)-B(2c-d))d\cos(e+fx)}{af}\\&\quad+\frac{(2A-3B)d^2\cos(e+fx)\sin(e+fx)}{2af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))}\end{aligned}$$

```
[Out] 1/2*(2*A*(2*c-d)*d+B*(2*c^2-4*c*d+3*d^2))*x/a+2*(A*(c-d)-B*(2*c-d))*d*cos(f
*x+e)/a/f+1/2*(2*A-3*B)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(A-B)*cos(f*x+e)*(c+d
*sin(f*x+e))^2/f/(a+a*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3056, 2813}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx\\&=\frac{x(-(d^2(2A-3B))+4Acd+2Bc(c-2d))}{2a}+\frac{2d(A(c-d)-B(2c-d))\cos(e+fx)}{af}\\&\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}+\frac{d^2(2A-3B)\sin(e+fx)\cos(e+fx)}{2af}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/(2*a) + (2*(A*(c - d) - B
*(2*c - d))*d*Cos[e + f*x])/(a*f) + ((2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e + f
*x])/(2*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[
e + f*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))} \\
&\quad+\frac{\int (c+d\sin(e+fx))(a(B(c-2d)+2Ad)-a(2A-3B)d\sin(e+fx))\,dx}{a^2} \\
&= \frac{(2Bc(c-2d)+4Acd-(2A-3B)d^2)\,x}{2a}+\frac{2(A(c-d)-B(2c-d))d\cos(e+fx)}{af} \\
&\quad+\frac{(2A-3B)d^2\cos(e+fx)\sin(e+fx)}{2af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)+2(2A(2c-d)d+B(2c^2-4cd+\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[2*(e + f*x)]))/(4*a*f*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{2\left(A\,c^{2}-2Acd+A\,d^{2}-B\,c^{2}+2cdB-d^{2}B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^{2}}{2}+\left(-A\,d^{2}-2cdB+d^{2}B\right)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^{2}}{2}-}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}}}{af}$
default	$\frac{-\frac{2\left(A\,c^{2}-2Acd+A\,d^{2}-B\,c^{2}+2cdB-d^{2}B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^{2}}{2}+\left(-A\,d^{2}-2cdB+d^{2}B\right)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^{2}}{2}-}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}}}{af}$
parallelrisch	$\frac{\left((-2fxA+3fxB-7A+7B)d^{2}+4c\left(\left(-fx-\frac{7}{2}\right)B+A(fx+2)\right)d-4\left(\left(-\frac{fx}{2}-1\right)B+A\right)c^{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left((-2fxA+3fxB-\right.}{}$
risch	$\frac{2xAcd}{a}-\frac{xA\,d^{2}}{a}+\frac{xB\,c^{2}}{a}-\frac{2xcdB}{a}+\frac{3x\,d^{2}B}{2a}-\frac{d^{2}{\mathrm e}^{i(fx+e)}A}{2af}-\frac{d\,{\mathrm e}^{i(fx+e)}Bc}{af}+\frac{d^{2}{\mathrm e}^{i(fx+e)}B}{2af}-\frac{d^{2}{\mathrm e}^{-i(fx+e}}{2af}$
norman	$\frac{\frac{\left(2A\,c^{2}-4Acd-2B\,c^{2}-2d^{2}B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(2A\,c^{2}-4Acd+2A\,d^{2}-2B\,c^{2}+4cdB-3d^{2}B\right)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(6A\,c^{2}-12Acd+2A\,d^{2}-\right.}{}}{}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-(A*c^2-2*A*c*d+A*d^2-B*c^2+2*B*c*d-B*d^2)/(tan(1/2*f*x+1/2*e)+1)+(1
/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(-A*d^2-2*B*c*d+B*d^2)*tan(1/2*f*x+1/2*e)^2-1
/2*B*tan(1/2*f*x+1/2*e)*d^2-A*d^2-2*c*d*B+d^2*B)/(1+tan(1/2*f*x+1/2*e)^2)^2
+1/2*(4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+3*B*d^2)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(139) = 278.

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \frac{Bd^2\cos(fx+e)^3 - 2(A-B)c^2 + 4(A-B)cd - 2(A-B)d^2 + (2Bc^2 + 4(A-B)cd - (2A-3B)d^2)f}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 +
 (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d^2
)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 - (2*
B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos(f*x
 + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4*(A -
 B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x + e))*si
n(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5763 vs. 2(117) = 234.

Time = 2.04 (sec) , antiderivative size = 5763, normalized size of antiderivative = 40.30

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-4*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*t
an(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c
*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*
x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan
(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d*f*x*tan(e/2 +
f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
```

```
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) + 8*A*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*
c*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 16*A*c*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 +
f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*f*x*tan(e/2 + f*x/2)**3/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2
*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/
2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*tan(e/2
 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f
*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
 2*a*f) - 4*A*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*A*d**2*tan(e/2 + f*x/2)**2/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 8*A*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2
)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
```

+ 4*B*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e
/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*
a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*
tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4
*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/
2) + 2*a*f) + 2*B*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) + 4*B*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2
)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/
2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*c**2*tan(e/2 + f*x/2)
2/(2*a*f*tan(e/2 + f*x/2)5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) +
 4*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c*d*f*
x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**
4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
 f*x/2) + 2*a*f) - 8*B*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2
)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4
*B*c*d*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*t
an(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2
*a*f) - 8*B*c*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(
e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2
*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c*d*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2
 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 24*B*c*d*tan(e/2
+ f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*
tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
2*a*f) - 8*B*c*d*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/
2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a
*f*tan(e/2 + f*x/2) + 2*a*f) - 16*B*c*d/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2*f*x*tan(e/2 + f*x/2)**5/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2
*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2

```
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/
2 + f*x/2) + 2*a*f) + 6*B*d**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d**2*f*x*tan(e/2 +
f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) + 3*B*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d**2*tan(e/2 + f*x/2)**4/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d
**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 10*B*d**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*d**2*tan(e/2 + f*x/2)/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*
d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f),
 Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.24

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \frac{Bd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-4Bcd\left(\cdots\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4*B*c*d*((sin(f*x + e)/(cos(f*x
```

```
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) + 2*B*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 4*A*c*d*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.50

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\frac{(2Bc^2+4Acd-4Bcd-2Ad^2+3Bd^2)(fx+e)}{a} - \frac{4(Ac^2-Bc^2-2Acd+2Bcd+Ad^2-Bd^2)}{a(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)} + \frac{2\left(Bd^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^3-4Bcd\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-2\cdots}{\cdots}}{2f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*c
^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e) +
1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 - 2*
A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*tan(1
/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*a))/f
```

Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{x\,(2\,B\,c^2-2\,A\,d^2+3\,B\,d^2+4\,A\,c\,d-4\,B\,c\,d)}{2\,a}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3(2\,A\,d^2-3\,B\,d^2+4\,B\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4(2\,A\,c^2+2\,A\,d^2-2\,B\,c^2-3\,B\,d^2-4\,A\,c\,d+4\cdots}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\cdots\right.\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

```
[Out] (x*(2*B*c^2 - 2*A*d^2 + 3*B*d^2 + 4*A*c*d - 4*B*c*d))/(2*a) - (tan(e/2 + (f
*x)/2)^3*(2*A*d^2 - 3*B*d^2 + 4*B*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 + 2*
A*d^2 - 2*B*c^2 - 3*B*d^2 - 4*A*c*d + 4*B*c*d) + tan(e/2 + (f*x)/2)^2*(4*A*
c^2 + 6*A*d^2 - 4*B*c^2 - 5*B*d^2 - 8*A*c*d + 12*B*c*d) + 2*A*c^2 + 4*A*d^2
 - 2*B*c^2 - 4*B*d^2 + tan(e/2 + (f*x)/2)*(2*A*d^2 - B*d^2 + 4*B*c*d) - 4*A
*c*d + 8*B*c*d)/(f*(a + a*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2
*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2)^4 + a*tan(e/2 + (f*x)/2)^5))
```

3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 67

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(B(c-d)+Ad)x}{a} - \frac{Bd\cos(e+fx)}{af} - \frac{(A-B)(c-d)\cos(e+fx)}{af(1+\sin(e+fx))}$$

```
[Out] (B*(c-d)+A*d)*x/a-B*d*cos(f*x+e)/a/f-(A-B)*(c-d)*cos(f*x+e)/a/f/(1+sin(f*x+
e))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2814, 2727}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)(c-d)\cos(e+fx)}{af(\sin(e+fx)+1)} + \frac{x(Ad+B(c-d))}{a} - \frac{Bd\cos(e+fx)}{af}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B*(c - d) + A*d)*x)/a - (B*d*Cos[e + f*x])/(a*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(a*f*(1 + Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{a+a\sin(e+fx)}\,dx\\
&= -\frac{Bd\cos(e+fx)}{af}+\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}\\
&= \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}+((A-B)(c-d))\int \frac{1}{a+a\sin(e+fx)}\,dx\\
&= \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}-\frac{(A-B)(c-d)\cos(e+fx)}{f(a+a\sin(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left((B(c-d)+Ad)(e+fx)-Bd\cos(e+fx)\right)+(2Ac \cdots\right.}{af(1+\sin(e+fx))}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d)
*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*(-
2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])
)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2(Ac-dA-Bc+dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+2(dA+Bc-dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
default	$\frac{-\frac{2(Ac-dA-Bc+dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+2(dA+Bc-dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
parallelrisch	$\frac{\left((2cfx-2dfx-4c+3d)B+4\left(\left(\frac{fx}{2}-1\right)d+c\right)A\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+((2cfx-2dfx-3d)B+2Adfx)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-dB\left(\cos\left(\frac{3fx}{2}+\cdots\right.\right.}{2af\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$
risch	$\frac{xdA}{a}+\frac{xBc}{a}-\frac{xdB}{a}-\frac{Bd\,e^{i(fx+e)}}{2af}-\frac{Bd\,e^{-i(fx+e)}}{2af}-\frac{2Ac}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2dA}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2Bc}{fa\left(e^{i(fx+e)}+i\right)}-\cdots$
norman	$\frac{\frac{(dA+Bc-dB)x}{a}+\frac{(dA+Bc-dB)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{(dA+Bc-dB)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(dA+Bc-dB)x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{2Ac-2dA-2Bc\cdots}{af}}{\cdots}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f/a*(-(A*c-A*d-B*c+B*d)/(tan(1/2*f*x+1/2*e)+1)-d*B/(1+tan(1/2*f*x+1/2*e)^
2)+(A*d+B*c-B*d)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.30

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{Bd\cos(fx+e)^2-(Bc+(A-B)d)fx+(A-B)c-(A-B)d-((Bc+(A-B)d)fx-(A-B)c-}{af\cos(fx+e)+}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B*
c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A - B
)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1307 vs. $2(49) = 98$.

Time = 1.04 (sec) , antiderivative size = 1307, normalized size of antiderivative = 19.51

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/
2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*tan(e/2 + f*x/2)**
3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2
 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e
/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 +
 a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 +
f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x
/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*
x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*ta
n(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
 B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**
2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*ta
```

```
n(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2
+ f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/
2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f
*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*B*d*
tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*
f*tan(e/2 + f*x/2) + a*f) - 2*B*d*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*B*d/(a*f*tan(e
/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(
f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(67) = 134$.

Time = 0.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{2\left(Bd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Bc\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\cdots\right.\right.}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*d*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + A*c/(a + a
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$=\frac{\frac{(Bc+Ad-Bd)(fx+e)}{a}-\frac{2\left(Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac-Bc}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)a}}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] ((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1/
2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^2
+ B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/2*e
)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx=\frac{x\,(A\,d+B\,c-B\,d)}{a}$$
$$-\frac{(2\,A\,c-2\,A\,d-2\,B\,c+2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,B\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,c-2\,A\,d-2\,B\,c+4\,B\,d}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] (x*(A*d + B*c - B*d))/a - (2*A*c - 2*A*d - 2*B*c + 4*B*d + tan(e/2 + (f*x)/
2)^2*(2*A*c - 2*A*d - 2*B*c + 2*B*d) + 2*B*d*tan(e/2 + (f*x)/2))/(f*(a + a*
tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx=\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] B*x/a-(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2814, 2727}

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx=\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] (B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{Bx}{a} - (-A+B)\int \frac{1}{a+a\sin(e+fx)}\,dx$$
$$= \frac{Bx}{a} - \frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. $2(35) = 70$.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(B(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2A+B(-2+e+fx))\sin\left(\frac{1}{2}(e+fx\right.\right.}{af(1+\sin(e+fx))}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*A
 + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	si
derivativedivides	$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$	42
default	$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$	42
parallelrisch	$\frac{fxB+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(fxB+2A-2B)}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	47
risch	$\frac{Bx}{a}-\frac{2A}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2B}{fa\left(e^{i(fx+e)}+i\right)}$	54
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{Bx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{Bx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(-2B+2A)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{(-2B+2A)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	13

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/f/a*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \frac{Bfx+(Bfx-A+B)\cos(fx+e)+(Bfx+A-B)\sin(fx+e)-A+B}{af\cos(fx+e)+af\sin(fx+e)+af}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] (B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A +
B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(26) = 52.

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \begin{cases} -\frac{2A}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{Bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{Bfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2B}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f\neq 0 \\ \frac{x(A+B\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
[Out] Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f*
tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*tan
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(35) = 70.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \frac{2\left(B\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{A}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] 2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx=\frac{\frac{(fx+e)B}{a}-\frac{2\,(A-B)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] ((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx=\frac{B\,x}{a}-\frac{2\,A-2\,B}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x)),x)
[Out] (B*x)/a - (2*A - 2*B)/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 101

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

```
[Out] -(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))+2*(-A*d+B*c)*arctan((d+c*tan(1/2
*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3057, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2(Bc-Ad)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)
*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{\int\frac{a(Bc-Ad)}{c+d\sin(e+fx)}\,dx}{a^2(c-d)}\\
&= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)}\\
&= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(2(Bc-Ad))\text{Subst}\left(\int\frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)f}
\end{aligned}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} - \frac{(4(Bc-Ad))\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)f}$$

$$= \frac{2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$$
$$= \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\sqrt{c^2-d^2}\sin\left(\frac{1}{2}(e+fx)\right)+(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+f\right.}{\sqrt{c^2-d^2}}\right.\right.}{a(c-d)\sqrt{c^2-d^2}f(1+\sin(e+fx))}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e +
f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Sin[e
 + f*x]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2(-dA+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}}{af}$
default	$\frac{-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2(-dA+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}}{af}$
risch	$-\frac{2A}{f(c-d)a\left(e^{i(fx+e)}+i\right)}+\frac{2B}{f(c-d)a\left(e^{i(fx+e)}+i\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}+c^2-d^2}{\sqrt{-c^2+d^2}\,d}\right)dA}{\sqrt{-c^2+d^2}\,(c-d)fa}+\frac{\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+}}{\sqrt{-c}}\right.}{\sqrt{-c^2+d^2}\,(c-}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f/a*(-(A-B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)+1/(c-d)*(-A*d+B*c)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(96) = 192$.

Time = 0.28 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.89

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$$
$$= \left[-\frac{2\,(A-B)c^2-2\,(A-B)d^2+(Bc-Ad+(Bc-Ad)\cos{(fx+e)}+(Bc-Ad)\sin{(fx+e)})\sqrt{-c^2+\cdots}}{2\,((ac^3-ac^2d-acd^2+a\cdots}\right.$$
$$-\frac{(A-B)c^2-(A-B)d^2+(Bc-Ad+(Bc-Ad)\cos{(fx+e)}+(Bc-Ad)\sin{(fx+e)})\sqrt{c^2-d^2}\,\mathrm{arc}\cdots}{(ac^3-ac^2d-acd^2+ad^3)f\cos{(fx+e)}+(ac^3-ac^2d\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x + e
) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d
*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)*c^2
- (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x
 + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2
*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d + (B*c -
 A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*s
in(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 - (A - B)*d
^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^
2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)
*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))}\,dx = \frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(Bc-Ad)}{(ac-ad)\sqrt{c^2-d^2}} - \frac{A-B}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A - B
)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\frac{(A\,d-B\,c)\,(2\,a\,d^2-2\,a\,c\,d)}{a\,\sqrt{c+d}\,(c-d)^{3/2}} - \frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,(a\,c-a\,d)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}}{2\,A\,d-2\,B\,c}\right)\,(A\,d-B\,c)}{a\,f\,\sqrt{c+d}\,(c-d)^{3/2}} - \frac{2\,(A-B)}{f\,\left(a+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(c-d)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*atan((((A*d - B*c)*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2))
- (2*c*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a*c - a*d))/(a*(c + d)^(1/2)*(c - d)
^(3/2)))/(2*A*d - 2*B*c))*(A*d - B*c))/(a*f*(c + d)^(1/2)*(c - d)^(3/2)) -
(2*(A - B))/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{3/2}f} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))}$$

```
[Out] -2*(A*d*(2*c+d)-B*(c^2+c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^
(1/2))/a/(c-d)/(c^2-d^2)^(3/2)/f+d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)
^2/(c+d)/f/(c+d*sin(f*x+e))-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*
sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{3057, 2833, 12, 2739, 632, 210\}$

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))^2}\,dx$$
$$= -\frac{2(Ad(2c+d) - B(c^2+cd+d^2))\arctan\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}} + \frac{d(B(2c+d) - A(c+2d))\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(3/2)*f) + (d*(B*(2*c + d) - A*(c +
 2*d))*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])) - ((A - B
)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

integral

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{\int \frac{a(2Ad-B(c+d))-a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{a^2(c-d)}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{\int \frac{a(Ad(2c+d)-B(c^2+cd+d^2))}{c+d\sin(e+fx)}\,dx}{a^2(c-d)^2(c+d)}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{(Ad(2c+d)-B(c^2+cd+d^2))\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)^2(c+d)}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{(2(Ad(2c+d)-B(c^2+cd+d^2)))\operatorname{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)^2(c+d)f}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} + \frac{(4(Ad(2c+d)-B(c^2+cd+d^2)))\operatorname{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)^2(c+d)f}$$

$$= -\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}f} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2(-Ad(2c+d)+B(c^2+cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2-d}}\right.}{(c+d)\sqrt{c^2-d}}\right.}{a(c-d)^2f(1+\sin(e+fx))}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-(
A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d^2]
) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c
+ d)*(c + d*Sin[e + f*x]))))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{2(A-B)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2Acd+A\,d^2-B\,c^2-cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}{2\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}\right.}{(c-d)^2}}{af}$
default	$\frac{-\frac{2(A-B)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2Acd+A\,d^2-B\,c^2-cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}{2\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}\right.}{(c-d)^2}}{af}$
risch	Expression too large to display

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-(A-B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)-1/(c-d)^2*((d^2*(A*d-B*c)/(c+d
)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1
/2*f*x+1/2*e)+c)+(2*A*c*d+A*d^2-B*c^2-B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(176) = 352$.

Time = 0.31 (sec) , antiderivative size = 1538, normalized size of antiderivative = 8.50

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3*
d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (
B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A
 - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A -
 B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A
 - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((A -
B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*x + e
) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d + (
2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x +
 e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f
*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4
- a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f -
 ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos
(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (
(A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B)*c^3*d + (2*A - B
)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (B*c^3 - 2*(A
 - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A - B)*c*d^2
- (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A - B)*c*d^2)*c
os(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 +
(B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (
(A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*
x + e) - ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d
```

```
+ (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*
x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6
)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d
^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*
f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*
cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(176) = 352$.

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.35

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{2\left(\frac{(Bc^2-2\,Acd+Bcd-Ad^2+Bd^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{c^2-d^2}} - \frac{Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+Ac^2}{}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 2*((B*c^2 - 2*A*c*d + B*c*d - A*d^2 + B*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^3
- a*c^2*d - a*c*d^2 + a*d^3)*sqrt(c^2 - d^2)) - (A*c^3*tan(1/2*f*x + 1/2*e)
^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c^2*
d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + A*d^3*tan(1/2*f
*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x + 1/2*e) - 3*B*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e) - 3*B*c*d^2*tan(1/2*f*x + 1/2*e) + A*
d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*c^3 + A*c^2*d - 2*B*c^2*d + A*c*d^2)/(
(a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1
/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2
*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\frac{\left(2\,a\,c^3\,d-2\,a\,c^2\,d^2-2\,a\,c\,d^3+2\,a\,d^4\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}+\frac{2\,c\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^3-a\,c^2\,d-a\,c\,d^2+a\,d^3\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}}{2\,B\,c^2-2\,A\,d^2+2\,B\,d^2-4\,A\,c\,d+2\,B\,c\,d}\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}-\frac{\frac{2\,\left(A\,c^2+A\,d^2-B\,c^2+A\,c\,d-2\,B\,c\,d\right)}{\left(c+d\right)\,{\left(c-d\right)}^2}+\frac{2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d^2+2\,A\,c\,d-3\,B\,c\,d\right)}{c\,{\left(c-d\right)}^2}+\frac{2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(A\,c^3+A\,d^3-B\,c^3+A\,c^2\,d-B\,c\,d^2-B\,c^2\,d\right)}{c\,\left(c+d\right)\,{\left(c-d\right)}^2}}{f\,\left(a\,c\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(a\,c+2\,a\,d\right)\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(a\,c+2\,a\,d\right)\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,c\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*atan((((2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d)*(B*c^2 - A*d^2 +
B*d^2 - 2*A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)) + (2*c*tan(e/2 +
(f*x)/2)*(a*c^3 + a*d^3 - a*c*d^2 - a*c^2*d)*(B*c^2 - A*d^2 + B*d^2 - 2*A*c
*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(2*B*c^2 - 2*A*d^2 + 2*B*d^2
- 4*A*c*d + 2*B*c*d))*(B*c^2 - A*d^2 + B*d^2 - 2*A*c*d + B*c*d))/(a*f*(c +
d)^(3/2)*(c - d)^(5/2)) - ((2*(A*c^2 + A*d^2 - B*c^2 + A*c*d - 2*B*c*d))/((
c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*d^2 + 2*A*c*d - 3*B*c*d))/(c*(
c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*c^3 + A*d^3 - B*c^3 + A*c^2*d - B*c*
d^2 - B*c^2*d))/(c*(c + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2)^2*(a*c
+ 2*a*d) + tan(e/2 + (f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)/2)^3))
```

3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 283

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx \\
&= -\frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{5/2}f} \\
&\quad-\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} \\
&\quad-\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} \\
&\quad-\frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

```
[Out] -(3*A*d*(2*c^2+2*c*d+d^2)-B*(2*c^3+4*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(
1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*A*c+3*A
*d-3*B*c-2*B*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2-(A-B)*cos(f
*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2*A*c^2+9*A*c*d+4*
A*d^2-5*B*c^2-6*B*c*d-4*B*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+
e))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$
$$= -\frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}$$
$$-\frac{d(2Ac^2+9Acd+4Ad^2-5Bc^2-6Bcd-4Bd^2)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}$$
$$-\frac{d(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2af(c-d)^2(c+d)(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -(((3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(5/2
)*f)) - (d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*S
in[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*A*c^2 - 5*B*c^2 + 9*A*c*d - 6*
B*c*d + 4*A*d^2 - 4*B*d^2)*Cos[e + f*x])/(2*a*(c - d)^3*(c + d)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\int \frac{a(3Ad-B(c+2d))-2a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{a^2(c-d)}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} + \frac{\int \frac{-2a\left(2(A-B)d^2+c(3Ad-B(c+2d))\right)+ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2a^2(c-d)^2(c+d)}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))} - \frac{\int \frac{a(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))}{c+d\sin(e+fx)}\,dx}{2a^2(c-d)^3(c+d)^2}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))} - \frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\int \frac{1}{c+d\sin(e+fx)}\,dx}{2a(c-d)^3(c+d)^2}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))} - \frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)^3(c+d)^2f}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))} + \frac{(2(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3)))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\right.\right.}{a(c-d)^3(c+d)^2f}$$

$$= -\frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}f}$$
$$- \frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2}$$
$$- \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2}$$
$$- \frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2(-3Ad(2c^2+2cd+d^2)+B(2c^3+4c^2d+7cd^2+2d^3))}{(c+}\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-3
*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[
(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) + (d*(
-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*a*(c - d)^3*
f*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{2\left(\frac{d^{2}\left(7c^{2}dA+2d^{2}cA-2A\,d^{3}-5B\,c^{3}-2c^{2}dB\right)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^{2}+2cd+d^{2}\right)}+\frac{d\left(6A\,c^{4}d+2A\,c^{3}d^{2}+11A\,c^{2}d^{3}+4Ac\,d^{4}-2A\,d^{5}-4B\,c^{5}-2B\,c^{4}d-9B\right.}{2\left(c^{2}+2cd+d^{2}\right)c^{2}}\right.}{\left(\left(\tan\right.\right.}$
default	$-\frac{2\left(\frac{d^{2}\left(7c^{2}dA+2d^{2}cA-2A\,d^{3}-5B\,c^{3}-2c^{2}dB\right)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^{2}+2cd+d^{2}\right)}+\frac{d\left(6A\,c^{4}d+2A\,c^{3}d^{2}+11A\,c^{2}d^{3}+4Ac\,d^{4}-2A\,d^{5}-4B\,c^{5}-2B\,c^{4}d-9B\right.}{2\left(c^{2}+2cd+d^{2}\right)c^{2}}\right.}{\left(\left(\tan\right.\right.}$
risch	Expression too large to display

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-1/(c-d)^3*((1/2*d^2*(7*A*c^2*d+2*A*c*d^2-2*A*d^3-5*B*c^3-2*B*c^2*d)
/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(6*A*c^4*d+2*A*c^3*d^2+11*A*c
^2*d^3+4*A*c*d^4-2*A*d^5-4*B*c^5-2*B*c^4*d-9*B*c^3*d^2-4*B*c^2*d^3-2*B*c*d^
4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(17*A*c^2*d+6*A*c*d^2-2
*A*d^3-11*B*c^3-6*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1
/2*d*(6*A*c^2*d+2*A*c*d^2-A*d^3-4*B*c^3-2*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2))
/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(6*A*c^2*d+6*A*c*d
^2+3*A*d^3-2*B*c^3-4*B*c^2*d-7*B*c*d^2-2*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2)^(
1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-(A-B)/(c-d)^
3/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(274) = 548$.

Time = 0.37 (sec) , antiderivative size = 3303, normalized size of antiderivative = 11.67

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(4*(A - B)*c^6 - 12*(A - B)*c^4*d^2 + 12*(A - B)*c^2*d^4 - 4*(A - B)*d
^6 - 2*((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3
*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + 2*(4*(A - 2*B)*c^5*d +
 4*(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A
 - B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 - (2*B*c^5 - 2*(3*A - 4*B)*c^
4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 -
```

(3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4
 - (3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 1
8*(A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 +
 (2*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2
*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*
A - 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11
*B)*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A -
 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B
)*c^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*(A
- B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (7
*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(f*x + e) - 2*
(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*d^6 - ((
2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*(3*A - 2
*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d + (14*A - 13
*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A - 7*B)*c*d^
5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
)), 1/2*(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*
d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*
(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + (4*(A - 2*B)*c^5*d + 4*
(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A -
B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 + (2*B*c^5 - 2*(3*A - 4*B)*c^4*d
 - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3
*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4 -
(3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 18*(
A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + (2
*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2*d^
3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*A -
 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)
*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*
B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B)*c
^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x + e

```
))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x +
e))) + (2*(A - B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)
*c^3*d^3 - (7*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(
f*x + e) - (2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A -
B)*d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 -
 3*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d +
 (14*A - 13*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A
- 7*B)*c*d^5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c
^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 +
a*d^9)*f*cos(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^
4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x +
e)^2 - (a*c^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2
*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4
*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^
8 + a*d^9)*f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^
3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^
7*d^2 - 3*a*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 +
 a*c*d^8)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6
*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*
sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(274) = 548.

Time = 0.38 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\frac{(2Bc^3-6Ac^2d+4Bc^2d-6Acd^2+7Bcd^2-3Ad^3+2Bd^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5)\sqrt{c^2-d^2}} - \frac{2(A-B)}{(ac^3-3ac^2d+3acd^2-ad^3)(\mathrm{t}}}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*B*c^3 - 6*A*c^2*d + 4*B*c^2*d - 6*A*c*d^2 + 7*B*c*d^2 - 3*A*d^3 + 2*B*d
^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a
*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) - 2*(A - B)/((a*c^3 - 3*a*c^2*d + 3*a*c*d^
2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)) + (5*B*c^4*d^2*tan(1/2*f*x + 1/2*e)^
3 - 7*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*c^3*d^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^5*tan(1/2*f*x + 1/2*e)^3 +
4*B*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B
*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*
c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 11*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*B*
c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 4*A*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d
^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^6*tan(1/2*f*x + 1/2*e)^2 + 11*B*c^4*d^2*t
an(1/2*f*x + 1/2*e) - 17*A*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*B*c^3*d^3*tan(1
/2*f*x + 1/2*e) - 6*A*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^4*tan(1/2*f*
x + 1/2*e) + 2*A*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^5*d - 6*A*c^4*d^2 + 2*B
*c^4*d^2 - 2*A*c^3*d^3 + B*c^3*d^3 + A*c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5
*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 - a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d
*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 1076, normalized size of antiderivative = 3.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\frac{A\,d^4-2\,A\,c^4+2\,B\,c^4-8\,A\,c^2\,d^2+4\,B\,c^2\,d^2-2\,A\,c\,d^3-4\,A\,c^3\,d+B\,c\,d^3+8\,B\,c^3\,d}{(c+d)\,(c^2-d^2)\,(c^2-2\,c\,d+d^2)}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A\,d^6-13\,A\,c^2\,d^4-17\,A\,c^3\,d^3-22\,A\,c^4\,d^2+4\,B\,c^2}{c^2\,(c^2-2\,c\,d+d^2)\,(}}{}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{\left(-2\,a\,c^5\,d+2\,a\,c^4\,d^2+4\,a\,c^3\,d^3-4\,a\,c^2\,d^4-2\,a\,c\,d^5+2\,a\,d^6\right)\,\left(2\,B\,c^3-3\,A\,d^3+2\,B\,d^3-6\,A\,c\,d^2-6\,A\,c^2\,d+7\,B\,c\,d^2+4\,B\,c^2\,d\right)}{2\,a\,(c+d)^{5/2}\,(c-d)^{7/2}}-\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^5-a\,c^4\right.}{}}{2\,B\,c^3-3\,A\,d^3+2\,B\,d^3-6\,A\,c\,d^2-6\,A\,c^2\,d+7\,B\,c\,d^2+4}\right.}{}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

```
[Out] ((A*d^4 - 2*A*c^4 + 2*B*c^4 - 8*A*c^2*d^2 + 4*B*c^2*d^2 - 2*A*c*d^3 - 4*A*c
^3*d + B*c*d^3 + 8*B*c^3*d)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (ta
n(e/2 + (f*x)/2)^3*(2*A*d^6 - 13*A*c^2*d^4 - 17*A*c^3*d^3 - 22*A*c^4*d^2 +
4*B*c^2*d^4 + 19*B*c^3*d^3 + 23*B*c^4*d^2 - 2*A*c*d^5 - 8*A*c^5*d + 2*B*c*d
^5 + 12*B*c^5*d))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (
tan(e/2 + (f*x)/2)^2*(2*A*d^5 - 4*A*c^5 + 4*B*c^5 - 21*A*c^2*d^3 - 14*A*c^3
*d^2 + 14*B*c^2*d^3 + 17*B*c^3*d^2 - 4*A*c*d^4 - 4*A*c^4*d + 2*B*c*d^4 + 8*
B*c^4*d))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) + (tan(e/2 + (f*x)/2)^4*(2*
A*c^5 - 2*A*d^5 - 2*B*c^5 + 7*A*c^2*d^3 + 2*A*c^3*d^2 - 2*B*c^2*d^3 - 7*B*c
^3*d^2 + 2*A*c*d^4 + 4*A*c^4*d - 4*B*c^4*d))/(c*(c^2 - 2*c*d + d^2)*(c*d^2
- c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(2*A*d^5 - 27*A*c^2*d^3 - 22*A*
c^3*d^2 + 15*B*c^2*d^3 + 29*B*c^3*d^2 - 5*A*c*d^4 - 8*A*c^4*d + 4*B*c*d^4 +
 12*B*c^4*d))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f
*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3*(2*a*c^2 + 4*
a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c*d) + tan(e/2 +
 (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5)) - (atan((((2*a
*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5 - 2*a*c^5*d)*(2*
B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))
/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (c*tan(e/2 + (f*x)/2)*(a*c^5 - a*d^5 +
 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^
3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))/(a*(c + d)^(5/2)*(c - d
)^(7/2)))/(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2
+ 4*B*c^2*d))*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*
d^2 + 4*B*c^2*d))/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx &= \frac{d(2A(3c-2d)d+B(6c^2-12cd+7d^2))\,x}{2a^2} \\ &\quad+\frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f} \\ &\quad+\frac{d^2(B(4c-21d)+2A(c+6d))\cos(e+fx)\sin(e+fx)}{6a^2f} \\ &\quad-\frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} \\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

```
[Out] 1/2*d*(2*A*(3*c-2*d)*d+B*(6*c^2-12*c*d+7*d^2))*x/a^2+2/3*d*(A*(c^2+6*c*d-5*
d^2)+B*(2*c^2-15*c*d+8*d^2))*cos(f*x+e)/a^2/f+1/6*d^2*(B*(4*c-21*d)+2*A*(c+
6*d))*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*(2*B*(c-4*d)+A*(c+5*d))*cos(f*x+e)*(c
+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))
^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3056, 2813}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx\\&=\frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f}\\&\quad+\frac{dx(2Ad(3c-2d)+B(6c^2-12cd+7d^2))}{2a^2}\\&\quad+\frac{d^2(2A(c+6d)+B(4c-21d))\sin(e+fx)\cos(e+fx)}{6a^2f}\\&\quad-\frac{(A(c+5d)+2B(c-4d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(\sin(e+fx)+1)}\\&\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*a^2) + (2*d*(A*(c
^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d + 8*d^2))*Cos[e + f*x])/(3*a^2*f) +
 (d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f)
 - ((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a
^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3
*f*(a + a*Sin[e + f*x])^2)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx))^2(a(Ac+2Bc+3Ad-3Bd)-a(2A-5B)d\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= -\frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} + \frac{\int (c+d\sin(e+fx))\left(a^2d(9Bc+10Ad-16Bd)-a^2d(B(4c-21d)+2A(c+6d))\sin(e+fx)\right)}{3a^4}$$

$$= \frac{d(2A(3c-2d)d+B(6c^2-12cd+7d^2))\,x}{2a^2} + \frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f} + \frac{d^2(B(4c-21d)+2A(c+6d))\cos(e+fx)\sin(e+fx)}{6a^2f} - \frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 547 vs. 2(228) = 456.

Time = 2.41 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.40

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(8Ad(6c^2+d^2(5-6e-6fx)+3cd(-4+3e+3fx))+B(16c^3+\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(8*A*d*(6*c^2 + d^2*(5 - 6*e - 6*
f*x) + 3*c*d*(-4 + 3*e + 3*f*x)) + B*(16*c^3 + 24*c^2*d*(-4 + 3*e + 3*f*x)
- 24*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x)))*Cos[(e + f*x)/
```

```
2] - (4*A*(4*c^3 + 24*c^2*d + d^3*(41 - 12*e - 12*f*x) + 6*c*d^2*(-10 + 3*e
 + 3*f*x)) + B*(32*c^3 + 24*c^2*d*(-10 + 3*e + 3*f*x) - 12*c*d^2*(-41 + 12*
e + 12*f*x) + d^3*(-239 + 84*e + 84*f*x)))*Cos[(3*(e + f*x))/2] + 3*(d^2*(1
2*B*c + 4*A*d - 5*B*d)*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] +
2*(8*A*c^3 + 8*B*c^3 + 24*A*c^2*d - 72*B*c^2*d - 72*A*c*d^2 + 108*B*c*d^2 +
 36*A*d^3 - 50*B*d^3 + 48*B*c^2*d*e + 48*A*c*d^2*e - 96*B*c*d^2*e - 32*A*d^
3*e + 56*B*d^3*e + 48*B*c^2*d*f*x + 48*A*c*d^2*f*x - 96*B*c*d^2*f*x - 32*A*
d^3*f*x + 56*B*d^3*f*x + d*(8*A*d*(3*c*(e + f*x) - 2*d*(1 + e + f*x)) + B*(
24*c^2*(e + f*x) - 48*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x)))*Cos[e
+ f*x] + 2*d^2*(-6*B*c - 2*A*d + 3*B*d)*Cos[2*(e + f*x)] + B*d^3*Cos[3*(e +
 f*x)])*Sin[(e + f*x)/2])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.49

method	result
derivativedivides	$-\frac{2\left(A\,c^{3}-3d^{2}cA+2A\,d^{3}-3c^{2}dB+6d^{2}cB-3d^{3}B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^{3}+6c^{2}dA-6d^{2}cA+2A\,d^{3}+2B\,c^{3}-6c^{2}dB+6d^{2}cB-2d^{3}B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(2A\,c^{3}-6c^{2}dA+\right.}{}$
default	$-\frac{2\left(A\,c^{3}-3d^{2}cA+2A\,d^{3}-3c^{2}dB+6d^{2}cB-3d^{3}B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^{3}+6c^{2}dA-6d^{2}cA+2A\,d^{3}+2B\,c^{3}-6c^{2}dB+6d^{2}cB-2d^{3}B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(2A\,c^{3}-6c^{2}dA+\right.}{}$
parallelrisch	$\left(\left(720fxA-1260fxB+696A-993B\right)d^{3}-1080\left(fxA-2fxB+\frac{7}{15}A-\frac{29}{15}B\right)c\,d^{2}-72c^{2}\left(15fxB+A+7B\right)d+216\left(A-\frac{B}{9}\right)c^{3}\right)\,\mathrm{co}$
risch	$\frac{3x\,d^{2}Ac}{a^{2}}-\frac{2x\,d^{3}A}{a^{2}}+\frac{3xdB\,c^{2}}{a^{2}}-\frac{6x\,d^{2}cB}{a^{2}}+\frac{7x\,d^{3}B}{2a^{2}}+\frac{id^{3}B\,\mathrm{e}^{2i(fx+e)}}{8a^{2}f}-\frac{d^{3}\mathrm{e}^{i(fx+e)}A}{2a^{2}f}-\frac{3d^{2}\mathrm{e}^{i(fx+e)}Bc}{2a^{2}f}+\frac{d^{3}}{}$
norman	Expression too large to display

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(-(A*c^3-3*A*c*d^2+2*A*d^3-3*B*c^2*d+6*B*c*d^2-3*B*d^3)/(tan(1/2*f*
x+1/2*e)+1)-1/2*(-2*A*c^3+6*A*c^2*d-6*A*c*d^2+2*A*d^3+2*B*c^3-6*B*c^2*d+6*B
*c*d^2-2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c^3-6*A*c^2*d+6*A*c*d^2-2
*A*d^3-2*B*c^3+6*B*c^2*d-6*B*c*d^2+2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^3+d*((1/
2*B*tan(1/2*f*x+1/2*e)^3*d^2+(-A*d^2-3*B*c*d+2*B*d^2)*tan(1/2*f*x+1/2*e)^2-
1/2*B*tan(1/2*f*x+1/2*e)*d^2-A*d^2-3*c*d*B+2*d^2*B)/(1+tan(1/2*f*x+1/2*e)^2
)^2+1/2*(6*A*c*d-4*A*d^2+6*B*c^2-12*B*c*d+7*B*d^2)*arctan(tan(1/2*f*x+1/2*e
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(218) = 436.

Time = 0.27 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = -\frac{3\,Bd^3\cos\left(fx+e\right)^4-2\,(A-B)c^3+6\,(A-B)c^2d-6\,(A-B)cd^2+2\,(A-B)d^3+6\,(3\,Bcd^2+(A-\ldots}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/6*(3*B*d^3*cos(f*x + e)^4 - 2*(A - B)*c^3 + 6*(A - B)*c^2*d - 6*(A - B)*
c*d^2 + 2*(A - B)*d^3 + 6*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^3 + 6*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - (2*(A + 2*B)*c^3 + 6*(2
*A - 5*B)*c^2*d - 6*(5*A - 11*B)*c*d^2 + (22*A - 31*B)*d^3 + 3*(6*B*c^2*d +
 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e)^2 - (2*(2*A + B)*c^
3 + 6*(A - 4*B)*c^2*d - 6*(4*A - 13*B)*c*d^2 + 2*(13*A - 19*B)*d^3 - 3*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e) + (3*B*d^3*
cos(f*x + e)^3 + 2*(A - B)*c^3 - 6*(A - B)*c^2*d + 6*(A - B)*c*d^2 - 2*(A -
 B)*d^3 + 6*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - 3*(6*B*
c*d^2 + (2*A - 3*B)*d^3)*cos(f*x + e)^2 - (2*(A + 2*B)*c^3 + 6*(2*A - 5*B)*
c^2*d - 6*(5*A - 14*B)*c*d^2 + 4*(7*A - 10*B)*d^3 - 3*(6*B*c^2*d + 6*(A - 2
*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14612 vs. 2(216) = 432.

Time = 7.60 (sec) , antiderivative size = 14612, normalized size of antiderivative = 64.09

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 1
8*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*ta
n(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x
/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c**3*tan(e/2 + f*x/2
)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**
2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
```

```
 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
 6*a**2*f) - 32*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*A*c**3*tan(e/2 + f*x/
2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a*
*2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/
2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) - 28*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c**3*tan(e/2 + f*x
/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2
*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
+ f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) - 8*A*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 36*A*c**2*d*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c
**2*d*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 72*A*c**2*d*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2
4*A*c**2*d*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*ta
n(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x
/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18
*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 36*A*c**2*d*tan(e/2 + f*x/2)/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 12*A*c**2*d/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6
+ 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f
*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
f*x/2) + 6*a**2*f) + 18*A*c*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2
+ f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5
 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 54*A*c*d**
2*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
```

4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a*
*2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/
2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)*
*3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f)
 + 126*A*c*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 126*A*c*d**2*f*x*tan(e/2 +
f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) + 90*A*c*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f
*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*t
an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 54*A*c*d**2*f
*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 18*A*c*d**2*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 36*A*c*d**2*tan(e/2 + f
*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30
*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan
(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/
2) + 6*a**2*f) + 108*A*c*d**2*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2
)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a
2*f*tan(e/2 + f*x/2)4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e
/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 120*A*c*d**2*tan(
e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)*
*6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**
2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
 + f*x/2) + 6*a**2*f) + 216*A*c*d**2*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2
+ f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5
 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 132*A*c*d*
*2*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) + 108*A*c*d**2*tan(e/2 + f*x/2)/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
2*f*tan(e/2 + f*x/2)2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 48*A*c
*d**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a*

```
*2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/
2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) - 12*A*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)*
*7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**
2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2
 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 36*A*d**3*f*x*tan(e
/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**
6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2
*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
+ f*x/2) + 6*a**2*f) - 60*A*d**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2
+ f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5
 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 84*A*d**3*
f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
 f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
+ 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) - 84*A*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 6
0*A*d**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*
tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f
*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 +
18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 36*A*d**3*f*x*tan(e/2 + f*x/2)/(6*
a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(
e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2
)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*
f) - 12*A*d**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/
2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*
a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) - 24*A*d**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2
+ f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5
 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 72*A*d**3*
tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) - 112*A*d**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/
2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)*
*5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**
2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 168*A*d*
*3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) - 128*A*d**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan
```

(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 96*A*
d**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f
*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*t
an(e/2 + f*x/2) + 6*a**2*f) - 40*A*d**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*B*c**3*tan(e/2 + f*x/2)*
*5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) - 4*B*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*B*c**3*tan(e/2 + f*x/2)*
*3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) - 8*B*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*B*c**3*tan(e/2 + f*x/2)/
(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*t
an(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*
x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a*
*2*f) - 4*B*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
6 + 30*a2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a*
*2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) + 18*B*c**2*d*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
2*f*tan(e/2 + f*x/2)2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 54*B*c
2*d*f*x*tan(e/2 + f*x/2)6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan
(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/
2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*
a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*B*c**2*d*f*x*tan(e/2 + f*x/2)**5/(
6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*ta
n(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x
/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**
2*f) + 126*B*c**2*d*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 126*B*c**2*d*f*x*tan(e/

2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*B*c**2*d*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 54*B*c**2*d*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 18*B*c**2*d*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 36*B*c**2*d*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 108*B*c**2*d*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 120*B*c**2*d*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 216*B*c**2*d*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 132*B*c**2*d*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 108*B*c**2*d*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 48*B*c**2*d/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 36*B*c*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 108*B*c*d**2*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f

*tan(e/2 + f*x/2) + 6*a**2*f) - 180*B*c*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a**
2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**
3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f)
- 252*B*c*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a
2*f*tan(e/2 + f*x/2)6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e
/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)
2 + 18*a2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 252*B*c*d**2*f*x*tan(e/2 + f
*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30
*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan
(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/
2) + 6*a**2*f) - 180*B*c*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f
*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*t
an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 108*B*c*d**2*
f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 36*B*c*d**2*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 72*B*c*d**2*tan(e/2 +
f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 216*B*c*d**2*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/
2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*
a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(
e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 336*B*c*d**2*tan
(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
6 + 30*a2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a*
*2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) - 504*B*c*d**2*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2
 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**
5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2
*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 384*B*c*d
2*tan(e/2 + f*x/2)2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
 f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
+ 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) - 288*B*c*d**2*tan(e/2 + f*x/2)/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 120*B
*c*d**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*
a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(

e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2
) + 6*a**2*f) + 21*B*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2
)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a
2*f*tan(e/2 + f*x/2)4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e
/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 63*B*d**3*f*x*tan
(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
6 + 30*a2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a*
*2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) + 105*B*d**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e
/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)
5 + 42*a2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a*
*2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 147*B*d
3*f*x*tan(e/2 + f*x/2)4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e
/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)
4 + 42*a2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a*
*2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 147*B*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a
2*f*tan(e/2 + f*x/2)7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e
/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)
3 + 30*a2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f
) + 105*B*d**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a
2*f*tan(e/2 + f*x/2)6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e
/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)
2 + 18*a2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 63*B*d**3*f*x*tan(e/2 + f*x/
2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) + 21*B*d**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 42*B*d**3*tan(e/2 + f*x/2)**6/(6*a**2*f*ta
n(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x
/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30
*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 126*
B*d**3*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/
2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)*
*4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**
2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 196*B*d**3*tan(e/2 + f*x/2)**4/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 2
52*B*d**3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan
(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/
2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*
a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 194*B*d**3*tan(e/2 + f*x/2)**2/(6*a**
2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2

```
 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**
3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f)
+ 150*B*d**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan
(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/
2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*
a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 64*B*d**3/(6*a**2*f*tan(e/2 + f*x/2)*
*7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**
2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2
 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B
*sin(e))*(c + d*sin(e))**3/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. 2(218) = 436.

Time = 0.36 (sec) , antiderivative size = 1382, normalized size of antiderivative = 6.06

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*d^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 12*B*c*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) - 4*A*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^2) + 6*B*c^2*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1
```

```
)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*A*c*d^2*((9*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^2) - 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3) - 2*B*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 6*A*c^2*d*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(218) = 436$.

Time = 0.32 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.07

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{3\left(6\,Bc^2d+6\,Acd^2-12\,Bcd^2-4\,Ad^3+7\,Bd^3\right)(fx+e)}{a^2} + \frac{6\left(Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,Bd^3\tan\left(\frac{1}{2}\,f\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/a
^2 + 6*(B*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2
*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*tan(1/2*f*x + 1/2*e)^2 - B*d^3*tan(
1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((tan(1/2*f*x + 1/2*e)^2
+ 1)^2*a^2) - 4*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*tan(1/2*f*x + 1
/2*e)^2 - 9*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e
)^2 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*A
*c^3*tan(1/2*f*x + 1/2*e) + 3*B*c^3*tan(1/2*f*x + 1/2*e) + 9*A*c^2*d*tan(1/
2*f*x + 1/2*e) - 27*B*c^2*d*tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*tan(1/2*f*x +
 1/2*e) + 45*B*c*d^2*tan(1/2*f*x + 1/2*e) + 15*A*d^3*tan(1/2*f*x + 1/2*e) -
 21*B*d^3*tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 12*B*c^2*d -
 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) +
 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.91

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{d\,\mathrm{atan}\left(\frac{d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(6Bc^2-4Ad^2+7Bd^2+6Acd-12Bcd)}{7Bd^3-4Ad^3+6Acd^2-12Bcd^2+6Bc^2d}\right)(6Bc^2-4Ad^2+7Bd^2+6Acd-12Bcd)}{a^2f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2Ac^3+16Ad^3+2Bc^3-25Bd^3-18Acd^2+6Ac^2d+48Bcd^2-18Bc^2d)+\frac{4Ac^3}{3}}{}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d*atan((d*tan(e/2 + (f*x)/2)*(6*B*c^2 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B
*c*d))/(7*B*d^3 - 4*A*d^3 + 6*A*c*d^2 - 12*B*c*d^2 + 6*B*c^2*d))*(6*B*c^2 -
 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B*c*d))/(a^2*f) - (tan(e/2 + (f*x)/2)*(2*
A*c^3 + 16*A*d^3 + 2*B*c^3 - 25*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 48*B*c*d^2
 - 18*B*c^2*d) + (4*A*c^3)/3 + (20*A*d^3)/3 + (2*B*c^3)/3 - (32*B*d^3)/3 +
tan(e/2 + (f*x)/2)^6*(2*A*c^3 + 4*A*d^3 - 7*B*d^3 - 6*A*c*d^2 + 12*B*c*d^2
- 6*B*c^2*d) + tan(e/2 + (f*x)/2)^5*(2*A*c^3 + 12*A*d^3 + 2*B*c^3 - 21*B*d^
3 - 18*A*c*d^2 + 6*A*c^2*d + 36*B*c*d^2 - 18*B*c^2*d) + tan(e/2 + (f*x)/2)^
3*(4*A*c^3 + 28*A*d^3 + 4*B*c^3 - 42*B*d^3 - 36*A*c*d^2 + 12*A*c^2*d + 84*B
*c*d^2 - 36*B*c^2*d) + tan(e/2 + (f*x)/2)^4*((16*A*c^3)/3 + (56*A*d^3)/3 +
(2*B*c^3)/3 - (98*B*d^3)/3 - 20*A*c*d^2 + 2*A*c^2*d + 56*B*c*d^2 - 20*B*c^2
*d) + tan(e/2 + (f*x)/2)^2*((14*A*c^3)/3 + (64*A*d^3)/3 + (4*B*c^3)/3 - (97
*B*d^3)/3 - 22*A*c*d^2 + 4*A*c^2*d + 64*B*c*d^2 - 22*B*c^2*d) - 8*A*c*d^2 +
 2*A*c^2*d + 20*B*c*d^2 - 8*B*c^2*d)/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2
*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/
2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*
tan(e/2 + (f*x)/2)))
```

3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 132

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= \frac{d(2B(c-d)+Ad)x}{a^2}+\frac{(A-4B)d^2\cos(e+fx)}{3a^2f}\\ &\quad-\frac{(c-d)(2B(c-3d)+A(c+3d))\cos(e+fx)}{3a^2f(1+\sin(e+fx))}\\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

```
[Out] d*(2*B*(c-d)+A*d)*x/a^2+1/3*(A-4*B)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)*(2*B*(c-
3*d)+A*(c+3*d))*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*s
in(f*x+e))^2/f/(a+a*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3056, 3047, 3102, 2814, 2727}

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= -\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{dx(Ad+2B(c-d))}{a^2}\\ &\quad+\frac{d^2(A-4B)\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*B*(c - d) + A*d)*x)/a^2 + ((A - 4*B)*d^2*Cos[e + f*x])/(3*a^2*f) - ((
c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx))(a(2B(c-d)+A(c+2d))-a(A-4B)d\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{ac(2B(c-d)+A(c+2d))+(-a(A-4B)cd+ad(2B(c-d)+A(c+2d)))\sin(e+fx)-a(A-4B)d^2\sin^2(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{a^2c(2B(c-d)+A(c+2d))+3a^2d(2B(c-d)+Ad)\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^3}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{((c-d)(2B(c-3d)+A(c+3d)))\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)(2B(c-3d)+A(c+3d))\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(132) = 264.

Time = 1.33 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(Ad(4c+d(-4+3e+3fx))+B(2c^2+d^2(5-6e-6fx)+2cd(-\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x)) + B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e + f*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x)) + 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d - 6*A*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*A*d^2*f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1 + e + f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12*a^2*f*(1 + Sin[e + f*x])^2)

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{2\left(A\,c^{2}-A\,d^{2}-2cdB+2d^{2}B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^{2}+4Acd-2A\,d^{2}+2B\,c^{2}-4cdB+2d^{2}B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(2A\,c^{2}-4Acd+2A\,d^{2}-2B\,c^{2}+4cdB-2d^{2}B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+2d\left(\right.}{a^{2}f}$
default	$\frac{-\frac{2\left(A\,c^{2}-A\,d^{2}-2cdB+2d^{2}B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^{2}+4Acd-2A\,d^{2}+2B\,c^{2}-4cdB+2d^{2}B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(2A\,c^{2}-4Acd+2A\,d^{2}-2B\,c^{2}+4cdB-2d^{2}B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+2d\left(\right.}{a^{2}f}$
risch	$\frac{x\,d^{2}A}{a^{2}}+\frac{2xdBc}{a^{2}}-\frac{2x\,d^{2}B}{a^{2}}-\frac{B\,d^{2}\mathrm{e}^{i(fx+e)}}{2a^{2}f}-\frac{B\,d^{2}\mathrm{e}^{-i(fx+e)}}{2a^{2}f}-\frac{2\left(10cdB+3iA\,c^{2}\mathrm{e}^{i(fx+e)}-6A\,d^{2}\mathrm{e}^{2i(fx+e)}+9B\,c\right.}{}$
parallelrisch	$\left(\left((-18fxA+36fxB-30A+78B)d^{2}+12((-3fx-5)B+A)cd+18\left(A+\frac{B}{3}\right)c^{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+((6fxA-12fxB-2A+5B)d^{2}\right.$
norman	$\frac{\frac{d(dA+2Bc-2dB)x}{a}+\frac{d(dA+2Bc-2dB)x\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4A\,c^{2}+4Acd-8A\,d^{2}+2B\,c^{2}-16cdB+20d^{2}B}{3af}-\frac{\left(2A\,c^{2}-2A\,d^{2}-4cdB+4d^{2}\right.}{af}}{}$

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 2/f/a^2*(-(A*c^2-A*d^2-2*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c^2+4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c^2-4*A*c*d+2*A*d^2-2*B*c^2+4*B*c*d-2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^3+d*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+2*B*c-2*B*d)*arctan(tan(1/2*f*x+1/2*e))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(126) = 252.

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.84

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{3\,Bd^2\cos{(fx+e)}^3-(A-B)c^2+2\,(A-B)cd-(A-B)d^2+6\,(2\,Bcd+(A-2\,B)d^2)fx-((A+2\,B}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5*A
 - 11*B)*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e)^2 - ((2*A + B)
*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x
)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d - (A
 - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A - 5*B
)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e))*s
in(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*
cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5358 vs. 2(121) = 242.

Time = 4.01 (sec) , antiderivative size = 5358, normalized size of antiderivative = 40.59

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e/2
+ f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c**2/(3*a**2*f*tan(e/2 + f*
```

x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*A*
c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d*tan(e/2 + f*x/2)**2/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 12*A*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d/(3*a**2*f*tan
(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2
)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 3*A*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 3*A*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*A*d**2*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
 14*A*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 8*A*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
+ f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/2 + f*x/2)**3/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) - 2*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta

n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 2*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B*c*d*f*x*tan(e/2 +
f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
3 + 12*a2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
+ 24*B*c*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*
f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*B*c*d*f*x*tan(e/2 +
f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2
+ f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
+ 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
6*B*c*d*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*B*c*d*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2
+ f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
+ 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
36*B*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e
/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)
2 + 9*a2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 28*B*c*d*tan(e/2 + f*x/2)**2/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 36*B*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 16*B*c*d/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f*x*
tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
2*f*tan(e/2 + f*x/2) + 3*a2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a
2*f*tan(e/2 + f*x/2)5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/
2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**
5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f

```
*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2
)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*B*d**
2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*
x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9
*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*d**2*tan(e/2 + f*x/2)**3/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 44*B*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 48*B*d**2*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12
*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(
e/2 + f*x/2) + 3*a**2*f) - 20*B*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*s
in(e))*(c + d*sin(e))**2/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(126) = 252$.

Time = 0.32 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.30

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*
B*c*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*d^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
```

```
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*c*d*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(126) = 252.

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(2\,Bcd+Ad^2-2\,Bd^2)(fx+e)}{a^2} - \frac{6\,Bd^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2} - \frac{2\left(3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bd^2}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(2*B*c*d + A*d^2 - 2*B*d^2)*(f*x + e)/a^2 - 6*B*d^2/((tan(1/2*f*x +
1/2*e)^2 + 1)*a^2) - 2*(3*A*c^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c*d*tan(1/2*f*
x + 1/2*e)^2 - 3*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*d^2*tan(1/2*f*x + 1/2*e
)^2 + 3*A*c^2*tan(1/2*f*x + 1/2*e) + 3*B*c^2*tan(1/2*f*x + 1/2*e) + 6*A*c*d
*tan(1/2*f*x + 1/2*e) - 18*B*c*d*tan(1/2*f*x + 1/2*e) - 9*A*d^2*tan(1/2*f*x
 + 1/2*e) + 15*B*d^2*tan(1/2*f*x + 1/2*e) + 2*A*c^2 + B*c^2 + 2*A*c*d - 8*B
*c*d - 4*A*d^2 + 7*B*d^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 16.49 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{2\,d\,\mathrm{atan}\left(\frac{2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d+2\,B\,c-2\,B\,d)}{2\,A\,d^2-4\,B\,d^2+4\,B\,c\,d}\right)\,(A\,d+2\,B\,c-2\,B\,d)}{a^2\,f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A\,c^2-6\,A\,d^2+2\,B\,c^2+12\,B\,d^2+4\,A\,c\,d-12\,B\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{10\,A\,c^2}{3}-\frac{14\,A\,d^2}{3}+\right.}{f\,\left(a\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*d*atan((2*d*tan(e/2 + (f*x)/2)*(A*d + 2*B*c - 2*B*d))/(2*A*d^2 - 4*B*d^2
 + 4*B*c*d))*(A*d + 2*B*c - 2*B*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(2*A*c^
2 - 6*A*d^2 + 2*B*c^2 + 12*B*d^2 + 4*A*c*d - 12*B*c*d) + tan(e/2 + (f*x)/2)
^2*((10*A*c^2)/3 - (14*A*d^2)/3 + (2*B*c^2)/3 + (44*B*d^2)/3 + (4*A*c*d)/3
- (28*B*c*d)/3) + (4*A*c^2)/3 - (8*A*d^2)/3 + (2*B*c^2)/3 + (20*B*d^2)/3 +
tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*A*d^2 + 4*B*d^2 - 4*B*c*d) + tan(e/2 + (f
*x)/2)*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 16*B*d^2 + 4*A*c*d - 12*B*c*d) + (4*A
*c*d)/3 - (16*B*c*d)/3)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(e/2 + (f
*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 + a^2 + 3*
a^2*tan(e/2 + (f*x)/2)))
```

3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{Bdx}{a^2} - \frac{(Ac+2Bc+2Ad-5Bd)\cos(e+fx)}{3a^2f(1+\sin(e+fx))} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

```
[Out] B*d*x/a^2-1/3*(A*c+2*A*d+2*B*c-5*B*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(
A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3098, 2814, 2727}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)} + \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (B*d*x)/a^2 - ((A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((A - B)*(c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx\\
&= -\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{\int \frac{-a(2B(c-d)+A(c+2d))-3aBd\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}\\
&= \frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(Ac+2Bc+2Ad-5Bd)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\
&= \frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(Ac+2Bc+2Ad-5Bd)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.

Time = 1.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c +
2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3
*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[
e + f*x])^2)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{2(Ac-dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2dA+2Bc-2dB}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2dA-2Bc+2dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2dB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}$
default	$\frac{-\frac{2(Ac-dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2dA+2Bc-2dB}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2dA-2Bc+2dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2dB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}$
parallelrisch	$\frac{3Bx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)df+((9fx+6)dB-6Ac)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+((9dfx-6c+18d)B-6A(c+d))\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+(3dfx-2c+8}{3fa^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
risch	$\frac{Bdx}{a^2}-\frac{2\left(-Ac-2dA+3iAc\,e^{i(fx+e)}+3id\,e^{i(fx+e)}A-2Bc+5dB+3iBc\,e^{i(fx+e)}-9iBd\,e^{i(fx+e)}+3Ad\,e^{2i(fx+e)}+3Bc\,e^{2i}\right.}{3fa^2\left(e^{i(fx+e)}+i\right)^3}$
norman	$\frac{\frac{xdB}{a}+\frac{xdB\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4Ac+2dA+2Bc-8dB}{3af}-\frac{(2Ac-2dB)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(16Ac+2dA+2Bc-20dB)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(14}{}}{}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^2*(-(A*c-B*d)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c+2*A*d+2*B*c-2*B*d)/(
tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c-2*A*d-2*B*c+2*B*d)/(tan(1/2*f*x+1/2*e)+1
)^3+d*B*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(81) = 162.

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.45

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{6\,Bdfx-(3\,Bdfx+(A+2\,B)c+(2\,A-5\,B)d)\cos\left(fx+e\right)^2-(A-B)c+(A-B)d+(3\,Bdfx-(2}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2
- (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x +
 e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2*A
- 5*B)*d)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
 + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. 2(83) = 166.

Time = 2.11 (sec) , antiderivative size = 1062, normalized size of antiderivative = 12.49

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c*tan
(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*d/(3*a**2*f*tan(e/2
 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B*c/(3*a**2
*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/
2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*B*d*f*x*tan
```

(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/(3*a**2*f*tan(e/2 + f*x
/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 6*B*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan
(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*B*d*tan(e/2 +
 f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*B*d/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f,
0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a)**2, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.34

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(Bd\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx}{(\cos(fx+e)}}}{3f}$$

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")

[Out] 2/3*(B*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*c*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - B*c*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*d*(3
*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{\frac{3(fx+e)Bd}{a^2} - \frac{2\left(3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 9Bd\tan\left(\frac{1}{2}\right.}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*d/a^2 - 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 3*B*d*tan(1/2*
f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) +
3*A*d*tan(1/2*f*x + 1/2*e) - 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c + A*d
 - 4*B*d)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{B\,d\,x}{a^2} - \frac{(2\,A\,c-2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + (2\,A\,c+2\,A\,d+2\,B\,c-6\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) + \frac{4\,A\,c}{3} + \frac{2\,A\,d}{3} + \frac{2\,B\,c}{3} - \frac{8\,B}{3}}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (B*d*x)/a^2 - ((4*A*c)/3 + (2*A*d)/3 + (2*B*c)/3 - (8*B*d)/3 + tan(e/2 + (f
*x)/2)*(2*A*c + 2*A*d + 2*B*c - 6*B*d) + tan(e/2 + (f*x)/2)^2*(2*A*c - 2*B*
d))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3)
```

3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx=-\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}$$

```
[Out] -1/3*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(A+2*B)*cos(f*x+e)/f/(a^2+a^
2*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2829, 2727}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx=-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e +
 f*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(A+2B)\int\frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx=-\frac{\cos(e+fx)(2A+B+(A+2B)\sin(e+fx))}{3a^2f(1+\sin(e+fx))^2}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
[Out] -1/3*(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e +
f*x])^2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{-6A\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-6A-6B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-2B}{3f\,a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$	60
risch	$-\frac{2\left(-A+3iA\,e^{i(fx+e)}+3iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2B\right)}{3f\,a^2\left(e^{i(fx+e)}+i\right)^3}$	68
derivativedivides	$\frac{-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(-2B+2A)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^2f}$	70
default	$\frac{-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(-2B+2A)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^2f}$	70
norman	$\frac{-\frac{4A+2B}{3af}-\frac{2A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2(5A+B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(2A+2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2(A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$	139

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-6*A*tan(1/2*f*x+1/2*e)^2+(-6*A-6*B)*tan(1/2*f*x+1/2*e)-4*A-2*B)/f/a^2
/(tan(1/2*f*x+1/2*e)+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{(A+2\,B)\cos\left(fx+e\right)^2+(2\,A+B)\cos\left(fx+e\right)+((A+2\,B)\cos\left(fx+e\right)-A+B)\sin\left(fx+e\right)+A-}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right)}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*x
 + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(56) = 112.

Time = 1.12 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.72

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$=\begin{cases}-\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{6A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2}\end{cases}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*
a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f),
Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(61) = 122.

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(\frac{A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{B\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2A+B\right)}{3a^2f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*f
*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{5A}{2}+\frac{B}{2}-\frac{A\cos(e+fx)}{2}+\frac{B\cos(e+fx)}{2}+\frac{3A\sin(e+fx)}{2}+\frac{3B\sin(e+fx)}{2}\right)}{3a^2f\left(\frac{3\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}-\frac{\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{2}\right)}$$

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((5*A)/2 + B/2 - (A*cos(e + f*x))/2 + (B*cos(e + f*x
))/2 + (3*A*sin(e + f*x))/2 + (3*B*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)*c
os(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/2))
```

3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 152

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{2d(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}f} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

```
[Out] -1/3*(A*(c-4*d)+B*(2*c+d))*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))-1/3*(A-B
)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2-2*d*(-A*d+B*c)*arctan((d+c*tan(1/2*
f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3057, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{2d(Bc-Ad)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c
 - d)^2*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/(3*
a^2*(c - d)^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(
a + a*Sin[e + f*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2Bc+A(c-3d))-a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx}{3a^2(c-d)}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} + \frac{\int -\frac{3a^2d(Bc-Ad)}{c+d\sin(e+fx)}\,dx}{3a^4(c-d)^2}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{(d(Bc-Ad))\int \frac{1}{c+d\sin(e+fx)}\,dx}{a^2(c-d)^2}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{(2d(Bc-Ad))\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^2(c-d)^2f}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} + \frac{(4d(Bc-Ad))\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^2(c-d)^2f}$$

$$= -\frac{2d(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}f} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.51

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d) +
 B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6
*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]))/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{2d(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^2\sqrt{c^2-d^2}}-\frac{2(-2B+2A)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-2dA+dB)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^2f}$
default	$\frac{\frac{2d(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^2\sqrt{c^2-d^2}}-\frac{2(-2B+2A)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-2dA+dB)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^2f}$
risch	$\frac{\frac{2Ac}{3}-\frac{8dA}{3}-2iAc\,e^{i(fx+e)}+6id\,e^{i(fx+e)}A+2Ad\,e^{2i(fx+e)}+\frac{4Bc}{3}+\frac{2dB}{3}-2iBc\,e^{i(fx+e)}-2iBd\,e^{i(fx+e)}-2Bc\,e^{2i(fx+e)}}{\left(e^{i(fx+e)}+i\right)^3(c-d)^2 f\,a^2}-$

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 2/f/a^2*(d*(A*d-B*c)/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1/3*(-2*B+2*A)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3-1/2*(2*B-2*A)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2-(A*c-2*A*d+B*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(143) = 286.

Time = 0.30 (sec) , antiderivative size = 1285, normalized size of antiderivative = 8.45

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 + 2*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos

```
(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e
) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2
*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d +
 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 - (A - B)*c^2*d
- (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B
)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A
*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 +
(B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(
f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^3 - (5*A - 2*B
)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos(f*x + e) - ((A - B)*c^3 -
(A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^
2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^
4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^
2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^
4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*si
n(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.64

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{2\left(\frac{3\,(Bcd-Ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^2-2\,a^2cd+a^2d^2)\sqrt{c^2-d^2}}+\frac{3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{}\right.}{3\,f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((
c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x +
1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*
c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*x + 1
/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(tan(
1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.74 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.99

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,(A\,d-B\,c)\,\left(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3\right)}{a^2\,\sqrt{c+d}\,{(c-d)}^{5/2}}+\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,\left(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2\right)}{a^2\,\sqrt{c+d}\,{(c-d)}^{5/2}}}{2\,A\,d^2-2\,B\,c\,d}\right)\,(A\,d-B\,c)}{a^2\,f\,\sqrt{c+d}\,{(c-d)}^{5/2}} - \frac{\frac{2\,(2\,A\,c-5\,A\,d+B\,c+2\,B\,d)}{3\,{(c-d)}^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,c-3\,A\,d+B\,c+B\,d)}{{(c-d)}^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(A\,c-2\,A\,d+B\,d)}{{(c-d)}^2}}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d*atan(((d*(A*d - B*c)*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c
+ d)^(1/2)*(c - d)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^2*c^2
+ a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*A*d^2 - 2*B*c
*d))*(A*d - B*c))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*A*c - 5*A*d
```

```
+ B*c + 2*B*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*c - 3*A*d + B*c +
B*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(A*c - 2*A*d + B*d))/(c - d)^2)/(
f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a^2*tan(
e/2 + (f*x)/2)))
```

3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 275

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx &= \frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f}\\ &\quad-\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))}\\ &\quad-\frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))}\\ &\quad-\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\end{aligned}$$

```
[Out] -1/3*d*(A*(c^2-6*c*d-10*d^2)+B*(2*c^2+9*c*d+4*d^2))*cos(f*x+e)/a^2/(c-d)^3/
(c+d)/f/(c+d*sin(f*x+e))-1/3*(A*c-6*A*d+2*B*c+3*B*d)*cos(f*x+e)/a^2/(c-d)^2
/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*
x+e))^2/(c+d*sin(f*x+e))+2*d*(A*d*(3*c+2*d)-B*(2*c^2+2*c*d+d^2))*arctan((d+
c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$$
$$=\frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d*(A*d*(3*c + 2*d) - B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(A*(
c^2 - 6*c*d - 10*d^2) + B*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/(3*a^2*(c
- d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos
[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - ((
A - B)*Cos[e + f*x])/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} \\
&\quad -\frac{\int \frac{-a(A(c-4d)+B(2c+d))-2a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx}{3a^2(c-d)} \\
&= -\frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} \\
&\quad -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} \\
&\quad +\frac{\int \frac{-2a^2d(3Bc-5Ad+2Bd)+a^2d(Ac+2Bc-6Ad+3Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{3a^4(c-d)^2}
\end{aligned}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int -\frac{3a^2d(Ad(3c+2d)-B(2c^2+2cd+d^2))}{c+d\sin(e+fx)}\,dx}{3a^4(c-d)^3(c+d)}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} + \frac{(d(Ad(3c+2d)-B(2c^2+2cd+d^2)))\int \frac{1}{c+d\sin(e+fx)}\,dx}{a^2(c-d)^3(c+d)}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} + \frac{(2d(Ad(3c+2d)-B(2c^2+2cd+d^2)))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^2(c-d)^3(c+d)f}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{(4d(Ad(3c+2d)-B(2c^2+2cd+d^2)))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^2(c-d)^3(c+d)f}$$

$$= \frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}f} - \frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 7.00 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.14

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d) +
 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
(6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)
*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1
 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{2d\left(\frac{\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(3Acd+2A\,d^2-2B\,c^2-2cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^3}-\frac{2(-2B-}{3(c-d)^2\left(\tan\left(\frac{f}{}\right.\right.}}{a^2f}$
default	$\frac{\frac{2d\left(\frac{\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(3Acd+2A\,d^2-2B\,c^2-2cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^3}-\frac{2(-2B-}{3(c-d)^2\left(\tan\left(\frac{f}{}\right.\right.}}{a^2f}$
risch	Expression too large to display

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(1/(c-d)^3*d*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)
/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(3*A*c*d+2*A*d^2-
2*B*c^2-2*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2)))-1/3*(-2*B+2*A)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3
-1/2*(2*B-2*A)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-(A*c-3*A*d+2*B*d)/(c-d)^3/(
tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(264) = 528$.

Time = 0.36 (sec) , antiderivative size = 3123, normalized size of antiderivative = 11.36

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*d
^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - 2*((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c
^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*co
s(f*x + e)^3 + 2*((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (
A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B
*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2
*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*B*c^3*d
 - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^
2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*
```

(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e) - 2*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e)), 1/3*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^3 + ((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e) - ((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7

```
 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*
d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 +
3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(
f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d
^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*c
os(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^
2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{2\left(\frac{3\left(2Bc^2d-3Acd^2+2Bcd^2-2Ad^3+Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)\sqrt{c^2-d^2}}+\frac{3\left(Bcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Ad\right.}{(a^2c^5-2a^2c^4d+2a^2c^2d^3-a^2cd^4)\left(c\right.}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2))
+ 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*d^2
- A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*
f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x + 1/2*
e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*
tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*f*x + 1/
2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/((a^2*c^3
- 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 16.73 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.07

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,\left(-2\,a^2\,c^4\,d+4\,a^2\,c^3\,d^2-4\,a^2\,c\,d^4+2\,a^2\,d^5\right)\,\left(2\,B\,c^2-2\,A\,d^2+B\,d^2-3\,A\,c\,d+2\,B\,c\,d\right)}{a^2\,{(c+d)}^{3/2}\,{(c-d)}^{7/2}}-\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^2\,c^4-2\,a^2\,c^3\,d+2\,a^2\,c\,d^3-a^2\,d^4\right)\,\left(2\,B\,c^2\right.}{a^2\,{(c+d)}^{3/2}\,{(c-d)}^{7/2}}}{2\,B\,d^3-4\,A\,d^3-6\,A\,c\,d^2+4\,B\,c\,d^2+4\,B\,c^2\,d}\right.}{a^2\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}} - \frac{\frac{2\,\left(2\,A\,c^3-3\,A\,d^3+B\,c^3-8\,A\,c\,d^2-6\,A\,c^2\,d+8\,B\,c\,d^2+6\,B\,c^2\,d\right)}{3\,(c+d)\,(c-d)\,\left(c^2-2\,c\,d+d^2\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(5\,A\,c^3-9\,A\,d^3+B\,c^3-30\,A\,c\,d^2-11\,A\,c^2\,d+27\,B\,c\,d^2+17\,B\,c\right.}{3\,c\,(c-d)\,\left(c^2-2\,c\,d+d^2\right)}}{f\,\left(a^2\,c+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\right.\right.}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d*atan(((d*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2)*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c - d)^(7/2
)) - (2*c*d*tan(e/2 + (f*x)/2)*(a^2*c^4 - a^2*d^4 + 2*a^2*c*d^3 - 2*a^2*c^3
*d)*(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c
- d)^(7/2)))/(2*B*d^3 - 4*A*d^3 - 6*A*c*d^2 + 4*B*c*d^2 + 4*B*c^2*d))*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*f*(c + d)^(3/2)*(c - d)^(7
/2)) - ((2*(2*A*c^3 - 3*A*d^3 + B*c^3 - 8*A*c*d^2 - 6*A*c^2*d + 8*B*c*d^2 +
 6*B*c^2*d))/(3*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)^2*(5*A*c^3 - 9*A*d^3 + B*c^3 - 30*A*c*d^2 - 11*A*c^2*d + 27*B*c*d^2 + 17*
B*c^2*d))/(3*c*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*c^
4 - 3*A*d^4 + B*c^4 - 9*A*c^2*d^2 + 8*B*c^2*d^2 - 7*A*c*d^3 - 2*A*c^3*d + 7
*B*c*d^3 + 4*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2
 + (f*x)/2)*(3*A*c^4 - 3*A*d^4 + 3*B*c^4 - 27*A*c^2*d^2 + 30*B*c^2*d^2 - 25
*A*c*d^3 - 8*A*c^3*d + 13*B*c*d^3 + 14*B*c^3*d))/(3*c*(c + d)*(c - d)*(c^2
```

```
- 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^4 - A*d^4 - 3*A*c^2*d^2 + 2*
B*c^2*d^2 - 2*A*c^3*d + B*c*d^3 + 2*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c
*d + d^2)))/(f*(a^2*c + tan(e/2 + (f*x)/2)*(3*a^2*c + 2*a^2*d) + tan(e/2 +
(f*x)/2)^4*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c + 6*a^2*d) +
 tan(e/2 + (f*x)/2)^3*(4*a^2*c + 6*a^2*d) + a^2*c*tan(e/2 + (f*x)/2)^5))
```

3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 386

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx &= \frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}f}\\ &- \frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}\\ &- \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2}\\ &- \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\\ &- \frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))}\end{aligned}$$

```
[Out] -1/6*d*(A*(2*c^2-16*c*d-21*d^2)+B*(4*c^2+19*c*d+12*d^2))*cos(f*x+e)/a^2/(c-
d)^3/(c+d)/f/(c+d*sin(f*x+e))^2-1/3*(A*c-8*A*d+2*B*c+5*B*d)*cos(f*x+e)/a^2/
(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a
+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(A*(2*c^3-16*c^2*d-59*c*d^2-32*d^
3)+B*(4*c^3+37*c^2*d+44*c*d^2+20*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+
d*sin(f*x+e))+d*(A*d*(12*c^2+16*c*d+7*d^2)-B*(6*c^3+12*c^2*d+13*c*d^2+4*d^3
))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d)^2/f/(
c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$$
$$=\frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\arctan\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}$$
$$-\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sin(e+fx))}$$
$$-\frac{(Ac-8Ad+2Bc+5Bd)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3)
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^
2*Sqrt[c^2 - d^2]*f) - (d*(A*(2*c^2 - 16*c*d - 21*d^2) + B*(4*c^2 + 19*c*d
+ 12*d^2))*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2)
 - ((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(a
 + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) - (d*(A*(2*c^3 - 16*c^2*d - 59
*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f*x])/
(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} \\
&\quad -\frac{\int \frac{-a(A(c-5d)+2B(c+d))-3a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx}{3a^2(c-d)} \\
&= -\frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} \\
&\quad -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} \\
&\quad +\frac{\int \frac{-3a^2d(3Bc-7Ad+4Bd)+2a^2d(Ac+2Bc-8Ad+5Bd)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3a^4(c-d)^2}
\end{aligned}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a^2d(Ad(19c+16d)-B(9c^2+16cd+10d^2))-a^2d(2Ac^2+4Bc^2-16Acd+19Bcd-21Ad^2+12Bd^2)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{6a^4(c-d)^3(c+d)}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{3a^2d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))}{c+d\sin(e+fx)}\,dx}{6a^4(c-d)^4(c+d)^2}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))} + \frac{(d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3)))\int \frac{1}{c+d\sin(e+fx)}\,dx}{2a^2(c-d)^4(c+d)^2}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))} + \frac{(d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3)))\operatorname{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e\right.\right.}{a^2(c-d)^4(c+d)^2f}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$
$$-\frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))}$$
$$-\frac{(2d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3)))\operatorname{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2$$
$$a^2(c-d)^4(c+d)^2f$$

$$=\frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}f}$$
$$-\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$
$$-\frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1257 vs. 2(386) = 772.

Time = 10.90 (sec) , antiderivative size = 1257, normalized size of antiderivative = 3.26

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$$

$$=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{48d(-Ad(12c^2+16cd+7d^2)+B(6c^3+12c^2d+13cd^2+4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(\mathrm{co}}{\sqrt{c^2-d^2}}\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-48*d*(-(A*d*(12*c^2 + 16*c*d + 7*
d^2)) + B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 -
d^2] + ((-(A*d*(96*c^4 + 524*c^3*d + 776*c^2*d^2 + 487*c*d^3 + 112*d^4)) +
```

```
B*(48*c^5 + 240*c^4*d + 536*c^3*d^2 + 701*c^2*d^3 + 400*c*d^4 + 70*d^5))*Co
s[(e + f*x)/2] - (A*(16*c^5 - 80*c^4*d - 536*c^3*d^2 - 1028*c^2*d^3 - 695*c
*d^4 - 134*d^5) + B*(32*c^5 + 224*c^4*d + 728*c^3*d^2 + 893*c^2*d^3 + 482*c
*d^4 + 98*d^5))*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x))/2] -
12*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 15*
A*c*d^4*Cos[(5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] + 6*A*d^5*Co
s[(5*(e + f*x))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*Cos[(7*(e +
 f*x))/2] + 8*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos[(7*(e + f*x
))/2] + 59*B*c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*(e + f*x))/2]
 + 76*B*c*d^4*Cos[(7*(e + f*x))/2] - 52*A*d^5*Cos[(7*(e + f*x))/2] + 34*B*d
^5*Cos[(7*(e + f*x))/2] + 48*A*c^5*Sin[(e + f*x)/2] + 48*B*c^5*Sin[(e + f*x
)/2] - 224*A*c^4*d*Sin[(e + f*x)/2] + 416*B*c^4*d*Sin[(e + f*x)/2] - 872*A*
c^3*d^2*Sin[(e + f*x)/2] + 992*B*c^3*d^2*Sin[(e + f*x)/2] - 1144*A*c^2*d^3*
Sin[(e + f*x)/2] + 967*B*c^2*d^3*Sin[(e + f*x)/2] - 685*A*c*d^4*Sin[(e + f*
x)/2] + 496*B*c*d^4*Sin[(e + f*x)/2] - 168*A*d^5*Sin[(e + f*x)/2] + 126*B*d
^5*Sin[(e + f*x)/2] + 48*B*c^4*d*Sin[(3*(e + f*x))/2] - 132*A*c^3*d^2*Sin[(
3*(e + f*x))/2] + 96*B*c^3*d^2*Sin[(3*(e + f*x))/2] - 204*A*c^2*d^3*Sin[(3*
(e + f*x))/2] + 207*B*c^2*d^3*Sin[(3*(e + f*x))/2] - 165*A*c*d^4*Sin[(3*(e
+ f*x))/2] + 174*B*c*d^4*Sin[(3*(e + f*x))/2] - 66*A*d^5*Sin[(3*(e + f*x))/
2] + 42*B*d^5*Sin[(3*(e + f*x))/2] - 16*A*c^4*d*Sin[(5*(e + f*x))/2] - 32*B
*c^4*d*Sin[(5*(e + f*x))/2] + 116*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 224*B*c^
3*d^2*Sin[(5*(e + f*x))/2] + 412*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 409*B*c^2
*d^3*Sin[(5*(e + f*x))/2] + 403*A*c*d^4*Sin[(5*(e + f*x))/2] - 286*B*c*d^4*
Sin[(5*(e + f*x))/2] + 114*A*d^5*Sin[(5*(e + f*x))/2] - 78*B*d^5*Sin[(5*(e
+ f*x))/2] + 15*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 21*A*c*d^4*Sin[(7*(e + f*x
))/2] + 12*B*c*d^4*Sin[(7*(e + f*x))/2] - 12*A*d^5*Sin[(7*(e + f*x))/2] + 6
*B*d^5*Sin[(7*(e + f*x))/2])/(c + d*Sin[e + f*x])^2))/(48*a^2*(c - d)^4*(c
+ d)^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{2(-2B+2A)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-4dA+3dB)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{d^2\left(9c^2dA+4d^2cA-2A\,d^3-7B\,c^3-4c^2d\right.}{2c\left(c^2+2cd+d^2\right)}\right.}{}}{}$
default	$\frac{-\frac{2(-2B+2A)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-4dA+3dB)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{d^2\left(9c^2dA+4d^2cA-2A\,d^3-7B\,c^3-4c^2d\right.}{2c\left(c^2+2cd+d^2\right)}\right.}{}}{}$
risch	Expression too large to display

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(-1/3*(-2*B+2*A)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3-1/2*(2*B-2*A)/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^2-(A*c-4*A*d+3*B*d)/(c-d)^4/(tan(1/2*f*x+1/2*e)
+1)+d/(c-d)^4*((1/2*d^2*(9*A*c^2*d+4*A*c*d^2-2*A*d^3-7*B*c^3-4*B*c^2*d)/c/(
c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(8*A*c^4*d+4*A*c^3*d^2+15*A*c^2*d
^3+8*A*c*d^4-2*A*d^5-6*B*c^5-4*B*c^4*d-13*B*c^3*d^2-8*B*c^2*d^3-2*B*c*d^4)/
(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(23*A*c^2*d+12*A*c*d^2-2*A
*d^3-17*B*c^3-12*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1/
2*d*(8*A*c^2*d+4*A*c*d^2-A*d^3-6*B*c^3-4*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2))/
(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(12*A*c^2*d+16*A*c*
d^2+7*A*d^3-6*B*c^3-12*B*c^2*d-13*B*c*d^2-4*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2
)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2456 vs. $2(373) = 746$.

Time = 0.45 (sec) , antiderivative size = 4997, normalized size of antiderivative = 12.95

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/12*(4*(A - B)*c^7 - 4*(A - B)*c^6*d - 12*(A - B)*c^5*d^2 + 12*(A - B)*c
^4*d^3 + 12*(A - B)*c^3*d^4 - 12*(A - B)*c^2*d^5 - 4*(A - B)*c*d^6 + 4*(A -
 B)*d^7 - 2*(2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^
3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*co
s(f*x + e)^4 - 2*(4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*
```

c^4*d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c
*d^6 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + 2*(2*(A + 2*B)*c^7 - 6*(2*A - 3*
B)*c^6*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*
c^3*d^4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7
)*cos(f*x + e)^2 + 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c
^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6
 + (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^
6)*cos(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)
*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^
5*d - 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4
 - (76*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 1
2*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(
10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2
*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A -
 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A
 - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A
 - 3*B)*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*
d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3
d^3 - 3(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*co
s(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*co
s(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^
2 - d^2)) + 4*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 19*B)*c^5*d^2
 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A - 21*B)*c^2*d
^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - 2*(2*(A - B)*
c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6*(A - B)*c
^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2*(A + 2*B)
*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A + 17*B)*c^
2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 - (4*(A + 2
*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(3*A + B)*c
^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A - 8*B)*d^7)
*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A - 18*B)*c
^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21*A - 22*B)
*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))*sin(f*x +
 e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2
*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 1
2*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3
 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4
 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*
c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c
^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2
*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a
^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5
 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3

+ 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^
5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f
*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d
^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2
+ 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x +
e)), -1/6*(2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)
*c^4*d^3 + 6*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A -
B)*d^7 - (2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*
d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(
f*x + e)^4 - (4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*c^4*
d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c*d^6
+ (43*A - 28*B)*d^7)*cos(f*x + e)^3 + (2*(A + 2*B)*c^7 - 6*(2*A - 3*B)*c^6
d - 12(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*c^3*d^
4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7)*cos(
f*x + e)^2 - 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3
- 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 + (6*
B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos
(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)*c^2*d
^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^5*d -
12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4 - (76
*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A -
2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A -
7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2*B)*c^
4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*
c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*
B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A - 3*B)
*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*d^6)*c
os(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 -
3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x
+ e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2
- d^2)*cos(f*x + e))) + 2*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 1
9*B)*c^5*d^2 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A -
21*B)*c^2*d^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - (
2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6
*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2
*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A
+ 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 -
(4*(A + 2*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(
3*A + B)*c^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A -
8*B)*d^7)*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A
- 18*B)*c^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21
*A - 22*B)*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))
*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^
5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^
4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c

```
^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f
*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^
2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^
8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2
+ 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e
) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c
^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^
2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f
*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*
a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a
^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10
*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a
^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)
*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. 2(373) = 746.

Time = 0.37 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.36

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3
- 7*A*d^4 + 4*B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*t
an(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^
4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(c^2 - d^2
)) + 3*(7*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*tan(1/2*f*x + 1/2*
e)^3 + 4*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*tan(1/2*f*x + 1/2*e
)^3 + 2*A*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*tan(1/2*f*x + 1/2*e)^2
 - 8*A*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^2
- 4*A*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^2
- 15*A*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*tan(1/2*f*x + 1/2*e)^2
- 8*A*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*A
*d^7*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*tan(1/2*f*x + 1/2*e) - 23*A*c^3*
d^4*tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*tan(1/2*f*x + 1/2*e) - 12*A*c^2*d^5
*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*A*c*d^6*tan(1/
2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4*A*c^3*d^4 + B*
c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3
- a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*
d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 12*A*d*t
an(1/2*f*x + 1/2*e)^2 + 9*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x +
1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 21*A*d*tan(1/2*f*x + 1/2*e) + 15*B*d*
tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 11*A*d + 8*B*d)/((a^2*c^4 - 4*a^2*c^3*
d + 6*a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (d*atan(((d*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c^2*d^5 - 8*a^2*
c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*
```

A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(2*a^2*(c + d)^(5/2)*(c - d)^(9/2)) + (c*d*tan(e/2 + (f*x)/2)*(2*a^2*c*d^5 - a^2*d^6 - a^2*c^6 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*(c + d)^(5/2)*(c - d)^(9/2)))/(4*B*d^4 - 7*A*d^4 - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3 + 6*B*c^3*d))*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*f*(c + d)^(5/2)*(c - d)^(9/2)) - ((tan(e/2 + (f*x)/2)^5*(2*A*c^6 + 2*A*d^6 + 2*B*c^6 - 23*A*c^2*d^4 - 40*A*c^3*d^3 - 38*A*c^4*d^2 + 6*B*c^2*d^4 + 43*B*c^3*d^3 + 40*B*c^4*d^2 - 4*A*c*d^5 - 4*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^5 - 3*c^4*d - 3*c*d^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (4*A*c^5 + 3*A*d^5 + 2*B*c^5 - 46*A*c^2*d^3 - 40*A*c^3*d^2 + 28*B*c^2*d^3 + 52*B*c^3*d^2 - 12*A*c*d^4 - 14*A*c^4*d + 3*B*c*d^4 + 20*B*c^4*d)/(3*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^3*(6*A*c^6 + 9*A*d^6 + 6*B*c^6 - 177*A*c^2*d^4 - 212*A*c^3*d^3 - 102*A*c^4*d^2 + 105*B*c^2*d^4 + 215*B*c^3*d^3 + 150*B*c^4*d^2 - 33*A*c*d^5 - 16*A*c^5*d + 9*B*c*d^5 + 40*B*c^5*d))/(3*c^2*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)*(6*A*c^5 + 6*A*d^5 + 6*B*c^5 - 160*A*c^2*d^3 - 114*A*c^3*d^2 + 97*B*c^2*d^3 + 156*B*c^3*d^2 - 33*A*c*d^4 - 20*A*c^4*d + 12*B*c*d^4 + 44*B*c^4*d))/(3*c*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^2*(14*A*c^7 + 6*A*d^7 + 4*B*c^7 - 232*A*c^2*d^5 - 583*A*c^3*d^4 - 532*A*c^4*d^3 - 226*A*c^5*d^2 + 124*B*c^2*d^5 + 412*B*c^3*d^4 + 595*B*c^4*d^3 + 352*B*c^5*d^2 - 6*A*c*d^6 - 16*A*c^6*d + 6*B*c*d^6 + 82*B*c^6*d))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^4*(16*A*c^7 + 18*A*d^7 + 2*B*c^7 - 303*A*c^2*d^5 - 522*A*c^3*d^4 - 502*A*c^4*d^3 - 220*A*c^5*d^2 + 156*B*c^2*d^5 + 453*B*c^3*d^4 + 538*B*c^4*d^3 + 328*B*c^5*d^2 - 48*A*c*d^6 - 14*A*c^6*d + 18*B*c*d^6 + 80*B*c^6*d))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^6*(2*A*c^6 + 2*A*d^6 - 9*A*c^2*d^4 - 8*A*c^3*d^3 - 14*A*c^4*d^2 + 4*B*c^2*d^4 + 13*B*c^3*d^3 + 12*B*c^4*d^2 - 4*A*c*d^5 - 4*A*c^5*d + 6*B*c^5*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/2 + (f*x)/2)*(3*a^2*c^2 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^2*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + tan(e/2 + (f*x)/2)^5*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + tan(e/2 + (f*x)/2)^3*(7*a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^4*(7*a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(3*a^2*c^2 + 4*a^2*c*d) + a^2*c^2 + a^2*c^2*tan(e/2 + (f*x)/2)^7))

3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 225

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(c-d)\left(3B(c^2+6cd-15d^2)+A(2c^2+7cd+15d^2)\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3}$$

```
[Out] d^2*(3*B*(c-d)+A*d)*x/a^3+1/15*d^2*(3*B*(c-9*d)+A*(2*c+7*d))*cos(f*x+e)/a^3
/f-1/15*(c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*cos(f*x+e)/f/
(a^3+a^3*sin(f*x+e))-1/15*(3*B*(c-3*d)+2*A*(c+2*d))*cos(f*x+e)*(c+d*sin(f*x
+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+
a*sin(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3056, 3047, 3102, 2814, 2727}

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx\\ &=-\frac{(c-d)\left(A(2c^2+7cd+15d^2)+3B(c^2+6cd-15d^2)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}\\ &\quad+\frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}\\ &\quad+\frac{d^2x(Ad+3B(c-d))}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}\\ &\quad-\frac{(2A(c+2d)+3B(c-3d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a\sin(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*(3*B*(c - d) + A*d)*x)/a^3 + (d^2*(3*B*(c - 9*d) + A*(2*c + 7*d))*Cos[
e + f*x])/(15*a^3*f) - ((c - d)*(3*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*
c*d + 15*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((3*B*(c - 3
*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a*f*(a + a*Si
n[e + f*x])^2) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(5*f*(a + a*
Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} \\
&\quad+\frac{\int \frac{(c+d\sin(e+fx))^2(a(2Ac+3Bc+3Ad-3Bd)-a(A-6B)d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{5a^2} \\
&= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} \\
&\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} \\
&\quad+\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(3B\left(c^2+5cd-6d^2\right)+A\left(2c^2+5cd+8d^2\right)\right)-a^2d(3B(c-9d)+A(2c+7d))\sin(e+fx)\right)}{a+a\sin(e+fx)}\,dx}{15a^4} \\
&= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} \\
&\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} \\
&\quad+\frac{\int \frac{a^2c\left(3B\left(c^2+5cd-6d^2\right)+A\left(2c^2+5cd+8d^2\right)\right)+\left(-a^2cd(3B(c-9d)+A(2c+7d))+a^2d\left(3B\left(c^2+5cd-6d^2\right)+A\left(2c^2+5cd+8d^2\right)\right)\right)\sin(e+}{a+a\sin(e+fx)}}{15a^4}
\end{aligned}$$

$$= \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{a^3c(3B(c^2+5cd-6d^2)+A(2c^2+5cd+8d^2))+15a^3d^2(3B(c-d)+Ad)\sin(e+fx)}{a+a\sin(e+fx)}dx}{15a^5}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} + \frac{((c-d)(3B(c^2+6cd-15d^2)+A(2c^2+7cd+15d^2)))\int \frac{1}{a+a\sin(e+fx)}dx}{15a^2}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(c-d)(3B(c^2+6cd-15d^2)+A(2c^2+7cd+15d^2))\cos(e+fx)}{15f(a^3+a^3\sin(e+fx))} - \frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3}$$

Mathematica [A] (verified)

Time = 6.90 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^3\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^3\left(\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2
*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e + f*x)
```

```
/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*(2*c^
2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
)/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{-\frac{2\left(A\,c^3-A\,d^3-3d^2cB+3d^3B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^3+6c^2dA-2A\,d^3+2B\,c^3-6d^2cB+4d^3B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A\,c^3-18c^2dA+12d^2cA-2A\,d^3-6B\,c^3+12c^2dB\cdots\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{\cdots}$
default	$\frac{-\frac{2\left(A\,c^3-A\,d^3-3d^2cB+3d^3B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^3+6c^2dA-2A\,d^3+2B\,c^3-6d^2cB+4d^3B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A\,c^3-18c^2dA+12d^2cA-2A\,d^3-6B\,c^3+12c^2dB\cdots\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{\cdots}$
parallelrisch	$\frac{\left((-300fxA+900fxB-540A+1755B)d^3+180c(-5fxB+A-9B)d^2+180c^2d(A+B)+180c^3\left(A+\frac{B}{3}\right)\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left((30\cdots\right.}{\cdots}$
risch	$\frac{x\,d^3A}{a^3}+\frac{3x\,d^2Bc}{a^3}-\frac{3x\,d^3B}{a^3}-\frac{B\,d^3e^{i(fx+e)}}{2a^3f}-\frac{B\,d^3e^{-i(fx+e)}}{2a^3f}-\frac{2(9c^2dA+21d^2cA+21c^2dB-96d^2cB+72d^3B+2\cdots}{\cdots}$
norman	Expression too large to display

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-(A*c^3-A*d^3-3*B*c*d^2+3*B*d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A*
c^3+6*A*c^2*d-2*A*d^3+2*B*c^3-6*B*c*d^2+4*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1
/3*(8*A*c^3-18*A*c^2*d+12*A*c*d^2-2*A*d^3-6*B*c^3+12*B*c^2*d-6*B*c*d^2)/(ta
n(1/2*f*x+1/2*e)+1)^3-1/4*(-8*A*c^3+24*A*c^2*d-24*A*c*d^2+8*A*d^3+8*B*c^3-2
4*B*c^2*d+24*B*c*d^2-8*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A*c^3-12*A*c^
2*d+12*A*c*d^2-4*A*d^3-4*B*c^3+12*B*c^2*d-12*B*c*d^2+4*B*d^3)/(tan(1/2*f*x+
1/2*e)+1)^5+d^2*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+3*B*c-3*B*d)*arctan(tan
(1/2*f*x+1/2*e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(217) = 434.

Time = 0.27 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{15\,Bd^3\cos(fx+e)^4-3\,(A-B)c^3+9\,(A-B)c^2d-9\,(A-B)cd^2+3\,(A-B)d^3+((2\,A+3\,B)c^3+3\cdots}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(15*B*d^3*cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A - B
)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A -
 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos
(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x - (2*(2*A + 3*B)*c^3 + 3*(
6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^2 + (
A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((3*A + 2*B)*c^3 + 3*(2*A + 3*B)*c^2*
d + 9*(A - 6*B)*c*d^2 - 9*(2*A - 7*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*
f*x)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A - B)*c^
2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x
- ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 - 2*(16*A -
 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((2*A +
 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A - 62*B)*d^3
- 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^3*f*co
s(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a
^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11456 vs. $2(206) = 412$.

Time = 14.03 (sec) , antiderivative size = 11456, normalized size of antiderivative = 50.92

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f
*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e/2 +
 f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3
*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 110*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2
+ f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**
5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 100*A
*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 94*A*c**3*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(
```

e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x
/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a
3*f) - 40*A*c3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c**3/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 9
0*A*c**2*d*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)**
4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/
2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 180*A*c**2*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 108*A*c**2*d
*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f
*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4
+ 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 - 18*A*c**2*d/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 120*A*c*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 60*A*c*d**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 132*A*c*d**2*tan(e/2 + f*x/2
)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a
3*f*tan(e/2 + f*x/2)5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan
(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 60*A*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)
7 + 75*a3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d**2/(

15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 15*A*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
3*f*tan(e/2 + f*x/2)4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*A*d**3*f*x*
tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*
x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*
f*tan(e/2 + f*x/2) + 15*a**3*f) + 165*A*d**3*f*x*tan(e/2 + f*x/2)**5/(15*a*
*3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e
/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/
2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 225*A*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225*A*d**3*f*x*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 165*A*d**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*
f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
+ f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)*
*3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) + 75*A*d**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*A*d**3*f*x/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 + 30*A*d**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*A*d**3*tan(e/2 + f*x/2)*
*5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**
3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e
/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) + 320*A*d**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)
7 + 75*a3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 340*A*d**3*ta
n(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2

25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 334*A*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
+ 190*A*d**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*ta
n(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f
*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
+ 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 44*A*d**3/(15*a**3*f*tan(e/2 +
f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
+ 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**
3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*
*3*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 +
f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**
4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**3*tan(e/2 + f*x/2)**4/(15*a**3
*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
+ f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)
3 + 165*a3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 60*B*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 36*B*c**3*tan(e/2 + f*x/2
)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a
3*f*tan(e/2 + f*x/2)5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan
(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 30*B*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**3/(15*a*
*3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e
/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/
2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) - 120*B*c**2*d*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 7
5*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*
tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*B*c**2*d*tan(e/2
+ f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6
+ 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**
3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 132*B*c**2*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e
/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2
)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 16
5*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60
*B*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e

/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/
2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*B*c**2*d/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 45*B*c*d
2*f*x*tan(e/2 + f*x/2)7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225*B*c*d**2*f*x*tan(e/2 + f*x/2)
6/(15*a3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) + 495*B*c*d**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 +
 f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 675*B*
c*d**2*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 675*B*c*d**2*f*x*tan(e/2 + f*x
/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165
*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t
an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) + 495*B*c*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)
5 + 225*a3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225
*B*c*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 45*B*c*d**2*f*x/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
90*B*c*d**2*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
 f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c*d**2*tan(e/2 + f*x/2)
5/(15*a3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) + 960*B*c*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x
/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 2
25*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f

*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1020*B*c*d
2*tan(e/2 + f*x/2)3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)*
*4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
3*f*tan(e/2 + f*x/2) + 15*a3*f) + 1002*B*c*d**2*tan(e/2 + f*x/2)**2/(15
*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*ta
n(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f
*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15
*a**3*f) + 570*B*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 7
5*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*
tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
 f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 132*B*c*d**2/(15*a**
3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/
2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2
)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) - 45*B*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 7
5*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*
tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
 f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*B*d**3*f*x*tan(e
/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 495*B*d**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 - 675*B*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a
3*f*tan(e/2 + f*x/2)6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan
(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 675*B*d**3*f*x*tan(e/2
+ f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6
+ 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**
3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 495*B*d**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
225*B*d**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
 f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 45*B*d**3*f*x/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 9
0*B*d**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan

```
(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*
x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 450*B*d**3*tan(e/2 + f*x/2)**5/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 960*B*d**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7
+ 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3
*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1200*B*d**3*tan(e
/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 1134*B*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
630*B*d**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 144*B*d**3/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0))
, (x*(A + B*sin(e))*(c + d*sin(e))**3/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(217) = 434$.

Time = 0.34 (sec) , antiderivative size = 1682, normalized size of antiderivative = 7.48

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^3} dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1
1*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
```

e)/(cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - A*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^2*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*c^2*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(217) = 434$.

Time = 0.32 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.52

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{\frac{30\,Bd^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3} - \frac{15\,(3\,Bcd^2+Ad^3-3\,Bd^3)(fx+e)}{a^3} + \frac{2\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Ad^3\tan\left(\frac{1}{2}\,fx+\right.}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(30*B*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3
- 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^2*
tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*tan(1/2
*f*x + 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*tan(1/2*f*x +
1/2*e)^3 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*tan(1/2*f*x + 1/
2*e)^3 - 75*A*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*tan(1/2*f*x + 1/2*e)^3
 + 40*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 45*A
*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*A*c*
d^2*tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*A*d^3
*tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^3*tan(1
/2*f*x + 1/2*e) + 15*B*c^3*tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*tan(1/2*f*x +
1/2*e) + 30*B*c^2*d*tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*tan(1/2*f*x + 1/2*e)
- 285*B*c*d^2*tan(1/2*f*x + 1/2*e) - 95*A*d^3*tan(1/2*f*x + 1/2*e) + 240*B*
d^3*tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*B*c^2*d + 6*A*
c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5
))/f
```

Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.64

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d+3\,B\,c-3\,B\,d)}{2\,A\,d^3-6\,B\,d^3+6\,B\,c\,d^2}\right)(A\,d+3\,B\,c-3\,B\,d)}{a^3\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5(4\,A\,c^3-10\,A\,d^3+2\,B\,c^3+30\,B\,d^3+6\,A\,c^2\,d-30\,B\,c\,d^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{8\,A\,c^3}{3}-\frac{38\,A\,d}{3}\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)

[Out] (2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(A*d + 3*B*c - 3*B*d))/(2*A*d^3 - 6*B
*d^3 + 6*B*c*d^2))*(A*d + 3*B*c - 3*B*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^5*(
4*A*c^3 - 10*A*d^3 + 2*B*c^3 + 30*B*d^3 + 6*A*c^2*d - 30*B*c*d^2) + tan(e/2
 + (f*x)/2)*((8*A*c^3)/3 - (38*A*d^3)/3 + 2*B*c^3 + 42*B*d^3 + 4*A*c*d^2 +
6*A*c^2*d - 38*B*c*d^2 + 4*B*c^2*d) + (14*A*c^3)/15 - (44*A*d^3)/15 + (2*B*
c^3)/5 + (48*B*d^3)/5 + tan(e/2 + (f*x)/2)^4*((22*A*c^3)/3 - (64*A*d^3)/3 +
 2*B*c^3 + 64*B*d^3 + 8*A*c*d^2 + 6*A*c^2*d - 64*B*c*d^2 + 8*B*c^2*d) + tan
(e/2 + (f*x)/2)^3*((20*A*c^3)/3 - (68*A*d^3)/3 + 4*B*c^3 + 80*B*d^3 + 4*A*c
*d^2 + 12*A*c^2*d - 68*B*c*d^2 + 4*B*c^2*d) + tan(e/2 + (f*x)/2)^2*((94*A*c
^3)/15 - (334*A*d^3)/15 + (12*B*c^3)/5 + (378*B*d^3)/5 + (44*A*c*d^2)/5 + (
36*A*c^2*d)/5 - (334*B*c*d^2)/5 + (44*B*c^2*d)/5) + tan(e/2 + (f*x)/2)^6*(2
*A*c^3 - 2*A*d^3 + 6*B*d^3 - 6*B*c*d^2) + (4*A*c*d^2)/5 + (6*A*c^2*d)/5 - (
44*B*c*d^2)/5 + (4*B*c^2*d)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan
(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)
^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*ta
n(e/2 + (f*x)/2)))
```

3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 164

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx &= \frac{Bd^2x}{a^3}-\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2}\\ &\quad-\frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2))\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}\\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3}\end{aligned}$$

```
[Out] B*d^2*x/a^3-1/15*(c-d)*(B*(3*c-7*d)+2*A*(c+d))*cos(f*x+e)/a/f/(a+a*sin(f*x+
e))^2-1/15*(B*(3*c^2+14*c*d-29*d^2)+2*A*(c^2+3*c*d+2*d^2))*cos(f*x+e)/f/(a^
3+a^3*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e)
)^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{3056, 3047, 3098, 2814, 2727\}$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{(2A(c^2+3cd+2d^2)+B(3c^2+14cd-29d^2))\cos(e+fx)}{15f\,(a^3\sin(e+fx)+a^3)}$$
$$+\frac{Bd^2x}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$
$$-\frac{(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*d^2*x)/a^3 - ((c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 + 14*c*d - 29*d^2) + 2*A*(c^2 + 3*c*
d + 2*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((A - B)*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} \\
&\quad + \frac{\int \frac{(c+d\sin(e+fx))(a(B(3c-2d)+2A(c+d))+5aBd\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{5a^2} \\
&= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} \\
&\quad + \frac{\int \frac{ac(B(3c-2d)+2A(c+d))+(5aBcd+ad(B(3c-2d)+2A(c+d)))\sin(e+fx)+5aBd^2\sin^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2} \\
&= -\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} \\
&\quad - \frac{\int \frac{-a^2(B(3c^2+14cd-14d^2)+2A(c^2+3cd+2d^2))-15a^2Bd^2\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{15a^4} \\
&= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} \\
&\quad + \frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2))\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2} \\
&= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} \\
&\quad - \frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2))\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 514 vs. 2(164) = 328.

Time = 0.81 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.13

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(30(2Ad(c+d)+B(c^2+4cd+d^2(-9+5e+5fx)))\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d
+ d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2)
+ B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] - 15
*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*A*c^
2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x)/2]
+ 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*Sin[(e
 + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f*x)/2]
+ 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 90*B*d^2*
Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*f*x*Sin[(
3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(e + f*x))/
2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/2] - 14*A*d
^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*d^2*e*Sin[(5
*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f*(1 + Sin[e +
 f*x])^3)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.46

method	result
parallelrisch	$\frac{15Bx\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2f+((75fx+30)B\,d^2-30A\,c^2)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left((150fx+150)B\,d^2-60Acd-60c^2\left(A+\frac{B}{2}\right)\right)\left(\mathrm{t}\right.}{}$
derivativedivides	$\frac{-\frac{2\left(A\,c^2-d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^2+4Acd+2B\,c^2-2d^2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-8A\,c^2+16Acd-8A\,d^2+8B\,c^2-16cdB+8d^2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4A\,c^2-8Acd+4A\,d^2-4B\,c^2+8\right.}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}}{a^3f}$
default	$\frac{-\frac{2\left(A\,c^2-d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^2+4Acd+2B\,c^2-2d^2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-8A\,c^2+16Acd-8A\,d^2+8B\,c^2-16cdB+8d^2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4A\,c^2-8Acd+4A\,d^2-4B\,c^2+8\right.}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}}{a^3f}$
risch	$\frac{B\,d^2x}{a^3}-\frac{2\left(14cdB+2A\,c^2+3B\,c^2-32d^2B+30iA\,d^2\mathrm{e}^{3i(fx+e)}+15iB\,c^2\mathrm{e}^{3i(fx+e)}+30Bcd\,\mathrm{e}^{4i(fx+e)}-135iB\,d^2\mathrm{e}^{3i(fx+e)}\right.}{}$
norman	$\frac{x\,d^2B}{a}+\frac{x\,d^2B\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{14A\,c^2+12Acd+4A\,d^2+6B\,c^2+8cdB-44d^2B}{15fa}-\frac{\left(2A\,c^2-2d^2B\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{\left(4A\,c^2+4Acd\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/15*(15*B*x*tan(1/2*f*x+1/2*e)^5*d^2*f+((75*f*x+30)*B*d^2-30*A*c^2)*tan(1/
2*f*x+1/2*e)^4+((150*f*x+150)*B*d^2-60*A*c*d-60*c^2*(A+1/2*B))*tan(1/2*f*x+
1/2*e)^3+((150*B*f*x-40*A+290*B)*d^2-60*(A+4/3*B)*c*d-80*c^2*(A+3/8*B))*tan
(1/2*f*x+1/2*e)^2+((75*B*f*x-20*A+190*B)*d^2-60*c*(A+2/3*B)*d-40*c^2*(A+3/4
*B))*tan(1/2*f*x+1/2*e)+(15*B*f*x-4*A+44*B)*d^2-12*c*(A+2/3*B)*d-14*c^2*(A+
3/7*B))/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(158) = 316$.

Time = 0.26 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.63

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{60\,Bd^2fx-(15\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+7\,B)cd-(7\,A-32\,B)d^2)\cos\left(fx+e\right)^3-3\,(A-B}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d -
 (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B)*d
^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*c*d
- 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A - B)
*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B)*c*d
```

```
+ (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*
(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(
f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3
*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. $2(151) = 302$.

Time = 7.85 (sec) , antiderivative size = 3468, normalized size of antiderivative = 21.15

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*
tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c**2*tan(e/2 + f*x/2)**2/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 40*A*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)
**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*
a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A
*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*
a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 60*A*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2
+ f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*A*d**2*t
an(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*A*d**2*tan(e/2 + f*x/2)/(15*a*
*3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e
/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) - 4*A*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/
```

2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)
3/(15*a3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a*
*3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 30*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*
tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 +
 f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 80*B*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*c*d*ta
n(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
3*f*tan(e/2 + f*x/2) + 15*a3*f) - 8*B*c*d/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d
2*f*x*tan(e/2 + f*x/2)5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*d**2*f*x*tan(e/2 +
f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**3/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 7
5*B*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d**2*f*x/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) + 30*B*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5
 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**
3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d
2*tan(e/2 + f*x/2)3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 290*B*d**2*tan(e/2 + f*x/2)*
*2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) + 190*B*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 +

```
f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
+ 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
+ 44*B*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4
+ 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e
))**2/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. 2(158) = 316.

Time = 0.32 (sec) , antiderivative size = 1132, normalized size of antiderivative = 6.90

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 4*B*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*d^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 6*A*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
```

```
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(158) = 316$.

Time = 0.31 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)Bd^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^
2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1
/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e
)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 2
0*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^
2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*tan(1/2*f
*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x + 1/2*e)
 - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4*B*c*d +
2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 16.84 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.74

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{B\,d^2\,x}{a^3} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{16\,A\,c^2}{3}+\frac{8\,A\,d^2}{3}+2\,B\,c^2-\frac{58\,B\,d^2}{3}+4\,A\,c\,d+\frac{16\,B\,c\,d}{3}\right)+\frac{14\,A\,c^2}{15}+\frac{4\,A\,d^2}{15}+\frac{2\,B\,c^2}{5}-\frac{44\,B\,d^2}{15}+\mathrm{t}}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\,\right.}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (B*d^2*x)/a^3 - (tan(e/2 + (f*x)/2)^2*((16*A*c^2)/3 + (8*A*d^2)/3 + 2*B*c^2
 - (58*B*d^2)/3 + 4*A*c*d + (16*B*c*d)/3) + (14*A*c^2)/15 + (4*A*d^2)/15 +
(2*B*c^2)/5 - (44*B*d^2)/15 + tan(e/2 + (f*x)/2)^3*(4*A*c^2 + 2*B*c^2 - 10*
```

```
B*d^2 + 4*A*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*B*d^2) + tan(e/2 + (f*
x)/2)*((8*A*c^2)/3 + (4*A*d^2)/3 + 2*B*c^2 - (38*B*d^2)/3 + 4*A*c*d + (8*B*
c*d)/3) + (4*A*c*d)/5 + (8*B*c*d)/15)/(f*(10*a^3*tan(e/2 + (f*x)/2)^2 + 10*
a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)
/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 127

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2Ac+3Bc+3Ad+7Bd)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -1/5*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c+3*A*d+3*B*c-8*
B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*A*c+3*A*d+3*B*c+7*B*d)*cos(f
*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3047, 3098, 2829, 2727}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)} - \frac{(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) - ((2*A*c +
3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*
A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc+3Ad-3Bd)-5aBd\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(2Ac+3Bc+3Ad+7Bd)\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2Ac+3Bc+3Ad+7Bd)\cos(e+fx)}{15f(a^3+a^3\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15(Ad+B(c+2d))\cos\left(\frac{1}{2}(e+fx)\right)-5(2Ac+3Bc+3Ad+4Bd)\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*x
)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*A*
c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x] +
 (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*a
^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{-30A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+((-60c-30d)A-30Bc)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left((-80c-30d)A-30B\left(c+\frac{4d}{3}\right)\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(}{15f\,a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
derivativedivides	$\frac{-\frac{2(8Ac-6dA-6Bc+4dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(4Ac-4dA-4Bc+4dB)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+2dA+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-8Ac+8dA+8Bc-8dB}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}}{a^3f}$
default	$\frac{-\frac{2(8Ac-6dA-6Bc+4dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(4Ac-4dA-4Bc+4dB)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+2dA+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-8Ac+8dA+8Bc-8dB}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}}{a^3f}$
risch	$-\frac{2\left(3Bc+2Ac+15Bd\,e^{4i(fx+e)}-15iBc\,e^{i(fx+e)}+30iBd\,e^{3i(fx+e)}+15iBc\,e^{3i(fx+e)}-10iAc\,e^{i(fx+e)}-20iBd\,e^{i(fx+e)}-\right.}{15f\,a^3\left(e^{i(f\right.}}$
norman	$\frac{-\frac{14Ac+6dA+6Bc+4dB}{15fa}-\frac{2Ac\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(2Ac+dA+Bc)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(34Ac+11dA+11Bc+14dB)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-}{}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/15*(-30*A*tan(1/2*f*x+1/2*e)^4*c+((-60*c-30*d)*A-30*B*c)*tan(1/2*f*x+1/2*
e)^3+((-80*c-30*d)*A-30*B*(c+4/3*d))*tan(1/2*f*x+1/2*e)^2+((-40*c-30*d)*A-3
0*B*(c+2/3*d))*tan(1/2*f*x+1/2*e)+(-14*c-6*d)*A-6*B*(c+2/3*d))/f/a^3/(tan(1
/2*f*x+1/2*e)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(121) = 242$.

Time = 0.25 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.13

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{((2A+3B)c+(3A+7B)d)\cos(fx+e)^3-(2(2A+3B)c+(6A-B)d)\cos(fx+e)^2-3(A-B)c}{15\left(a^3f\cos(fx+e)^3\right.}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c +
(6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*c
+ (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x +
e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*cos(f*
x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^
3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) -
 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(121) = 242$.

Time = 4.37 (sec) , antiderivative size = 1819, normalized size of antiderivative = 14.32

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
```

```
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(
e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)/(15*a**3*f*t
an(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f
*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15
*a**3*f) - 6*A*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2
)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*
f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*d*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*B*d*tan(e/2 + f*x/2)/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 4*B*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
d*sin(e))/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(121) = 242$.

Time = 0.23 (sec) , antiderivative size = 733, normalized size of antiderivative = 5.77

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.65

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^3}\,dx =$$
$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^4 + 30\,Ac\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 + 15\,Bc\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 + 15\,Ad\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/2*
f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/2*e
)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*B*c*
tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*x + 1
/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 14.02 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{53\,A\,c}{4}+3\,A\,d+3\,B\,c+\frac{13\,B\,d}{4}-4\,A\,c\,\cos\left(e+f\,x\right)+\frac{3\,A\,d\,\cos(e+f\,x)}{2}+\frac{3\,B\,c\,\cos(e+f\,x)}{2}+B\,d\,c\right.}{}$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A*c)/4 + 3*A*d + 3*B*c + (13*B*d)/4 - 4*A*c*cos(
e + f*x) + (3*A*d*cos(e + f*x))/2 + (3*B*c*cos(e + f*x))/2 + B*d*cos(e + f*
x) + (25*A*c*sin(e + f*x))/2 + (15*A*d*sin(e + f*x))/2 + (15*B*c*sin(e + f*
x))/2 + (5*B*d*sin(e + f*x))/2 - (9*A*c*cos(2*e + 2*f*x))/4 - (3*A*d*cos(2*
e + 2*f*x))/2 - (3*B*c*cos(2*e + 2*f*x))/2 - (9*B*d*cos(2*e + 2*f*x))/4 - (
5*A*c*sin(2*e + 2*f*x))/4 + (5*B*d*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1
/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/
2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2A+3B)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -1/5*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A+3*B)*cos(f*x+e)/a/f/(a
+a*sin(f*x+e))^2-1/15*(2*A+3*B)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2829, 2729, 2727}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)} - \frac{(2A+3B)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2} - \frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) - ((2*A + 3*B)*Cos[e
 + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A + 3*B)*Cos[e + f*x])/(15*f
*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{(2A+3B)\int\frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}\\
&= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}+\frac{(2A+3B)\int\frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}\\
&= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2A+3B)\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)\,(7A+3B+(6A+9B)\sin(e+fx)+(2A+3B)\sin^2(e+fx))}{15a^3f(1+\sin(e+fx))^3}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
[Out] -1/15*(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Sin
[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{2 i \left(20 i A\,{\mathrm e}^{2 i \left(f x +e \right)}+15 i B\,{\mathrm e}^{2 i \left(f x +e \right)}+15 B\,{\mathrm e}^{3 i \left(f x +e \right)}-2 i A -10 A\,{\mathrm e}^{i \left(f x +e \right)}-3 i B -15 B\,{\mathrm e}^{i \left(f x +e \right)}\right)}{15 f \,a^{3} \left({\mathrm e}^{i \left(f x +e \right)}+i \right)^{5}}$
parallelrisch	$\frac{-30 A \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+\left(-60 A -30 B \right) \left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+\left(-80 A -30 B \right) \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)+\left(-40 A -30 B \right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)-14 A}{15 f \,a^{3} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{2 A}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2 \left(8 A -6 B \right)}{3 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-8 A +8 B}{2 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2 \left(4 A -4 B \right)}{5 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-4 A +2 B}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}}{a^{3} f}$
default	$\frac{-\frac{2 A}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2 \left(8 A -6 B \right)}{3 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-8 A +8 B}{2 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2 \left(4 A -4 B \right)}{5 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-4 A +2 B}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}}{a^{3} f}$
norman	$\frac{-\frac{14 A +6 B}{15 f a}-\frac{2 A \left(\tan^{6}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{f a}-\frac{\left(94 A +36 B \right) \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 f a}-\frac{\left(20 A +12 B \right) \left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 f a}-\frac{\left(22 A +6 B \right) \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 f a}-\frac{(8A+6}{}}{\left(1+\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right) a^{2} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*I*(20*I*A*exp(2*I*(f*x+e))+15*I*B*exp(2*I*(f*x+e))+15*B*exp(3*I*(f*x+
e))-2*I*A-10*A*exp(I*(f*x+e))-3*I*B-15*B*exp(I*(f*x+e)))/f/a^3/(exp(I*(f*x+
e))+I)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(2\,A+3\,B)\cos\left(fx+e\right)^3-2\,(2\,A+3\,B)\cos\left(fx+e\right)^2-3\,(3\,A+2\,B)\cos\left(fx+e\right)-((2\,A+3\,B)\cos(}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\right.\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A +
 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x +
e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(87) = 174$.

Time = 2.37 (sec) , antiderivative size = 1015, normalized size of antiderivative = 9.95

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 40*A*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 30*B*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan
(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(96) = 192$.

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.79

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3}\,dx =$$

$$-\frac{2\left(\frac{A\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3B\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx\cdots}{(\cos(fx+e\cdots}\right.}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin\cdots}{(\cos(f\cdots}}\right.}{15f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*B*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right.}{15\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*tan
(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/2*
e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.47

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = \frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{53\,A}{4}+3\,B-4\,A\,\cos\left(e+f\,x\right)+\frac{3\,B\,\cos(e+f\,x)}{2}+\frac{25\,A\,\sin(e+f\,x)}{2}+\frac{15\,B\,\sin(e+f\,x)}{2}-\frac{9\,A\,\cos(2\,e+2\,f\,x)}{4}\right.}{15\,a^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}-\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}\right)}$$

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A)/4 + 3*B - 4*A*cos(e + f*x) + (3*B*cos(e + f*x
))/2 + (25*A*sin(e + f*x))/2 + (15*B*sin(e + f*x))/2 - (9*A*cos(2*e + 2*f*x
```

```
))/4 - (3*B*cos(2*e + 2*f*x))/2 - (5*A*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*
2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f
*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 229

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx &= \frac{2d^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} \\ &\quad - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} \\ &\quad - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))}\end{aligned}$$

```
[Out] -1/5*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c-7*A*d+3*B*c+2*
B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/15*(B*(3*c^2-16*c*d-2*d^2)
+A*(2*c^2-9*c*d+22*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))+2*d^2*(-
A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^3/f/(c^
2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{3057, 12, 2739, 632, 210\}$

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$
$$= \frac{2d^2(Bc-Ad)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}} - \frac{(A(2c^2-9cd+22d^2)+B(3c^2-16cd-2d^2))\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)} - \frac{(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*d^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(
c - d)^3*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Si
n[e + f*x])^3) - ((2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/(15*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 - 16*c*d - 2*d^2) + A*(2*c^2 -
9*c*d + 22*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc-5Ad)-2a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx}{5a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} + \frac{\int \frac{a^2(Bc(3c-13d)+A(2c^2-7cd+15d^2))+a^2d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx}{15a^4(c-d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))} - \frac{\int -\frac{15a^3d^2(Bc-Ad)}{c+d\sin(e+fx)}\,dx}{15a^6(c-d)^3}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))} + \frac{(d^2(Bc-Ad))\int \frac{1}{c+d\sin(e+fx)}\,dx}{a^3(c-d)^3}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))} + \frac{(2d^2(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^3(c-d)^3f}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))} - \frac{(4d^2(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^3(c-d)^3f}$$

$$= \frac{2d^2(Bc - Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f(a^3+a^3\sin(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 502 vs. 2(229) = 458.

Time = 4.75 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.19

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15Bc^2\cos\left(\frac{1}{2}(e+fx)\right)-15Acd\cos\left(\frac{1}{2}(e+fx)\right)-75Bcd\cos\left(\frac{1}{2}(e\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c*
d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2]
- 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c*d*
Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*(e +
 f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2] + 15*
B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e + f*x)
/2] + 145*A*d^2*Sin[(e + f*x)/2] - 20*B*d^2*Sin[(e + f*x)/2] - (60*d^2*(-(B
*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/Sqrt[c^2 - d^2] - 15*B*c*d*Sin[(3*(e + f*x))/2] +
 15*A*d^2*Sin[(3*(e + f*x))/2] - 2*A*c^2*Sin[(5*(e + f*x))/2] - 3*B*c^2*Sin
[(5*(e + f*x))/2] + 9*A*c*d*Sin[(5*(e + f*x))/2] + 16*B*c*d*Sin[(5*(e + f*x
))/2] - 22*A*d^2*Sin[(5*(e + f*x))/2] + 2*B*d^2*Sin[(5*(e + f*x))/2]))/(30*
a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{2d^2(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^3\sqrt{c^2-d^2}}-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6dA+2Bc-4dB}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^3f}$
default	$\frac{-\frac{2d^2(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^3\sqrt{c^2-d^2}}-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6dA+2Bc-4dB}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^3f}$
risch	$-\frac{34Bcd\,e^{2i(fx+e)}}{3}+2Bcd\,e^{4i(fx+e)}+\frac{38iA\,d^2e^{i(fx+e)}}{3}+2iAcd\,e^{3i(fx+e)}-6iAcd\,e^{i(fx+e)}-10iA\,d^2e^{3i(fx+e)}+\frac{32cdB}{15}-\frac{4A\,c^2}{15}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^3*(-d^2*(A*d-B*c)/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1/4*(-8*A+8*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4
-1/5*(4*A-4*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+6*A*d+2*B*c-4*B*d
)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-10*A*d-6*B*c+8*B*d)/(c-d)^2/(
tan(1/2*f*x+1/2*e)+1)^3-(A*c^2-3*A*c*d+3*A*d^2-B*d^2)/(c-d)^3/(tan(1/2*f*x+
1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(218) = 436$.

Time = 0.33 (sec) , antiderivative size = 2292, normalized size of antiderivative = 10.01

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4
- 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16
*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (18*A
 + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A - 4*B)
*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x
+ e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x +
 e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2
- A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
```

```
e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d + 5*(3*A - B)
*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) - 2*(3*(A - B
)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^
4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A
 - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B)*c^3*d + 5*(3*
A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos(f*x + e))*sin(f
*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*
d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
+ 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*
a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*
x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3
 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a
^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3
*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)
*f)*sin(f*x + e)), 1/15*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3
- 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d
^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + (2*(2*A + 3*B)
*c^4 - (18*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 -
(29*A - 4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d
^3)*cos(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^
3)*cos(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 +
 2*(B*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c
*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^4 - (
11*A + 9*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*
d^4)*cos(f*x + e) - (3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*
(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 +
 (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4
 - (9*A + 11*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A -
2*B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*
a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*
d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d +
2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2
*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3
*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(218) = 436$.

Time = 0.32 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \frac{2\left(\frac{15\,(Bcd^2-Ad^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^3c^3-3\,a^3c^2d+3\,a^3cd^2-a^3d^3)\sqrt{c^2-d^2}}-\frac{15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,Ad^2\tan\left(\frac{1}{2}\,fx\right.}{}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="giac")
[Out] 2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
- 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*
```

```
d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan
(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan(1/2*f
*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*f*x + 1
/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2
 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 185
*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*
tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*tan(1/2*f*x
 + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f*x + 1/2*e)
- 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d - 11*B*c*d +
 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(
1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 17.22 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.58

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$
$$= \frac{2\,d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,(A\,d-B\,c)\,\left(-2\,a^3\,c^3\,d+6\,a^3\,c^2\,d^2-6\,a^3\,c\,d^3+2\,a^3\,d^4\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}-\frac{2\,c\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,\left(a^3\,c^3-3\,a^3\,c^2\,d+3\,a^3\,c\,d^2-a^3\,d^3\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}}{2\,A\,d^3-2\,B\,c\,d^2}\right)\,(A\,d-B}{a^3\,f\,\sqrt{c+d}\,(c-d)^{7/2}}$$
$$-\frac{\frac{2\,(7\,A\,c^2+32\,A\,d^2+3\,B\,c^2-7\,B\,d^2-24\,A\,c\,d-11\,B\,c\,d)}{15\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,A\,c^2+23\,A\,d^2+3\,B\,c^2-4\,B\,d^2-15\,A\,c\,d-11\,B\,c\,d)}{3\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f}{}\right.}{}}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\right.}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d^2*atan(((d^2*(A*d - B*c)*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a^
3*c^2*d^2))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/2)
*(A*d - B*c)*(a^3*c^3 - a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c + d)^
(1/2)*(c - d)^(7/2)))/(2*A*d^3 - 2*B*c*d^2))*(A*d - B*c))/(a^3*f*(c + d)^(1
/2)*(c - d)^(7/2)) - ((2*(7*A*c^2 + 32*A*d^2 + 3*B*c^2 - 7*B*d^2 - 24*A*c*d
 - 11*B*c*d))/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(4*A
*c^2 + 23*A*d^2 + 3*B*c^2 - 4*B*d^2 - 15*A*c*d - 11*B*c*d))/(3*(c - d)*(c^2
 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2*A*c^2 + 9*A*d^2 + B*c^2 - 2*B
*d^2 - 7*A*c*d - 3*B*c*d))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*
x)/2)^2*(8*A*c^2 + 37*A*d^2 + 3*B*c^2 - 8*B*d^2 - 27*A*c*d - 13*B*c*d))/(3*
(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^2 + 3*A*d^2 - B
*d^2 - 3*A*c*d))/((c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*tan(e/2 + (f*x)/
2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e
/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx \\
&= -\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}f} \\
&\quad -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} \\
&\quad -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} \\
&\quad -\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} \\
&\quad -\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))\,(c+d\sin(e+fx))}
\end{aligned}$$

```
[Out] -1/15*d*(B*(3*c^3-23*c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*d
^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)/(
c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))-1/15*(2*A*c-9*A*d+3*B*c+4*B*d)*c
os(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/15*(B*(3*c^2-23
*c*d-15*d^2)+A*(2*c^2-12*c*d+45*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x
+e))/(c+d*sin(f*x+e))-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^4/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\arctan\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}} - \frac{(A(2c^2-12cd+45d^2)+B(3c^2-23cd-15d^2))\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))} - \frac{d(A(2c^3-12c^2d+43cd^2+72d^3)+B(3c^3-23c^2d-63cd^2-22d^3))\cos(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sin(e+fx))} - \frac{(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^2*(A*d*(4*c + 3*d) - B*(3*c^2 + 3*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(
B*(3*c^3 - 23*c^2*d - 63*c*d^2 - 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 +
 72*d^3))*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e +
f*x])) - ((2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(
a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) - ((B*(3*c^2 - 23*c*d - 15*d^2)
 + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Si
n[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} \\
&\quad - \frac{\int \frac{-a(2A(c-3d)+B(3c+d))-3a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx}{5a^2(c-d)} \\
&= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} \\
&\quad - \frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} \\
&\quad + \frac{\int \frac{a^2(B(3c^2-17cd-7d^2)+A(2c^2-8cd+27d^2))+2a^2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx}{15a^4(c-d)^2}
\end{aligned}$$

$$=-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}-\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}-\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}-\frac{\int\frac{-2a^3d^2(Ac+24Bc-36Ad+11Bd)-a^3d\left(B\left(3c^2-23cd-15d^2\right)+A\left(2c^2-12cd+45d^2\right)\right)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{15a^6(c-d)^3}$$

$$=-\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}-\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}-\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}+\frac{\int-\frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)}{c+d\sin(e+fx)}dx}{15a^6(c-d)^4(c+d)}$$

$$=-\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}-\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}-\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}-\frac{(d^2(Ad(4c+3d)-B(3c^2+3cd+d^2)))\int\frac{1}{c+d\sin(e+fx)}dx}{a^3(c-d)^4(c+d)}$$

$$=-\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}-\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}-\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}-\frac{(2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2)))\,\text{Subst}\left(\int\frac{1}{c+2dx+cx^2}dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^3(c-d)^4(c+d)f}$$

$$= -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$
$$-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}$$
$$-\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}$$
$$-\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}$$
$$+\frac{(4d^2(Ad(4c+3d)-B(3c^2+3cd+d^2)))\operatorname{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a^3(c-d)^4(c+d)f}$$

$$= -\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}f}$$
$$-\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$
$$-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}$$
$$-\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}$$
$$-\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1253 vs. 2(381) = 762.

Time = 12.28 (sec) , antiderivative size = 1253, normalized size of antiderivative = 3.29

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{2d^2(3Bc^2-4Acd+3Bcd-3Ad^2+Bd^2)\arctan\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)+c\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{(c-d)^4(c+d)\sqrt{c^2-d^2}f(a+a\sin(e+fx))^3}$$
$$+\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(60Bc^4\cos\left(\frac{1}{2}(e+fx)\right)-80Ac^3d\cos\left(\frac{1}{2}(e+fx)\right)-390Bc^3d\cos\left(\frac{1}{2}\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] (2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*x
)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a + a*
Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos[(e
```

```
+ f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2] + 54
0*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430*A*c*d
^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(e + f*x
)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] - 60*B*c^
4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*c^3*d*Cos
[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c^2*d^2*Cos
[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c*d^3*Cos[(3
*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos[(3*(e + f*x
))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*(e + f*x))/2]
 + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x))/2] + 30*B*d
^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6*B*c^3*d*Cos[(7
*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c^2*d^2*Cos[(7*(e
 + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^3*Cos[(7*(e + f*x
))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*(e + f*x))/2] + 80
*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 340*A*c^3*d*Sin[(e +
f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d^2*Sin[(e + f*x)/2] - 1
520*B*c^2*d^2*Sin[(e + f*x)/2] + 2140*A*c*d^3*Sin[(e + f*x)/2] - 1435*B*c*d
^3*Sin[(e + f*x)/2] + 975*A*d^4*Sin[(e + f*x)/2] - 340*B*d^4*Sin[(e + f*x)/
2] - 90*B*c^3*d*Sin[(3*(e + f*x))/2] + 120*A*c^2*d^2*Sin[(3*(e + f*x))/2] -
 390*B*c^2*d^2*Sin[(3*(e + f*x))/2] + 540*A*c*d^3*Sin[(3*(e + f*x))/2] - 31
5*B*c*d^3*Sin[(3*(e + f*x))/2] + 285*A*d^4*Sin[(3*(e + f*x))/2] - 150*B*d^4
*Sin[(3*(e + f*x))/2] - 8*A*c^4*Sin[(5*(e + f*x))/2] - 12*B*c^4*Sin[(5*(e +
 f*x))/2] + 28*A*c^3*d*Sin[(5*(e + f*x))/2] + 62*B*c^3*d*Sin[(5*(e + f*x))/
2] - 52*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 362*B*c^2*d^2*Sin[(5*(e + f*x))/2]
 - 568*A*c*d^3*Sin[(5*(e + f*x))/2] + 553*B*c*d^3*Sin[(5*(e + f*x))/2] - 55
5*A*d^4*Sin[(5*(e + f*x))/2] + 190*B*d^4*Sin[(5*(e + f*x))/2] - 15*B*c*d^3*
Sin[(7*(e + f*x))/2] + 15*A*d^4*Sin[(7*(e + f*x))/2]))/(120*(c - d)^4*(c +
d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]))
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{2d^2\left(\frac{\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(4Acd+3A\,d^2-3B\,c^2-3cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^4}-\frac{-8A\cdots}{2(c-d)^2\left(\tan\left(\cdots\right.\right.}}{a\cdots}$
default	$\frac{-\frac{2d^2\left(\frac{\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(4Acd+3A\,d^2-3B\,c^2-3cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^4}-\frac{-8A\cdots}{2(c-d)^2\left(\tan\left(\cdots\right.\right.}}{a\cdots}$
risch	Expression too large to display

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-d^2/(c-d)^4*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c
)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(4*A*c*d+3*A*d^2
-3*B*c^2-3*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/4*(-8*A+8*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^
4-1/5*(4*A-4*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+8*A*d+2*B*c-6*
B*d)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-12*A*d-6*B*c+10*B*d)/(c-d)
^3/(tan(1/2*f*x+1/2*e)+1)^3-(A*c^2-4*A*c*d+6*A*d^2-3*B*d^2)/(c-d)^4/(tan(1/
2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2201 vs. $2(368) = 736$.

Time = 0.42 (sec) , antiderivative size = 4486, normalized size of antiderivative = 11.77

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/30*(6*(A - B)*c^6 - 12*(A - B)*c^5*d - 6*(A - B)*c^4*d^2 + 24*(A - B)*c
^3*d^3 - 6*(A - B)*c^2*d^4 - 12*(A - B)*c*d^5 + 6*(A - B)*d^6 - 2*((2*A + 3
*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*
d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - 2*((2*A +
 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*B)*c^
3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^
6)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A -
7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A -
```

104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^6 - (11*A + 9*B)*c^5*d + (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 + (47*A + 28*B)*c^2*d^4 - 71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e) - 2*(3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 + 5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 + 2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e)), -1/15*(3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 - ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - ((2*A + 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A -

18*B)*c^4*d^2 + (147*A - 152*B)*c^3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^6)*cos(f*x + e)^3 + (2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A - 7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A - 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^6 - (11*A + 9*B)*c^5*d + (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 + (47*A + 28*B)*c^2*d^4 - 71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e) - (3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 + 5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 + 2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e))]

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(368) = 736.

Time = 0.35 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.95

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \frac{2\left(\frac{15\,(3\,Bc^2d^2-4\,Acd^3+3\,Bcd^3-3\,Ad^4+Bd^4)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^3c^5-3\,a^3c^4d+2\,a^3c^3d^2+2\,a^3c^2d^3-3\,a^3cd^4+a^3d^5)\sqrt{c^2-d^2}}+\frac{15\,(Bcd^4\tan(\frac{1}{2}\,fx\cdots}{(a^3c^6-3\,a^3c^5d+2\,a^3c^4d^2+2\,a^3c^3d^3-\cdots}\cdots\right.}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
[Out] 2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor(
1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3
*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) - A*d
```

```
^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*
a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1
/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*B
*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*ta
n(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*d*tan(1/2*
f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*tan(1/2*f*x +
 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)
^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x + 1/2*e)^2 +
420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*
c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 110*A*c*d*tan(1/
2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*tan(1/2*f*x + 1/
2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 34*A*c*d - 16*B
*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3
*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 1349, normalized size of antiderivative = 3.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d^2*atan(((d^2*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(2*a^3*d^
6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^3*c^3*d^3 - 6*a^3*c^4*d
^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(3*B*c
^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*d^4
- 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c - d)^
(9/2)))/(2*B*d^4 - 6*A*d^4 + 6*B*c^2*d^2 - 8*A*c*d^3 + 6*B*c*d^3))*(3*B*c^2
 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))/(a^3*f*(c + d)^(3/2)*(c - d)^(9/2)
) - ((2*(7*A*c^4 + 15*A*d^4 + 3*B*c^4 + 38*A*c^2*d^2 - 48*B*c^2*d^2 + 72*A*
c*d^3 - 27*A*c^3*d - 47*B*c*d^3 - 13*B*c^3*d))/(15*(c + d)*(c - d)*(3*c*d^2
 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(5*A*c^4 + 15*A*d^4 + 3*
B*c^4 + 19*A*c^2*d^2 - 45*B*c^2*d^2 + 84*A*c*d^3 - 18*A*c^3*d - 52*B*c*d^3
- 11*B*c^3*d))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 +
 (f*x)/2)*(20*A*c^5 + 15*A*d^5 + 15*B*c^5 + 346*A*c^2*d^3 + 106*A*c^3*d^2 -
 286*B*c^2*d^3 - 221*B*c^3*d^2 + 219*A*c*d^4 - 76*A*c^4*d - 79*B*c*d^4 - 59
*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(
e/2 + (f*x)/2)^5*(2*A*c^5 + 5*A*d^5 + B*c^5 + 24*A*c^2*d^3 + 4*A*c^3*d^2 -
16*B*c^2*d^3 - 13*B*c^3*d^2 + 13*A*c*d^4 - 6*A*c^4*d - 11*B*c*d^4 - 3*B*c^4
*d))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*
x)/2)^4*(11*A*c^5 + 30*A*d^5 + 3*B*c^5 + 162*A*c^2*d^3 + 4*A*c^3*d^2 - 139*
B*c^2*d^3 - 84*B*c^3*d^2 + 135*A*c*d^4 - 27*A*c^4*d - 84*B*c*d^4 - 11*B*c^4
```

```
*d))/(3*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (
f*x)/2)^2*(47*A*c^5 + 75*A*d^5 + 18*B*c^5 + 812*A*c^2*d^3 + 88*A*c^3*d^2 -
757*B*c^2*d^3 - 463*B*c^3*d^2 + 690*A*c*d^4 - 137*A*c^4*d - 305*B*c*d^4 - 6
8*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan
(e/2 + (f*x)/2)^6*(A*c^5 + A*d^5 + 6*A*c^2*d^3 + 2*A*c^3*d^2 - 3*B*c^2*d^3
- 3*B*c^3*d^2 - 3*A*c^4*d - B*c*d^4))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d
 + c^3 - d^3)))/(f*(a^3*c + tan(e/2 + (f*x)/2)*(5*a^3*c + 2*a^3*d) + tan(e/
2 + (f*x)/2)^6*(5*a^3*c + 2*a^3*d) + tan(e/2 + (f*x)/2)^2*(11*a^3*c + 10*a^
3*d) + tan(e/2 + (f*x)/2)^5*(11*a^3*c + 10*a^3*d) + tan(e/2 + (f*x)/2)^3*(1
5*a^3*c + 20*a^3*d) + tan(e/2 + (f*x)/2)^4*(15*a^3*c + 20*a^3*d) + a^3*c*ta
n(e/2 + (f*x)/2)^7))
```

3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 508

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$$
$$= -\frac{d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}f}$$
$$-\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}$$
$$-\frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}$$
$$-\frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f(a^3+a^3\sin(e+fx))(c+d\sin(e+fx))^2}$$
$$-\frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))\cos(e+f}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))}$$

```
[Out] -1/30*d*(3*B*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)+A*(4*c^3-30*c^2*d+146*c*d^2+1
95*d^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))^2-1/5*(A-B)*cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2-1/15*(2*A*c-11*A*d+3*B*c
+6*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/15*(
3*B*(c^2-10*c*d-12*d^2)+A*(2*c^2-15*c*d+76*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+
a^3*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/30*d*(3*B*(2*c^4-20*c^3*d-119*c^2*d^2-
130*c*d^3-48*d^4)+A*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4))*cos(f*x
+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))-d^2*(A*d*(20*c^2+30*c*d+13*d^2)-
3*B*(4*c^3+8*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2
)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$$
$$= -\frac{d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3))\arctan\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}$$
$$-\frac{(A(2c^2-15cd+76d^2)+3B(c^2-10cd-12d^2))\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(4c^3-30c^2d+146cd^2+195d^3)+3B(2c^3-20c^2d-57cd^2-30d^3))\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)+3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4))\cos(e+}{30a^3f(c-d)^5(c+d)^2(c+d\sin(e+fx))}$$
$$-\frac{(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^2*(A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2
*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^5*(c
+ d)^2*Sqrt[c^2 - d^2]*f)) - (d*(3*B*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3)
 + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3))*Cos[e + f*x])/(30*a^3*(c - d
)^4*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f
*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2) - ((2*A*c + 3*B*c - 11*A*d
+ 6*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[
e + f*x])^2) - ((3*B*(c^2 - 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))
*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2) - (d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4) + A*(4
*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(2Ac+3Bc-7Ad+2Bd)-4a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx}{5a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} + \frac{\int \frac{a^2(3B(c^2-7cd-6d^2)+A(2c^2-9cd+43d^2))+3a^2d(A(2c-11d)+3B(c+2d))\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx}{15a^4(c-d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))\,(c+d\sin(e+fx))^2} - \frac{\int \frac{-3a^3d^2(2Ac+33Bc-65Ad+30Bd)-2a^3d(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{15a^6(c-d)^3}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))\,(c+d\sin(e+fx))^2} + \frac{\int \frac{2a^3d^2(2Ac^2+93Bc^2-165Acd+150Bcd-152Ad^2+72Bd^2)+a^3d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3)}{(c+d\sin(e+fx))^2}}{30a^6(c-d)^4(c+d)}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))\,(c+d\sin(e+fx))^2} - \frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))} - \frac{\int \frac{15a^3d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3))}{c+d\sin(e+fx)}\,dx}{30a^6(c-d)^5(c+d)^2}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^2} - \frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))\,\mathrm{co}}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))} - \frac{(d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3)))\int\frac{1}{c+d\sin(e+fx)}\,dx}{2a^3(c-d)^5(c+d)^2}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^2} - \frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))\,\mathrm{co}}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))} - \frac{(d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3)))\,\mathrm{Subst}\left(\int\frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e\right.\right.}{a^3(c-d)^5(c+d)^2f}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^2} - \frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))\,\mathrm{co}}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))} + \frac{(2d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3)))\,\mathrm{Subst}\left(\int\frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+\right.}{a^3(c-d)^5(c+d)^2f}$$

$$= -\frac{d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}f} - \frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f(a^3+a^3\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 8.64 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)+6(-A+B)(c-d)^2\left(\cos\left(\frac{1}{2}(e\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - d)
*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2 - 19*
c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 +
(30*d^2*(-(A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2
 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d)*d^3*(B*
c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)*(c
+ d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 + 6*c*d + 2*d
^2))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*(c +
d*Sin[e + f*x]))))/(30*a^3*(c - d)^5*f*(1 + Sin[e + f*x])^3)
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{-8A+8B}{2(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+10dA+2Bc-8dB}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-14dA-6Bc+12dB)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(A c^2-5A\right.}{(c-d)^5\left(\right.}}{}$
default	$\frac{-\frac{-8A+8B}{2(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+10dA+2Bc-8dB}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-14dA-6Bc+12dB)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(A c^2-5A\right.}{(c-d)^5\left(\right.}}{}$
risch	Expression too large to display

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A+8*B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*B)/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+10*A*d+2*B*c-8*B*d)/(c-d)^4/(tan(
1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-14*A*d-6*B*c+12*B*d)/(c-d)^4/(tan(1/2*f*x+1/
2*e)+1)^3-(A*c^2-5*A*c*d+10*A*d^2-6*B*d^2)/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)-d
^2/(c-d)^5*((1/2*d^2*(11*A*c^2*d+6*A*c*d^2-2*A*d^3-9*B*c^3-6*B*c^2*d)/c/(c^
2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(10*A*c^4*d+6*A*c^3*d^2+19*A*c^2*d^
3+12*A*c*d^4-2*A*d^5-8*B*c^5-6*B*c^4*d-17*B*c^3*d^2-12*B*c^2*d^3-2*B*c*d^4)
/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(29*A*c^2*d+18*A*c*d^2-2*
A*d^3-23*B*c^3-18*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1
/2*d*(10*A*c^2*d+6*A*c*d^2-A*d^3-8*B*c^3-6*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2)
)/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(20*A*c^2*d+30*A*
c*d^2+13*A*d^3-12*B*c^3-24*B*c^2*d-21*B*c*d^2-6*B*d^3)/(c^2+2*c*d+d^2)/(c^2
-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3599 vs. $2(493) = 986$.

Time = 0.56 (sec) , antiderivative size = 7283, normalized size of antiderivative = 14.34

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(493) = 986$.

Time = 0.43 (sec) , antiderivative size = 1224, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(12*B*c^3*d^2 - 20*A*c^2*d^3 + 24*B*c^2*d^3 - 30*A*c*d^4 + 21*B*c*
d^4 - 13*A*d^5 + 6*B*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan
((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a
^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^
3*d^7)*sqrt(c^2 - d^2)) + 15*(9*B*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 11*A*c^3
*d^5*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*A*c^2*
d^6*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*B*c^5*d^3
*tan(1/2*f*x + 1/2*e)^2 - 10*A*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^4*d^4
*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^3*d^5
```

```
*tan(1/2*f*x + 1/2*e)^2 - 19*A*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*B*c^2*d^
6*tan(1/2*f*x + 1/2*e)^2 - 12*A*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^7*ta
n(1/2*f*x + 1/2*e)^2 + 2*A*d^8*tan(1/2*f*x + 1/2*e)^2 + 23*B*c^4*d^4*tan(1/
2*f*x + 1/2*e) - 29*A*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*B*c^3*d^5*tan(1/2*f
*x + 1/2*e) - 18*A*c^2*d^6*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^6*tan(1/2*f*x +
 1/2*e) + 2*A*c*d^7*tan(1/2*f*x + 1/2*e) + 8*B*c^5*d^3 - 10*A*c^4*d^4 + 6*B
*c^4*d^4 - 6*A*c^3*d^5 + B*c^3*d^5 + A*c^2*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a
^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 -
a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) -
 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 150
*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 90*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*
tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 195*A*c*d*tan(1/
2*f*x + 1/2*e)^3 - 75*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*A*d^2*tan(1/2*f*x
+ 1/2*e)^3 - 300*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*
e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*A*c*d*tan(1/2*f*x + 1/2*e)^2 -
 135*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 745*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 420*
B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan
(1/2*f*x + 1/2*e) - 145*A*c*d*tan(1/2*f*x + 1/2*e) - 105*B*c*d*tan(1/2*f*x
+ 1/2*e) + 485*A*d^2*tan(1/2*f*x + 1/2*e) - 270*B*d^2*tan(1/2*f*x + 1/2*e)
+ 7*A*c^2 + 3*B*c^2 - 44*A*c*d - 21*B*c*d + 127*A*d^2 - 72*B*d^2)/((a^3*c^5
 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3*d^5)*(
tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 2387, normalized size of antiderivative = 4.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)
```

```
[Out] ((15*A*d^6 - 14*A*c^6 - 6*B*c^6 - 404*A*c^2*d^4 - 420*A*c^3*d^3 - 92*A*c^4*
d^2 + 234*B*c^2*d^4 + 450*B*c^3*d^3 + 222*B*c^4*d^2 - 90*A*c*d^5 + 60*A*c^5
*d + 15*B*c*d^5 + 30*B*c^5*d)/(15*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^
3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^7*(2*A*d^8 - 4*A*c^8 - 2*B*c^8
- 49*A*c^2*d^6 - 141*A*c^3*d^5 - 200*A*c^4*d^4 - 122*A*c^5*d^3 + 2*A*c^6*d^
2 + 12*B*c^2*d^6 + 95*B*c^3*d^5 + 187*B*c^4*d^4 + 146*B*c^5*d^3 + 58*B*c^6*
d^2 - 2*A*c*d^7 + 10*A*c^7*d + 2*B*c*d^7 + 6*B*c^7*d))/(c^2*(c - d)*(2*c*d
+ c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x
)/2)^6*(30*A*d^8 - 28*A*c^8 - 6*B*c^8 - 759*A*c^2*d^6 - 1707*A*c^3*d^5 - 19
60*A*c^4*d^4 - 870*A*c^5*d^3 + 62*A*c^6*d^2 + 336*B*c^2*d^6 + 1257*B*c^3*d^
5 + 1893*B*c^4*d^4 + 1350*B*c^5*d^3 + 414*B*c^6*d^2 - 114*A*c*d^7 + 54*A*c^
7*d + 30*B*c*d^7 + 18*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4
*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(60*A*d^8 - 32
```

*A*c^8 - 18*B*c^8 - 1857*A*c^2*d^6 - 3763*A*c^3*d^5 - 3560*A*c^4*d^4 - 1294
*A*c^5*d^3 + 70*A*c^6*d^2 + 900*B*c^2*d^6 + 2859*B*c^3*d^5 + 3705*B*c^4*d^4
 + 2358*B*c^5*d^3 + 678*B*c^6*d^2 - 270*A*c*d^7 + 62*A*c^7*d + 60*B*c*d^7 +
 42*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 +
 d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(30*A*d^8 - 108*A*c^8 - 42*B*c^8
 - 2501*A*c^2*d^6 - 8725*A*c^3*d^5 - 10616*A*c^4*d^4 - 4810*A*c^5*d^3 + 10*
A*c^6*d^2 + 1056*B*c^2*d^6 + 5235*B*c^3*d^5 + 9891*B*c^4*d^4 + 7770*B*c^5*d
^3 + 2370*B*c^6*d^2 - 30*A*c*d^7 + 290*A*c^7*d + 30*B*c*d^7 + 150*B*c^7*d))
/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (
tan(e/2 + (f*x)/2)^3*(150*A*d^8 - 140*A*c^8 - 90*B*c^8 - 7945*A*c^2*d^6 - 1
9441*A*c^3*d^5 - 18600*A*c^4*d^4 - 6898*A*c^5*d^3 + 210*A*c^6*d^2 + 3660*B*
c^2*d^6 + 13311*B*c^3*d^5 + 19455*B*c^4*d^4 + 12618*B*c^5*d^3 + 3570*B*c^6*
d^2 - 570*A*c*d^7 + 314*A*c^7*d + 150*B*c*d^7 + 246*B*c^7*d))/(15*c^2*(c +
d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x
)/2)*(30*A*d^7 - 40*A*c^7 - 30*B*c^7 - 1901*A*c^2*d^5 - 3400*A*c^3*d^4 - 20
18*A*c^4*d^3 - 190*A*c^5*d^2 + 921*B*c^2*d^5 + 2655*B*c^3*d^4 + 2778*B*c^4*
d^3 + 1050*B*c^5*d^2 - 195*A*c*d^6 + 154*A*c^6*d + 60*B*c*d^6 + 126*B*c^6*d
))/(15*c*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) - (
tan(e/2 + (f*x)/2)^8*(2*A*c^7 - 2*A*d^7 + 11*A*c^2*d^5 + 20*A*c^3*d^4 + 30*
A*c^4*d^3 + 2*A*c^5*d^2 - 6*B*c^2*d^5 - 21*B*c^3*d^4 - 24*B*c^4*d^3 - 12*B*
c^5*d^2 + 6*A*c*d^6 - 6*A*c^6*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c
^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^4*(300*A*d^7 - 204
*A*c^7 - 66*B*c^7 - 10235*A*c^2*d^5 - 14330*A*c^3*d^4 - 7254*A*c^4*d^3 - 31
6*A*c^5*d^2 + 5460*B*c^2*d^5 + 12675*B*c^3*d^4 + 10764*B*c^4*d^3 + 3666*B*c
^5*d^2 - 1650*A*c*d^6 + 614*A*c^6*d + 300*B*c*d^6 + 276*B*c^6*d))/(15*c^2*(
c + d)*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)))/(f*(tan(e/2 +
(f*x)/2)*(5*a^3*c^2 + 4*a^3*c*d) + tan(e/2 + (f*x)/2)^2*(12*a^3*c^2 + 4*a^3
*d^2 + 20*a^3*c*d) + tan(e/2 + (f*x)/2)^7*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*
c*d) + tan(e/2 + (f*x)/2)^3*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/
2 + (f*x)/2)^6*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (f*x)/2)^
4*(26*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^5*(26*a^3*c^2
 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(5*a^3*c^2 + 4*a^3*c*d)
+ a^3*c^2 + a^3*c^2*tan(e/2 + (f*x)/2)^9)) - (d^2*atan(((d^2*(12*B*c^3 - 13
*A*d^3 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(2*a^
3*d^8 - 6*a^3*c*d^7 - 2*a^3*c^7*d + 2*a^3*c^2*d^6 + 10*a^3*c^3*d^5 - 10*a^3
*c^4*d^4 - 2*a^3*c^5*d^3 + 6*a^3*c^6*d^2))/(2*a^3*(c + d)^(5/2)*(c - d)^(11
/2)) - (c*d^2*tan(e/2 + (f*x)/2)*(12*B*c^3 - 13*A*d^3 + 6*B*d^3 - 30*A*c*d^
2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(a^3*c^7 - a^3*d^7 + 3*a^3*c*d^6
- 3*a^3*c^6*d - a^3*c^2*d^5 - 5*a^3*c^3*d^4 + 5*a^3*c^4*d^3 + a^3*c^5*d^2))
/(a^3*(c + d)^(5/2)*(c - d)^(11/2)))/(6*B*d^5 - 13*A*d^5 - 20*A*c^2*d^3 + 2
4*B*c^2*d^3 + 12*B*c^3*d^2 - 30*A*c*d^4 + 21*B*c*d^4))*(12*B*c^3 - 13*A*d^3
 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d))/(a^3*f*(c
+ d)^(5/2)*(c - d)^(11/2))

3.286 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 256

$$\begin{aligned}\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx &= \frac{4a(c+d)(Bc-9Ad-8Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} \\ &+ \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} \\ &+ \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105af} \\ &+ \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} \\ &- \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

```
[Out] 4/105*d*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+4/31
5*a*(c+d)*(-9*A*d+B*c-8*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(
f*x+e))^(1/2)+2/63*a*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(
a+a*sin(f*x+e))^(1/2)-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*sin(f*
x+e))^(1/2)+8/315*(5*c-d)*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3060, 2849, 2840, 2830, 2725}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$=\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{4d(c+d)(-9Ad+Bc-8Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105af}$$
$$+\frac{8(5c-d)(c+d)(-9Ad+Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}$$
$$-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (4*a*(c + d)*(B*c - 9*A*d - 8*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/
(315*d*f*Sqrt[a + a*Sin[e + f*x]]) + (8*(5*c - d)*(c + d)*(B*c - 9*A*d - 8*
B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*d*(c + d)*(B*c - 9
*A*d - 8*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*a*f) + (2*a*(B*
c - 9*A*d - 8*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{(9aAd-B(ac-8ad))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3\,dx}{9ad} \\
&= \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{(2(c+d)(9aAd-B(ac-8ad)))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\,dx}{21ad}
\end{aligned}
$$

$$= \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105af} + \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} + \frac{(4(c+d)(9aAd-B(ac-8ad)))\int\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(5c^2+3d^2)+a(5c-d)d\sin(e+fx)\right.}{105a^2d}$$

$$= \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} + \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105af} + \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} + \frac{(2(c+d)(15c^2+10cd+7d^2)(9aAd-B(ac-8ad)))\int\sqrt{a+a\sin(e+fx)}\,dx}{315ad}$$

$$= \frac{4a(c+d)(Bc-9Ad-8Bd)(15c^2+10cd+7d^2)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} + \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} + \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105af} + \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.19

$$\int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(2520Ac^3+1680Bc^3+5040Ac^2d+4788Bc^2}{}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^3,x]
```

[Out] -1/1260*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
2520*A*c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 4104
*B*c*d^2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^2 +
162*c*d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*B*c^
3*Sin[e + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x] + 20
16*A*c*d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e + f*x
] + 752*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3*Sin[3*
(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]))

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(35B\left(\cos^4(fx+e)\right)d^3+\left(-45A\,d^3-135d^2cB-40d^3B\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+\left(-189d^2cA-54A\,d^3-\right.}{}$
parts	$\frac{2A\,c^3(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2c^2(3dA+Bc)(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2d^2(dA+3Bc)(1+\sin(fx+\right.}{}$

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)

[Out] 2/315*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d^3+(-45*A*d^3-135
*B*c*d^2-40*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-189*A*c*d^2-54*A*d^3-189*B*c^2
*d-162*B*c*d^2-118*B*d^3)*cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^3+1
05*B*c^3+252*B*c^2*d+351*B*c*d^2+104*B*d^3)*sin(f*x+e)+315*A*c^3+630*c^2*d*
A+693*d^2*c*A+198*A*d^3+210*B*c^3+693*c^2*d*B+594*d^2*c*B+211*d^3*B)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.82

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{2\left(35\,Bd^3\cos\left(fx+e\right)^5-5\left(27\,Bcd^2+(9\,A+B)d^3\right)\cos\left(fx+e\right)^4+105\,(3\,A+B)c^3+63\,(5\,A+7\,B)c^2\right.}{}$$

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")

[Out] -2/315*(35*B*d^3*cos(f*x + e)^5 - 5*(27*B*c*d^2 + (9*A + B)*d^3)*cos(f*x +
e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A + 27*B)*c*d^2 + (

```
81*A + 107*B)*d^3 - (189*B*c^2*d + 27*(7*A + 6*B)*c*d^2 + 2*(27*A + 59*B)*d
^3)*cos(f*x + e)^3 + (105*B*c^3 + 63*(5*A + B)*c^2*d + 9*(7*A + 36*B)*c*d^2
 + 2*(54*A + 13*B)*d^3)*cos(f*x + e)^2 + (105*(3*A + 2*B)*c^3 + 63*(10*A +
11*B)*c^2*d + 99*(7*A + 6*B)*c*d^2 + (198*A + 211*B)*d^3)*cos(f*x + e) - (3
5*B*d^3*cos(f*x + e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A
 + 27*B)*c*d^2 + (81*A + 107*B)*d^3 + 5*(27*B*c*d^2 + (9*A + 8*B)*d^3)*cos(
f*x + e)^3 - 3*(63*B*c^2*d + 9*(7*A + B)*c*d^2 + (3*A + 26*B)*d^3)*cos(f*x
+ e)^2 - (105*B*c^3 + 63*(5*A + 4*B)*c^2*d + 9*(28*A + 39*B)*c*d^2 + 13*(9*
A + 8*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f
*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$=\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}(A+B\sin\left(e+fx\right))\left(c+d\sin\left(e+fx\right)\right)^3\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**3, x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$=\int (B\sin\left(fx+e\right)+A)\sqrt{a\sin\left(fx+e\right)+a}(d\sin\left(fx+e\right)+c)^3\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(236) = 472$.

Time = 0.40 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.15

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi +
9/2*f*x + 9/2*e) + 630*(8*A*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 12*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c*d^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*B*c^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 6*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(6*B*c^2*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 3*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*
pi + 5/2*f*x + 5/2*e) + 45*(6*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 2*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx\\&=\int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^3\,dx\end{aligned}$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,
 x)
```

3.287 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 192

$$\begin{aligned}\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= \frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{105df\sqrt{a+a\sin(e+fx)}} \\ &\quad+\frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} \\ &\quad+\frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} \\ &\quad-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

```
[Out] 2/35*d*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+2/105*a*(-7
*A*d+B*c-6*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/2)
-2/7*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)+4/105*(5*
c-d)*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3060, 2840, 2830, 2725}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{2a(15c^2+10cd+7d^2)\,(-7Ad+Bc-6Bd)\cos(e+fx)}{105df\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{2d(-7Ad+Bc-6Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}$$
$$+\frac{4(5c-d)(-7Ad+Bc-6Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}$$
$$-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*(B*c - 7*A*d - 6*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(105*d*f
*Sqrt[a + a*Sin[e + f*x]]) + (4*(5*c - d)*(B*c - 7*A*d - 6*B*d)*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(105*f) + (2*d*(B*c - 7*A*d - 6*B*d)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{(7aAd-B(ac-6ad))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\,dx}{7ad} \\
&= \frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} \\
&\quad-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{(2(7aAd-B(ac-6ad)))\int\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(5c^2+3d^2)+a(5c-d)d\sin(e+fx)\right)\,dx}{35a^2d} \\
&= \frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} \\
&\quad+\frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} \\
&\quad-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{((15c^2+10cd+7d^2)(7aAd-B(ac-6ad)))\int\sqrt{a+a\sin(e+fx)}\,dx}{105ad} \\
&= \frac{2a(Bc-7Ad-6Bd)(15c^2+10cd+7d^2)\cos(e+fx)}{105df\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} \\
&\quad+\frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} \\
&\quad-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(420Ac^2+280Bc^2+560Acd+532Bcd+266\ldots}{21\ldots}$$

[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2,x]

[Out] -1/210*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
20*A*c^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*d*
(14*B*c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140*c^
2 + 224*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]))

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(-15B\left(\cos^2(fx+e)\right)\sin(fx+e)d^2+\left(-21A\,d^2-42cdB-18d^2B\right)\left(\cos^2(fx+e)\right)+\left(70Acd+28A\,d^2+35B\ldots\right.}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2A\,c^2(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2c(2dA+Bc)(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2d(dA+2Bc)(1+\sin(fx+e)\ldots}{15\,\mathrm{co}\ldots}$

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)

[Out] 2/105*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)*d^2+(-
21*A*d^2-42*B*c*d-18*B*d^2)*cos(f*x+e)^2+(70*A*c*d+28*A*d^2+35*B*c^2+56*B*c
*d+39*B*d^2)*sin(f*x+e)+105*A*c^2+140*A*c*d+77*A*d^2+70*B*c^2+154*c*d*B+66*
d^2*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.59

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{2\left(15\,Bd^2\cos\left(fx+e\right)^4+3\left(14\,Bcd+(7\,A+6\,B)d^2\right)\cos\left(fx+e\right)^3-35\left(3\,A+B\right)c^2-14\left(5\,A+7\,B\right)cd-\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x + e
)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c^2
 + 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)*c^
2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2*cos
(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^2 - 3
*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*B)*c*d
 + (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^2\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**2, x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^2\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.81

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{\sqrt{2}\left(15\,Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+105\,\left(8\,Ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 105*(8*A*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 8*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*si
n(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(4*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*
(4*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi +
5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^2\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
 x)
```

3.288 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 118

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af}$$

```
[Out] -2/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*A*c+5*A*d+5*B*c+7
*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/15*(5*A*d+5*B*c-2*B*d)*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3047, 3102, 2830, 2725}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{2(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f} - \frac{2a(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(15*A*c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(15*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \sqrt{a+a\sin(e+fx)}\left(Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)\right)\,dx$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} + \frac{2\int\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(5Ac+3Bd)+\frac{1}{2}a(5Bc+5Ad-2Bd)\sin(e+fx)\right)\,dx}{5a}$$

$$
\begin{aligned}
&= -\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} \\
&\quad -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} \\
&\quad +\frac{1}{15}(15Ac+5Bc+5Ad+7Bd)\int\sqrt{a+a\sin(e+fx)}\,dx \\
&= -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} \\
&\quad -\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} \\
&\quad -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$
\int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(30Ac+20Bc+20Ad+19Bd-3Bd\cos(2(}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}
$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
),x]
[Out] -1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30
*A*c + 20*B*c + 20*A*d + 19*B*d - 3*B*d*Cos[2*(e + f*x)] + 2*(5*B*c + 5*A*d
 + 4*B*d)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(3B\left(\sin^{2}(fx+e)\right)d+5A\sin(fx+e)d+5B\sin(fx+e)c+4B\sin(fx+e)d+15Ac+10dA+10Bc+8dB\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2Ac(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(dA+Bc)(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2dB(1+\sin(fx+e))a(\sin(fx+}{15\cos(fx+}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+5
*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*d*A+10*B*c+8*d*B)/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{2\left(3\,Bd\cos\left(fx+e\right)^{3}-\left(5\,Bc+\left(5\,A+B\right)d\right)\cos\left(fx+e\right)^{2}-5\left(3\,A+B\right)c-\left(5\,A+7\,B\right)d-\left(5\left(3\,A+2\,B\right)\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*B*d*cos(f*x + e)^3 - (5*B*c + (5*A + B)*d)*cos(f*x + e)^2 - 5*(3*A
+ B)*c - (5*A + 7*B)*d - (5*(3*A + 2*B)*c + (10*A + 11*B)*d)*cos(f*x + e) -
 (3*B*d*cos(f*x + e)^2 - 5*(3*A + B)*c - (5*A + 7*B)*d + (5*B*c + (5*A + 4*
B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.58

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{\sqrt{2}\left(3\,Bd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)+30\left(2\,Ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*f
*x + 5/2*e) + 30*(2*A*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*c*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*
B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x +
 3/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)), x
)
```

3.289 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

```
[Out] -2/3*a*(3*A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*B*cos(f*x+e)*(a+a*si
n(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2830, 2725}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(3A+B)\int\sqrt{a+a\sin(e+fx)}\,dx\\ &= -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\begin{aligned}&\int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx\\ &=-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(3A+2B+B\sin(e+fx))}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\end{aligned}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*A +
 2*B + B*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(B\sin(fx+e)+3A+2B)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	58
parts	$\frac{2A(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	96

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2\left(B\cos\left(fx+e\right)^{2}+\left(3\,A+2\,B\right)\cos\left(fx+e\right)+\left(B\cos\left(fx+e\right)-3\,A-B\right)\sin\left(fx+e\right)+3\,A+B\right)\sqrt{a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(B*cos(f*x + e)^2 + (3*A + 2*B)*cos(f*x + e) + (B*cos(f*x + e) - 3*A -
 B)*sin(f*x + e) + 3*A + B)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$=\frac{\sqrt{2}\left(B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+3\left(2A\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\mathrm{sg}\right.}{3f}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x +
3/2*e) + 3*(2*A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$=\int (A+B\sin(e+fx))\sqrt{a+a\sin(e+fx)}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 100

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}f} - \frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e)
)^(1/2))*a^(1/2)/d^(3/2)/f/(c+d)^(1/2)-2*a*B*cos(f*x+e)/d/f/(a+a*sin(f*x+e)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3060, 2852, 214}

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}} - \frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{(-aBc+aAd)\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{ad}\\
&= -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{(2a(Bc-Ad))\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{df}\\
&= \frac{2\sqrt{a}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}f}-\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.75 (sec) , antiderivative size = 903, normalized size of antiderivative = 9.03

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{(2-2i)B\sqrt{d}\cos\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)}{f}+\frac{(-Bc+Ad)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\frac{\operatorname{RootSum}\left[-d+2ice^{ie}\#1^2+de^{2ie}\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x]),x]
```

```
[Out] ((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f +
 ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d +
 (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*
f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt
[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I
)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)
/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sq
rt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*
(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e
]))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((-(
B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] +
(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt
[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + S
qrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #
1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f
*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 +
 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*
e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/(Sqr
t[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((2 - 2*I)*B
*Sqrt[d]*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2])/f)*Sqrt[a*(1 + Sin[e + f*x])])
/(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(A \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)ad-B \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)ac+\sqrt{-a(\sin(fx+}\right.}{d\sqrt{a(c+d)d}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(A*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a*d-B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a*c+(-a*(sin(f*x+e)-1))^(1/2)*B*(a*(c+d)*d)^(1/2))/d/(a*(c+d)*
d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 651, normalized size of antiderivative = 6.51

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left[-\frac{(Bc-Ad+(Bc-Ad)\cos(fx+e)+(Bc-Ad)\sin(fx+e))\sqrt{\frac{a}{cd+d^2}}\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2\,acd-ad^2-}{}\right.}{}\right.$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*cos(f*x + e) - B*sin(f
*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))
```

```
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f*x + e) - B*si
n(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{d\sin(fx+e)+c}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{\sqrt{2}\left(\frac{2\,B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d}+\frac{\sqrt{2}\left(Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{-cd-d^2}d}\right.}{f}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] sqrt(2)*(2*B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/d + sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/s
qrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{c+d\,\sin(e+f\,x)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),
 x)
```

3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 126

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{a}(Ad+B(c+2d))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

```
[Out] -(A*d+B*(c+2*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/
f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3059, 2852, 214}

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))} - \frac{\sqrt{a}(Ad+B(c+2d))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] -((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])]))/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c -
A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad + -\frac{(-aAd-B(ac+2ad))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2d(ac+ad)} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad - \frac{(a(Ad+B(c+2d)))\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{d(c+d)f}
\end{aligned}$$

$$= -\frac{\sqrt{a}(Ad + B(c+2d))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.02 (sec) , antiderivative size = 901, normalized size of antiderivative = 7.15

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{a(1+\sin(e+fx))}\left(\frac{(Ad+B(c+2d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\frac{\text{RootSum}\left[-d+2ice^{ie}\#1^2+de^{2ie}\#1^4\&,\frac{(1+i)\cdots}{\cdots}\right]\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] + I
*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((
2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)
*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqr
t[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
 + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sq
rt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)
*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(Cos[e] + I
*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(Cos[e/2] +
 I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E
```

```
^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 +
 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*
#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*
#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^
((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*
E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e
] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(3/2)*(Cos[e] + I*
(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqrt[d]*(-(B*c) + A*d)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x]))))/
(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(110) = 220$.

Time = 0.84 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.17

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)ad(dA+Bc+2dB)+A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a}{d(c+d)(c+d\sin(fx+e))}$

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(A*d+B*c+2*B*d)+A*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+B*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a*c^2+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a*c*d+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d-B*(a-a*sin(f*x
+e))^(1/2)*(a*(c+d)*d)^(1/2)*c)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(110) = 220$.

Time = 0.85 (sec) , antiderivative size = 1012, normalized size of antiderivative = 8.03

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*cos
(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c*d
```

```
 + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*
c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*
cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
+ 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*c
os(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B*c - A*d)*co
s(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 +
d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 +
 d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*
x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d
^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3
*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e)
 - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c - A*d + (B*c -
A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c
*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*
c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)
*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(Bc\text{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+Ad\text{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+2Bd\text{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\arctan\left(\frac{\sqrt{2}d\sin(-\frac{1}{4}\pi+\frac{1}{2}f}{\sqrt{-cd-d^2}}\right.}{(cd+d^2)\sqrt{-cd-d^2}}\right.}{2f}$$

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c*d + d^2)*sqrt(-c*d - d^2)) - 2*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)*(c*d + d^2)))/f

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\sin(e+fx))\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx$$

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^2, x)

3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 192

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx &= -\frac{\sqrt{a}(3Ad+B(c+4d))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f}\\ &\quad+\frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\\ &\quad-\frac{a(3Ad+B(c+4d))\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\end{aligned}$$

```
[Out] -1/4*(3*A*d+B*(c+4*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*
sin(f*x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e
)/d/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/4*a*(3*A*d+B*(c+4*d
))*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used

$= \{3059, 2851, 2852, 214\}$

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= -\frac{\sqrt{a}(3Ad+B(c+4d))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}} - \frac{a(3Ad+B(c+4d))\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))} + \frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] -1/4*(Sqrt[a]*(3*A*d + B*(c + 4*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(5/2)*f) + (a*(B*
c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e
 + f*x])^2) - (a*(3*A*d + B*(c + 4*d))*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[
a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad + -\frac{(-3aAd-B(ac+4ad))\int\frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx}{4d(ac+ad)} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad - \frac{a(3Ad+B(c+4d))\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad + \frac{(3Ad+B(c+4d))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{8d(c+d)^2} \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad - \frac{a(3Ad+B(c+4d))\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad - \frac{(a(3Ad+B(c+4d)))\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{4d(c+d)^2f} \\
&= -\frac{\sqrt{a}(3Ad+B(c+4d))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} \\
&\quad + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad - \frac{a(3Ad+B(c+4d))\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.53 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.04

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{a(1+\sin(e+fx))}\left(\frac{(3Ad+B(c+4d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\frac{\text{RootSum}\left[-d+2ice^{ie}\#1^2+de^{2ie}\#1^4\&,\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*(((3*A*d + B*(c + 4*d))*(Cos[e/2]
 + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*
E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^(
(-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)
*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d
]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((
I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))
*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2)*(Cos[e]
 + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4*d))*(Cos[
e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*
I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x
+ (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d
]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*
c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/S
qrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c
 + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt
```

```
[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(5/2)*(Cos[e
] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*Sqrt[d]*(-(B*c)
+ A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x
])^2) - ((2 - 2*I)*Sqrt[d]*(3*A*d + B*(c + 4*d))*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(d^(3/2)*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(168) = 336$.

Time = 1.09 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.27

method	result
default	$\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2d^2(3dA+Bc+4dB)\left(\cos^2(fx+e)\right)-2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2cd(3dA+Bc+4dB)+3A\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/a*(arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(3*A*d
+B*c+4*B*d)*cos(f*x+e)^2-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c
*d+a*d^2)^(1/2))*a^2*c*d*(3*A*d+B*c+4*B*d)+3*A*(a-a*sin(f*x+e))^(3/2)*(a*(c
+d)*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3
+B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^(3/2)*
(a*(c+d)*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1
/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^
2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-4*B*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*(a-a*sin(f
*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)
^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-3*B*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d
)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(
c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(168) = 336.

Time = 1.29 (sec) , antiderivative size = 1750, normalized size of antiderivative = 9.11

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
- (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c
*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d
^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A +
3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d +
d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*
d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x +
e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 +
(c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(
c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
 + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^
2 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2
*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3
*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*
d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 +
d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*c
os(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
 + 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(
f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))
, 1/8*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 -
 (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c*
d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d^
2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3
*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d +
d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-
a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2
 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2*
A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3*
A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d
```

```
^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d
^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*co
s(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
+ 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f
*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(d\sin(fx+e)+c)^3}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(168) = 336$.

Time = 0.34 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\left(Bc\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4Bd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.}{\sqrt{-cd-d^2}}\right.}{(c^2d+2cd^2+d^3)\sqrt{-cd-d^2}}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*
A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))
/((c^2*d + 2*c*d^2 + d^3)*sqrt(-c*d - d^2)) + 2*(2*B*c*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*A*d^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + B*c^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*c*d*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*c*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*d^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*d^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c^2*d
 + 2*c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
3, x)
```

3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 374

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{8a(5c-d)(c+d)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465df}$$
$$+\frac{4(c+d)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f}$$
$$+\frac{2a^2(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df}$$

```
[Out] 4/1155*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f
*x+e))^(3/2)/f+4/3465*a^2*(c+d)*(15*c^2+10*c*d+7*d^2)*(11*A*(c-17*d)*d-3*B*
(c^2-9*c*d+56*d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/693*a^2*(11*A
*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+
a*sin(f*x+e))^(1/2)+2/99*a^2*(3*B*(c-4*d)-11*A*d)*cos(f*x+e)*(c+d*sin(f*x+e
))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)+8/3465*a*(5*c-d)*(c+d)*(11*A*(c-17*d)*d-3
*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a*B*cos(f
*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3055, 3060, 2849, 2840, 2830, 2725}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{2a^2(11Ad(c-17d)-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}} + \frac{4a^2(c+d)\,(15c^2+10cd+7d^2)\,(11Ad(c-17d)-3B(c^2-9cd+56d^2))\cos(e+fx)}{3465d^2f\sqrt{a\sin(e+fx)+a}} + \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a\sin(e+fx)+a}} + \frac{4(c+d)\,(11Ad(c-17d)-3B(c^2-9cd+56d^2))\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{1155f} + \frac{8a(5c-d)(c+d)\,(11Ad(c-17d)-3B(c^2-9cd+56d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465df} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (4*a^2*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*
c*d + 56*d^2))*Cos[e + f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (8*a*(
5*c - d)*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f) + (4*(c + d)*(11*A*(c - 17*d)*d -
3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*
f) + (2*a^2*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(
c + d*Sin[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(3*B*(
c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[
e + f*x])^4)/(11*d*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df} + \frac{2\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3\left(\frac{1}{2}a(11Ad+B(c+8d))-\frac{1}{2}a(3B(c-4d)-11Ad)\sin(e+\right.}{11d}$$

$$= \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df} - \frac{(a(11A(c-17d)d-3B(c^2-9cd+56d^2)))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3\,dx}{99d^2}$$

$$= \frac{2a^2(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df} - \frac{(2a(c+d)(11A(c-17d)d-3B(c^2-9cd+56d^2)))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\,d}{231d^2}$$

$$= \frac{4(c+d)(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f} + \frac{2a^2(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df} - \frac{(4(c+d)(11A(c-17d)d-3B(c^2-9cd+56d^2)))\int\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(5c^2+3d^2)+a(5c\right.}{1155d^2}$$

$$= \frac{8a(5c-d)(c+d)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465df}$$
$$+ \frac{4(c+d)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f}$$
$$+ \frac{2a^2(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+ \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}}$$
$$- \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df}$$
$$- \frac{\left(2a(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\right)\int\sqrt{a+a\sin(e+fx)}\,dx}{3465d^2}$$

$$= \frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+ \frac{8a(5c-d)(c+d)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465df}$$
$$+ \frac{4(c+d)\left(11A(c-17d)d-3B(c^2-9cd+56d^2)\right)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f}$$
$$+ \frac{2a^2(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+ \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}}$$
$$- \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df}$$

Mathematica [A] (verified)

Time = 4.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.04

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(92400Ac^3+72072Bc^3+216216Ac^2d+195624$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] -1/27720*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(92400*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A*c
*d^2 + 177474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 + 35
```

```
1*c*d + 137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*Cos[2
*(e + f*x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 18480*A*
c^3*Sin[e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f*x] +
100188*B*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B*c*d^2*
Sin[e + f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x] - 5940*B
*c^2*d*Sin[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*c*d^2*Sin
[3*(e + f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e + f*x)] +
 315*B*d^3*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.83

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(315B(\cos^4(fx+e))\sin(fx+e)d^3+(385A\,d^3+1155d^2cB+735d^3B)(\cos^4(fx+e))+(-1485d^2cA-9}{}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)d^2(dA+3Bc)(35(\sin^4(fx+e))+85(\sin^3(fx+e))+102(\sin^2(fx+e))+136\sin(fx+e)+272)}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1}{}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^3
+(385*A*d^3+1155*B*c*d^2+735*B*d^3)*cos(f*x+e)^4+(-1485*A*c*d^2-935*A*d^3-1
485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-2079*A*c^2*d
-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*B*c^2*d-5676*B*c*d^2-2478*B*d^3)*co
s(f*x+e)^2+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3+6633
*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+5775*A*c^3+14553*c^2*d*A+14157
*d^2*c*A+4499*A*d^3+4851*B*c^3+14157*c^2*d*B+13497*d^2*c*B+4431*d^3*B)/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{2\left(315\,Bad^3\cos\left(fx+e\right)^6+35\left(33\,Bacd^2+(11\,A+21\,B)ad^3\right)\cos\left(fx+e\right)^5+924\,(5\,A+3\,B)ac^3+396\,(\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d^
3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 132
```

```
*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33*(9
*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*c^3
+ 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1239*B
)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a*c^2*d
 + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e)^2 + (
231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A + 409*B)*a
*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(f*x + e)^
5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A + 47*B)*a
*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12*B)*a*d^3)*
cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (187*A + 294*
B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c^2*d + 33*(24
*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 + (231*(5*A + 9*
B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c*d^2 + 17*(143*
A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*c
os(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int \left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^3\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x))**3, x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{3}{2}}(d\sin\left(fx+e\right)+c)^3\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(350) = 700$.

Time = 0.45 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.02

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/4*
pi + 11/2*f*x + 11/2*e) + 6930*(24*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 16*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*A*a*c^2*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 42*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 42*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 11*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f
*x + 1/2*e) + 2310*(8*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*B*a*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*A*a*c^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 30*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*A*a
*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a*c*d^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 10*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a
*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) +
693*(8*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*a*c^2*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 36*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 13*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x
 + 5/2*e) + 495*(12*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*
c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a*c*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 6*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a*d
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e) + 38
5*(6*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*si
n(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\left(c+d\,\sin\left(e+f\,x\right)\right)^3\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,
 x)
```

3.294 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 294

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{2a^2(15c^2+10cd+7d^2)\,(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)}{315d^2f\sqrt{a+a\sin(e+fx)}}+\frac{4a(5c-d)\,(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315df}+\frac{2(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f}+\frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df}$$

```
[Out] 2/105*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/
2)/f+2/315*a^2*(15*c^2+10*c*d+7*d^2)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*
cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/63*a^2*(-9*A*d+3*B*c-10*B*d)*cos(
f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+4/315*a*(5*c-d)*(3*A
*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/9
*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3055, 3060, 2840, 2830, 2725}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\frac{2a^2(15c^2+10cd+7d^2)\,(3Ad(c-13d)-B(c^2-7cd+34d^2))\cos(e+fx)}{315d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2(3Ad(c-13d)-B(c^2-7cd+34d^2))\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105f}+\frac{4a(5c-d)\,(3Ad(c-13d)-B(c^2-7cd+34d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315df}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (2*a^2*(15*c^2 + 10*c*d + 7*d^2)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^
2))*Cos[e + f*x])/(315*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (4*a*(5*c - d)*(3*
A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(315*d*f) + (2*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(3*B*c - 9*A*d - 10*B*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d^2*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*
f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df} \\
&\quad+\frac{2\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\left(\frac{1}{2}a(9Ad+B(c+6d))-\frac{1}{2}a(3Bc-9Ad-10Bd)\sin(e+f\right.}{9d} \\
&= \frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df} \\
&\quad-\frac{(a(3A(c-13d)d-B(c^2-7cd+34d^2)))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\,dx}{21d^2}
\end{aligned}$$

$$= \frac{2(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df} - \frac{(2(3A(c-13d)d-B(c^2-7cd+34d^2)))\int\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(5c^2+3d^2)+a(5c-d)d\sin(e\right.}{105d^2}$$

$$= \frac{4a(5c-d)(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315df} + \frac{2(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df} - \frac{(a(15c^2+10cd+7d^2)(3A(c-13d)d-B(c^2-7cd+34d^2)))\int\sqrt{a+a\sin(e+fx)}\,dx}{315d^2}$$

$$= \frac{2a^2(15c^2+10cd+7d^2)(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)}{315d^2f\sqrt{a+a\sin(e+fx)}} + \frac{4a(5c-d)(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315df} + \frac{2(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df}$$

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(4200Ac^2+3276Bc^2+6552Acd+5928Bcd+2}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(4200*A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B*d
^2 - 4*(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e + f*
x)] + 35*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*Sin[e
 + f*x] + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d^2*Si
n[e + f*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 90*A*d^
2*Sin[3*(e + f*x)] - 170*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(35B\left(\cos^4(fx+e)\right)d^2+\left(-45A\,d^2-90cdB-85d^2B\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+\left(-126Acd-117A\,d^2\right.}{}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)d(dA+2Bc)\left(15\left(\sin^3(fx+e)\right)+39\left(\sin^2(fx+e)\right)+52\sin(fx+e)+104\right)}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+\sin(fx+e))a^2(\sin(f}{5\,c}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d^2+(-45*A*d^2-9
0*B*c*d-85*B*d^2)*cos(f*x+e)^2*sin(f*x+e)+(-126*A*c*d-117*A*d^2-63*B*c^2-23
4*B*c*d-172*B*d^2)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2+40
2*B*c*d+221*B*d^2)*sin(f*x+e)+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+858*c
*d*B+409*d^2*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2\left(35\,Bad^2\cos\left(fx+e\right)^5-5\left(18\,Bacd+(9\,A+10\,B)ad^2\right)\cos\left(fx+e\right)^4+84\,(5\,A+3\,B)ac^2+24\,(21\,A+\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a*d^2*cos(f*x + e)^5 - 5*(18*B*a*c*d + (9*A + 10*B)*a*d^2)*cos
(f*x + e)^4 + 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*
B)*a*d^2 - (63*B*a*c^2 + 18*(7*A + 13*B)*a*c*d + (117*A + 172*B)*a*d^2)*cos
(f*x + e)^3 + (21*(5*A + 6*B)*a*c^2 + 6*(42*A + 43*B)*a*c*d + (129*A + 134*
```

```
B)*a*d^2)*cos(f*x + e)^2 + (21*(25*A + 21*B)*a*c^2 + 6*(147*A + 143*B)*a*c*
d + (429*A + 409*B)*a*d^2)*cos(f*x + e) - (35*B*a*d^2*cos(f*x + e)^4 + 84*(
5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*B)*a*d^2 + 5*(18*B
*a*c*d + (9*A + 17*B)*a*d^2)*cos(f*x + e)^3 - 3*(21*B*a*c^2 + 6*(7*A + 8*B)
*a*c*d + (24*A + 29*B)*a*d^2)*cos(f*x + e)^2 - (21*(5*A + 9*B)*a*c^2 + 6*(6
3*A + 67*B)*a*c*d + (201*A + 221*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\int (a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x))**2, x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^2\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
+ 9/2*f*x + 9/2*e) + 630*(12*A*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
8*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 16*A*a*c*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 14*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*A*
a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*A*a*c^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
12*A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 5*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*
a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) +
 126*(2*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*a*c*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(4*B*a*c*d*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 3*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7
/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,
 x)
```

3.295 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 165

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af}$$

```
[Out] -2/35*(7*A*d+7*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*d*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f-8/105*a^2*(35*A*c+21*A*d+21*B*c+19*B*d)*co
s(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a*(35*A*c+21*A*d+21*B*c+19*B*d)*cos
(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3102, 2830, 2726, 2725}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{8a^2(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}} - \frac{2(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f} - \frac{2a(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*a*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(105*f) - (2*(7*B*c + 7*A*d - 2*B*d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(7*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int (a+a\sin(e+fx))^{3/2}\left(Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)\right)\,dx$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af} + \frac{2\int (a+a\sin(e+fx))^{3/2}\left(\frac{1}{2}a(7Ac+5Bd)+\frac{1}{2}a(7Bc+7Ad-2Bd)\sin(e+fx)\right)\,dx}{7a}$$

$$= -\frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af} + \frac{1}{35}(35Ac+21Bc+21Ad+19Bd)\int (a+a\sin(e+fx))^{3/2}\,dx$$

$$= -\frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af} + \frac{1}{105}(4a(35Ac+21Bc+21Ad+19Bd))\int \sqrt{a+a\sin(e+fx)}\,dx$$

$$= -\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af}$$

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(700Ac+546Bc+546Ad+494Bd-6(7Bc+}{210f\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(700*A*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2*(
e + f*x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*d*S
in[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(15B(\sin^3(fx+e))d+21A(\sin^2(fx+e))d+21B(\sin^2(fx+e))c+39B(\sin^2(fx+e))d+35A\sin(fx+e}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(dA+Bc)(\sin^2(fx+e)+3\sin(fx+e)+6)}{5\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Ac(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*B*sin(f*x+e)^3*d+21*A*sin(f*x+e
)^2*d+21*B*sin(f*x+e)^2*c+39*B*sin(f*x+e)^2*d+35*A*sin(f*x+e)*c+63*A*sin(f*
x+e)*d+63*B*sin(f*x+e)*c+52*B*sin(f*x+e)*d+175*A*c+126*d*A+126*B*c+104*d*B)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.56

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{2\left(15\,Bad\cos\left(fx+e\right)^{4}+3\left(7\,Bac+(7\,A+13\,B)ad\right)\cos\left(fx+e\right)^{3}-28\,(5\,A+3\,B)ac-$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x + e
)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42*A
 + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*d)*
cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A + 19
*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B)*a*c
 + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{3}{2}}(d\sin\left(fx+e\right)+c)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.73

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{\sqrt{2}\left(15\,Bad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+105\,(12\,Aac\mathrm{sgn}(\cos$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 105*(12*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B*a
*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 7*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 35*(4*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 5*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x +
 3/2*e) + 21*(2*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)), x
)
```

3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}$$

```
[Out] -2/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-8/15*a^2*(5*A+3*B)*cos(f*x+e)/f/
(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2830, 2726, 2725}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}} - \frac{2a(5A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(5*A + 3*B)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*A + 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*B*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5A+3B)\int(a+a\sin(e+fx))^{3/2}\,dx\\
&= -\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}\\
&\quad-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}\\
&\quad+\frac{1}{15}(4a(5A+3B))\int\sqrt{a+a\sin(e+fx)}\,dx\\
&= -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}\\
&\quad-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(50A+39B-3B\cos(2(e+fx))+2(5A+9B)}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
[Out] -1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
50*A + 39*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 9*B)*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(-3B\left(\cos^2(fx+e)\right)+\sin(fx+e)(5A+9B)+25A+21B\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	77
parts	$\frac{2A(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(\sin^2(fx+e)+3\sin(fx+e)+6\right)}{5\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	118

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2+sin(f*x+e)*(5*A+9
*B)+25*A+21*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \frac{2\left(3\,Ba\cos(fx+e)^3-(5\,A+6\,B)a\cos(fx+e)^2-(25\,A+21\,B)a\cos(fx+e)-4\,(5}{15\,(f\,\mathrm{co}}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
[Out] 2/15*(3*B*a*cos(f*x + e)^3 - (5*A + 6*B)*a*cos(f*x + e)^2 - (25*A + 21*B)*a
*cos(f*x + e) - 4*(5*A + 3*B)*a - (3*B*a*cos(f*x + e)^2 + (5*A + 9*B)*a*cos
(f*x + e) - 4*(5*A + 3*B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(
f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int \left(a(\sin{(e+fx)}+1)\right)^{\frac{3}{2}}\left(A+B\sin{(e+fx)}\right)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(3\,Ba\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)+30\left(3\,Aa\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
[Out] 1/30*sqrt(2)*(3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*f
*x + 5/2*e) + 30*(3*A*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*A*a*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 153

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2a^{3/2}(c-d)(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}f} + \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}$$

```
[Out] -2*a^(3/2)*(c-d)*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/
(a+a*sin(f*x+e))^(1/2))/d^(5/2)/f/(c+d)^(1/2)+2/3*a^2*(-3*A*d+3*B*c-4*B*d)*
cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*B*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3055, 3060, 2852, 214}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2a^{3/2}(c-d)(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}} + \frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (-2*a^(3/2)*(c - d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*
c - 3*A*d - 4*B*d)*Cos[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*
B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(Bc+3Ad)-\frac{1}{2}a(3Bc-3Ad-4Bd)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{3d}$$

$$= \frac{2a^2(3Bc - 3Ad - 4Bd)\cos(e + fx)}{3d^2 f\sqrt{a + a\sin(e + fx)}} - \frac{2aB\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{3df} + \frac{(a(c - d)(Bc - Ad))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{d^2}$$

$$= \frac{2a^2(3Bc - 3Ad - 4Bd)\cos(e + fx)}{3d^2 f\sqrt{a + a\sin(e + fx)}} - \frac{2aB\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{3df} - \frac{(2a^2(c - d)(Bc - Ad))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{d^2 f}$$

$$= -\frac{2a^{3/2}(c - d)(Bc - Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c + d}f} + \frac{2a^2(3Bc - 3Ad - 4Bd)\cos(e + fx)}{3d^2 f\sqrt{a + a\sin(e + fx)}} - \frac{2aB\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{3df}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.35 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.87

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = \frac{(a(1 + \sin(e + fx)))^{3/2}\left(-6\sqrt{d}(-2Bc + 2Ad + 3Bd)\text{co}\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e +
 f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] + (3*(c - d)*(B*c - A*d)*((c +
d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 +
 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]])
 - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f
*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 +
Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqr
t[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]
*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Lo
g[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c +
d)^(3/2) + (3*(c - d)*(B*c - A*d)*(-((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)
/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 &
, (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*
```

x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &]))/(c + d)^(3/2) + 6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*(e + f*x))/2]))/(6*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(131) = 262$.

Time = 1.06 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.91

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3A\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)a^2cd+3A\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)a^2d^2-B(-a(s\right.}{}$

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -2/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*d^2-B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d+3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d+3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d-3*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c+6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(131) = 262.

Time = 0.85 (sec) , antiderivative size = 880, normalized size of antiderivative = 5.75

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \Bigg[-\frac{3\,(Bac^2-(A+B)acd+Aad^2+(Bac^2-(A+B)ac}{}$$

$$-\frac{3\,(Bac^2-(A+B)acd+Aad^2+(Bac^2-(A+B)acd+Aad^2)\cos{(fx+e)}+(Bac^2-(A+B)acd+Aa}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*
a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqr
t(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a
*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)
*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
- 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(
f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*a*d*cos(f*x + e)^2 - 3*
B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d
*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1/3*(3*(B*a*c^2
 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*cos(f*x +
e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt(-a/(c*d + d^2))
*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*
d + d^2))/(a*cos(f*x + e))) + 2*(B*a*d*cos(f*x + e)^2 - 3*B*a*c + (3*A + 4*
B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d*cos(f*x + e) + 3
*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*co
s(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{d\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(131) = 262.

Time = 0.31 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.79

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\left(Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Aacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Aad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.\right.\right.}{\sqrt{-cd-d^2}d^2}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
[Out] -1/3*sqrt(2)*sqrt(a)*(3*sqrt(2)*(B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) - A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*d*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sq
rt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*
d^2) + 2*(2*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 + 3*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) - 6*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e))/d^3)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)),
 x)
```

3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2} dx$

Optimal result

Integrand size = 37, antiderivative size = 191

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2} dx = -\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f} - \frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] -a^(3/2)*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^(
1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^2*(-A*d+3*
B*c+2*B*d)*cos(f*x+e)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)+a*(-A*d+B*c)*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3054, 3060, 2852, 214}

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2} dx = -\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}} - \frac{a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -((a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqr
t[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d
)^(3/2)*f)) - (a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[
a + a*Sin[e + f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} \\
&\quad + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\frac{1}{2}a(Bc-3Ad-2Bd)+\frac{1}{2}a(3Bc-Ad+2Bd)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{d(c+d)} \\
&= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} \\
&\quad + \frac{\left(a\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\right)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2d^2(c+d)} \\
&= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} \\
&\quad - \frac{\left(a^2\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\right)\operatorname{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{d^2(c+d)f} \\
&= -\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f} \\
&\quad - \frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.59 (sec) , antiderivative size = 922, normalized size of antiderivative = 4.83

$$
\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{(a(1+\sin(e+fx)))^{3/2}\left(-8B\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right) + \frac{(Ad(c\cdots}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((A*d*(c + 3
*d) + B*(-3*c^2 - 3*c*d + 2*d^2))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (
-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/
```

4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &]))/(c + d)^(5/2) + ((A*d*(c + 3*d) + B*(-3*c^2 - 3*c*d + 2*d^2))*(-((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &]))/(c + d)^(5/2) + 8*B*Sqrt[d]*Sin[(e + f*x)/2] - (4*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(173) = 346$.

Time = 0.97 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.10

method	result
default	$a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\Big(-\sin(fx+e)d\Big(A\,\mathrm{arctanh}\Big(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\Big)acd+3A\,\mathrm{arctanh}\Big(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\Big)a\,d^2-3B$

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d)-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2+3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^3+3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d^2-3*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2-B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*

```
d)^(1/2)*c*d)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(173) = 346.

Time = 1.00 (sec) , antiderivative size = 1428, normalized size of antiderivative = 7.48

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^
3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a
*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c
^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2 - (A + B)*a*c
*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 -
(A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2
*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x
+ e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d
^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c^3 - (A - 6*B)*a*c^2*d
- (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2
- (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A
 + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*
a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B
)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f
*x + e))) - 2*(3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B
*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e)
 - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x
```

```
 + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f -
((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(173) = 346$.

Time = 0.33 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{\sqrt{2}\left(\frac{4\,Ba\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d^2}+\frac{\sqrt{2}(3\,Bac^2\cdots}{\cdots}\cdots\right)}{\cdots}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/d^2 + sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A
*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*c*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a*d^2
```

```
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^2 + d^3)*sqrt(-c*d - d^2)) - 2*(B*a*c^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a*c*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*a*
c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + A*
a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
(c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{3/2}}{\left(c+d\,\sin(e+f\,x)\right)^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
2, x)
```

3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 221

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(c-5d)d+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

```
[Out] -1/4*a^(3/2)*(A*d*(c+7*d)+3*B*(c^2+3*c*d+4*d^2))*arctanh(cos(f*x+e)*a^(1/2)
*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+1/4*a^2*
(A*(c-5*d)*d+B*(3*c^2+5*c*d-4*d^2))*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e
))/(a+a*sin(f*x+e))^(1/2)+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2
)/d/(c+d)/f/(c+d*sin(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used

$= \{3054, 3059, 2852, 214\}$

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}} + \frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}$$

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/4*(a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*ArcTanh[(Sqrt[a]*
Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c
+ d)^(5/2)*f) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*
(c + d)*f*(c + d*Sin[e + f*x])^2) + (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d
- 4*d^2))*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*
Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\text{integral} = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\frac{1}{2}a(Bc-5Ad-4Bd)+\frac{1}{2}a(3Bc+Ad+4Bd)\sin(e+fx)\right)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(c-5d)d+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(a(Ad(c+7d)+3B(c^2+3cd+4d^2)))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{8d^2(c+d)^2}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(c-5d)d+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(a^2(Ad(c+7d)+3B(c^2+3cd+4d^2)))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{4d^2(c+d)^2f}$$

$$= -\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(c-5d)d+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.84 (sec) , antiderivative size = 957, normalized size of antiderivative = 4.33

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{(a(1+\sin(e+fx)))^{3/2}\left(\frac{(Ad(c+7d)+3B(c^2+3cd+4d^2))\left((c+d)(e+\right.}{}\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))
*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c + 4
*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*
x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan
[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log
[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
+ c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f
*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c
+ d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])
)/(c + d)^(7/2) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(-((c + d)*(
e + f*x - 2*Log[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*
c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) -
d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)
/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan
[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d
]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1
^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-
#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^
(7/2) - (8*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]*(A*d*(c + 7*d) + B*(-5
*c^2 - 7*c*d + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c
 + d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(197) = 394$.

Time = 1.34 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.05

method	result
default	$-\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2d^2\left(Acd+7A\,d^2+3B\,c^2+9cdB+12d^2B\right)\left(\cos^2(fx+e)\right)+2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right.\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*(-arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*d
+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*cos(f*x+e)^2+2*sin(f*x+e)*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9*B*
c*d+12*B*d^2)+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c
^3*d+7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+
A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+7*A*arcta
nh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4-A*(a-a*sin(f*x+e))
^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-7*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*d^3+3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4+9*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d+15*B*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+9*B*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+12*B*arctanh((a-a*sin(
f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+5*B*(a-a*sin(f*x+e))^(3/2)*(a*
(c+d)*d)^(1/2)*c^2*d+7*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-4*B
*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3-A*(a-a*sin(f*x+e))^(1/2)*(a*(
c+d)*d)^(1/2)*a*c^2*d+8*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2+
9*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3-3*B*(a-a*sin(f*x+e))^(1/
2)*(a*(c+d)*d)^(1/2)*a*c^3-12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*
c^2*d-5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2+4*B*(a-a*sin(f*x
+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e)
)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/cos(f*x+e)/(a+a*sin(f*x+
e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(197) = 394$.

Time = 1.49 (sec) , antiderivative size = 2208, normalized size of antiderivative = 9.99

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A
 + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3
+ (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2*d^
2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*a*c^
4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A +
 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*
B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2
 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*a*c^3*d
+ (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^3 + (A + 2*B)*a
*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*B*a*c^2*d - (A - 7*
B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 + (A + 7*B)*a*c
^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x + e) - (3*B*a*c^3 + (A + 2*
B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 - (5*B*a*c^2*d - (A
- 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5
*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d
^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c
*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 +
2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*
d^5 + d^6)*f)*sin(f*x + e)), 1/8*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
+ 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^
2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3
*d + (2*A + 21*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*
cos(f*x + e)^2 + (3*B*a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 +
(A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15
*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12
*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*
x + e)^2 + 2*(3*B*a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos
(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e
) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -
2*(3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^
3 + (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 + (A + 7*B)*a*c^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x +
 e) - (3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*
a*d^3 - (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f
```

```
*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4
*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4
*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(
f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*
c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^3}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(197) = 394$.

Time = 0.36 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.82

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
[Out] -1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 7*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*B
*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi +
```

```
1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c^2*d^2 + 2*c*d^3 + d^4)*sqrt(-c*d - d
^2)) - 2*(10*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - 2*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4
*pi + 1/2*f*x + 1/2*e)^3 + 14*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 14*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 8*B*a*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a*c^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a*c^2*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 12*B*a*c^2*d*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 8*A*a*c*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*B*a
*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 9*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 4*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e))/((c^2*d^2 + 2*c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
c - d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
3, x)
```

3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 534

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e}{45045d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)\sqrt{a+}}{45045d^2f}$$
$$-\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(a+a\sin(e+}{15015df}$$
$$-\frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{9009d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df}$$

```
[Out] -4/15015*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d
^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/13*a*B*cos(f*x+e)*(a+
a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^4/d/f-4/45045*a^3*(c+d)*(15*c^2+10*c*d
+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d
^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/9009*a^3*(13*A*d*(3*c^2-38*c
*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(c+d*sin(f*
x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/1287*a^3*(-39*A*c*d+299*A*d^2+15*B*c
```

```
^2-75*B*c*d+280*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^3/f/(a+a*sin(f*x+e))
^(1/2)-8/45045*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-1
50*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/143
*a^2*(-13*A*d+5*B*c-16*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^
(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3055, 3060, 2849, 2840, 2830, 2725}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{2a^3(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{9009d^3f\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)}{45045d^3f\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{45045d^2f}$$
$$+\frac{2a^2(-13Ad+5Bc-16Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{143d^2f}$$
$$-\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{15015df}$$
$$-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(13*A*d*(3*c^2 - 38*c*d + 355*d^2
) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x])/(45045*d^3
*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(13*A*d*(3*c^2 - 38
*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(45045*d^2*f) - (4*a*(c + d)*(13*A*d*(3*c^2 -
38*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^(3/2))/(15015*d*f) - (2*a^3*(13*A*d*(3*c^2 - 38*c
*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(9009*d^3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*(15
*B*c^2 - 39*A*c*d - 75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(1287*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 13*
```

```
A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)
/(143*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e
+ f*x])^4)/(13*d*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

```
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df} + \frac{2\int(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\left(\frac{1}{2}a(3Bc+13Ad+8Bd)-\frac{1}{2}a(5Bc-13Ad-16Bd)\sin(e}{13d}$$

$$= \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df} + \frac{4\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3\left(\frac{1}{4}a^2(13Ad(c+19d)-B(5c^2-9cd-216d^2))+\frac{1}{4}a^2(}{143d^2}$$

$$= -\frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df} + \frac{(a^2(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)))\int\sqrt{a+a\sin(e+fx)}(c}{1287d^3}$$

$$= -\frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e}{9009d^3f\sqrt{a+a\sin(e+fx)}} - \frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df} + \frac{(2a^2(c+d)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)))\int\sqrt{a+a\sin}}{3003d^3}$$

$$= -\frac{4a(c+d)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(a+}{15015df} - \frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e}{9009d^3f\sqrt{a+a\sin(e+fx)}} - \frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df} + \frac{(4a(c+d)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)))\int\sqrt{a+a\sin(e}}{15015d^3}$$

$$= -\frac{8a^2(5c-d)(c+d)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+}{45045d^2f} - \frac{4a(c+d)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(a+}{15015df} - \frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e}{9009d^3f\sqrt{a+a\sin(e+fx)}} - \frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df} + \frac{(2a^2(c+d)(15c^2+10cd+7d^2)(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-418}{45045d^3}$$

$$= -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d\right.}{45045d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+}{45045d^2f}$$
$$-\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(a+a}{15015df}$$
$$-\frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e}{9009d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 1565, normalized size of antiderivative = 2.93

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{Bd^3\cos\left(\frac{13}{2}(e+fx)\right)(a(1+\sin(e+fx)))^{5/2}}{416f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(40Ac^3+30Bc^3+90Ac^2d+78Bc^2d+78Acd^2+69Bcd^2+23Ad^3+21Bd^3)\left(\left(-\frac{1}{16}-\frac{i}{16}\right)\cos\left(\frac{1}{2}(e+fx)\right)\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(40Ac^3+30Bc^3+90Ac^2d+78Bc^2d+78Acd^2+69Bcd^2+23Ad^3+21Bd^3)\left(\left(-\frac{1}{16}+\frac{i}{16}\right)\cos\left(\frac{1}{2}(e+fx)\right)\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(80Ac^3+88Bc^3+264Ac^2d+240Bc^2d+240Acd^2+228Bcd^2+76Ad^3+71Bd^3)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(80Ac^3+88Bc^3+264Ac^2d+240Bc^2d+240Acd^2+228Bcd^2+76Ad^3+71Bd^3)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(16Ac^3+40Bc^3+120Ac^2d+144Bc^2d+144Acd^2+150Bcd^2+50Ad^3+51Bd^3)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(16Ac^3+40Bc^3+120Ac^2d+144Bc^2d+144Acd^2+150Bcd^2+50Ad^3+51Bd^3)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(4Bc^3+12Ac^2d+30Bc^2d+30Acd^2+39Bcd^2+13Ad^3+15Bd^3)(a(1+\sin(e+fx)))^{5/2}\left(\left(\frac{1}{224}+\frac{i}{224}\right)\mathrm{co}\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(4Bc^3+12Ac^2d+30Bc^2d+30Acd^2+39Bcd^2+13Ad^3+15Bd^3)(a(1+\sin(e+fx)))^{5/2}\left(\left(\frac{1}{224}-\frac{i}{224}\right)\mathrm{co}\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(6Bc^2+6Acd+15Bcd+5Ad^2+7Bd^2)(a(1+\sin(e+fx)))^{5/2}\left(\left(-\frac{1}{288}-\frac{i}{288}\right)d\cos\left(\frac{9}{2}(e+fx)\right)+\left(\frac{1}{288}-\right.\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(6Bc^2+6Acd+15Bcd+5Ad^2+7Bd^2)(a(1+\sin(e+fx)))^{5/2}\left(\left(-\frac{1}{288}+\frac{i}{288}\right)d\cos\left(\frac{9}{2}(e+fx)\right)+\left(\frac{1}{288}+\right.\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(6Bc+2Ad+5Bd)(a(1+\sin(e+fx)))^{5/2}\left(\left(-\frac{1}{704}+\frac{i}{704}\right)d^2\cos\left(\frac{11}{2}(e+fx)\right)-\left(\frac{1}{704}+\frac{i}{704}\right)d^2\sin\left(\frac{11}{2}(e+\right.\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$+\frac{(6Bc+2Ad+5Bd)(a(1+\sin(e+fx)))^{5/2}\left(\left(-\frac{1}{704}-\frac{i}{704}\right)d^2\cos\left(\frac{11}{2}(e+fx)\right)-\left(\frac{1}{704}-\frac{i}{704}\right)d^2\sin\left(\frac{11}{2}(e+\right.\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$
$$-\frac{Bd^3(a(1+\sin(e+fx)))^{5/2}\sin\left(\frac{13}{2}(e+fx)\right)}{416f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] (B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78*B
*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*Cos
[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2
))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90
*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1
/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3
+ 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d
^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos[(3*(e + f
*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240
*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[(3*(e + f*x))/
2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 +
120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3
)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e + f*x))/2] - (1/3
20 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150
*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 + I/32
0)*Cos[(5*(e + f*x))/2] - (1/320 - I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d + 30*B*c^2*d + 3
0*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((1/224 + I/224)*Cos[(7*(e + f*x))/2] + (1/224 - I/224)*Sin[(7*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d +
30*B*c^2*d + 30*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e +
 f*x]))^(5/2)*((1/224 - I/224)*Cos[(7*(e + f*x))/2] + (1/224 + I/224)*Sin[(
7*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 +
 6*A*c*d + 15*B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/
288 - I/288)*d*Cos[(9*(e + f*x))/2] + (1/288 - I/288)*d*Sin[(9*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 + 6*A*c*d + 15*
B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/288 + I/288)*d
*Cos[(9*(e + f*x))/2] + (1/288 + I/288)*d*Sin[(9*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e
+ f*x]))^(5/2)*((-1/704 + I/704)*d^2*Cos[(11*(e + f*x))/2] - (1/704 + I/704
)*d^2*Sin[(11*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) +
 ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/704 - I/704)*d^
2*Cos[(11*(e + f*x))/2] - (1/704 - I/704)*d^2*Sin[(11*(e + f*x))/2]))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - (B*d^3*(a*(1 + Sin[e + f*x]))^(5/2
)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] (verified)

Time = 168.51 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(-3465B\left(\cos^6(fx+e)\right)d^3+\left(4095A\,d^3+12285d^2cB+11970d^3B\right)\left(\cos^4(fx+e)\right)\sin(fx+e)+(15015c}{}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)d^2(dA+3Bc)\left(63\left(\sin^5(fx+e)\right)+224\left(\sin^4(fx+e)\right)+355\left(\sin^3(fx+e)\right)+426\left(\sin^2(fx+e)\right)+568\sin(f}{693\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/45045*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-3465*B*cos(f*x+e)^6*d^3+(4095*A
*d^3+12285*B*c*d^2+11970*B*d^3)*cos(f*x+e)^4*sin(f*x+e)+(15015*A*c*d^2+1456
0*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4+(-19305*A*c^2
*d-55770*A*c*d^2-31265*A*d^3-6435*B*c^3-55770*B*c^2*d-93795*B*c*d^2-44860*B
*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^2-568
10*A*d^3-25740*B*c^3-123981*B*c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f*x+e)^
2+(42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3+181038
*B*c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)+138138*A*c^3+373230*c^2*d*A
+359502*d^2*c*A+116090*A*d^3+124410*B*c^3+359502*c^2*d*B+348270*d^2*c*B+113
818*d^3*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.62

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/45045*(3465*B*a^2*d^3*cos(f*x + e)^7 - 315*(39*B*a^2*c*d^2 + (13*A + 27*B
)*a^2*d^3)*cos(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)
*a^2*c^2*d - 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3
- 35*(429*B*a^2*c^2*d + 39*(11*A + 32*B)*a^2*c*d^2 + 4*(104*A + 205*B)*a^2*
d^3)*cos(f*x + e)^5 + 5*(1287*B*a^2*c^3 + 429*(9*A + 19*B)*a^2*c^2*d + 39*(
209*A + 320*B)*a^2*c*d^2 + 2*(2080*A + 2813*B)*a^2*d^3)*cos(f*x + e)^4 + (1
287*(7*A + 20*B)*a^2*c^3 + 429*(180*A + 289*B)*a^2*c^2*d + 39*(3179*A + 437
0*B)*a^2*c*d^2 + (56810*A + 72109*B)*a^2*d^3)*cos(f*x + e)^3 - (429*(77*A +
 85*B)*a^2*c^3 + 429*(255*A + 263*B)*a^2*c^2*d + 39*(2893*A + 2965*B)*a^2*c
*d^2 + (38545*A + 39113*B)*a^2*d^3)*cos(f*x + e)^2 - 2*(429*(161*A + 145*B)
*a^2*c^3 + 429*(435*A + 419*B)*a^2*c^2*d + 39*(4609*A + 4465*B)*a^2*c*d^2 +
```

```
(58045*A + 56909*B)*a^2*d^3)*cos(f*x + e) - (3465*B*a^2*d^3*cos(f*x + e)^6
 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)*a^2*c^2*d - 1248*(143*A
+ 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3 + 315*(39*B*a^2*c*d^2 + (
13*A + 38*B)*a^2*d^3)*cos(f*x + e)^5 - 35*(429*B*a^2*c^2*d + 39*(11*A + 23*
B)*a^2*c*d^2 + (299*A + 478*B)*a^2*d^3)*cos(f*x + e)^4 - 5*(1287*B*a^2*c^3
+ 429*(9*A + 26*B)*a^2*c^2*d + 507*(22*A + 37*B)*a^2*c*d^2 + (6253*A + 8972
*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(429*(7*A + 15*B)*a^2*c^3 + 429*(45*A + 53*
B)*a^2*c^2*d + 39*(583*A + 655*B)*a^2*c*d^2 + (8515*A + 9083*B)*a^2*d^3)*co
s(f*x + e)^2 + 2*(429*(49*A + 65*B)*a^2*c^3 + 429*(195*A + 211*B)*a^2*c^2*d
 + 39*(2321*A + 2465*B)*a^2*c*d^2 + (32045*A + 33181*B)*a^2*d^3)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e)
 + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)^3\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(506) = 1012.

Time = 0.55 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.90

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/1441440*sqrt(2)*(3465*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
13/4*pi + 13/2*f*x + 13/2*e) + 180180*(40*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 30*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*A*a^2*
c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 78*B*a^2*c^2*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 78*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
69*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*A*a^2*d^3*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 21*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(80*A*a^2*c^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 88*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 264*
A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 240*B*a^2*c^2*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 240*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 228*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 76*A*a^2*d^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 71*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 9009*(16*A*a^2*c^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 40*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 120*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 144*B*a^2*c^2*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 144*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 150*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 50*A*
a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 51*B*a^2*d^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 12870*(4*B*a^2*c^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 30*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*A*a^2
*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 39*B*a^2*c*d^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 13*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
5*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/
2*e) + 10010*(6*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a^2*c
*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*B*a^2*c*d^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 5*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*
a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-9/4*pi + 9/2*f*x + 9/2*e)
 + 4095*(6*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*d^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sin(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^3\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,
 x)
```

3.301 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 429

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2a^3(15c^2+10cd+7d^2)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)}{3465d^3f\sqrt{a+a\sin(e+fx)}} - \frac{4a^2(5c-d)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465d^2f} - \frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155df} + \frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df}$$

```
[Out] -2/1155*a*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))
*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/11*a*B*cos(f*x+e)*(a+a*sin(f*x+e))
^(3/2)*(c+d*sin(f*x+e))^3/d/f-2/3465*a^3*(15*c^2+10*c*d+7*d^2)*(11*A*d*(c^2
-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)/d^3/f/(a+a
*sin(f*x+e))^(1/2)+2/693*a^3*(11*A*(3*c-19*d)*d-B*(15*c^2-65*c*d+194*d^2))*
cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-4/3465*a^2*(5*c-
d)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/99*a^2*(-11*A*d+5*B*c-14*B*d)*cos(f*x+e
)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3055, 3060, 2840, 2830, 2725}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{2a^3(11Ad(3c-19d)-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a\sin(e+fx)+a}} - \frac{2a^3(15c^2+10cd+7d^2)\,(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)}{3465d^3f\sqrt{a\sin(e+fx)+a}} - \frac{4a^2(5c-d)\,(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465d^2f} + \frac{2a^2(-11Ad+5Bc-14Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{99d^2f} - \frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{1155df} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (-2*a^3*(15*c^2 + 10*c*d + 7*d^2)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^
3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x])/(3465*d^3*f*Sqrt[a + a*S
in[e + f*x]]) - (4*a^2*(5*c - d)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3
 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(3465*d^2*f) - (2*a*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d +
 169*c*d^2 - 710*d^3))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*d*f)
+ (2*a^3*(11*A*(3*c - 19*d)*d - B*(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(693*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B
*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^3)/(99*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])^3)/(11*d*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df} + \frac{2\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2\left(\frac{1}{2}a(11Ad+3B(c+2d))-\frac{1}{2}a(5Bc-11Ad-14Bd)\sin(\right.}{11d}$$

$$= \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df} + \frac{4\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\left(\frac{1}{4}a^2(11Ad(c+15d)-B(5c^2-11cd-138d^2))-\frac{1}{4}a\right.}{99d^2}$$

$$= \frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df} + \frac{(a^2(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)))\int\sqrt{a+a\sin(e+fx)}(c+d\,\mathrm{s}}{231d^3}$$

$$= -\frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a+a\sin(e+fx))}{1155df} + \frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df} + \frac{(2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)))\int\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(5c^2\right.}{1155d^3}$$

$$= -\frac{4a^2(5c-d)(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{3465d^2f} - \frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a+a\sin(e+fx))}{1155df} + \frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df} + \frac{(a^2(15c^2+10cd+7d^2)(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)))\int\sqrt{a+a}}{3465d^3}$$

$$
= -\frac{2a^3(15c^2+10cd+7d^2)\,(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+}{3465d^3f\sqrt{a+a\sin(e+fx)}}
- \frac{4a^2(5c-d)\,(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)\sqrt{a+a}}{3465d^2f}
- \frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a+a\sin(e+fx}{1155df}
+ \frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}}
+ \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f}
- \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df}
$$

Mathematica [A] (verified)

Time = 7.73 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.76

$$
\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =
-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(164472Ac^2+137280Bc^2+274560Acd+248}{}
$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -1/27720*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x
])]*(164472*A*c^2 + 137280*B*c^2 + 274560*A*c*d + 248732*B*c*d + 124366*A*d
^2 + 114640*B*d^2 - 8*(11*A*(63*c^2 + 360*c*d + 254*d^2) + 2*B*(990*c^2 + 2
794*c*d + 1625*d^2))*Cos[2*(e + f*x)] + 70*d*(22*B*c + 11*A*d + 32*B*d)*Cos
[4*(e + f*x)] + 51744*A*c^2*Sin[e + f*x] + 66660*B*c^2*Sin[e + f*x] + 13332
0*A*c*d*Sin[e + f*x] + 137104*B*c*d*Sin[e + f*x] + 68552*A*d^2*Sin[e + f*x]
 + 69890*B*d^2*Sin[e + f*x] - 1980*B*c^2*Sin[3*(e + f*x)] - 3960*A*c*d*Sin[
3*(e + f*x)] - 11440*B*c*d*Sin[3*(e + f*x)] - 5720*A*d^2*Sin[3*(e + f*x)] -
 8675*B*d^2*Sin[3*(e + f*x)] + 315*B*d^2*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 36.06 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.60

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(315B(\cos^4(fx+e))\sin(fx+e)d^2+(385A\,d^2+770cdB+1120d^2B)(\cos^4(fx+e))+(-990Acd-1430A}{}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)d(dA+2Bc)(35(\sin^4(fx+e))+130(\sin^3(fx+e))+219(\sin^2(fx+e))+292\sin(fx+e)+584)}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+s}{}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^2
+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-990*A*c*d-1430*A*d^2-495*B
*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(f*x+e)+(-693*A*c^2-3960*A*c*d-
3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+(3234*A*c^2+8580*
A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*sin(f*x+e)+10626*A*c^2+1
9140*A*c*d+9218*A*d^2+9570*B*c^2+18436*c*d*B+8930*d^2*B)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{2\left(315\,Ba^2d^2\cos\left(fx+e\right)^6+35\left(22\,Ba^2cd+(11\,A+32\,B)a^2d^2\right)\cos\left(fx+e\right)^5+1056\,(7\,A+5\,B)a^2c^2+70}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a^
2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*c*
d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c*d
+ (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 + 22*(
180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (33*(7
7*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*a^2*d^
2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*B)*a^2*
c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(f*x + e)
^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(143*A + 125
*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x + e)^4 - 5*
(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d^2)*cos(f*x
```

```
 + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d + (583*A +
655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 + 22*(195*A +
211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)^2\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/
4*pi + 11/2*f*x + 11/2*e) + 6930*(40*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 60*A*a^2*c*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 52*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x
```

```
 + 1/2*e)) + 26*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*B*a^2*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 231
0*(20*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 22*B*a^2*c^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 44*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 40*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*A*a^2*d^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 19*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 693*(8*A*a^2*c^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 20*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 40*
A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*B*a^2*c*d*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + 24*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
25*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5
/2*e) + 495*(4*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*a^2*c*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 10*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 13*B*a^2*d
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e) + 38
5*(4*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*sin(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^2\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
 x)
```

3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 212

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \\ -\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}} \\ -\frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} \\ -\frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} \\ -\frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f} \\ -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}\end{aligned}$$

```
[Out] -2/105*a*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-
2/63*(9*A*d+9*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*B*d*cos(f*
x+e)*(a+a*sin(f*x+e))^(7/2)/a/f-64/315*a^3*(21*A*c+15*A*d+15*B*c+13*B*d)*co
s(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/315*a^2*(21*A*c+15*A*d+15*B*c+13*B*d)*
cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3047, 3102, 2830, 2726, 2725}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{64a^3(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}} - \frac{16a^2(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f} - \frac{2(9Ad+9Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{63f} - \frac{2a(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105f} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x])/(315*f*Sqrt[a +
a*Sin[e + f*x]]) - (16*a^2*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(315*f) - (2*a*(21*A*c + 15*B*c + 15*A*d + 13*B*
d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) - (2*(9*B*c + 9*A*d - 2
*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(63*f) - (2*B*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a+a\sin(e+fx))^{5/2}\left(Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)\right)\,dx\\
&= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}\\
&\quad+\frac{2\int (a+a\sin(e+fx))^{5/2}\left(\frac{1}{2}a(9Ac+7Bd)+\frac{1}{2}a(9Bc+9Ad-2Bd)\sin(e+fx)\right)\,dx}{9a}\\
&= -\frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f}\\
&\quad-\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}\\
&\quad+\frac{1}{21}(21Ac+15Bc+15Ad+13Bd)\int (a+a\sin(e+fx))^{5/2}\,dx\\
&= -\frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f}\\
&\quad-\frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f}\\
&\quad-\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}\\
&\quad+\frac{1}{105}(8a(21Ac+15Bc+15Ad+13Bd))\int (a+a\sin(e+fx))^{3/2}\,dx
\end{aligned}$$

$$= -\frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} - \frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} - \frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af} + \frac{1}{315}\left(32a^2(21Ac+15Bc+15Ad+13Bd)\right)\int\sqrt{a+a\sin(e+fx)}\,dx$$

$$= -\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} - \frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} - \frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}$$

Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(7476Ac+6240Bc+6240Ad+5653Bd-4(6$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(7476*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180*A
*d + 254*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[e +
 f*x] + 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e + f*
x] - 90*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3*(e +
 f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 7.57 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(35B(\cos^4(fx+e))d+(-45dA-45Bc-130dB)(\cos^2(fx+e))\sin(fx+e)+(-63Ac-180dA-180Bc}{315\cos(fx+e)\sqrt{a+a\sin(fx+e}}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(dA+Bc)(3(\sin^3(fx+e))+12(\sin^2(fx+e))+23\sin(fx+e)+46)}{21\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Ac(1+\sin(fx+e))a^3(\sin(fx-}{15\cos(fx}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d+(-45*A*d-45*B*
c-130*B*d)*cos(f*x+e)^2*sin(f*x+e)+(-63*A*c-180*A*d-180*B*c-289*B*d)*cos(f*
x+e)^2+(294*A*c+390*A*d+390*B*c+422*B*d)*sin(f*x+e)+966*A*c+870*d*A+870*B*c
+838*d*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.70

$$\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{2\left(35\,Ba^2d\cos\left(fx+e\right)^5-5\left(9\,Ba^2c+(9\,A+19\,B)a^2d\right)\cos\left(fx+e\right)^4+96\,(7\,A+5\,B)a^2c+32\,(15\,A+1\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*d*cos(f*x + e)^5 - 5*(9*B*a^2*c + (9*A + 19*B)*a^2*d)*cos(
f*x + e)^4 + 96*(7*A + 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d - (9*(7*A + 20*B
)*a^2*c + (180*A + 289*B)*a^2*d)*cos(f*x + e)^3 + (3*(77*A + 85*B)*a^2*c +
(255*A + 263*B)*a^2*d)*cos(f*x + e)^2 + 2*(3*(161*A + 145*B)*a^2*c + (435*A
 + 419*B)*a^2*d)*cos(f*x + e) - (35*B*a^2*d*cos(f*x + e)^4 + 96*(7*A + 5*B)
*a^2*c + 32*(15*A + 13*B)*a^2*d + 5*(9*B*a^2*c + (9*A + 26*B)*a^2*d)*cos(f*
x + e)^3 - 3*(3*(7*A + 15*B)*a^2*c + (45*A + 53*B)*a^2*d)*cos(f*x + e)^2 -
2*(3*(49*A + 65*B)*a^2*c + (195*A + 211*B)*a^2*d)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{5}{2}}(d\sin\left(fx+e\right)+c)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(192) = 384.

Time = 0.36 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.91

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{\sqrt{2}\left(35\,Ba^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)+630\,\left(20\,Aa^2c\mathrm{sgn}(\mathrm{co}\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
[Out] 1/2520*sqrt(2)*(35*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
+ 9/2*f*x + 9/2*e) + 630*(20*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
15*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*A*a^2*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 13*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 210*(10*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/
```

```
2*e)) + 11*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*A*a^2*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(2*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 5*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a^2*d*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*B*a^2*c*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 2*A*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*d
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(
a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)), x
)
```

3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}$$

```
[Out] -2/35*a*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/f-64/105*a^3*(7*A+5*B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/
2)-16/105*a^2*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}} - \frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f} - \frac{2a(7A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(7*A + 5*B)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}+\frac{1}{7}(7A+5B)\int(a+a\sin(e+fx))^{5/2}\,dx\\
&= -\frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}\\
&\quad-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}\\
&\quad+\frac{1}{35}(8a(7A+5B))\int(a+a\sin(e+fx))^{3/2}\,dx\\
&= -\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}\\
&\quad-\frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}\\
&\quad-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}\\
&\quad+\frac{1}{105}\left(32a^2(7A+5B)\right)\int\sqrt{a+a\sin(e+fx)}\,dx
\end{aligned}$$

$$= -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}$$

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(1246A+1040B-6(7A+20B)\cos(2(e+fx)}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(1246*A + 1040*B - 6*(7*A + 20*B)*Cos[2*(e + f*x)] + (392*A + 505*B)*Sin[
e + f*x] - 15*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
)
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(-15B\left(\cos^2(fx+e)\right)\sin(fx+e)+(-21A-60B)\left(\cos^2(fx+e)\right)+(98A+130B)\sin(fx+e)+322A+290\right.}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2A(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(3\left(\sin^2(fx+e)\right)+14\sin(fx+e)+43\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(3\left(\sin^3(fx+e)\right)+12(\right.}{21\cos(fx+e)\sqrt{a+a\sin(fx+}}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(-21
*A-60*B)*cos(f*x+e)^2+(98*A+130*B)*sin(f*x+e)+322*A+290*B)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{2\left(15\,Ba^2\cos\left(fx+e\right)^4+3\left(7\,A+20\,B\right)a^2\cos\left(fx+e\right)^3-(77\,A+85\,B)a^2\cos\left(fx+e\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(f*x + e)^4 + 3*(7*A + 20*B)*a^2*cos(f*x + e)^3 - (77*A
+ 85*B)*a^2*cos(f*x + e)^2 - 2*(161*A + 145*B)*a^2*cos(f*x + e) - 32*(7*A +
 5*B)*a^2 + (15*B*a^2*cos(f*x + e)^3 - 3*(7*A + 15*B)*a^2*cos(f*x + e)^2 -
2*(49*A + 65*B)*a^2*cos(f*x + e) + 32*(7*A + 5*B)*a^2)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int \left(a(\sin\left(e+fx\right)+1)\right)^{\frac{5}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(15\,Ba^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+525\left(4\,Aa^2\mathrm{sgn}\left(\cos\left(\right.\right.\right.\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 525*(4*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*
(10*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 218

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx=\frac{2a^{5/2}(c-d)^2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c+d}f}$$
$$+\frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df}$$

```
[Out] -2/5*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f+2*a^(5/2)*(c-d)^2*(-A*d+B*c)
*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(
7/2)/f/(c+d)^(1/2)+2/15*a^3*(5*A*(3*c-7*d)*d-B*(15*c^2-35*c*d+32*d^2))*cos(
f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/15*a^2*(-5*A*d+5*B*c-8*B*d)*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used

$= \{3055, 3060, 2852, 214\}$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2a^{5/2}(c-d)^2(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{7/2}f\sqrt{c+d}}$$
$$+\frac{2a^3(5Ad(3c-7d)-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15d^2f}$$
$$-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (2*a^(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2)*Sqrt[c + d]*f) + (2*a^3*(5*A
*(3*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(15*d^3*f*Sqrt
[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(15*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df} + \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}\left(\frac{1}{2}a(3Bc+5Ad)-\frac{1}{2}a(5Bc-5Ad-8Bd)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{5d}$$

$$= \frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df} + \frac{4\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\frac{1}{4}a^2(Bc(5c-17d)-5Ad(c+3d))-\frac{1}{4}a^2\left(5A(3c-7d)d-B\left(15c^2-35cd+32d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{15d^2}$$

$$= \frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df} - \frac{(a^2(c-d)^2(Bc-Ad))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{d^3}$$

$$= \frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df} + \frac{(2a^3(c-d)^2(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{d^3f}$$

$$= \frac{2a^{5/2}(c-d)^2(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c+d}f} + \frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f} - \frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.38 (sec) , antiderivative size = 992, normalized size of antiderivative = 4.55

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{(a(1+\sin(e+fx)))^{5/2}\left(-30\sqrt{d}(Ad(-2c+5d)+B(2c^2\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 - 5
*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[(3
*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] - (15*(c - d)^2*(B*c - A*
d)*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c +
 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e +
f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 + T
an[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*L
og[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#
1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e +
 f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[
c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &
]))/(c + d)^(3/2) + (15*(c - d)^2*(B*c - A*d)*((c + d)*(e + f*x - 2*Log[Sec
[(e + f*x)/4]^2]) - Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 +
c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Ta
n[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*L
og[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 +
c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e +
 f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]
*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
```

```
]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(3/2) + 30*Sqrt[d]*(
A*d*(-2*c + 5*d) + B*(2*c^2 - 5*c*d + 5*d^2))*Sin[(e + f*x)/2] - 5*d^(3/2)*
(-2*B*c + 2*A*d + 5*B*d)*Sin[(3*(e + f*x))/2] - 3*B*d^(5/2)*Sin[(5*(e + f*x
))/2]))/(30*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(192) = 384$.

Time = 2.94 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.49

method	result
default	$\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3B(a-a\sin(fx+e))^{\frac{5}{2}}\sqrt{a(c+d)d}\,d^2+5A(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{a(c+d)d}\,a\,d^2-15A\,\operatorname{arctanh}\left(\right.\right.}{}$

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)*
(a*(c+d)*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2-15
*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+e))^(
3/2)*(a*(c+d)*d)^(1/2)*a*c*d+20*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*
a*d^2+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^3-30
*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+15*B*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2+15*A*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-45*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d
)*d)^(1/2)*a^2*d^2-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c^2+45
*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-60*B*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a^2*d^2)/d^3/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(192) = 384$.

Time = 1.30 (sec) , antiderivative size = 1314, normalized size of antiderivative = 6.03

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^
3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*cos
```

```
(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*
d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 -
2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 +
 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x +
 e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*
cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d
 + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)
*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f
*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B
*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32
*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 - (15*B*a
^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x + e) - (3
*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A +
32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x + e) +
d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A
*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d
^3)*cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 -
 A*a^2*d^3)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) +
 2*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35
*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 -
(15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x +
e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (
35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x
+ e) + d^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{d\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(192) = 384.

Time = 0.33 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{15\sqrt{2}(Ba^2c^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-Aa^2c^2d\mathrm{sgn}(\cos(-\frac{1}{4}\pi\right.}{}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/15*sqrt(2)*sqrt(a)*(15*sqrt(2)*(B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^2*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*a^2*d^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d^3) + 2*(12*B*a^2*d^4*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 10*B*a^2*c*d^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*
A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 - 40*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^3 + 15*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e) - 15*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 45*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 45*A*a^2*d^4*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 60*B*a^2*d^4*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^5)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)),
 x)
```

3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 265

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\operatorname{arctanh}}{d^{7/2}(c+d)^{3/2}f}$$
$$-\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1
/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(3/2)/f+a*(-A
*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+d)/f/(c+d*sin(f*x+e))-1/3*a^
3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*cos(f*x+e)/d^3/(c+d)/f/(a+a*sin(f
*x+e))^(1/2)-1/3*a^2*(-3*A*d+5*B*c+2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)
/d^2/(c+d)/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3054, 3055, 3060, 2852, 214}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\operatorname{arctanh}\left(\right.}{d^{7/2}f(c+d)^{3/2}}$$
$$-\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3f(c+d)\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3d^2f(c+d)}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2 + 5*c*d - 2*d^2))*ArcTanh[(Sqr
t[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2
)*(c + d)^(3/2)*f) - (a^3*(3*A*d*(3*c + d) - B*(15*c^2 - 5*c*d - 14*d^2))*C
os[e + f*x])/(3*d^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*(5*B*c - 3*A
*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d^2*(c + d)*f) + (a*(
B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c + d*Sin
[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
```

```
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^{3/2}\left(-\frac{1}{2}a(3Bc-5Ad-2Bd)+\frac{1}{2}a(5Bc-3Ad+2Bd)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= -\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\frac{1}{4}a^2\left(3A(c-5d)d-B\left(5c^2-7cd+6d^2\right)\right)+\frac{1}{4}a^2\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{3d^2(c+d)}$$

$$= -\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} - \frac{(a^2(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}dx}{2d^3(c+d)}$$

$$= -\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} + \frac{(a^3(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right))\,\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{d^3(c+d)f}$$

$$= \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}(c+d)^{3/2}f} - \frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.43 (sec) , antiderivative size = 1002, normalized size of antiderivative = 3.78

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{(a(1+\sin(e+fx)))^{5/2}\left(-12\sqrt{d}(-4Bc+2Ad+5Bd)\,\text{co}\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-12*Sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e
+ f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] + (3*(c - d)*(-(A*d*(3*c + 5*d
)) + B*(5*c^2 + 5*c*d - 2*d^2))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^
2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(
c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(
e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*
Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan
[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d -
 c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + (3*(c - d)*(-(A*d*(3*c + 5
*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(-((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)
/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 &
, (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*
x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 +
Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[c
+ d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]
*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/
(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + 12*Sqrt[d]*(-4*B*c +
2*A*d + 5*B*d)*Sin[(e + f*x)/2] - (12*(c - d)^2*Sqrt[d]*(-(B*c) + A*d)*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])) - 4*B*d^(
3/2)*Sin[(3*(e + f*x))/2]))/(12*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(241) = 482$.

Time = 10.18 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	932

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(-9*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d-6*A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+15*A*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+
d)*d)^(1/2)*c*d-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2+15*a^2*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3-21*B*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+6*B*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3+6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+
d)*d)^(1/2)*a*c*d+6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2-12*B*(
```

```
a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2+6*B*(a-a*sin(f*x+e))^(1/2)*(a
*(c+d)*d)^(1/2)*a*c*d+18*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2)-
9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-6*A*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+15*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-2*B*(a-a*sin(f*x+e
))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/
2)*c*d^2+15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4
-21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+6*B
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+9*A*(a-a*s
in(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*A*(a-a*sin(f*x+e))^(1/2)*(a*(c
+d)*d)^(1/2)*a*d^3-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3+12*B
*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+15*B*(a-a*sin(f*x+e))^(1/
2)*(a*(c+d)*d)^(1/2)*a*c*d^2)/d^3/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(241) = 482$.

Time = 1.49 (sec) , antiderivative size = 2046, normalized size of antiderivative = 7.72

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 +
(3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*
d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^
2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*co
s(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2
 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c
^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*
d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*
d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d
 + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e)
 + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*
cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^3 -
(9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2
*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*
```

```
a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10
*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*
a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a
^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*
(A + B)*a^2*c*d^2 - (3*A + 8*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a))/((c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*c
os(f*x + e) - (c^2*d^3 + 2*c*d^4 + d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) +
 (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin(f*x + e)), 1/6*(3*(5*B*a^2*c^4 - (3*A - 5
*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*
B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*
A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*
B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A
- 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A +
 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 +
(5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arcta
n(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^
2))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A -
 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*
x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*
cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 -
(3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d
+ 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d
^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*(A + B)*a^2*c*d^2 - (3*A + 8*B)*a
^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^4 + d^5
)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^2*d^3 + 2*c*d^4
+ d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) + (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin
(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(241) = 482$.

Time = 0.36 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.25

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/6*sqrt(2)*sqrt(a)*(3*sqrt(2)*(5*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*A*a^2*c*d^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 5*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a^2*d^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^3 + d^4)*sqrt(-c*d - d^2)) - 6*(B*a^2*c^
3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a^
2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
- 2*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) - A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e))/((c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 - c - d)) + 4*(2*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 + 6*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^6)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
2,x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
2, x)
```

3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 308

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{7/2}(c+d)^{5/2}f} + \frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))}$$

```
[Out] -1/4*a^(5/2)*(A*d*(3*c^2+10*c*d+19*d^2)-B*(15*c^3+30*c^2*d+7*c*d^2-20*d^3))
*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(
7/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+
d)/f/(c+d*sin(f*x+e))^2+1/4*a^3*(3*A*d*(c+3*d)-B*(15*c^2+25*c*d+4*d^2))*cos
(f*x+e)/d^3/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*(A*d*(c+7*d)-B*(5*c^2+
7*c*d-4*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/(c+d*sin(f*x+
e))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3054, 3060, 2852, 214}

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx =$$
$$-\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{7/2}f(c+d)^{5/2}}$$
$$+\frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3f(c+d)^2\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4d^2f(c+d)^2(c+d\sin(e+fx))}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}$$

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/4*(a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d
^2 - 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a +
a*Sin[e + f*x]])]])/(d^(7/2)*(c + d)^(5/2)*f) + (a^3*(3*A*d*(c + 3*d) - B*(1
5*c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(4*d^3*(c + d)^2*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4
*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*d^2*(c + d)^2*f*(c + d*Sin
[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad+\frac{\int \frac{(a+a\sin(e+fx))^{3/2}\left(-\frac{1}{2}a(3Bc-7Ad-4Bd)+\frac{1}{2}a(5Bc-Ad+4Bd)\sin(e+fx)\right)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)} \\
&= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad-\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))} \\
&\quad+\frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{4}a^2\left(Ad(c+19d)-B\left(5c^2+3cd-20d^2\right)\right)-\frac{1}{4}a^2\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{2d^2(c+d)^2} \\
&= \frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad-\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))} \\
&\quad+\frac{(a^2(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3)))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{8d^3(c+d)^2}
\end{aligned}$$

$$= \frac{a^3(3Ad(c+3d) - B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(Ad(c+7d) - B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{(a^3(Ad(3c^2+10cd+19d^2) - B(15c^3+30c^2d+7cd^2-20d^3)))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(}{\sqrt{a+a\sin}}\right.}{4d^3(c+d)^2f}$$

$$= -\frac{a^{5/2}(Ad(3c^2+10cd+19d^2) - B(15c^3+30c^2d+7cd^2-20d^3))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{7/2}(c+d)^{5/2}f} + \frac{a^3(3Ad(c+3d) - B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(Ad(c+7d) - B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 12.86 (sec) , antiderivative size = 1046, normalized size of antiderivative = 3.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+}{}\right.}{}$$

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3
+ 30*c^2*d + 7*c*d^2 - 20*d^3))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^
2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(
c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(
e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*
Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan
```

```
[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d -
 c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(7/2) + ((A*d*(3*c^2 + 10*c*d + 19
*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(-((c + d)*(e + f*x - 2*L
og[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*
#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-
#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sq
rt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + T
an[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt
[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e +
 f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(7/2) - (4*Sq
rt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^4 - 3*A*c^3*d + 20*B*c^
3*d - 8*A*c^2*d^2 - B*c^2*d^2 + 9*A*c*d^3 + 10*B*c*d^3 + 2*A*d^4 + 4*B*d^4
- 4*B*d^2*(c + d)^2*Cos[2*(e + f*x)] + d*(A*d*(-5*c^2 - 6*c*d + 11*d^2) + B
*(25*c^3 + 34*c^2*d + c*d^2 + 4*d^3))*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[
e + f*x])^2)))/(16*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(280) = 560$.

Time = 63.26 (sec) , antiderivative size = 1587, normalized size of antiderivative = 5.15

method	result	size
default	Expression too large to display	1587

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*a*(-7*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x
+e)^2*a^2*c*d^4+6*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*
sin(f*x+e)*a^2*c^3*d^2+13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d
^4+15*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^4+10*A*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+19*A*arctanh((-a*(si
n(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3+4*B*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+20*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+5*A*(-a*(sin(f*x+e)-1))^(3/2)*(a
*(c+d)*d)^(1/2)*c^2*d^2+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d
)^(1/2))*sin(f*x+e)*a^2*c^2*d^3+38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c*d^4+8*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d
)*d)^(1/2)*sin(f*x+e)^2*a*d^4-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^4*d+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*
d)^(1/2)*c*d^3-15*a^2*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*B*c^5-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-11*A*(-a*(sin(f
*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-9*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d
```

```
)*d)^(1/2)*c^3*d-2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+15
*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3-30*B*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^4*d-7*B*arctanh((-a*(sin(f*x+e)
-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+40*B*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c*d^4+16*B*(-a*(sin(f*x+e)-1))^
(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c*d^3+16*B*(-a*(sin(f*x+e)-1))^(1/2)
*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^3*d+32*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*sin(f*x+e)*a*c^2*d^2+16*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)
^(1/2)*sin(f*x+e)*a*c*d^3-60*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2-14*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3-15*B*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-30*B*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-13*B*(-a*(sin
(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3+10*A*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4-3*A*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a
*(c+d)*d)^(1/2)*a*c^2*d^2+3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a
*c*d^3+29*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-3*B*(-a*(si
n(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2+8*B*(-a*(sin(f*x+e)-1))^(1/2
)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d^2+20*B*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^2*c^4*d)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))
/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^3/cos(f*x+e)/(a+a*sin(f*x+e
))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(280) = 560$.

Time = 1.73 (sec) , antiderivative size = 3046, normalized size of antiderivative = 9.89

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2
- 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2
*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^
4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B
)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19
*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d
 - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a
```

^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2 - 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e)), -1/8*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*

```
A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^
3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2
- 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^
2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*
c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2
*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a
^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25
*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B
)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5
+ 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)
*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos
(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5
 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(
f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e
))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(d\sin(fx+e)+c)^3}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(280) = 560$.

Time = 0.41 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.91

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/8*sqrt(2)*(16*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)/d^3 + sqrt(2)*(15*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a^2*c^2*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 19*A*a^2*d
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d -
 d^2))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(-c*d - d^2)) - 2*(18*B*a^2*c^3*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^
2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 + 4*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 - 12*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 - 30*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 22*A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*a^2*d^4*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 7*B*a^2*c^4*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*a^2*c^3*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 13*B
*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 13*A*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e) + 11*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 13*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 13*A*a^2*d^4*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*a^2*d^4*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c^2*d^3 + 2*c*d^4
 + d^5)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3, x)
```

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 284

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\sqrt{2}(A-B)(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}\\ &\quad-\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}\\ &\quad-\frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af}\\ &\quad-\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}}\\ &\quad-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

```
[Out] -(A-B)*(c-d)^3*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
))*2^(1/2)/f/a^(1/2)-4/105*(7*A*d*(21*c^2-12*c*d+7*d^2)+B*(36*c^3-63*c^2*d+
144*c*d^2-37*d^3))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/35*(7*A*d+6*B*c-B*
d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-2/7*B*cos(f*x+e)*
(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)-2/105*d*(7*A*(9*c-d)*d+B*(24*c^
2-15*c*d+31*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3062, 3047, 3102, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{\sqrt{2}(A-B)(c-d)^3\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$
$$-\frac{2d(7Ad(9c-d)+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105af}$$
$$-\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{2(7Ad+6Bc-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(7*A*d*(21*c^2 - 12*c*d + 7*d^2) +
B*(36*c^3 - 63*c^2*d + 144*c*d^2 - 37*d^3))*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*d*(7*A*(9*c - d)*d + B*(24*c^2 - 15*c*d + 31*d^2))*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*a*f) - (2*(6*B*c + 7*A*d - B*d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(35*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*C
os[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(7Ac-Bc+6Bd)+\frac{1}{2}a(6Bc+7Ad-Bd)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{7a}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{4\int \frac{(c+d\sin(e+fx))\left(\frac{1}{4}a^2\left(35Ac^2-11Bc^2-7Acd+55Bcd+28Ad^2-4Bd^2\right)+\frac{1}{4}a^2\left(7A(9c-d)d+B\left(24c^2-15cd+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} d}{35a^2}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{4\int \frac{\frac{1}{4}a^2c\left(35Ac^2-11Bc^2-7Acd+55Bcd+28Ad^2-4Bd^2\right)+\left(\frac{1}{4}a^2d\left(35Ac^2-11Bc^2-7Acd+55Bcd+28Ad^2-4Bd^2\right)+\frac{1}{4}a^2c(7A(9c-d}{\sqrt{a+a\sin(e+fx)}}}{35a^2}$$

$$= -\frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af}$$

$$-\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{8\int \frac{-\frac{1}{8}a^3\left(B\left(33c^3-189c^2d+27cd^2-31d^3\right)-7A\left(15c^3-3c^2d+21cd^2-d^3\right)\right)+\frac{1}{4}a^3\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(36c^3-63c^2d+144cd^2-\right.\right.}{\sqrt{a+a\sin(e+fx)}}}{105a^3}$$

$$= -\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af}$$

$$-\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}$$

$$+\left((A-B)(c-d)^3\right)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af} - \frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} - \frac{(2(A-B)(c-d)^3)\,\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af} - \frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.32

$$\int\frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((840+840i)(-1)^{3/4}(A-B)(c-d)^3\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((840 + 840*I)*(-1)^(3/4)*(A - B)*(c
 - d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 105*(4*A*
d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^2*d + 24*c*d^2 - 5*d^3))*Cos[(e
 + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c^2 - 6*c*d + 5*d^2))*Cos[(3*(e
+ f*x))/2] + 21*d^2*(6*B*c + 2*A*d - B*d)*Cos[(5*(e + f*x))/2] + 15*B*d^3*C
```

```
os[(7*(e + f*x))/2] + 105*(4*A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^
2*d + 24*c*d^2 - 5*d^3))*Sin[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c
^2 - 6*c*d + 5*d^2))*Sin[(3*(e + f*x))/2] + 21*d^2*(-2*A*d + B*(-6*c + d))*
Sin[(5*(e + f*x))/2] + 15*B*d^3*Sin[(7*(e + f*x))/2]))/(420*f*Sqrt[a*(1 + S
in[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(259) = 518$.

Time = 3.56 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.97

method	result
parts	$-\frac{A c^3(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{c^2(3dA+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1}}{a\cos(fx}$
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(105A\,a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^3-315A\,a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^2\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -A*c^3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2
*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f+c^2*(3*A*d+B*c)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-2*(a-a*sin(f*x+e))^
(1/2))/a/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/15*d^2*(A*d+3*B*c)*(1+sin(f*
x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))-6*(a-a*sin(f*x+e))^(5/2)+10*a*(a-a*sin(f*x+e))
^(3/2)-30*a^2*(a-a*sin(f*x+e))^(1/2))/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)
/f-1/105*d^3*B*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(105*a^(7/2)*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-30*(a-a*sin(f*x+e))^(
7/2)+84*(a-a*sin(f*x+e))^(5/2)*a-140*a^2*(a-a*sin(f*x+e))^(3/2))/a^4/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f+c*d*(A*d+B*c)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-
1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))+2*(a-a*sin(f*x+e))^(3/2))/a^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(259) = 518$.

Time = 0.29 (sec) , antiderivative size = 629, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{\frac{105\sqrt{2}\left((A-B)ac^3-3\,(A-B)ac^2d+3\,(A-B)acd^2-(A-B)ad^3+\left((A-B)ac^3-3\,(A-B)ac^2d+3\,(A-B)acd^2-(A-B)ad^3\right)\cos(fx+e)+\left((A-B)ac^3-\right.\right.}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 -
 (A - B)*a*d^3 + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 - (
A - B)*a*d^3)*cos(f*x + e) + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)
*a*c*d^2 - (A - B)*a*d^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
n(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(15*B*d^3*cos(f*x + e
)^4 - 105*B*c^3 - 105*(3*A - 2*B)*c^2*d + 21*(10*A - 17*B)*c*d^2 - (119*A -
 92*B)*d^3 + 3*(21*B*c*d^2 + (7*A - B)*d^3)*cos(f*x + e)^3 - (105*B*c^2*d +
 21*(5*A - 4*B)*c*d^2 - 4*(7*A - 16*B)*d^3)*cos(f*x + e)^2 - (105*B*c^3 + 1
05*(3*A - B)*c^2*d - 21*(5*A - 16*B)*c*d^2 + 2*(56*A - 23*B)*d^3)*cos(f*x +
 e) + (15*B*d^3*cos(f*x + e)^3 + 105*B*c^3 + 105*(3*A - 2*B)*c^2*d - 21*(10
*A - 17*B)*c*d^2 + (119*A - 92*B)*d^3 - 3*(21*B*c*d^2 + (7*A - 6*B)*d^3)*co
s(f*x + e)^2 - (105*B*c^2*d + 21*(5*A - B)*c*d^2 - (7*A - 46*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin
(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^3}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) +
 a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(259) = 518.

Time = 0.41 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.95

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\frac{105\sqrt{2}\left(A\sqrt{a}c^3-B\sqrt{a}c^3-3A\sqrt{a}c^2d+3B\sqrt{a}c^2d+3A\sqrt{a}cd^2-3B\sqrt{a}cd^2-A\sqrt{a}d^3+B\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{105\sqrt{2}(A\sqrt{a}c^3-}{}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/210*(105*sqrt(2)*(A*sqrt(a)*c^3 - B*sqrt(a)*c^3 - 3*A*sqrt(a)*c^2*d + 3*B
*sqrt(a)*c^2*d + 3*A*sqrt(a)*c*d^2 - 3*B*sqrt(a)*c*d^2 - A*sqrt(a)*d^3 + B*
sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))) - 105*sqrt(2)*(A*sqrt(a)*c^3 - B*sqrt(a)*c^3 - 3*A*sqrt(a
)*c^2*d + 3*B*sqrt(a)*c^2*d + 3*A*sqrt(a)*c*d^2 - 3*B*sqrt(a)*c*d^2 - A*sqr
t(a)*d^3 + B*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*(120*B*a^(13/2)*d^3*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^7 - 252*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^5 - 84*A*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 168*B*a^(13/2)*d^
3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 210*B*a^(13/2)*c^2*d*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 + 210*A*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 2
10*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 70*A*a^(13/2)*d^3*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 140*B*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^3 - 105*B*a^(13/2)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 315*A*a^(13
/2)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 315*B*a^(13/2)*c*d^2*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) - 105*A*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a
^7*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2
), x)
```

3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 200

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx
&= -\frac{\sqrt{2}(A-B)(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} \\
&\quad -\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} \\
&\quad -\frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} \\
&\quad -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

```
[Out] -(A-B)*(c-d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
))*2^(1/2)/f/a^(1/2)-4/15*(5*A*(3*c-d)*d+B*(6*c^2-7*c*d+7*d^2))*cos(f*x+e)/
f/(a+a*sin(f*x+e))^(1/2)-2/5*B*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x
+e))^(1/2)-2/15*d*(5*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3062, 3047, 3102, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{\sqrt{2}(A-B)(c-d)^2\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{4(5Ad(3c-d)+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}} - \frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(5*A*(3*c - d)*d + B*(6*c^2 - 7*c*d
 + 7*d^2))*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*(4*B*c + 5*
A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*a*f) - (2*B*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(5Ac-Bc+4Bd)+\frac{1}{2}a(4Bc+5Ad-Bd)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{5a}$$

$$= -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}ac(5Ac-Bc+4Bd)+\left(\frac{1}{2}ac(4Bc+5Ad-Bd)+\frac{1}{2}ad(5Ac-Bc+4Bd)\right)\sin(e+fx)+\frac{1}{2}ad(4Bc+5Ad-Bd)\sin^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{5a}$$

$$= -\frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{4\int \frac{\frac{1}{4}a^2(5A(3c^2+d^2)-B(3c^2-16cd+d^2))+\frac{1}{2}a^2(5A(3c-d)d+B(6c^2-7cd+7d^2))\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}dx}{15a^2}$$

$$= -\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \left((A-B)(c-d)^2\right)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}dx$$

$$= -\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{(2(A-B)(c-d)^2)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.23

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((60+60i)(-1)^{3/4}(A-B)(c-d)^2\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\text{t}\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c -
 d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 30*(A*(4*c
- d)*d + 2*B*(c^2 - c*d + d^2))*Cos[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c +
d))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 30*(A*(4*c - d)*d
 + 2*B*(c^2 - c*d + d^2))*Sin[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c + d))*Si
n[(3*(e + f*x))/2] - 3*B*d^2*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[a*(1 + Sin[e
 + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(179) = 358.

Time = 2.62 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.98

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(15A\,a^{\frac{5}{2}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^2-30A\,a^{\frac{5}{2}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)cd+\right.}{}$
parts	$-\frac{A\,c^2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{c(2dA+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}}{a\cos(fx}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*A*a^(5/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*a^(5/2)*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*a^(5/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*a^(5/2)*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*a^(5/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a*sin(f*
x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x+e))^(3/
```

```
2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*(a-a*sin(f*x+e))^(1/2)*A*a^2*
c*d+30*(a-a*sin(f*x+e))^(1/2)*B*a^2*c^2+30*(a-a*sin(f*x+e))^(1/2)*B*a^2*d^2
)/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{\frac{15\sqrt{2}\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2+\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2\right)\cos(fx+e)+\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2\right)\sin(}{\sqrt{a}}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*((A - B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2 + ((A -
B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*cos(f*x + e) + ((A - B)*a*c^2 -
 2*(A - B)*a*c*d + (A - B)*a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos
(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x +
 e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(3*B*d^2*cos(
f*x + e)^3 - 15*B*c^2 - 10*(3*A - 2*B)*c*d + (10*A - 17*B)*d^2 - (10*B*c*d
+ (5*A - 4*B)*d^2)*cos(f*x + e)^2 - (15*B*c^2 + 10*(3*A - B)*c*d - (5*A - 1
6*B)*d^2)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - 15*B*c^2 - 10*(3*A - 2*B
)*c*d + (10*A - 17*B)*d^2 + (10*B*c*d + (5*A - B)*d^2)*cos(f*x + e))*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*
f)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^2}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) +
 a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(179) = 358$.

Time = 0.35 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\frac{15\sqrt{2}\left(A\sqrt{a}c^2-B\sqrt{a}c^2-2A\sqrt{a}cd+2B\sqrt{a}cd+A\sqrt{a}d^2-B\sqrt{a}d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{15\sqrt{2}\left(A\sqrt{a}c^2-B\sqrt{a}c^2-2A\sqrt{a}cd+2B\sqrt{a}cd\right.}{a\mathrm{sgn}\left(\cos\left(-\right.\right.}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/30*(15*sqrt(2)*(A*sqrt(a)*c^2 - B*sqrt(a)*c^2 - 2*A*sqrt(a)*c*d + 2*B*sqr
t(a)*c*d + A*sqrt(a)*d^2 - B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(a)*c^2
- B*sqrt(a)*c^2 - 2*A*sqrt(a)*c*d + 2*B*sqrt(a)*c*d + A*sqrt(a)*d^2 - B*sqr
t(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) + 4*sqrt(2)*(12*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^5 - 20*B*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^(9/2)*d^2*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^3 + 15*B*a^(9/2)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 30*A*a^(9/2)*
c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/(a^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2
),x)
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2
), x)
```

3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sqrt{2}(A-B)(c-d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

```
[Out] -(A-B)*(c-d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))
*2^(1/2)/f/a^(1/2)-2/3*(3*A*d+3*B*c-2*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1
/2)-2/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3102, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sqrt{2}(A-B)(c-d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{2(3Ad+3Bc-2Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (2*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])
/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{Ac + (Bc + Ad)\sin(e + fx) + Bd\sin^2(e + fx)}{\sqrt{a + a\sin(e + fx)}}\,dx$$

$$= -\frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + \frac{2\int \frac{\frac{1}{2}a(3Ac+Bd)+\frac{1}{2}a(3Bc+3Ad-2Bd)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a}$$

$$= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + ((A-B)(c-d))\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} - \frac{(2(A-B)(c-d))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-6-6i)(-1)^{3/4}(A-B)(c-d)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\right.\right.\right.}{3f\sqrt{a(1+\sin}}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -1/3*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)*
(c - d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x])))/(
f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(113) = 226$.

Time = 2.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(3A\,a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c-3A\,a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)d-3B\,a^{\frac{3}{2}}\right.}{3a^2\,c}$
parts	$-\frac{Ac(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}-\frac{(dA+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-\sqrt{\right.}}{a\cos(fx+e)}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*a^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-3*A*a^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-3*B*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c+3*B*a^(3/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-2*B*(a-a*sin(f*x+e))^(3/2)*d+6*(
a-a*sin(f*x+e))^(1/2)*A*a*d+6*(a-a*sin(f*x+e))^(1/2)*B*a*c)/a^2/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(113) = 226$.

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.33

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{\frac{3\sqrt{2}((A-B)ac-(A-B)ad+((A-B)ac-(A-B)ad)\cos(fx+e)+((A-B)ac-(A-B)ad)\sin(fx+e))\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{}}{}}{\cos(fx+e)^2-(\cos(}\right.}{\sqrt{a}}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*cos
(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e)^2
- (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos
(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*d*co
s(f*x + e)^2 + 3*B*c + (3*A - 2*B)*d + (3*B*c + (3*A - B)*d)*cos(f*x + e) +
 (B*d*cos(f*x + e) - 3*B*c - (3*A - 2*B)*d)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1
)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)}{\sqrt{a\sin(fx+e)+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a
), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\frac{3\sqrt{2}(A\sqrt{a}c-B\sqrt{a}c-A\sqrt{a}d+B\sqrt{a}d)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}(A\sqrt{a}c-B\sqrt{a}c-A\sqrt{a}d+B\sqrt{a}d)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} -}{6f}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] 1/6*(3*sqrt(2)*(A*sqrt(a)*c - B*sqrt(a)*c - A*sqrt(a)*d + B*sqrt(a)*d)*log(
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
```

```
 - 3*sqrt(2)*(A*sqrt(a)*c - B*sqrt(a)*c - A*sqrt(a)*d + B*sqrt(a)*d)*log(-s
in(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
- 4*sqrt(2)*(2*B*a^(5/2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a^(5/2)*c
*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^(5/2)*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e))/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
 x)
```

3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=-\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/f/a^(1/2)-2*B*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2830, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=-\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+(A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx\\ &= -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{(2(A-B))\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\ &= -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int\frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}(A-B)\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+f\right.\right.\right.\right.}{f\sqrt{a(1+\sin(e+fx))}}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)A-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)B+2B\sqrt{a-}\right.}{a\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$-\frac{A(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{B(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{a}\,\sqrt{2}\,\mathrm{arc}\right.}{a\cos(fx+e)\sqrt{a}}$
risch	$-\frac{(-2iA+iB+B\,\mathrm{e}^{i(fx+e)})(\mathrm{e}^{i(fx+e)}+i)\sqrt{2}\,\mathrm{e}^{-i(fx+e)}}{f\sqrt{-a(-2\,\mathrm{e}^{i(fx+e)}+i\mathrm{e}^{2i(fx+e)}-i)\mathrm{e}^{-i(fx+e)}}}-\frac{2i(A-B)(\mathrm{e}^{i(fx+e)}+i)\left(\arctan\left(\frac{\sqrt{-i\mathrm{e}^{i(fx+e)}a}}{\sqrt{a}}\right)a\sqrt{-i\mathrm{e}^{i(fx+e)}a}+a^{\frac{3}{2}}\right)\sqrt{}}{f\,a^{\frac{3}{2}}\sqrt{-a(-2\,\mathrm{e}^{i(fx+e)}+i\mathrm{e}^{2i(fx+e)}-i)\mathrm{e}^{-i(fx+e)}}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*A-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*B+2*B*(a-a*sin(f*x+e))^(1/2))/a/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(68) = 136.

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.66

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\frac{\sqrt{2}((A-B)a\cos(fx+e)+(A-B)a\sin(fx+e)+(A-B)a)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+}{\sqrt{a}}}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{\sqrt{a}}}{2\,(af\cos(fx+e)+af\sin(fx+}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a)
*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*
sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e
) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) +
a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{A+B\sin{(e+fx)}}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)

[Out] Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{B\sin{(fx+e)}+A}{\sqrt{a\sin{(fx+e)}+a}}\,dx$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.81

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=\frac{\frac{4\sqrt{2}B\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2\,f}$$

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] 1/2*(4*sqrt(2)*B*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.91

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{A\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\sin(e+f\,x))}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}} - \frac{B\,\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|1\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{a+a\,\sin(e+f\,x)}{2\,a}}}{f\,\cos\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] - (A*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))
/(f*(a + a*sin(e + f*x))^(1/2)) - (B*(4*ellipticE(asin((2^(1/2)*(1 - sin(e
+ f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))
/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*cos(
e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} dx$

Optimal result

Integrand size = 37, antiderivative size = 136

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} dx = -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)/f/a^(1/2)-2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1
/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} dx = -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)} - \frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}$$

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*
```

```
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqr
t[d]*Sqrt[c + d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} + \frac{(Bc-Ad)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{(2(A-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} - \frac{(2(Bc-Ad))\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f}$$

$$= -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.09 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.55

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$= \frac{(-1)^{3/4}\left((4+4i)(A-B)\sqrt{d}\sqrt{c+d}\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\sqrt[4]{-1}(Bc-Ad)\text{F}\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
]),x]
```

```
[Out] ((-1)^(3/4)*((4 + 4*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)
^(3/4)*(-1 + Tan[(e + f*x)/4])] + (-1)^(1/4)*(B*c - A*d)*RootSum[c + 4*d*#1
 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqr
t[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 +
 Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#
1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &
 ] - (-1)^(1/4)*(B*c - A*d)*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1
^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Ta
n[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*
Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqr
t[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*
x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]))/(2*(c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a(c+d)d}\,A-2A\,\text{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)\sqrt{a}\,d-\right.}{(c-d)\sqrt{a(c+d)d}\,\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx}}$

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*A-2*A*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d-2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*B+2*B*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c)/(c-d)/(a*(c+d)*d)^(1/2)/a^(
1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(113) = 226$.

Time = 0.84 (sec) , antiderivative size = 744, normalized size of antiderivative = 5.47

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}(c + d\sin(e+fx))}\,dx$$

$$= \left[\frac{\sqrt{acd+ad^2}(Bc-Ad)\log\left(\frac{ad^2\cos(fx+e)^3 - ac^2 - 2acd - ad^2 - (6acd+7ad^2)\cos(fx+e)^2 - 4\sqrt{acd+ad^2}\left(d\cos(fx+e)^2 - (c+2d)\cos\right.}{d^2\cos(fx+e)^3 + (2cd+d^2)\cos}\right.}{} \right.$$

$$-\frac{2\sqrt{-acd-ad^2}(Bc-Ad)\arctan\left(\frac{\sqrt{-acd-ad^2}\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)-c-2d)}{2(acd+ad^2)\cos(fx+e)}\right) - \frac{\sqrt{2}((A-B)acd+(A-B)ad^2)\log\left(\right.}{}}{2(ac^2d-ad^3)f}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2
*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)
*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*si
n(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2
)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c
*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos
(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt
(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) -
2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) +
 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f), -1/2*
(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*
```

```
sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))
) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
- sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}(d\sin\left(fx+e\right)+c)}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(113) = 226$.

Time = 0.33 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.85

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx=$$
$$-\frac{\frac{2\sqrt{2}(B\sqrt{a}c-A\sqrt{a}d)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(\sqrt{2}ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}-\frac{(A\sqrt{a}-B\sqrt{a})\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\right.\right.}}{f}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*sqrt(2)*(B*sqrt(a)*c - A*sqrt(a)*d)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*
f*x + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - sqrt(2)*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2))
- (A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*
a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) + (A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)/(sqrt(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),
 x)
```

3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 207

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx &= -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2f}\\ &\quad+\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2\sqrt{d}(c+d)^{3/2}f}\\ &\quad-\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\end{aligned}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)^2/f/a^(1/2)+(A*d*(3*c+d)-B*(c^2+c*d+2*d^2))*arctanh(cos(f*x+e)*a^(
1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)^2/(c+d)^(3/2)/f/a^(1
/2)/d^(1/2)-(-A*d+B*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x
+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used

= {3063, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \frac{(Ad(3c+d)-B(c^2+cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)^2(c+d)^{3/2}} - \frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^2} - \frac{(Bc-Ad)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^2*f)) + ((A*d*(3*c + d) - B*(c^2 + c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(Sqrt[a]*(c - d)^2*Sqrt[d]*(c + d)^(3/2)*f) - ((B*c - A*d)*Cos[e +
f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad-\frac{\int\frac{-\frac{1}{2}a(A(2c+d)-B(c+2d))-\frac{1}{2}a(Bc-Ad)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{a\,(c^2-d^2)} \\
&= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}+\frac{(A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{(c-d)^2} \\
&\quad-\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2a(c-d)^2(c+d)} \\
&= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad-\frac{(2(A-B))\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)^2f} \\
&\quad+\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\,\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)^2(c+d)f}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2f} \\
&+ \frac{(Ad(3c+d)-B(c^2+cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2\sqrt{d}(c+d)^{3/2}f} \\
&- \frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.97 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.56

$$
\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx
$$

$$
= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((8+8i)(-1)^{3/4}(A-B)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+\right.\right.\right.\right.}{}
$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) + B*
(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d
*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) +
Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)
/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#
1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3
) & ]))/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*d + 2*d^2
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4
*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d
]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/
4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1
 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(Sqrt[d]*(
c + d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
))/((c + d)*(c + d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x]
)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(182) = 364.

Time = 1.00 (sec) , antiderivative size = 899, normalized size of antiderivative = 4.34

method	result	size
default	Expression too large to display	899

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(5/2)*(-sin(f*x+e)*d*(-3*A*a^(5/
2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d-A*a^(5/2)*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2+B*a^(5/2)*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^2+B*a^(5/2)*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d+2*B*a^(5/2)*arctanh((a-a*sin(f*x+e)
)^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+A*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d-B*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-B*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^
2*d)+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*
d+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2-B*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^3-B*arctanh(
(a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d-2*B*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+A*(a-a*sin(f*x+e))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(
1/2)*a^(3/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^2*c^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d
)*d)^(1/2)*a^(3/2)*c^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c
*d+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c^2+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a^2*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1
/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(182) = 364.

Time = 2.12 (sec) , antiderivative size = 2159, normalized size of antiderivative = 10.43

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")

[Out] [-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*
c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B
)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A
 - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c
*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) +
c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*
d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2
 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 +
(2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e)
 + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x +
e))) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3
 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*
cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*
cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 +
 (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*co
s(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x
 + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1
)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*
d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (B*c^3*d - A*c
^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^4*d
^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^3*d^3 + a*c*d
^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c
*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e) + (a*c^
5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f)*sin(f*x +
 e)), 1/2*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 -
 (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A
 - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d -
(4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*
d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*
d - a*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*
d^2)*cos(f*x + e))) + sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A
 - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A
 - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A -
 B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A -
 B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A -
 B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e)
 - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin
(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(B*c^3*d - A*c^2*d^2 -

```
 B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (
B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^
3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*
c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x
 + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)
*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(182) = 364$.

Time = 0.38 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.38

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{2}\left(\sqrt{2}B\sqrt{a}c^2-3\sqrt{2}A\sqrt{a}cd+\sqrt{2}B\sqrt{a}cd-\sqrt{2}A\sqrt{a}d^2+2\sqrt{2}B\sqrt{a}d^2\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(ac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-ac^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-acd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+ad^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-}}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/2*(sqrt(2)*(sqrt(2)*B*sqrt(a)*c^2 - 3*sqrt(2)*A*sqrt(a)*c*d + sqrt(2)*B*
sqrt(a)*c*d - sqrt(2)*A*sqrt(a)*d^2 + 2*sqrt(2)*B*sqrt(a)*d^2)*arctan(sqrt(
2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a*c^3*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) - a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a*c*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sqrt(-c*d - d^2)) - 2*(A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2
*sqrt(2)*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a*d^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))) + 2*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
 2*sqrt(2)*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*(B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2
*e) - A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a*c^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
), x)
```

3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 309

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx &= -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3f} \\ &\quad+\frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{a}(c-d)^3\sqrt{d}(c+d)^{5/2}f} \\ &\quad-\frac{(Bc-Ad)\cos(e+fx)}{2(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\ &\quad+\frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4(c^2-d^2)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\end{aligned}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)^3/f/a^(1/2)+1/4*(A*d*(15*c^2+10*c*d+7*d^2)-B*(3*c^3+6*c^2*d+19*c*d
^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^
(1/2))/(c-d)^3/(c+d)^(5/2)/f/a^(1/2)/d^(1/2)-1/2*(-A*d+B*c)*cos(f*x+e)/(c^2
-d^2)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*(A*d*(7*c+d)-B*(3*c^2
+c*d+4*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/
2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3063, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$$
$$=\frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{a}\sqrt{d}f(c-d)^3(c+d)^{5/2}}$$
$$-\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^3}$$
$$+\frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4f(c^2-d^2)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$
$$-\frac{(Bc-Ad)\cos(e+fx)}{2f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^3*f)) + ((A*d*(15*c^2 + 10*c*d + 7*d^2) - B*(3
*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[a]*(c - d)^3*Sqrt[d]*(c +
d)^(5/2)*f) - ((B*c - A*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a + a*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^2) + ((A*d*(7*c + d) - B*(3*c^2 + c*d + 4*d^2)
)*Cos[e + f*x])/(4*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(A(4c+d)-B(c+4d))-\frac{3}{2}a(Bc-Ad)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx}{2a\left(c^2-d^2\right)}\\ &= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4\left(c^2-d^2\right)^2 f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\\ &\quad + \frac{\int \frac{\frac{1}{4}a^2\left(8Ac^2-5Bc^2+9Acd-15Bcd+7Ad^2-4Bd^2\right)-\frac{1}{4}a^2\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{2a^2\left(c^2-d^2\right)^2}\end{aligned}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4(c^2-d^2)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(A-B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{(c-d)^3} - \frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{8a(c-d)^3(c+d)^2}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4(c^2-d^2)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(2(A-B))\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)^3f} + \frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{4(c-d)^3(c+d)^2f}$$

$$= -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3f} + \frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{a}(c-d)^3\sqrt{d}(c+d)^{5/2}f} - \frac{(Bc-Ad)\cos(e+fx)}{2(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4(c^2-d^2)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.96 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.76

$$\int\frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((32+32i)(-1)^{3/4}(A-B)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e\right.\right.\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((32 + 32*I)*(-1)^(3/4)*(A - B)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + ((A*d*(15*c^2 + 10*c*
d + 7*d^2) - B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(
e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d
*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)
/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + T
an[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c
 + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3
)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(Sqrt[d]*(c + d)^(5/2)) + ((-(A*d*(
15*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*
x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 +
c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1
+ Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c +
 d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 +
 Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e
+ f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(Sqrt[d]*(c + d)^(5/
2)) - (8*(c - d)^2*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c +
 d)*(c + d*Sin[e + f*x])^2) - (4*(c - d)*(-(A*d*(7*c + d)) + B*(3*c^2 + c*d
 + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e +
 f*x]))))/(16*(c - d)^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. $2(276) = 552$.

Time = 1.59 (sec) , antiderivative size = 2275, normalized size of antiderivative = 7.36

method	result	size
default	Expression too large to display	2275

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4*(-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^
5-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*a^4*c^3*d-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^2*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^3*d+4*B*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a
^4*c^2*d^2-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*d^4+4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*d^4-6*
B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^4*d+A*(-
```

a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4-5*B*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^4+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(7/2)*d^4+A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/
2)*d^4-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*d^4-4*B*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*d^5
+7*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x
+e)^2*d^5-19*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/
2)*c^3*d^2-4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/
2)*c^2*d^3+15*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9
/2)*c^4*d+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/
2)*c^3*d^2+7*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/
2)*c^2*d^3+16*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^2*d^2-16*A*(a*(c+d)*d)^(1/2)*2^(1/2)
arctanh(1/2(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^2*
d^2-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)*a^4*c*d^3+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/
2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^3*d+8*B*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))
*sin(f*x+e)*a^4*c*d^3-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c*d^3+4*B*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)
^2*a^4*c^2*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c*d^3-8*A*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^
3*d-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c^2*d^2-19*B*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c*d^4+30*A*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2+20*A*arctanh
((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3-
6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+
e)^2*c^2*d^3-7*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^2*d^
2+14*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f
*x+e)*c*d^4-6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9
/2)*sin(f*x+e)*c^4*d-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2-38*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3-8*B*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4+9*A*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d-A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(7/2)*c^2*d^2-9*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)
*a^(7/2)*c*d^3+B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d+
B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^2*d^2-B*(-a*(sin(f*
x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3+15*A*arctanh((-a*(sin(f*x+e)
-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c^2*d^3+10*A*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c*d^4-3*
B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)

```
^2*c^3*d^2+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d^3+3*
B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^3*d-2*B*(-a*(sin(f*
x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^2*d^2+3*B*(-a*(sin(f*x+e)-1))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d^3-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4+4*B*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4)*(-
a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/a^(9/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f
*x+e))^2/(c+d)^2/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(276) = 552$.

Time = 4.28 (sec) , antiderivative size = 4180, normalized size of antiderivative = 13.53

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A -
8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*
A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 -
(6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (35*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c
*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2
*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A
 - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B
)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 -
 (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d
^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c
^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)
*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*
cos(f*x + e)^2 + 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*
x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x
+ e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 8*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*
c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 +
 (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d
^5 + (A - B)*a*d^6)*cos(f*x + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3
*d^3 + 9*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e
)^2 + ((A - B)*a*c^5*d + 3*(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A -
```

B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*
a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4
+ 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2
d^4 + 3(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4
d^2 + 3(A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x
+ e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e)
- 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqr
t(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e
) - cos(f*x + e) - 2))/sqrt(a) + 4*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*
A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6
+ (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A
- 4*B)*d^6)*cos(f*x + e)^2 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^
3*d^3 + (11*A - B)*c^2*d^4 + (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B
*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4
- (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A
- B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9
)*f*cos(f*x + e)^3 + (2*a*c^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 +
6*a*c^3*d^6 + 3*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d
- 2*a*c^6*d^3 + 2*a*c^2*d^7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d
^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*
d^9)*f + ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2
- 2*(a*c^7*d^2 - 3*a*c^5*d^4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c
^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f)*sin(f*x + e)), 1/8*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d
- 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (
7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4
- (7*A - 4*B)*d^5)*cos(f*x + e)^3 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (3
5*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2
+ (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)
*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 -
3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*
(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3
- (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*
(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x +
e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt
(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x +
e))) - 4*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*
d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a
*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x
+ e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3*d^3 + 9*(A - B)*a*c^2*d^4
+ 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + ((A - B)*a*c^5*d + 3*
(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A - B)*a*c^2*d^4 + 3*(A - B)*a
*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d
^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A -

```
 B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 +
(A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4*d^2 + 3*(A - B)*a*c^3*d^3
+ 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x + e))*sin(f*x + e))*log(-(
cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x
 + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/
(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt
(a) + 2*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A -
 B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 + (3*B*c^4*d^2 - (7*A - B)*
c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e)^2
 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^3*d^3 + (11*A - B)*c^2*d^4
+ (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B*c^5*d - (9*A + 2*B)*c^4*d^
2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A -
4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c
*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^3 + (2*a*c
^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 + 6*a*c^3*d^6 + 3*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d - 2*a*c^6*d^3 + 2*a*c^2*d^
7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*
d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f + ((a*c^6*d^3 - 3*a*
c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^7*d^2 - 3*a*c^5*d^
4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^
6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f)*sin
(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(276) = 552$.

Time = 0.48 (sec) , antiderivative size = 876, normalized size of antiderivative = 2.83

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3} dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/8*(sqrt(2)*(3*sqrt(2)*B*sqrt(a)*c^3 - 15*sqrt(2)*A*sqrt(a)*c^2*d + 6*sqr
t(2)*B*sqrt(a)*c^2*d - 10*sqrt(2)*A*sqrt(a)*c*d^2 + 19*sqrt(2)*B*sqrt(a)*c*
d^2 - 7*sqrt(2)*A*sqrt(a)*d^3 + 4*sqrt(2)*B*sqrt(a)*d^3)*arctan(sqrt(2)*d*s
in(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a*c^5*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) - a*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a*c^3*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*a*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + a*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a*d^5*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - 8*(A*sqrt(a) - B*sqrt(a))*
log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^3*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) - 3*sqrt(2)*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*sqrt(2)*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 8*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 - 3*sqrt(2)*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a*c*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) + 4*(6*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 1
4*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 2*B*sqrt(a)*c*d^2*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^3 - 2*A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^3 + 8*B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*B*sqrt(a)*c^3*s
in(-1/4*pi + 1/2*f*x + 1/2*e) + 9*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e) - 4*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 8*A*sqrt(a)*c*d^
2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x
+ 1/2*e) - A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*sqrt(a)*d^3*s
in(-1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a*c^4*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - 2*sqrt(2)*a*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)
*a*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^2 - c - d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
), x)
```

3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 283

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)^2(3B(c-5d)+A(c+11d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} + \frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} + \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
^2*(3*B*(c-5*d)+A*(c+11*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin
(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/15*d*(15*A*c^2-120*A*c*d+65*A*d^2-99*B*
c^2+168*B*c*d-93*B*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+1/10*(5*A-9*B
)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^(1/2)+1/30*d^2*(15*A
*c-35*A*d-51*B*c+39*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3056, 3062, 3047, 3102, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)^2(A(c+11d)+3B(c-5d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d^2(15Ac-35Ad-51Bc+39Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{30a^2f} + \frac{d(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2)\cos(e+fx)}{15af\sqrt{a\sin(e+fx)+a}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}} + \frac{d(5A-9B)\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/2*((c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x
])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + (d*(15*A*c^2
- 99*B*c^2 - 120*A*c*d + 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(15
*a*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*C
os[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(30*a^2*f) + ((5*A - 9*B)*d*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(10*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^3)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(3B(c-2d)+A(c+6d))-\frac{1}{2}a(5A-9B)d\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{4}a^2\left(5A\left(c^2+7cd-4d^2\right)+3B\left(5c^2-13cd+12d^2\right)\right)-\frac{1}{4}a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{5a^3}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\frac{1}{4}a^2c\left(5A\left(c^2+7cd-4d^2\right)+3B\left(5c^2-13cd+12d^2\right)\right)+\left(-\frac{1}{4}a^2cd(15Ac-51Bc-35Ad+39Bd)+\frac{1}{4}a^2d\left(5A\left(c^2+7cd-4d^2\right)+3B\left(5c^2-13cd\right.\right.\right.}{\sqrt{a+a\sin(e+fx)}}}{5a^3}$$

$$= \frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} + \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{8}a^3\left(3B\left(15c^3-39c^2d+53cd^2-13d^3\right)+5A\left(3c^3+21c^2d-15cd^2+7d^3\right)\right)-\frac{1}{4}a^3d\left(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2\right)}{\sqrt{a+a\sin(e+fx)}}}{15a^4}$$

$$= \frac{d(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} + \frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} + \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{((c-d)^2(3B(c-5d)+A(c+11d)))\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= \frac{d(15Ac^2 - 99Bc^2 - 120Acd + 168Bcd + 65Ad^2 - 93Bd^2)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} + \frac{d^2(15Ac - 51Bc - 35Ad + 39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} + \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((c-d)^2(3B(c-5d)+A(c+11d)))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$

$$= -\frac{(c-d)^2(3B(c-5d)+A(c+11d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d(15Ac^2 - 99Bc^2 - 120Acd + 168Bcd + 65Ad^2 - 93Bd^2)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} + \frac{d^2(15Ac - 51Bc - 35Ad + 39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} + \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.42

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-30Ac^3\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*c
^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e + f*
x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 110*A
*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3*(e
+ f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e + f*x
))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2] + 30
*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*d^3*Cos
[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e + f*x)/2
```

```
] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*B*c^2*d*S
in[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[(e + f*x)/
2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (30 + 30*I)*
(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*x))/2] + 210*B
*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2] - 123*B*d^3*Sin
[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*d^3*Sin[(5*(e +
f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(e + f*x))/2]))/(6
0*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(256) = 512.

Time = 3.18 (sec) , antiderivative size = 817, normalized size of antiderivative = 2.89

method	result	size
parts	Expression too large to display	817
default	Expression too large to display	1030

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*A*c^3/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/4*c^2*(3*A*d+B*c)/a^(5/2)*(-3*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)-3*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+2*(a-a*sin(f*x+e))^(1/2)*a
^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/12*
d^2*(A*d+3*B*c)*(-8*(a-a*sin(f*x+e))^(3/2)*sin(f*x+e)*a^(1/2)-24*(a-a*sin(f
*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)-8*(a-a*sin(f*x+e))^(3/2)*a^(1/2)-30*(a-a
*sin(f*x+e))^(1/2)*a^(3/2)+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f+1/20*d^3*B*(-8*(a-a*sin(f*x+e))^(5/2)*a^(1/2)*sin(f*x+e)-80*
(a-a*sin(f*x+e))^(1/2)*a^(5/2)*sin(f*x+e)-8*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-
90*(a-a*sin(f*x+e))^(1/2)*a^(5/2)+75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f-3/4*c*d*(A*d+B*c)/a^(5/2)*(-7*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)+8*(a-a*sin(f*x+e))^(1/
2)*sin(f*x+e)*a^(1/2)-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a+10*(a-a*sin(f*x+e))^(1/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(256) = 512$.

Time = 0.29 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}(2(A+3B)c^3+6(3A-7B)c^2d-6(7A-11B)c}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/120*(15*sqrt(2)*(2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c
*d^2 + 2*(11*A - 15*B)*d^3 - ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A
- 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 + 3*(3*A
 - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e) + (2
*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c*d^2 + 2*(11*A - 15*
B)*d^3 + ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*
A - 15*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(12*B*d^
3*cos(f*x + e)^4 - 15*(A - B)*c^3 + 45*(A - B)*c^2*d - 45*(A - B)*c*d^2 + 1
5*(A - B)*d^3 + 4*(15*B*c*d^2 + (5*A - 3*B)*d^3)*cos(f*x + e)^3 - 4*(45*B*c
^2*d + 15*(3*A - 4*B)*c*d^2 - 4*(5*A - 9*B)*d^3)*cos(f*x + e)^2 - 15*((A -
B)*c^3 - 3*(A - 5*B)*c^2*d + 15*(A - B)*c*d^2 - (5*A - 9*B)*d^3)*cos(f*x +
e) + (12*B*d^3*cos(f*x + e)^3 + 15*(A - B)*c^3 - 45*(A - B)*c^2*d + 45*(A -
 B)*c*d^2 - 15*(A - B)*d^3 - 4*(15*B*c*d^2 + (5*A - 6*B)*d^3)*cos(f*x + e)^
2 - 60*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
- 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(256) = 512$.

Time = 0.40 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.30

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\frac{15\sqrt{2}\left(A\sqrt{a}c^3+3\,B\sqrt{a}c^3+9\,A\sqrt{a}c^2d-21\,B\sqrt{a}c^2d-21\,A\sqrt{a}cd^2+33\,B\sqrt{a}cd^2+11\,A}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/120*(15*sqrt(2)*(A*sqrt(a)*c^3 + 3*B*sqrt(a)*c^3 + 9*A*sqrt(a)*c^2*d - 21
*B*sqrt(a)*c^2*d - 21*A*sqrt(a)*c*d^2 + 33*B*sqrt(a)*c*d^2 + 11*A*sqrt(a)*d
^3 - 15*B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(a)*c^3 + 3*B*sqrt(a)*c^3
 + 9*A*sqrt(a)*c^2*d - 21*B*sqrt(a)*c^2*d - 21*A*sqrt(a)*c*d^2 + 33*B*sqrt(
a)*c*d^2 + 11*A*sqrt(a)*d^3 - 15*B*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 30*sqrt(2)*(A*sqr
t(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f
*x + 1/2*e) - 3*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*B*sqrt(a
)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/
2*f*x + 1/2*e) - 3*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*sqrt(
a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))) + 16*sqrt(2)*(12*B*a^(17/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^5 - 30*B*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^(17/
2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 45*B*a^(17/2)*c^2*d*sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 45*A*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 45
*B*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 15*A*a^(17/2)*d^3*sin(-1
/4*pi + 1/2*f*x + 1/2*e) + 30*B*a^(17/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(a^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2
),x)
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2
), x)
```

3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 203

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
*(A*c+7*A*d+3*B*c-11*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/3*d*(3*A*c-9*A*d-15*B*c+13*B*d)*cos(f*x+e
)/a/f/(a+a*sin(f*x+e))^(1/2)+1/6*(3*A-7*B)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3056, 3047, 3102, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)(Ac+7Ad+3Bc-11Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f} + \frac{d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{3af\sqrt{a\sin(e+fx)+a}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/2*((c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])
/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + (d*(3*A*c - 15*
B*c - 9*A*d + 13*B*d)*Cos[e + f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*
A - 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((A - B)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} \\
&\quad+\frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(Ac+3Bc+4Ad-4Bd)-\frac{1}{2}a(3A-7B)d\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2} \\
&= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} \\
&\quad+\frac{\int \frac{\frac{1}{2}ac(Ac+3Bc+4Ad-4Bd)+\left(-\frac{1}{2}a(3A-7B)cd+\frac{1}{2}ad(Ac+3Bc+4Ad-4Bd)\right)\sin(e+fx)-\frac{1}{2}a(3A-7B)d^2\sin^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2} \\
&= \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} \\
&\quad+\frac{\int \frac{-\frac{1}{4}a^2\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)-\frac{1}{2}a^2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a^3}
\end{aligned}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{((c-d)(Ac+3Bc+7Ad-11Bd))\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((c-d)(Ac+3Bc+7Ad-11Bd))\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$

$$= -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.73 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^2\sin\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(
-1)^(3/4)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(
```

```
3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 6*d
*(-4*B*c - 2*A*d + 3*B*d)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^2 - 2*B*d^2*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - 6*d*(-4*B*c - 2*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - 2*B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[(3*(
e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(180) = 360$.

Time = 2.59 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.01

method	result
parts	$-\frac{A c^2\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\right)\sqrt{-a(\sin(fx+}}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
default	$-\frac{\left(\sin(fx+e)\left(3A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2c^2+18A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2cd-21A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e}}{2\sqrt{a}}\right.\right.\right.}{}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*A*c^2/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/4*c*(2*A*d+B*c)/a^(5/2)*(-3*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)-3*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+2*(a-a*sin(f*x+e))^(1/2)*a^(
1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/4*d*(
A*d+2*B*c)/a^(5/2)*(7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a*sin(f*x+e)+7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a-8*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(1/2)-10*(a-a*sin(f*x+e))^(1
/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+
1/12*d^2*B*(-33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a^2*sin(f*x+e)+24*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+8*(a-a*sin(f*x
+e))^(3/2)*sin(f*x+e)*a^(1/2)-33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*a^2+30*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+8*(a-a*sin(f*x+e))^
(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e)
)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(180) = 360$.

Time = 0.27 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{3\sqrt{2}(2(A+3B)c^2+4(3A-7B)cd-2(7A-11B)d^2-((A+3B)c^2+2(3A-7B)cd-(7A-11B)\cdots}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2
- ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e)^2 + (
(A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e) + (2*(A
+ 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2 + ((A + 3*B)*c^2 + 2*(3
*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) + 4*(4*B*d^2*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
 B)*d^2 - 4*(6*B*c*d + (3*A - 4*B)*d^2)*cos(f*x + e)^2 - 3*((A - B)*c^2 - 2
*(A - 5*B)*c*d + 5*(A - B)*d^2)*cos(f*x + e) - (4*B*d^2*cos(f*x + e)^2 - 3*
(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 + 12*(2*B*c*d + (A - B)*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 -
 a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e)
)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^2}{(a\,(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1)
)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(180) = 360$.

Time = 0.35 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.27

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\left(A\sqrt{a}c^2+3B\sqrt{a}c^2+6A\sqrt{a}cd-14B\sqrt{a}cd-7A\sqrt{a}d^2+11B\sqrt{a}d^2\right)\log\left(\sin\left(-\frac{1}{4}\right.\right.}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*(A*sqrt(a)*c^2 + 3*B*sqrt(a)*c^2 + 6*A*sqrt(a)*c*d - 14*B*s
qrt(a)*c*d - 7*A*sqrt(a)*d^2 + 11*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt
(a)*c^2 + 3*B*sqrt(a)*c^2 + 6*A*sqrt(a)*c*d - 14*B*sqrt(a)*c*d - 7*A*sqrt(a
)*d^2 + 11*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 6*sqrt(2)*(A*sqrt(a)*c^2*sin(-1/4*pi +
1/2*f*x + 1/2*e) - B*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*A*sqrt(
a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 2*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f
*x + 1/2*e) + A*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*d^2*
sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 16*sqrt(2)*(2*B*a^(9/2)*d^2*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 - 6*B*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)
- 3*A*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*B*a^(9/2)*d^2*sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c+d\,\sin(e+f\,x)\right)^2}{\left(a+a\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2
), x)
```

3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 133

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(Ac+3Bc+3Ad-7Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*c+3*A*d+3*B*c-7
*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2
)/f*2^(1/2)-2*B*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3047, 3098, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(Ac+3Ad+3Bc-7Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -1/2*((A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]
*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((A - B)*(c - d)*Cos[e +
 f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)) - (2*B*d*Cos[e + f*x])/(a*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{Ac + (Bc + Ad)\sin(e + fx) + Bd\sin^2(e + fx)}{(a + a\sin(e + fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3B(c-d)+A(c+3d))-2aBd\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} + \frac{(Ac+3Bc+3Ad-7Bd)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{(Ac+3Bc+3Ad-7Bd)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$

$$= -\frac{(Ac+3Bc+3Ad-7Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4
)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f*x)/
2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(116) = 232$.

Time = 2.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.92

method	result
default	$-\frac{\left(\sin(fx+e)\left(A\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)ac+3A\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)ad+3B\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.\right.}{}$
parts	$-\frac{Ac\left(\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\right)\sqrt{-a(\sin(fx+}}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+8*B*
(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d-2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+10*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(116) = 232$.

Time = 0.31 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(((A+3B)c+(3A-7B)d)\cos(fx+e)^{2}-2(A+3B)c-2(3A-7B)d-((A+3B)c+(3A-7\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c
 - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A + 3
*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*sin(f
*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
```

```
)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
 + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c - (A - B
)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e) - (A - B
)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})}{(a\,(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)}{(a\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3
/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{%%{[268435456,0]:[1,0,-2]%%},[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(c+d\,\sin(e+f\,x)\right)}{\left(a+a\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
x)
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
 x)
```

3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=-\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A+3*B)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2829, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=-\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f
*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}+\frac{(A+3B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}\\
&= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}-\frac{(A+3B)\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}\\
&= -\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.72

$$\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A +
B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(72) = 144.

Time = 1.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

method	result
default	$-\frac{\left(\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a(A+3B)+A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a+3B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.}{4a^{\frac{5}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$-\frac{A\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\right)\sqrt{-a(\sin(fx+e)}}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*2^(1/2)*a*(A+3*B)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*A-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*B)*(-a*
(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(72) = 144.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left((A+3\,B)\cos\left(fx+e\right)^{2}-(A+3\,B)\cos\left(fx+e\right)-\left((A+3\,B)\cos\left(fx+e\right.\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*B
)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(72) = 144$.

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{2}(A\sqrt{a}+3B\sqrt{a})\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}(A\sqrt{a}+3B\sqrt{a})\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}}{8f}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] 1/8*(sqrt(2)*(A*sqrt(a) + 3*B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(a) + 3*B*sq
rt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))) - 2*sqrt(2)*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sq
rt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}}\,dx = \int \frac{A + B\,\sin(e + f\,x)}{(a + a\,\sin(e + f\,x))^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 187

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A(c-5d)+B(3c+d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^2f} + \frac{2\sqrt{d}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^2\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*(c-5*d)+B*(3*c+
d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/
(c-d)^2/f*2^(1/2)+2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/
2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^2/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3057, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A(c-5d)+B(3c+d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2} + \frac{2\sqrt{d}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -1/2*((A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^2*f) + (2*Sqrt[d]*(B*c
- A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e
 + f*x]])])/(a^(3/2)*(c - d)^2*Sqrt[c + d]*f) - ((A - B)*Cos[e + f*x])/(2*(
c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3Bc+A(c-4d))-\frac{1}{2}a(A-B)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{2a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(d(Bc-Ad))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a^2(c-d)^2} + \frac{(A(c-5d)+B(3c+d))\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a(c-d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(2d(Bc-Ad))\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a(c-d)^2f} - \frac{(A(c-5d)+B(3c+d))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a(c-d)^2f}$$

$$= -\frac{(A(c-5d)+B(3c+d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^2f} + \frac{2\sqrt{d}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^2\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.96 (sec) , antiderivative size = 781, normalized size of antiderivative = 4.18

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\right.}{}$$

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/
4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A*d)*
(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*
#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*L
og[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*S
qrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]
*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 +
Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-(B*c) + A*d)*(e + f*x -
 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#
1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + T
an[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]
*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sq
rt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f
x)/4]]#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &])*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(158) = 316.

Time = 0.98 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.34

method	result
default	$-\frac{\left(\sin(fx+e)\left(8A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{3}{2}}d^{2}-8B\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{3}{2}}cd+A\sqrt{a(c+d)d}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx}}{2\sqrt{a}}\right.\right.\right.}{}$

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)

[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(

```
c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a*d+2*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a*(
c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d-2*B*(a*(c+d)*d)^(1/2)*(a-a*s
in(f*x+e))^(1/2)*a^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(
1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(158) = 316$.

Time = 1.81 (sec) , antiderivative size = 1561, normalized size of antiderivative = 8.35

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c +
 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*c
- 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e))*s
qrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*
(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*
a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x + e)^2
+ (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*d)*cos(f
*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
 + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^
2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
 + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)
*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x +
e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d
^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c
*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*
d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2
*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^
2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*
d^2)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x +
e)^2 - 2*(A + 3*B)*c + 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
+ e) - (2*(A + 3*B)*c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
 + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
```

```
- (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(2*B*a*c - 2*A*a*d - (B*a*c -
A*a*d)*cos(f*x + e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d +
 (B*a*c - A*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/
2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/
(d*cos(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*d)*cos(
f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/
((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d +
a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 -
 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)
*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(158) = 316$.

Time = 0.36 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.52

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\frac{8\sqrt{2}(Bcd-Ad^2)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx\right.}{\sqrt{-cd-d^2}}\right.}{\left(\sqrt{2}a^{\frac{3}{2}}c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}a^{\frac{3}{2}}cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{}\right.}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/4*(8*sqrt(2)*(B*c*d - A*d^2)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^(3/2)*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 2*sqrt(2)*a^(3/2)*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*
a^(3/2)*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (A*sqr
t(a)*c + 3*B*sqrt(a)*c - 5*A*sqrt(a)*d + B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2
*sqrt(2)*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*c + 3*B*sqrt(a)*c - 5*A*sqrt(
a)*d + B*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
- 2*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*sin(-1/4*pi + 1/2
*f*x + 1/2*e))/((sqrt(2)*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2
)*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),
 x)
```

3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 292

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \\
&-\frac{(Ac+3Bc-9Ad+5Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} \\
&-\frac{\sqrt{d}(Ad(5c+3d)-B(3c^2+3cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^3(c+d)^{3/2}f} \\
&-\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} \\
&+\frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/4*(
A*c-9*A*d+3*B*c+5*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+
e))^(1/2))/a^(3/2)/(c-d)^3/f*2^(1/2)-(A*d*(5*c+3*d)-B*(3*c^2+3*c*d+2*d^2))*
arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1
/2)/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f+1/2*d*(B*(3*c+d)-A*(c+3*d))*cos(f*x+e)/a/
(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{d}(Ad(5c+3d)-B(3c^2+3cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}} - \frac{(Ac-9Ad+3Bc+5Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3} + \frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -1/2*((A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]
*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[2]*a^(3/2)*(c - d)^3*f) - (Sqrt[d]*(A*d*
(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f
*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(a^(3/2)*(c - d)^3*(c + d)^(3
/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c
+ d*Sin[e + f*x])) + (d*(B*(3*c + d) - A*(c + 3*d))*Cos[e + f*x])/(2*a*(c -
 d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(Ac+3Bc-6Ad+2Bd)-\frac{3}{2}a(A-B)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx}{2a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}a^2(A(c^2-7cd-6d^2)+B(3c^2+5cd+4d^2))-\frac{1}{2}a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{2a^3(c-d)^2(c+d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(Ac+3Bc-9Ad+5Bd)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a(c-d)^3} + \frac{(d(Ad(5c+3d)-B(3c^2+3cd+2d^2)))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2a^2(c-d)^3(c+d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(Ac+3Bc-9Ad+5Bd)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a(c-d)^3f} - \frac{(d(Ad(5c+3d)-B(3c^2+3cd+2d^2)))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a(c-d)^3(c+d)f}$$

$$= -\frac{(Ac+3Bc-9Ad+5Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} - \frac{\sqrt{d}(Ad(5c+3d)-B(3c^2+3cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^3(c+d)^{3/2}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.57 (sec) , antiderivative size = 904, normalized size of antiderivative = 3.10

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-B)(c-d)\sin\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-1)
^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-(A*d
*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 +
 Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*L
og[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*
x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[
-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*
#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c +
d)^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x
- 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*
#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 +
Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + S
qrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e +
f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*d)*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c
+ d*Sin[e + f*x]))))/(4*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. $2(259) = 518$.

Time = 1.40 (sec) , antiderivative size = 2049, normalized size of antiderivative = 7.02

method	result	size
default	Expression too large to display	2049

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f
*x+e)*c*d^2-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(
1/2)*2^(1/2)/a^(1/2))*a*c^2*d-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2-6*B*(-a*(sin(f*x+e)-1))^(1/
2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d+4*B*(-a*(sin(f*x+e)-1))^(1/2)
*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d+5*B*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c
*d^2+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^
(1/2)/a^(1/2))*sin(f*x+e)^2*a*d^3+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3-9*A*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)*a*d^3+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*d^3-9*A*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)^2*a*d^3+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*
sin(f*x+e)*c^2*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^
2+11*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)*a*c^2*d+13*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2-7*A*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin
(f*x+e)*a*c^2*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1
))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d-17*A*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^
2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2-6*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(1/2)*sin(f*x+e)*d^3+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(s
in(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^3+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a
*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^3+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^3+2*A*(-a*(sin(f*x+e)-
1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^2-4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*
```

```
(c+d)*d)^(1/2)*a^(1/2)*c^2*d+6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(1/2)*c*d^2-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(3/2)*sin(f*x+e)*c^3*d+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*c*d^3-24*B*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^2*d^2-20*B*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c*d^3-12*B*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*c^2*d^2-1
2*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+
e)^2*c*d^3+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3
/2)*sin(f*x+e)*c^2*d^2+32*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)
^(1/2))*a^(3/2)*sin(f*x+e)*c*d^3-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(1/2)*c^3+12*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2
))*a^(3/2)*sin(f*x+e)^2*d^4-8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4+12*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*d^4-8*B*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*d^4+12*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+20*A*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2-12*B*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^3*d-12*B*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2-8*B*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+2*A*(-a*(sin(f*x+e)-1))^(
1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3-4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d
)^(1/2)*a^(1/2)*d^3)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f
*x+e))/(c+d)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(259) = 518$.

Time = 4.26 (sec) , antiderivative size = 3403, normalized size of antiderivative = 11.65

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d
^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d
^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c
*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^
2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^
3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A
+ 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*
B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin
```

(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A + 3*B)*c*d^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*

```
cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8
*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A -
 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d
- (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5
*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*
sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*((A - B)*
c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A
 + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A
 + 3*B)*c*d^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A -
 B)*c*d^2 + (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^
2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*
d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
- a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*
x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4
 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(
f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d
^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a
^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(259) = 518$.

Time = 0.45 (sec) , antiderivative size = 866, normalized size of antiderivative = 2.97

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(3*sqrt(2)*B*sqrt(a)*c^2*d - 5*sqrt(2)*A*sqrt(a)*c*d^2 + 3*s
qrt(2)*B*sqrt(a)*c*d^2 - 3*sqrt(2)*A*sqrt(a)*d^3 + 2*sqrt(2)*B*sqrt(a)*d^3)
*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^2*c^
4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 2*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*d^4*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (A*sqrt(a)*c + 3*B*s
qrt(a)*c - 9*A*sqrt(a)*d + 5*B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^2
*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^2*c*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))) - (A*sqrt(a)*c + 3*B*sqrt(a)*c - 9*A*sqrt(a)*d + 5*B*sqrt(a)*d)*log(-
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) - 3*sqrt(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*sqrt(2)*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(2*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 - 6*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*A*sqrt
(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 2*B*sqrt(a)*d^2*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)
*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x
 + 1/2*e) + 6*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*sqrt(a)*d^
2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/
2*e))/((sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*c*d^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
2 - 3*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + c + d)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
), x)
```

3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 402

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A(c-13d)+3B(c+3d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4f} - \frac{\sqrt{d}(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{3/2}(c-d)^4(c+d)^{5/2}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{d(3B(3c^2+3cd+2d^2)-A(2c^2+15cd+7d^2))\cos(e+fx)}{4a(c-d)^3(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/4
*(A*(c-13*d)+3*B*(c+3*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1/2)-1/4*(A*d*(35*c^2+42*c*d+19*d^2)-3*B
*(5*c^3+10*c^2*d+13*c*d^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^
(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f+1/2*d*(
B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*s
in(f*x+e))^(1/2)+1/4*d*(3*B*(3*c^2+3*c*d+2*d^2)-A*(2*c^2+15*c*d+7*d^2))*cos
(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\sqrt{d}(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}$$
$$-\frac{(A(c-13d)+3B(c+3d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^4}$$
$$+\frac{d(3B(3c^2+3cd+2d^2)-A(2c^2+15cd+7d^2))\cos(e+fx)}{4af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$
$$+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] -1/2*((A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2
]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^4*f) - (Sqrt[d]*(A*d
*(35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*Ar
cTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])
])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d
)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + (d*(B*(2*c + d) -
A*(c + 2*d))*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^2) + (d*(3*B*(3*c^2 + 3*c*d + 2*d^2) - A*(2*c^2 + 15
*c*d + 7*d^2))*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e +
f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(Ac+3Bc-8Ad+4Bd)-\frac{5}{2}a(A-B)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx}{2a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{\int \frac{a^2\left(A\left(c^2-9cd-7d^2\right)+3B\left(c^2+2cd+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx}{4a^3(c-d)^2(c+d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-9Bc^2+15Acd-9Bcd+7Ad^2-6Bd^2)\cos(e+fx)}{4a(c-d)^3(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a^3\left(A\left(2c^3-20c^2d-35cd^2-19d^3\right)+3B\left(2c^3+7c^2d+11cd^2+4d^3\right)\right)-\frac{1}{2}a^3d\left(2Ac^2-9Bc^2+15Acd-9Bcd+7Ad^2-6Bd^2\right)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}}{4a^4(c-d)^3(c+d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-9Bc^2+15Acd-9Bcd+7Ad^2-6Bd^2)\cos(e+fx)}{4a(c-d)^3(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(A(c-13d)+3B(c+3d))\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a(c-d)^4} + \frac{(d(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3)))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{8a^2(c-d)^4(c+d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-9Bc^2+15Acd-9Bcd+7Ad^2-6Bd^2)\cos(e+fx)}{4a(c-d)^3(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(A(c-13d)+3B(c+3d))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a(c-d)^4f} - \frac{(d(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3)))\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(}{\sqrt{a+a\sin}}\right.}{4a(c-d)^4(c+d)^2f}$$

$$= -\frac{(A(c-13d)+3B(c+3d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4f} - \frac{\sqrt{d}(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{3/2}(c-d)^4(c+d)^{5/2}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(2Ac^2-9Bc^2+15Acd-9Bcd+7Ad^2-6Bd^2)\cos(e+fx)}{4a(c-d)^3(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 12.80 (sec) , antiderivative size = 1757, normalized size of antiderivative = 4.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \frac{(1+i)(Ac+3Bc-13Ad+9Bd)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^3\right.}{\left(2\sqrt[4]{-1}c^4-8\sqrt[4]{-1}c^3d+\right.}$$
$$+\frac{\sqrt{d}(-Ad(35c^2+42cd+19d^2)+3B(5c^3+10c^2d+13cd^2+4d^3))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+\text{R}\right.}{}$$
$$-\frac{\sqrt{d}(-Ad(35c^2+42cd+19d^2)+3B(5c^3+10c^2d+13cd^2+4d^3))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+\text{R}\right.}{}$$
$$+\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8Ac^4\cos\left(\frac{1}{2}(e+fx)\right)+8Bc^4\cos\left(\frac{1}{2}(e+fx)\right)-8Ac^3d\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]

[Out] ((1 + I)*(A*c + 3*B*c - 13*A*d + 9*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((2*(-1)^(1/4)*c^4 - 8*(-1)^(1/4)*c^3*d + 12*(-1)^(1/4)*c^2*d^2 - 8*(-1)^(1/4)*c*d^3 + 2*(-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(3/2)) + (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) - (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-8*A*c^4*Cos[(e + f*x)/2] + 8*B*c^4*Cos[(e + f*x)/2] - 8*A*c^3*d*Cos[(e + f*x)/2] + 26*B*c^3*d*Cos[(e + f*x)/2] - 22*A*c^2*d^2*Cos[(e + f*x)/2] + 6*B*c^2*d^2*Cos[(e + f*x)/2] - 10*A*c*d^3*Cos[(e + f*x)/2] + 4*B*c*d^3*Cos[(e + f*x)/2] + 4*B*d^4*Cos[(e + f*x)/2] - 8*A*c^3*d*Cos[(3*(e + f*x))/2] + 26*B*c^3*d*Cos[(3*(e + f*x))/2] - 40*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 31*B*c^2*d^2*Cos[(3*(e + f*x))/2] - 25*A*c*d^3*Cos[(3*(e + f*x))/2] + 13*B*c*d^3*Cos[(3*(e + f*x))/2] + A*d^4*Cos[(3*(e + f*x))/2] + 2*B*d^4*Cos[(3*(e + f*x))/2] + 2*A*c^2*d^2*Cos[(5*(e + f*x))/2] - 9*B*c^2*d^2*Cos[(5*(e + f*x))/2] + 15*A*c*d^3*Cos[(5*(e + f*x))/2] - 9*B*c*d^3*Cos[(5*(e + f*x))/2] + 7*A*d^4*Cos[(5*(e + f*x))/2] - 6*B*d^4*Cos[(5*(e + f*x))/2] + 8*A*c^4*Sin[(e + f*x)/2] - 8*B*c^4*Sin[(e + f*x)/2] + 8*A*c^3*d*Sin[(e + f*x)/2] - 26*B*c^3*d*Sin[(e + f*x)/2] + 22*A*c^2*d^2*Sin[(e + f*x)/2] - 6*B*c^2*d^2*Sin[(e + f*x)/2] + 10*A*c*d^3*Sin[(e + f*x)/2] - 4*B*c*d^3*Sin[(e + f*x)/2] - 4*B*d^4*Sin[(e + f*x)/2] - 8*A*c^3*d*Sin[(3*(e + f*x))/2] + 26*B*c^3*d*Sin[(3*(e + f*x))/2] - 40*A*c^2*d^2*Sin[(3*(e + f*x))/2] + 31*B*c^2*d^2*Sin[(3*(e + f*x))/2] - 25*A*c*d^3*Sin[(3*(e + f*x))/2] + 13*B*c*d^3*Sin[(3*(e + f*x))/2] + A*d^4*Sin[(3*(e + f*x))/2] + 2*B*d^4*Sin[(3*(e + f*x))/2] - 2*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 9*B*c^2*d^2*Sin[(5*(e + f*x))/2] - 15*A*c*d^3*Sin[(5*(e + f*x))/2] + 9*B*c*d^3*Sin[(5*(e + f*x))/2] - 7*A*d^4*Sin[(5*(e + f*x))/2] + 6*B*d^4*Sin[(5*(e + f*x))/2]))/(16*(c - d)^3*(c + d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c + d*Sin[e + f*x])^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4706 vs. $2(363) = 726$.

Time = 2.07 (sec) , antiderivative size = 4707, normalized size of antiderivative = 11.71

method	result	size
default	Expression too large to display	4707

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*(-a*(sin(f*x+e)-1))^(1/2)*(-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^3+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d
)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^4-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*
d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3+15*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d+9*B*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^3*a^2*d^5-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5+9*B*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5
+4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^4*d+1
7*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d^2-
A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3-17
*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4-13*
B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^4*d-7*B*
(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d^2+6*A*
(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4+A*(a*(
c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)
)*sin(f*x+e)*a^2*c^5-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^5-11*A*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d-2
5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^3*d^2-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)^2*d^6+35*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2+42*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^4-15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(5/2)*c^5*d-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2-39*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*
d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^4+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)
*d)^(1/2)*a^(3/2)*c^5-3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/
2)*d^5-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^5+4*B*(-a*
(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^5+5*A*(-a*(sin(f*x+e)-1))
^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^5-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)
```

*d)^(1/2)*a^(1/2)*d^5+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)^3*d^6-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^6+19*A*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^6-13*A*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3+
9*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3+
7*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4+2*
A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^3*d^2+
2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^
3-2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^
4-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^3*
d^2+21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^
(1/2)/a^(1/2))*a^2*c^3*d^2+9*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(s
in(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3+7*B*(-a*(sin(f*x+e)-1))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d^2-2*B*(-a*(sin(f*x+e)-1))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3-B*(-a*(sin(f*x+e)-1))^(3/2
)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5+2
A(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*sin(f*x+e)^2*a^2*c^4*d-61*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-51*A*(a
*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)^2*a^2*c*d^4+51*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^3-9*A*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)*a^2*c^4*d-47*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2-63*A*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^
2*c^2*d^3-26*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4+21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d+5
1*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2-21*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
(-a(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2+A*(a*(
c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)
)*sin(f*x+e)^3*a^2*c^3*d^2+18*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4+39*B*(a*(c+d)*d)
^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f
*x+e)^2*a^2*c*d^4+6*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4*d+33*B*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*
a^2*c^3*d^2+57*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-11*A*(a*(c+d)*d)^(1/2)*2^(1
/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2
*c^2*d^3-25*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/

2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d
^2+15*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^3+21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4-10
8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+
e)^2*c^2*d^4-63*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(5/2)*sin(f*x+e)^2*c*d^5+7*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^
(1/2)*c^3*d^2-2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d
^3-B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^4+3*B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^
2*c^5+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^
5+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d+11*A*(-a*(s
in(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^2+A*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3-13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*
(c+d)*d)^(1/2)*a^(3/2)*c*d^4+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(3/2)*sin(f*x+e)^2*d^5-15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^5*d-60*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
d/(a(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2-99*B*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3-90*B*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^4-24
*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e
)*c*d^5-3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*
d^5-11*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d-B*(-a*(s
in(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^2-2*A*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^5+35*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2+112*A*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^
3+103*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(
f*x+e)*c^2*d^4+38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*
a^(5/2)*sin(f*x+e)*c*d^5+35*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*
d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d^4+42*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^5-15*B*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^3*d^3-30*B*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*
d^4-39*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin
(f*x+e)^3*c*d^5+70*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)^2*c^3*d^3+119*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^4+80*A*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^5+7*B*(-a*(sin(f*x+e
)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3+3*B*(-a*(sin(f*x+e)-1))^(1/2)
(a(c+d)*d)^(1/2)*a^(3/2)*c*d^4+5*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^
(1/2)*a^(1/2)*sin(f*x+e)*d^5-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2
)*a^(1/2)*sin(f*x+e)*d^5-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a
^(1/2)*c^2*d^3+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^

```
4+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^5-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2
))*a^(5/2)*sin(f*x+e)^2*c^4*d^2-75*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^3)/a^(7/2)/(a*(c+d)*d)^(1/2)/(c
+d*sin(f*x+e))^2/(c+d)^2/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2789 vs. $2(362) = 724$.

Time = 8.46 (sec) , antiderivative size = 5864, normalized size of antiderivative = 14.59

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(362) = 724$.

Time = 0.60 (sec) , antiderivative size = 1263, normalized size of antiderivative = 3.14

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(15*sqrt(2)*B*sqrt(a)*c^3*d - 35*sqrt(2)*A*sqrt(a)*c^2*d^2 + 3
0*sqrt(2)*B*sqrt(a)*c^2*d^2 - 42*sqrt(2)*A*sqrt(a)*c*d^3 + 39*sqrt(2)*B*sqr
t(a)*c*d^3 - 19*sqrt(2)*A*sqrt(a)*d^4 + 12*sqrt(2)*B*sqrt(a)*d^4)*arctan(sq
rt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^2*c^6*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c^5*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - a^2*c^4*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*a^2*c^3*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*c^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 2*a^2*c*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a^2*d^6*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + 2*(A*sqrt(a)*c + 3*B*sqrt(a)
*c - 13*A*sqrt(a)*d + 9*B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c^3*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^2*c^2*d^2*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(A*sqrt(a)
*c + 3*B*sqrt(a)*c - 13*A*sqrt(a)*d + 9*B*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4
*sqrt(2)*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^2*c^2*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)) - 4*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*sin(-1/4*pi +
1/2*f*x + 1/2*e))/((sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3
*sqrt(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^2*c*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)) + 4*(14*B*sqrt(a)
*c^2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 22*A*sqrt(a)*c*d^3*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^3 + 10*B*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 10*A*sqrt(a)*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*sqrt(a)*d^4*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^3 - 9*B*sqrt(a)*c^3*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e) + 13*A*sqrt(a)*c^2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 12*B*sqrt(a)*c^
2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 16*A*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2
*f*x + 1/2*e) - 7*B*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*sqrt
(a)*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*sqrt(a)*d^4*sin(-1/4*pi + 1/2*
f*x + 1/2*e))/((sqrt(2)*a^2*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(
2)*a^2*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c^3*d^2*sg
```

```
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*a^2*c^2*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
- sqrt(2)*a^2*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
), x)
```

3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 308

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx &= \\ &-\frac{(c-d)\left(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} \\ &+\frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} \\ &+\frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} \\ &-\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} \\ &-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}}\end{aligned}$$

```
[Out] -1/16*(3*A*c+9*A*d+5*B*c-17*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin
(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(
5/2)-1/32*(c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*arctanh(1
/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/2
4*d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*cos(f*x+e)/a^2/f/(
a+a*sin(f*x+e))^(1/2)+1/48*d^2*(9*A*c+39*A*d+15*B*c-95*B*d)*cos(f*x+e)*(a+a
*sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3056, 3047, 3102, 2830, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{(c-d)\left(3A(c^2+6cd+25d^2)+B(5c^2+62cd-163d^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}$$
$$+\frac{d^2(9Ac+39Ad+15Bc-95Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{48a^3f}$$
$$+\frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(e+fx)}{24a^2f\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$
$$-\frac{(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/16*((c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*
ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2
]*a^(5/2)*f) + (d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*
d^2))*Cos[e + f*x])/(24*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(9*A*c + 15*
B*c + 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(48*a^3*f) -
((3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(16*
a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^3)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(3Ac+5Bc+6Ad-6Bd)-\frac{1}{2}a(3A-11B)d\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{4}a^2\left(B(5c^2+47cd-68d^2)+3A(c^2+3cd+12d^2)\right)-\frac{1}{4}a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}dx}{8a^4}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{4}a^2c\left(B(5c^2+47cd-68d^2)+3A(c^2+3cd+12d^2)\right)+\left(-\frac{1}{4}a^2cd(9Ac+15Bc+39Ad-95Bd)+\frac{1}{4}a^2d\left(B(5c^2+47cd-68d^2)+3A(c^2+3cd+\right.\right.}{\sqrt{a+a\sin(e+fx)}}}{8a^4}$$

$$= \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} - \frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{8}a^3\left(3A(3c^3+9c^2d+33cd^2-13d^3)+B(15c^3+141c^2d-219cd^2+95d^3)\right)-\frac{1}{4}a^3d\left(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2)\right)\sin(e}{\sqrt{a+a\sin(e+fx)}}}{12a^5}$$

$$= \frac{d\left(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2)\right)\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} + \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} - \frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\left((c-d)\left(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)\right)\right)\int\frac{1}{\sqrt{a+a\sin(e+fx)}}dx}{32a^2}$$

$$= \frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} + \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} - \frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} - \frac{((c-d)(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)))\,\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2f}$$

$$= -\frac{(c-d)(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} + \frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} + \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} - \frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.77 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)(c-d)^3\,\text{s}\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)/
2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c - d)
^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2 + 62
*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 16*B*d
^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*
```

```
I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2*A*d - 5*B
*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[(3*(e + f*x
))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. 2(281) = 562.

Time = 3.99 (sec) , antiderivative size = 1157, normalized size of antiderivative = 3.76

method	result	size
parts	Expression too large to display	1157
default	Expression too large to display	1438

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/32*A*c^3/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^2*sin(f*x+e)+6*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+6*2
^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*sin(
f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f-1/32*c^2*(3*A*d+B*c)*(-5*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*cos(f*x+e)^2+10*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+12*(a-a*sin(f*x+e))^
(1/2)*a^(5/2)-10*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+10*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)
/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/32*d^2*(A*d+3*B*c)/a^
(9/2)*(-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
cos(f*x+e)^2+64*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*cos(f*x+e)^2+150*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)-128*(a-a*si
n(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*a^2-204*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+42*(a-a*sin(
f*x+e))^(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f-1/96*d^3*B*(64*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*cos(
f*x+e)^2+384*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*cos(f*x+e)^2-489*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2-128*(a-a*sin
(f*x+e))^(3/2)*sin(f*x+e)*a^(1/2)-768*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(
3/2)+978*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*si
n(f*x+e)+46*(a-a*sin(f*x+e))^(3/2)*a^(1/2)-1092*(a-a*sin(f*x+e))^(1/2)*a^(3
/2)+978*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-
a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^
(1/2)/f-3/32*c*d*(A*d+B*c)/a^(9/2)*(-19*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
```

```
)^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)+38*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2-44*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+26*(
a-a*sin(f*x+e))^(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(282) = 564$.

Time = 0.30 (sec) , antiderivative size = 980, normalized size of antiderivative = 3.18

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A - 7
5*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d
 + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A + 5
*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3
)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75
*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 12*(5*A
 + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*
B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)
*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*
B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B)*c^2*d - 36*(
A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*d^3)*cos(f*x +
e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A - 53*B)*c*d^2 +
(53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(A - 9*B)*c^2*d -
27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (32*B*d^3*cos(f*x +
 e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c*d^2 - 12*(A - B)*d
^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*((3*A + 5*B)*c^3 + 3
*(5*A - 13*B)*c^2*d - 3*(13*A - 85*B)*c*d^2 + (85*A - 157*B)*d^3)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(282) = 564$.

Time = 0.55 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.44

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
[Out] 1/192*(3*sqrt(2)*(3*A*sqrt(a)*c^3 + 5*B*sqrt(a)*c^3 + 15*A*sqrt(a)*c^2*d +
57*B*sqrt(a)*c^2*d + 57*A*sqrt(a)*c*d^2 - 225*B*sqrt(a)*c*d^2 - 75*A*sqrt(a
)*d^3 + 163*B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(3*A*sqrt(a)*c^3 + 5*B*sqrt(a
)*c^3 + 15*A*sqrt(a)*c^2*d + 57*B*sqrt(a)*c^2*d + 57*A*sqrt(a)*c*d^2 - 225*
B*sqrt(a)*c*d^2 - 75*A*sqrt(a)*d^3 + 163*B*sqrt(a)*d^3)*log(-sin(-1/4*pi +
1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 6*sqrt(2)
*(3*A*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^3*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 + 15*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^3 - 39*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 39*A*sqrt(a)*
c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 63*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1
```

```
/2*f*x + 1/2*e)^3 + 21*A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 29*
B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c^3*sin(-1/4*p
i + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*A
*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 33*B*sqrt(a)*c^2*d*sin(-1/4
*pi + 1/2*f*x + 1/2*e) + 33*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)
- 57*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 19*A*sqrt(a)*d^3*sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 27*B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e
))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))) - 128*sqrt(2)*(2*B*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
 - 9*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^(13/2)*d^3*sin
(-1/4*pi + 1/2*f*x + 1/2*e) + 6*B*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^3}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
), x)
```

3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 219

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] -1/16*(c-d)*(3*A*c+5*A*d+5*B*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2
)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)-1/32*(B*
(5*c^2+38*c*d-75*d^2)+A*(3*c^2+10*c*d+19*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/
2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/4*(A-9*B)*d^2*cos(f*
x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used

$= \{3056, 3047, 3098, 2830, 2728, 212\}$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A(3c^2+10cd+19d^2)+B(5c^2+38cd-75d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f} + \frac{d^2(A-9B)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}} - \frac{(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/16*((B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[
(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2
)*f) - ((c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/(16*a*f*(a +
 a*Sin[e + f*x])^(3/2)) + ((A - 9*B)*d^2*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*
Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(4*f*(a + a*
Sin[e + f*x])^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} \\
&\quad+\frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(3Ac+5Bc+4Ad-4Bd)-\frac{1}{2}a(A-9B)d\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a^2} \\
&= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} \\
&\quad+\frac{\int \frac{\frac{1}{2}ac(3Ac+5Bc+4Ad-4Bd)+\left(-\frac{1}{2}a(A-9B)cd+\frac{1}{2}ad(3Ac+5Bc+4Ad-4Bd)\right)\sin(e+fx)-\frac{1}{2}a(A-9B)d^2\sin^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a^2} \\
&= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} \\
&\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} \\
&\quad-\frac{\int \frac{-\frac{1}{4}a^2\left(B\left(5c^2+38cd-39d^2\right)+A\left(3c^2+10cd+15d^2\right)\right)+a^2(A-9B)d^2\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{8a^4}
\end{aligned}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\int\frac{1}{\sqrt{a+a\sin(e+fx)}}dx}{32a^2}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\text{Subst}\left(\int\frac{1}{2a-x^2}dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2f}$$

$$= -\frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.48

$$\int\frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-11Ac^2\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*Cos[(e + f*x)/2] + 3*B*c^
2*Cos[(e + f*x)/2] + 6*A*c*d*Cos[(e + f*x)/2] + 10*B*c*d*Cos[(e + f*x)/2] +
 5*A*d^2*Cos[(e + f*x)/2] - 45*B*d^2*Cos[(e + f*x)/2] - 3*A*c^2*Cos[(3*(e +
 f*x))/2] - 5*B*c^2*Cos[(3*(e + f*x))/2] - 10*A*c*d*Cos[(3*(e + f*x))/2] +
26*B*c*d*Cos[(3*(e + f*x))/2] + 13*A*d^2*Cos[(3*(e + f*x))/2] - 69*B*d^2*Co
s[(3*(e + f*x))/2] + 16*B*d^2*Cos[(5*(e + f*x))/2] + 11*A*c^2*Sin[(e + f*x)
/2] - 3*B*c^2*Sin[(e + f*x)/2] - 6*A*c*d*Sin[(e + f*x)/2] - 10*B*c*d*Sin[(e
 + f*x)/2] - 5*A*d^2*Sin[(e + f*x)/2] + 45*B*d^2*Sin[(e + f*x)/2] + (2 + 2*
I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] +
```

```
Sin[(e + f*x)/2])^4 - 3*A*c^2*Sin[(3*(e + f*x))/2] - 5*B*c^2*Sin[(3*(e + f*
x))/2] - 10*A*c*d*Sin[(3*(e + f*x))/2] + 26*B*c*d*Sin[(3*(e + f*x))/2] + 13
*A*d^2*Sin[(3*(e + f*x))/2] - 69*B*d^2*Sin[(3*(e + f*x))/2] - 16*B*d^2*Sin[
(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(196) = 392.

Time = 3.22 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.89

method	result	size
parts	Expression too large to display	852
default	Expression too large to display	982

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/32*A*c^2/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^2*sin(f*x+e)+6*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+6*2
^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*sin(
f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f-1/32*c*(2*A*d+B*c)*(-5*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*cos(f*x+e)^2+10*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+12*(a-a*sin(f*x+e))^(1
/2)*a^(5/2)-10*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+10*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(
1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/32*d*(A*d+2*B*c)/a^(9/2
)*(-19*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(
f*x+e)^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
*sin(f*x+e)+38*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2-44*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+26*(a-a*sin(f*x+e))^(3/2)*a^(1/2))*(-
a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+
1/32*d^2*B/a^(9/2)*(-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*cos(f*x+e)^2+64*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*cos(f*x+e)^2+15
0*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e
)-128*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+150*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2-204*(a-a*sin(f*x+e))^(1/2)*a^(3/2)
+42*(a-a*sin(f*x+e))^(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e)
)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(196) = 392$.

Time = 0.29 (sec) , antiderivative size = 744, normalized size of antiderivative = 3.40

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{2}\left(\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos(fx+e)^3-4(3A+5B)c^2-8(5A+19B\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*
cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)*d
^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x +
 e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*
x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2 - ((
3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^2 + 2
*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos
(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*s
in(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 - 4*(A - B)*c
^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d
- (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A - 9*B)*c*d - 9*
(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A - B)*c^2 + 8*
(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d - (13*A
 - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(196) = 392.

Time = 0.43 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.42

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\frac{128\sqrt{2}Bd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}(3A\sqrt{a}c^2+5B\sqrt{a}c^2+10A\sqrt{a}cd+38B\sqrt{}}{a^3\operatorname{sgn}(}}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/64*(128*sqrt(2)*B*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(a^(5/2)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 + 10
*A*sqrt(a)*c*d + 38*B*sqrt(a)*c*d + 19*A*sqrt(a)*d^2 - 75*B*sqrt(a)*d^2)*lo
g(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))) - sqrt(2)*(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 + 10*A*sqrt(a)*c*d + 38*B
*sqrt(a)*c*d + 19*A*sqrt(a)*d^2 - 75*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt(2)*(3*
A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^2*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 + 10*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 26*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 13*A*sqrt(a)*d^2*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^3 + 21*B*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^3 - 5*A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c^2*si
n(-1/4*pi + 1/2*f*x + 1/2*e) - 6*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 22*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 11*A*sqrt(a)*d^2*sin
(-1/4*pi + 1/2*f*x + 1/2*e) - 19*B*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*
e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
),x)
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
), x)
```

3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 151

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3Ac+5Bc+5Ad+19Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c+5*A*d+5*B*
c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A*c+5*A*d+5*B*c+19*
B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)
/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3098, 2829, 2728, 212}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3Ac+5Ad+5Bc+19Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -1/16*((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)*f) - ((A - B)*(c - d)*Cos
[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*c + 5*B*c + 5*A*d - 13*
B*d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{Ac + (Bc + Ad)\sin(e + fx) + Bd\sin^2(e + fx)}{(a + a\sin(e + fx))^{5/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc+5Ad-5Bd)-4aBd\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(3Ac+5Bc+5Ad+19Bd)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}dx}{32a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(3Ac+5Bc+5Ad+19Bd)\text{Subst}\left(\int \frac{1}{2a-x^2}dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2f}$$

$$= -\frac{(3Ac+5Bc+5Ad+19Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)\sin\left(\frac{1}{2}\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2]
- 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*B*
c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(132) = 264$.

Time = 2.59 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.97

method	result
default	$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(3Ac+5dA+5Bc+19dB)\left(\cos^2(fx+e)\right)+2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2}{}$
parts	$-\frac{Ac\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\left(\cos^2(fx+e)\right)+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+6\sqrt{a-a\sin(fx+e)}}{32a^{\frac{9}{2}}\left(1+\sin(fx+e)\right)\cos(fx+e)\sqrt{a+a\sin}}$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32*(-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(3*
A*c+5*A*d+5*B*c+19*B*d)*cos(f*x+e)^2+2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)+6*A*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+10*A*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+20*A*(a-a*sin(f*x+
e))^(1/2)*a^(3/2)*c+12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-6*A*(a-a*sin(f*x+
e))^(3/2)*a^(1/2)*c-10*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d+10*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+38*B*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+12*B*(a-a*sin(f*x+e))^(1/
2)*a^(3/2)*c-44*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-10*B*(a-a*sin(f*x+e))^(3
/2)*a^(1/2)*c+26*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1
/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(132) = 264$.

Time = 0.27 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.55

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(((3A+5B)c+(5A+19B)d)\cos{(fx+e)}^3+3\left((3A\right.\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*(((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*
d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c + (5*
A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((3*A
+ 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(
```

```
f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin
(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2
*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
+ 4*(((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c - 4*(A -
 B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c - 4*(A - B)
*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5
/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(132) = 264.

Time = 0.35 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.41

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\frac{\sqrt{2}(3A\sqrt{a}c+5B\sqrt{a}c+5A\sqrt{a}d+19B\sqrt{a}d)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}(3\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*A*sqrt(a)*c + 5*B*sqrt(a)*c + 5*A*sqrt(a)*d + 19*B*sqrt(a)
*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))) - sqrt(2)*(3*A*sqrt(a)*c + 5*B*sqrt(a)*c + 5*A*sqrt(a)*d + 19*B*
sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*A*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 + 5*sqrt(2)*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*sqrt(2)*A*
sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 13*sqrt(2)*B*sqrt(a)*d*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*A*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1
/2*e) - 3*sqrt(2)*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(2)*A*
sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 11*sqrt(2)*B*sqrt(a)*d*sin(-1/4*
pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3A+5B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A+5*B)*cos(f*x+e)/a/
f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A+5*B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2829, 2729, 2728, 212}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3A+5B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(3A+5B)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/16*((3*A + 5*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[2]*a^(5/2)*f) - ((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e
+ f*x])^(5/2)) - ((3*A + 5*B)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}+\frac{(3A+5B)\int\frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}\\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}+\frac{(3A+5B)\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2}\\
&= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}\\
&\quad -\frac{(3A+5B)\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2f}\\
&= -\frac{(3A+5B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}\\
&\quad -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+4(-A+B)\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A
+ B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A + 5*B)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(107) = 214.

Time = 1.91 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.21

method	result
default	$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)\left(\cos^2(fx+e)\right)+2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)+20A\sqrt{\right.}{}$
parts	$-\frac{A\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\left(\cos^2(fx+e)\right)+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+6\sqrt{a-a\sin(fx+e)}\,\sin\right.}{32a^{\frac{9}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{a+a\sin(}}$

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*(3*
A+5*B)*cos(f*x+e)^2+2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*a^3*(3*A+5*B)+20*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-6*A*(a-
a*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-10*B*(a-a*s
in(f*x+e))^(3/2)*a^(3/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left((3\,A+5\,B)\cos\left(fx+e\right)^{3}+3\,(3\,A+5\,B)\cos\left(fx+e\right)^{2}-2\,(3\,A+5\,B)\,\mathrm{c}\right.}{}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 -
2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*co
s(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*B)*cos(
f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(107) = 214$.

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.87

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}}\,dx = \frac{\frac{\sqrt{2}(3A\sqrt{a}+5B\sqrt{a})\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}(3A\sqrt{a}+5B\sqrt{a})\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{\ }$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*A*sqrt(a) + 5*B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*sqrt(a) + 5
*B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^3 + 5*sqrt(2)*B*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*A*sqr
t(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(2)*B*sqrt(a)*sin(-1/4*pi + 1/2
*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}}\,dx = \int \frac{A + B\,\sin\left(e + f\,x\right)}{{\left(a + a\,\sin\left(e + f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 261

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^3f} - \frac{2d^{3/2}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^3\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c-11*A*d+5*B
*c+3*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)-1/32*(B*(5*c^2-34*c
*d-3*d^2)+A*(3*c^2-14*c*d+43*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(
a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^3/f*2^(1/2)-2*d^(3/2)*(-A*d+B*c)*arcta
nh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(
c-d)^3/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3057, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{(A(3c^2-14cd+43d^2)+B(5c^2-34cd-3d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3} - \frac{2d^{3/2}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}} - \frac{(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -1/16*((B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(
Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[2]*a^(5/2)
*(c - d)^3*f) - (2*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(5/2)*(c - d)^3*Sqrt[c + d]*
f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*
A*c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e
+ f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-8Ad)-\frac{3}{2}a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx}{4a^2(c-d)} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}} \\
&\quad + \frac{\int \frac{\frac{1}{4}a^2(Bc(5c-29d)+A(3c^2-11cd+32d^2))+\frac{1}{4}a^2d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{8a^4(c-d)^2} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}} \\
&\quad + \frac{(d^2(Bc-Ad))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a^3(c-d)^3} \\
&\quad + \frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2(c-d)^3}
\end{aligned}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}} - \frac{(2d^2(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^2(c-d)^3f} - \frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2(c-d)^3f}$$

$$= -\frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^3f} - \frac{2d^{3/2}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^3\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.74 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)^2\sin\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*
(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2) + A
*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (8*d^(3/2)*(-(B*c) + A*d
)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*
d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]
*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4
]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1
+ Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*
```

```
x)/2] + Sin[(e + f*x)/2])^4)/Sqrt[c + d] + (8*d^(3/2)*(B*c - A*d)*(e + f*x
- 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*
#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 +
Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + S
qrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e +
f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4)/Sqrt[c + d]))/(16*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. $2(228) = 456$.

Time = 1.32 (sec) , antiderivative size = 1418, normalized size of antiderivative = 5.43

method	result	size
default	Expression too large to display	1418

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32*((64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*
d^3-64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^
2-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c^2+14*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-43*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-5*B*(a*(c+d)*d)^(1/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+34*B*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
*c*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*d^2)*cos(f*x+e)^2+2*sin(f*x+e)*(-64*A*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+64*B*arctanh((a-a*sin(f*x+e))^
(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-14*A*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+4
3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2*d^2+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^2*c^2-34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)-128*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+128*B*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+6*A*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-28
*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c*d+86*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
```

```
1/2)*2^(1/2)/a^(1/2))*a^2*d^2+20*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)
*a^(3/2)*c^2-72*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d+52*A
*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^2-6*A*(a*(c+d)*d)^(1/2)
*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+28*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))
^(3/2)*a^(1/2)*c*d-22*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^
2+10*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^2-68*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-6*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+12*B*(a-a*sin(f*x+e))^(1/2
)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2+8*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(3/2)*c*d-20*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^2-10*
B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+4*B*(a*(c+d)*d)^(1/2
)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d+6*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))
^(3/2)*a^(1/2)*d^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+
d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(228) = 456$.

Time = 3.69 (sec) , antiderivative size = 2577, normalized size of antiderivative = 9.87

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*(((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*c
os(f*x + e)^3 - 4*(3*A + 5*B)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2
 + 3*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)
^2 - 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x +
e) - (4*(3*A + 5*B)*c^2 - 8*(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A +
 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A
 + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*s
qrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x +
e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) - 32*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)
*cos(f*x + e)^3 - 3*(B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d
^2)*cos(f*x + e) + (4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e
)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))
*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2
*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^
```

2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^
2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x
+ e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2
 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^
2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (1
1*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A - 3*B)*c*d + (15*
A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2
 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2
- a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*
d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f
*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3
*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 -
 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c
^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), 1/64*(sqrt(2)*(((3*A + 5*B)
c^2 - 2(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^3 - 4*(3*A + 5*B
)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2 + 3*((3*A + 5*B)*c^2 - 2*(7
*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 - 2*((3*A + 5*B)*c^2 - 2*
(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e) - (4*(3*A + 5*B)*c^2 - 8*
(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c
*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)
*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*
x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f
*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a
)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
64*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d
 - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*
d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*
cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e)))
 + 4*(4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*
(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A
 - 3*B)*c*d + (15*A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c
d + 4(A - B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^
2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*
c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e
)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx=\int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(228)=456$.

Time = 0.43 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
[Out] -1/32*(64*sqrt(2)*(B*c*d^2 - A*d^3)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^(5/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 3*sqrt(2)*a^(5/2)*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*sqrt(2)*a^(5/2)*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^(5/2
)*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - (3*A*sqrt(a)
*c^2 + 5*B*sqrt(a)*c^2 - 14*A*sqrt(a)*c*d - 34*B*sqrt(a)*c*d + 43*A*sqrt(a)
*d^2 - 3*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^
3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^2*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + (3*A*sqrt(
a)*c^2 + 5*B*sqrt(a)*c^2 - 14*A*sqrt(a)*c*d - 34*B*sqrt(a)*c*d + 43*A*sqrt(
```

```
a)*d^2 - 3*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)
*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^2*d*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*(3*A*
sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - 11*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 3*B*sq
rt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c*sin(-1/4*pi + 1/2*
f*x + 1/2*e) - 3*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 13*A*sqrt(a)*
d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*B*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/
2*e))/((sqrt(2)*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^3
*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),
x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),
 x)
```

3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 395

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^4f} + \frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^4(c+d)^{3/2}f} - \frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

```
[Out] d^(3/2)*(A*d*(7*c+5*d)-B*(5*c^2+5*c*d+2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^
(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1/4
*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))-1/16*(3*A
*c-15*A*d+5*B*c+7*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*s
in(f*x+e))-1/32*(B*(5*c^2-58*c*d-43*d^2)+A*(3*c^2-22*c*d+115*d^2))*arctanh(
1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/f*2^
(1/2)-1/16*d*(A*(3*c^2-16*c*d-35*d^2)+B*(5*c^2+32*c*d+11*d^2))*cos(f*x+e)/a
^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A(3c^2-22cd+115d^2)+B(5c^2-58cd-43d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^4} + \frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}} - \frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))} - \frac{(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -1/16*((B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2))*ArcTanh
[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[2]*a^(5/
2)*(c - d)^4*f) + (d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2))*Ar
cTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])
]])/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*
f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - ((3*A*c + 5*B*c - 15*A
*d + 7*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c +
 d*Sin[e + f*x])) - (d*(A*(3*c^2 - 16*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 1
1*d^2))*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} \\
&\quad - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-10Ad+2Bd)-\frac{5}{2}a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx}{4a^2(c-d)} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} \\
&\quad - \frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} \\
&\quad + \frac{\int \frac{\frac{1}{4}a^2\left(B\left(5c^2-43cd-22d^2\right)+A\left(3c^2-13cd+70d^2\right)\right)+\frac{3}{4}a^2d(3Ac+5Bc-15Ad+7Bd)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx}{8a^4(c-d)^2} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} \\
&\quad - \frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} \\
&\quad - \frac{d\left(A\left(3c^2-16cd-35d^2\right)+B\left(5c^2+32cd+11d^2\right)\right)\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad - \frac{\int \frac{-\frac{1}{4}a^3\left(B\left(5c^3-48c^2d-69cd^2-32d^3\right)+A\left(3c^3-16c^2d+77cd^2+80d^3\right)\right)-\frac{1}{4}a^3d\left(A\left(3c^2-16cd-35d^2\right)+B\left(5c^2+32cd+11d^2\right)\right)\sin(e+f}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}}{8a^5(c-d)^3(c+d)} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} \\
&\quad - \frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} \\
&\quad - \frac{d\left(A\left(3c^2-16cd-35d^2\right)+B\left(5c^2+32cd+11d^2\right)\right)\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad - \frac{\left(d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\right)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2a^3(c-d)^4(c+d)} \\
&\quad + \frac{\left(B\left(5c^2-58cd-43d^2\right)+A\left(3c^2-22cd+115d^2\right)\right)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2(c-d)^4}
\end{aligned}
$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(d^2(Ad(7c+5d)-B(5c^2+5cd+2d^2)))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^2(c-d)^4(c+d)f} - \frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2(c-d)^4f}$$

$$= -\frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^4f} + \frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^4(c+d)^{3/2}f} - \frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 15.73 (sec) , antiderivative size = 1680, normalized size of antiderivative = 4.25

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \frac{(1+i)\,(3Ac^2+5Bc^2-22Acd-58Bcd+115Ad^2-43Bc}{(16\sqrt[4]{-1}c^4-6}$$

$$+\frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+\text{RootSum}\left[c+4d\#1+2c\#1\right.\right.}{}$$

$$+\frac{d^{3/2}(-Ad(7c+5d)+B(5c^2+5cd+2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+\text{RootSum}\left[c+4d\#1+2c\right.\right.}{}$$

$$+\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-22Ac^3\cos\left(\frac{1}{2}(e+fx)\right)+6Bc^3\cos\left(\frac{1}{2}(e+fx)\right)+40Ac^2d\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 22*A*c*d - 58*B*c*d + 115*A*d^2 - 43*B*d^2)*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e +
 f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^4 - 6
4*(-1)^(1/4)*c^3*d + 96*(-1)^(1/4)*c^2*d^2 - 64*(-1)^(1/4)*c*d^3 + 16*(-1)^
(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(A*d*(7*c + 5*d) - B*
(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c +
 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]
) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e +
f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Lo
g[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x
)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*
#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3
/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-(A*d*(7*c + 5*d)) + B*(5*c
^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d
*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) -
Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)
/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#
1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3
) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2)*
f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-
22*A*c^3*Cos[(e + f*x)/2] + 6*B*c^3*Cos[(e + f*x)/2] + 40*A*c^2*d*Cos[(e +
f*x)/2] - 40*B*c^2*d*Cos[(e + f*x)/2] + 54*A*c*d^2*Cos[(e + f*x)/2] - 70*B*
c*d^2*Cos[(e + f*x)/2] + 24*A*d^3*Cos[(e + f*x)/2] + 8*B*d^3*Cos[(e + f*x)/
2] - 6*A*c^3*Cos[(3*(e + f*x))/2] - 10*B*c^3*Cos[(3*(e + f*x))/2] + 21*A*c^
2*d*Cos[(3*(e + f*x))/2] - 29*B*c^2*d*Cos[(3*(e + f*x))/2] + 54*A*c*d^2*Cos
[(3*(e + f*x))/2] - 86*B*c*d^2*Cos[(3*(e + f*x))/2] + 75*A*d^3*Cos[(3*(e +
f*x))/2] - 19*B*d^3*Cos[(3*(e + f*x))/2] + 3*A*c^2*d*Cos[(5*(e + f*x))/2] +
 5*B*c^2*d*Cos[(5*(e + f*x))/2] - 16*A*c*d^2*Cos[(5*(e + f*x))/2] + 32*B*c*
d^2*Cos[(5*(e + f*x))/2] - 35*A*d^3*Cos[(5*(e + f*x))/2] + 11*B*d^3*Cos[(5*
(e + f*x))/2] + 22*A*c^3*Sin[(e + f*x)/2] - 6*B*c^3*Sin[(e + f*x)/2] - 40*A
*c^2*d*Sin[(e + f*x)/2] + 40*B*c^2*d*Sin[(e + f*x)/2] - 54*A*c*d^2*Sin[(e +
 f*x)/2] + 70*B*c*d^2*Sin[(e + f*x)/2] - 24*A*d^3*Sin[(e + f*x)/2] - 8*B*d^
3*Sin[(e + f*x)/2] - 6*A*c^3*Sin[(3*(e + f*x))/2] - 10*B*c^3*Sin[(3*(e + f*
x))/2] + 21*A*c^2*d*Sin[(3*(e + f*x))/2] - 29*B*c^2*d*Sin[(3*(e + f*x))/2]
+ 54*A*c*d^2*Sin[(3*(e + f*x))/2] - 86*B*c*d^2*Sin[(3*(e + f*x))/2] + 75*A*
d^3*Sin[(3*(e + f*x))/2] - 19*B*d^3*Sin[(3*(e + f*x))/2] - 3*A*c^2*d*Sin[(5
*(e + f*x))/2] - 5*B*c^2*d*Sin[(5*(e + f*x))/2] + 16*A*c*d^2*Sin[(5*(e + f*
x))/2] - 32*B*c*d^2*Sin[(5*(e + f*x))/2] + 35*A*d^3*Sin[(5*(e + f*x))/2] -
11*B*d^3*Sin[(5*(e + f*x))/2]))/(64*(c - d)^3*(c + d)*f*(a*(1 + Sin[e + f*x
]))^(5/2)*(c + d*Sin[e + f*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4091 vs. 2(358) = 716.

Time = 2.02 (sec) , antiderivative size = 4092, normalized size of antiderivative = 10.36

method	result	size
default	Expression too large to display	4092

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/32*(-84*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)
*c^2*d^2+6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2
)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4-43*B*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1
/2))*sin(f*x+e)^3*a^2*d^4+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(si
n(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4-43*B*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x
+e)*a^2*d^4-19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d+93*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+115*A*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d
^3-53*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*a^2*c^3*d-101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(si
n(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2-43*B*(a*(c+d)*d)^(1/2)*2^(1
/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3-10*B*(
-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d-22*B*(-
a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^2-76*B*(
-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^3-32*B*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3+20*A*(
-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d-84*A*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^3-32*A*(-a
*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3+32*B*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^2+10*B
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)*a^2*c^4+115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^4+10*B*(-a*(sin(f*
x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^3-86*B*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)^2*a^2*d^4-6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(
f*x+e)*c^3*d+38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f
*x+e)*c^2*d^2+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f
*x+e)*c*d^3+230*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^4+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*
```

arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4
+12*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d+
116*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^
2-160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(
f*x+e)^3*d^5+64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(5/2)*sin(f*x+e)^3*d^5-160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d
)^(1/2))*a^(5/2)*sin(f*x+e)*d^5+32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(3/2)*d^4+12*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*
c^4-320*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*si
n(f*x+e)^2*d^5+128*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)^2*d^5+64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5+160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^2+160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
d/(a(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d
)*d)^(1/2)*a^(1/2)*c^4-10*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(
1/2)*c^4-160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/
2)*c*d^4+64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2
)*c*d^4+20*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4-224*A*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-101*
B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a
^(1/2))*sin(f*x+e)*a^2*c^3*d-255*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2-187*B*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))
*sin(f*x+e)*a^2*c*d^3-207*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^2-245*B*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)^2*a^2*c*d^3-35*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d+93*A*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*
a^2*c*d^3+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d-53*B*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*
d^2-101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^3-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d+55
A(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^2+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
(-a(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d-19*A*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)^3*a^2*c^2*d^2+167*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2+323*A*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)*a^2*c*d^3+301*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^3-43*B*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+

```
e)^2*a^2*c^3*d+160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)^2*c^3*d^2+480*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3+384*B*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4+22*B*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3+22*B*(-a*(sin(f*x+e)-1))^(3/2
)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4+38*A*(-a*(sin(f*x+e)-1))^(3/2)*(
a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-84*A*(-a*(sin(f*x+e)-1))^(
1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d-20*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d
)*d)^(1/2)*a^(3/2)*c^2*d^2-38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)
*a^(1/2)*sin(f*x+e)*d^4+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(
1/2)*c^2*d^2-38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3
+32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^4+
160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*
x+e)^3*c^2*d^3+160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)^3*c*d^4+52*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(3/2)*c^3*d+20*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^
2*d^2-84*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3+320*B*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c
^3*d^2+480*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)
*sin(f*x+e)*c^2*d^3+288*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(5/2)*sin(f*x+e)*c*d^4+148*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(3/2)*sin(f*x+e)*d^4-52*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/
2)*a^(3/2)*sin(f*x+e)*d^4-448*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^3-544*A*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4-224*A*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4+3*A*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^
2*c^4-22*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d+10*B*(
-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2-224*A*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3-6
08*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x
+e)^2*c*d^4+52*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3)
*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*si
n(f*x+e))/(c+d)/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2433 vs. $2(358) = 716$.

Time = 8.10 (sec) , antiderivative size = 5151, normalized size of antiderivative = 13.04

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
```

```
orithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(358) = 716$.

Time = 0.71 (sec) , antiderivative size = 1099, normalized size of antiderivative = 2.78

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/32*(16*sqrt(2)*(5*sqrt(2)*B*sqrt(a)*c^2*d^2 - 7*sqrt(2)*A*sqrt(a)*c*d^3
+ 5*sqrt(2)*B*sqrt(a)*c*d^3 - 5*sqrt(2)*A*sqrt(a)*d^4 + 2*sqrt(2)*B*sqrt(a)
*d^4)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a
^3*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c^4*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 2*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*a
^3*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c*d^4*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-
c*d - d^2)) - (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 22*A*sqrt(a)*c*d - 58*B*
```

```
sqrt(a)*c*d + 115*A*sqrt(a)*d^2 - 43*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*s
qrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3*c^2*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
 + (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 22*A*sqrt(a)*c*d - 58*B*sqrt(a)*c*d
 + 115*A*sqrt(a)*d^2 - 43*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3
*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3*c^2*d^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 64*(B*s
qrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*sqrt(a)*d^3*sin(-1/4*pi + 1
/2*f*x + 1/2*e))/((sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*
sqrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*a^3*c*d^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)) + 2*(3*A*s
qrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 - 19*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 11*B*sq
rt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c*sin(-1/4*pi + 1/2*
f*x + 1/2*e) - 3*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 21*A*sqrt(a)*
d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 13*B*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e))/((sqrt(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^
3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
),x)
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
), x)
```

3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 519

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^5f}$$
$$+\frac{d^{3/2}(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{5/2}(c-d)^5(c+d)^{5/2}f}$$
$$-\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}$$
$$-\frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}$$
$$-\frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

```
[Out] 1/4*d^(3/2)*(3*A*d*(21*c^2+30*c*d+13*d^2)-B*(35*c^3+70*c^2*d+67*c*d^2+20*d^
3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/
a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))
^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*A*c-19*A*d+5*B*c+11*B*d)*cos(f*x+e)/a/(c-
d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/32*(B*(5*c^2-82*c*d-115*
d^2)+3*A*(c^2-10*c*d+73*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*s
in(f*x+e))^(1/2))/a^(5/2)/(c-d)^5/f*2^(1/2)-1/16*d*(A*(3*c^2-20*c*d-31*d^2)
+B*(5*c^2+28*c*d+15*d^2))*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^2
/(a+a*sin(f*x+e))^(1/2)-1/16*d*(3*A*(c^3-7*c^2*d-37*c*d^2-21*d^3)+B*(5*c^3+
```

```
73*c^2*d+79*c*d^2+35*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e)
)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx =$$
$$-\frac{(3A(c^2-10cd+73d^2)+B(5c^2-82cd-115d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^5}$$
$$+\frac{d^{3/2}(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{5/2}f(c-d)^5(c+d)^{5/2}}$$
$$-\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$
$$-\frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$
$$-\frac{(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] -1/16*((B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d + 73*d^2))*ArcTanh
[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/
2)*(c - d)^5*f) + (d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2) - B*(35*c^3 +
70*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) -
 ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[
e + f*x])^2) - ((3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])/(16*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(A*(3*c^2 -
20*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x])/(16*a^2*(c -
d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (d*(3*A*(
c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^
3))*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} \\
&\quad -\frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-12Ad+4Bd)-\frac{7}{2}a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx}{4a^2(c-d)} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} \\
&\quad -\frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} \\
&\quad +\frac{\int \frac{\frac{1}{4}a^2\left(B\left(5c^2-57cd-60d^2\right)+A\left(3c^2-15cd+124d^2\right)\right)+\frac{5}{4}a^2d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx}{8a^4(c-d)^2} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} \\
&\quad -\frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} \\
&\quad -\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad -\frac{\int \frac{-\frac{1}{2}a^3\left(B\left(5c^3-62c^2d-113cd^2-70d^3\right)+3A\left(c^3-6c^2d+43cd^2+42d^3\right)\right)-\frac{3}{2}a^3d\left(A\left(3c^2-20cd-31d^2\right)+B\left(5c^2+28cd+15d^2\right)\right)\sin(e+fx}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}}{16a^5(c-d)^3(c+d)} \\
&= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} \\
&\quad -\frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} \\
&\quad -\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad -\frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} \\
&\quad +\frac{\int \frac{\frac{1}{2}a^4\left(B\left(5c^4-67c^3d-201c^2d^2-233cd^3-80d^4\right)+3A\left(c^4-7c^3d+47c^2d^2+99cd^3+52d^4\right)\right)+\frac{1}{2}a^4d\left(3A\left(c^3-7c^2d-37cd^2-21d^3\right)+B\left(5c^3-\right.\right.}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}}{16a^6(c-d)^4(c+d)^2}
\end{aligned}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\int\frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2(c-d)^5} - \frac{(d^2(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3)))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{8a^3(c-d)^5(c+d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\,\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{16a^2(c-d)^5f} + \frac{(d^2(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3)))\,\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a}{\sqrt{a}}\right.}{4a^2(c-d)^5(c+d)^2f}$$

$$= -\frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^5f} + \frac{d^{3/2}(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{5/2}(c-d)^5(c+d)^{5/2}f} - \frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.01 (sec) , antiderivative size = 2465, normalized size of antiderivative = 4.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^3),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 30*A*c*d - 82*B*c*d + 219*A*d^2 - 115*B*d^2)*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^5 -
80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 - 160*(-1)^(1/4)*c^2*d^3 + 80*
(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 + Sin[e + f*x]))^(5/2)) - (d^
(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2
+ 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*
#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 +
2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e
 + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c
*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 +
 Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(3
5*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]
 + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan
[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-
#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 +
 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c -
d)^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + S
in[(e + f*x)/2])*(-44*A*c^5*Cos[(e + f*x)/2] + 12*B*c^5*Cos[(e + f*x)/2] +
84*A*c^4*d*Cos[(e + f*x)/2] - 116*B*c^4*d*Cos[(e + f*x)/2] + 249*A*c^3*d^2*
Cos[(e + f*x)/2] - 433*B*c^3*d^2*Cos[(e + f*x)/2] + 385*A*c^2*d^3*Cos[(e +
f*x)/2] - 277*B*c^2*d^3*Cos[(e + f*x)/2] + 239*A*c*d^4*Cos[(e + f*x)/2] - 9
5*B*c*d^4*Cos[(e + f*x)/2] + 47*A*d^5*Cos[(e + f*x)/2] - 51*B*d^5*Cos[(e +
f*x)/2] - 12*A*c^5*Cos[(3*(e + f*x))/2] - 20*B*c^5*Cos[(3*(e + f*x))/2] + 4
0*A*c^4*d*Cos[(3*(e + f*x))/2] - 104*B*c^4*d*Cos[(3*(e + f*x))/2] + 261*A*c
^3*d^2*Cos[(3*(e + f*x))/2] - 581*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 781*A*c^
2*d^3*Cos[(3*(e + f*x))/2] - 665*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 579*A*c*d
^4*Cos[(3*(e + f*x))/2] - 299*B*c*d^4*Cos[(3*(e + f*x))/2] + 79*A*d^5*Cos[(
3*(e + f*x))/2] - 59*B*d^5*Cos[(3*(e + f*x))/2] + 12*A*c^4*d*Cos[(5*(e + f*
x))/2] + 20*B*c^4*d*Cos[(5*(e + f*x))/2] - 73*A*c^3*d^2*Cos[(5*(e + f*x))/2
```

```
] + 217*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 353*A*c^2*d^3*Cos[(5*(e + f*x))/2]
 + 397*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 419*A*c*d^4*Cos[(5*(e + f*x))/2] +
251*B*c*d^4*Cos[(5*(e + f*x))/2] - 127*A*d^5*Cos[(5*(e + f*x))/2] + 75*B*d^
5*Cos[(5*(e + f*x))/2] + 3*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 5*B*c^3*d^2*Cos
[(7*(e + f*x))/2] - 21*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 73*B*c^2*d^3*Cos[(7
*(e + f*x))/2] - 111*A*c*d^4*Cos[(7*(e + f*x))/2] + 79*B*c*d^4*Cos[(7*(e +
f*x))/2] - 63*A*d^5*Cos[(7*(e + f*x))/2] + 35*B*d^5*Cos[(7*(e + f*x))/2] +
44*A*c^5*Sin[(e + f*x)/2] - 12*B*c^5*Sin[(e + f*x)/2] - 84*A*c^4*d*Sin[(e +
 f*x)/2] + 116*B*c^4*d*Sin[(e + f*x)/2] - 249*A*c^3*d^2*Sin[(e + f*x)/2] +
433*B*c^3*d^2*Sin[(e + f*x)/2] - 385*A*c^2*d^3*Sin[(e + f*x)/2] + 277*B*c^2
*d^3*Sin[(e + f*x)/2] - 239*A*c*d^4*Sin[(e + f*x)/2] + 95*B*c*d^4*Sin[(e +
f*x)/2] - 47*A*d^5*Sin[(e + f*x)/2] + 51*B*d^5*Sin[(e + f*x)/2] - 12*A*c^5*
Sin[(3*(e + f*x))/2] - 20*B*c^5*Sin[(3*(e + f*x))/2] + 40*A*c^4*d*Sin[(3*(e
 + f*x))/2] - 104*B*c^4*d*Sin[(3*(e + f*x))/2] + 261*A*c^3*d^2*Sin[(3*(e +
f*x))/2] - 581*B*c^3*d^2*Sin[(3*(e + f*x))/2] + 781*A*c^2*d^3*Sin[(3*(e + f
*x))/2] - 665*B*c^2*d^3*Sin[(3*(e + f*x))/2] + 579*A*c*d^4*Sin[(3*(e + f*x)
)/2] - 299*B*c*d^4*Sin[(3*(e + f*x))/2] + 79*A*d^5*Sin[(3*(e + f*x))/2] - 5
9*B*d^5*Sin[(3*(e + f*x))/2] - 12*A*c^4*d*Sin[(5*(e + f*x))/2] - 20*B*c^4*d
*Sin[(5*(e + f*x))/2] + 73*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 217*B*c^3*d^2*S
in[(5*(e + f*x))/2] + 353*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 397*B*c^2*d^3*Si
n[(5*(e + f*x))/2] + 419*A*c*d^4*Sin[(5*(e + f*x))/2] - 251*B*c*d^4*Sin[(5*
(e + f*x))/2] + 127*A*d^5*Sin[(5*(e + f*x))/2] - 75*B*d^5*Sin[(5*(e + f*x))
/2] + 3*A*c^3*d^2*Sin[(7*(e + f*x))/2] + 5*B*c^3*d^2*Sin[(7*(e + f*x))/2] -
 21*A*c^2*d^3*Sin[(7*(e + f*x))/2] + 73*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 11
1*A*c*d^4*Sin[(7*(e + f*x))/2] + 79*B*c*d^4*Sin[(7*(e + f*x))/2] - 63*A*d^5
*Sin[(7*(e + f*x))/2] + 35*B*d^5*Sin[(7*(e + f*x))/2]))/(128*(c - d)^4*(c +
 d)^2*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7321 vs. $2(476) = 952$.

Time = 3.45 (sec) , antiderivative size = 7322, normalized size of antiderivative = 14.11

method	result	size
default	Expression too large to display	7322

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4135 vs. 2(476) = 952.

Time = 24.73 (sec) , antiderivative size = 8555, normalized size of antiderivative = 16.48

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2024 vs. 2(476) = 952.

Time = 1.00 (sec) , antiderivative size = 2024, normalized size of antiderivative = 3.90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1/32*(4*sqrt(2)*(35*sqrt(2)*B*sqrt(a)*c^3*d^2 - 63*sqrt(2)*A*sqrt(a)*c^2*d
^3 + 70*sqrt(2)*B*sqrt(a)*c^2*d^3 - 90*sqrt(2)*A*sqrt(a)*c*d^4 + 67*sqrt(2)
*B*sqrt(a)*c*d^4 - 39*sqrt(2)*A*sqrt(a)*d^5 + 20*sqrt(2)*B*sqrt(a)*d^5)*arc
tan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^3*c^7*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c^6*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + a^3*c^5*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*a^3*c^4*d^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*a^3*c^3*d^4*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) - a^3*c^2*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*a^3*c*d^
6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^3*d^7*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sqrt(-c*d - d^2)) - (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 30*A*sqr
t(a)*c*d - 82*B*sqrt(a)*c*d + 219*A*sqrt(a)*d^2 - 115*B*sqrt(a)*d^2)*log(si
n(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^5*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) - 5*sqrt(2)*a^3*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*
sqrt(2)*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*sqrt(2)*a^3*c^
2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*a^3*c*d^4*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))) + (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 30*A*sqrt(a)*c*d - 82*B*sqrt(a)
*c*d + 219*A*sqrt(a)*d^2 - 115*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x +
1/2*e) + 1)/(sqrt(2)*a^3*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*sqrt(2
)*a^3*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*sqrt(2)*a^3*c^3*d^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*sqrt(2)*a^3*c^2*d^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 5*sqrt(2)*a^3*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - sqrt(2)*a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*(12*A*sqrt(a)
*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 + 20*B*sqrt(a)*c^3*d^2*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^7 - 84*A*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^7 + 292*B*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 - 444*A*sqrt(a
)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 + 316*B*sqrt(a)*c*d^4*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^7 - 252*A*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 +
 140*B*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 - 12*A*sqrt(a)*c^4*d*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 20*B*sqrt(a)*c^4*d*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^5 + 52*A*sqrt(a)*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 252*B*s
qrt(a)*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 500*A*sqrt(a)*c^2*d^3*sin
(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 908*B*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^5 + 1196*A*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 804*B
*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 568*A*sqrt(a)*d^5*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^5 - 320*B*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^5 + 3*A*sqrt(a)*c^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^5*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*A*sqrt(a)*c^4*d*sin(-1/4*pi + 1/2*f*x +
1/2*e)^3 + 51*B*sqrt(a)*c^4*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 146*A*sqrt
(a)*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 434*B*sqrt(a)*c^3*d^2*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 - 710*A*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x +
1/2*e)^3 + 918*B*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 1057*A*
sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 665*B*sqrt(a)*c*d^4*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 - 399*A*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^3 + 231*B*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c^5
```

```
*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c^5*sin(-1/4*pi + 1/2*f*x + 1
/2*e) + 9*A*sqrt(a)*c^4*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 33*B*sqrt(a)*c^4
*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 86*A*sqrt(a)*c^3*d^2*sin(-1/4*pi + 1/2*
f*x + 1/2*e) - 206*B*sqrt(a)*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 290*A
*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 298*B*sqrt(a)*c^2*d^3*sin
(-1/4*pi + 1/2*f*x + 1/2*e) + 303*A*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1
/2*e) - 175*B*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 85*A*sqrt(a)*d
^5*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 53*B*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x
+ 1/2*e))/((sqrt(2)*a^3*c^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)
*a^3*c^5*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*c^4*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*sqrt(2)*a^3*c^3*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) - sqrt(2)*a^3*c^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
2*sqrt(2)*a^3*c*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^6*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 -
 c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 3*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
+ c + d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
), x)
```

3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$
$$-\frac{8\sqrt{2}a^2B\operatorname{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c}{\cdots}\right.}{f\sqrt{1+\sin(e+fx)}}$$
$$-\frac{4\sqrt{2}a^2(A-B)\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx\cdots}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -8*a^2*B*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e)
)*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+s
in(f*x+e))^(1/2)-4*a^2*(A-B)*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d
),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*
x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used

= {3066, 2863, 144, 143}

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+\right.}{f\sqrt{\sin(e+fx)+1}}$$

$$-\frac{8\sqrt{2}a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(\right.}{f\sqrt{\sin(e+fx)+1}}$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)*Ap
pellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx + \frac{B\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx}{a}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{(a^2B\cos(e+fx))\,\text{Subst}\left(\int \frac{(1+x)^{5/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{\left(a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1+x)^{5/2}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}}\,dx, x\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{8\sqrt{2}a^2B\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+f}{f\sqrt{1+\sin(e+fx)}} - \frac{4\sqrt{2}a^2(A-B)\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F]

$$\int (a+a\sin(fx+e))^2\,(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n\,dx$$

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2(d\sin(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2(d\sin(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^2\left(c+d\,\sin(e+f\,x)\right)^n\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 33, antiderivative size = 217

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = -\frac{4\sqrt{2}aB\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{f\sqrt{1+\sin(e+fx)}} - \frac{2\sqrt{2}a(A-B)\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -4*a*B*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)-2*a*(A-B)*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/
2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

$= \{3047, 3096, 2834, 144, 143, 2863\}$

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx = -\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2}(1-\sin(e+f\right.}{f\sqrt{\sin(e+fx)+1}} - \frac{4\sqrt{2}aB\cos(e+fx)(c+d\sin(e+fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2}(1-\sin(e+fx)), \frac{d(1}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*Appell
F1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)
]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c + d*Si
n[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
```

```
]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3096

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, I
nt[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*S
in[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C,
m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \int (c+d\sin(e+fx))^n \left(aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)\right) dx$$

$$= (a(A-B))\int (1+\sin(e+fx))(c+d\sin(e+fx))^n\,dx + (aB)\int (1+\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

$$= \frac{(a(A-B)\cos(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}(c+dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n \left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right) \mathrm{Subst}\left(\int \frac{\sqrt{1+x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}}\,dx, x\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$+ \frac{\left(aB\cos(e+fx)(c+d\sin(e+fx))^n \left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right) \mathrm{Subst}\left(\int \frac{(1+x)^{3/2}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}}\,dx, x, \mathrm{s}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{4\sqrt{2}aB\,\mathrm{AppellF1}\left(\frac{1}{2}, -\frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

$$- \frac{2\sqrt{2}a(A-B)\,\mathrm{AppellF1}\left(\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
 x]
```

Maple [F]

$$\int (a+a\sin(fx+e))(A+B\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

Giac [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(d\sin(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))\,(c + d\,\sin(e + f\,x))^n\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = -\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(\ldots}{c+\ldots}\right.}{af\sqrt{1+\sin(e+fx)}} - \frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\ldots}{\ldots}\right.}{\sqrt{2}af\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(
1+sin(f*x+e))^(1/2)-B*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/
2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))/
(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

= {3066, 2863, 144, 143, 2744}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}$$

$$-\frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(}{c+}\right.}{af\sqrt{\sin(e+fx)+1}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/2,
-n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]
*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx + \frac{B\int (c+d\sin(e+fx))^n\,dx}{a}$$

$$= \frac{((A-B)\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{(B\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \sin\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{\left(B\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(e+\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n}{af\sqrt{1+\sin(e+fx)}} - \frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx}{\sqrt{2}af\sqrt{1+\sin(e+fx)}}$$

Mathematica [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
), x]
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}dx$$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c+d\,\sin(e+f\,x)\right)^n}{a+a\,\sin(e+f\,x)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = -\frac{B\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+f}{c+d}\right.}{\sqrt{2}a^2f\sqrt{1+\sin(e+fx)}} - \frac{(A-B)\operatorname{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d}{}\right.}{2\sqrt{2}a^2f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/2*B*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+
sin(f*x+e))^(1/2)-1/4*(A-B)*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+d),
1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/
(c+d))^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used

$= \{3066, 2863, 144, 143\}$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}} - \frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]
))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 5/2
, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*
Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx}{a}$$

$$= \frac{((A-B)\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{(B\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}(1+x)^{5/2}}\,dx, x, \sin\ldots\right.}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{\left(B\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \sin(e+\ldots\right.}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{B\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+\ldots}{\ldots}\right.}{\sqrt{2}a^2 f\sqrt{1+\sin(e+fx)}} - \frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx\ldots}{2\sqrt{2}a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2, x]
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^2}dx$$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^2}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^2}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^2}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$
$$=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x
)
```

3.332 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 37, antiderivative size = 427

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)} + \frac{2a^2(A-B)(c-d(5+4n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2B(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a^2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))
^(1/2)+2*a^2*B*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(4*
n^2+16*n+15)/(a+a*sin(f*x+e))^(1/2)+2*a^2*(A-B)*(c-d*(5+4*n))*cos(f*x+e)*hy
pergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2
*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*B*(3*c^2-2*c*
d*(7+4*n)+d^2*(16*n^2+56*n+43))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-s
in(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d^2/f/(4*n^2+16*n+15)/(((c+d*sin(f*x+e
))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)
*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3066, 2842, 21, 2855, 72, 71, 3060}

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{2a^2(A-B)(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeon}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{2a^2B(3c^2-2cd(4n+7)+d^2(16n^2+56n+43))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hyperge}}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$
$$+\frac{2a^2B(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{n+1}}{df(2n+5)}$$

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*a^2*(A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) + (2*a^2*B*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^(1 + n))/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]
]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 +
 n))/(d*f*(5 + 2*n)) + (2*a^2*(A - B)*(c - d*(5 + 4*n))*Cos[e + f*x]*Hyperg
eometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d
))^n) - (2*a^2*B*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e
 + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c
+ d*Sin[e + f*x])^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (A-B)\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,dx \\
&\quad + \frac{B\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx}{a} \\
&= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)} \\
&\quad + \frac{(2(A-B))\int \frac{(c+d\sin(e+fx))^n\left(-\frac{1}{2}a^2(c-5d-4dn)-\frac{1}{2}a^2(c-5d-4dn)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{d(3+2n)} \\
&\quad + \frac{(2B)\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\left(\frac{1}{2}a^2(c+d(7+4n))-\frac{1}{2}a^2(3c-11d-4dn)\sin(e}{ad(5+2n)} \\
&= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)} \\
&\quad - \frac{(a(A-B)(c-d(5+4n)))\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx}{d(3+2n)} \\
&\quad + \frac{(aB(3c^2-2cd(7+4n)+d^2(43+56n+16n^2)))\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx}{d^2(3+2n)(5+2n)}
\end{aligned}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)}$$
$$-\frac{(a^3(A-B)(c-d(5+4n))\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{df(3+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(a^3B(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx\right.}{d^2f(3+2n)(5+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)}$$
$$-\frac{\left(a^3(A-B)(c-d(5+4n))\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+fx))}{-ac-ad}\right)^{-n}\right)\mathrm{Subst}\left(\int\right.}{df(3+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{\left(a^3B(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+}{-ac-ad}\right.\right.}{d^2f(3+2n)(5+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(}}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)}$$
$$+\frac{2a^2(A-B)(c-d(5+4n))\cos(e+fx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^2B(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\cos(e+fx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1}{}\right.}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.79 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.57

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left(-30(A+B)(c-d(5+4n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^n,x]
```

```
[Out] -1/15*(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*n
))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + 6*
B*d*(3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/(c
+ d))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hypergeom
etric2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e + f*
x]) + 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 +
 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [F]

$$\int (a+a\sin(fx+e))^{\frac{3}{2}}(A+B\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx=\int (A+B\sin(e+f\,x))\,(a+a\sin(e+f\,x))^{3/2}\,(c+d\sin(e+f\,x))^n\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,
x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,
 x)
```

3.333 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 37, antiderivative size = 167

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$- \frac{2a(Ad(3+2n)-B(c-2d(1+n)))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
-2*a*(A*d*(3+2*n)-B*(c-2*d*(1+n)))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(
1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*sin(f*x+e))/(c+d
))^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3060, 2855, 72, 71}

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \frac{2a\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$
$$- \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[
e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/
(c + d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

integral

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(aAd(3+2n)-B(ac-2ad(1+n)))\int\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx}{ad(3+2n)}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(aAd(3+2n)-B(ac-2ad(1+n)))\cos(e+fx))\text{Subst}\left(\int\frac{(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{df(3+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a(aAd(3+2n)-B(ac-2ad(1+n)))\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+fx))}{-ac-ad}\right)^{-n}\right)\text{Subst}\Big(}{df(3+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a(Bc-2Bd(1+n)-Ad(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [F]

$$\int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x]
```

Maple [F]

$$\int \sqrt{a+a\sin(fx+e)}\,(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n\,dx$$

```
[In] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)
^n, x)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int \sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))\,(c+d\sin(e+fx))^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**n, x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

Giac [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^n\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,
 x)
```

3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 220

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{(A-B)\operatorname{AppellF1}\left(1+n,\frac{1}{2},1,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx)}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*B*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(
f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-(A-B)*Appel
lF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)
*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)/f/(1+n)/(1-sin
(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used

$= \{3066, 2867, 142, 141, 2855, 72, 71\}$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{1}{2},1,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d}{}\right.}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -(((A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c
+ d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)
]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[
a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (
d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

integral

$$= (A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx}{a}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$
$$+ \frac{\left(aB\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+fx))}{-ac-ad}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(\frac{c}{c+d}+\frac{dx}{c+d}\right)^n}{\sqrt{a-ax}}dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$=$$
$$-\frac{(A-B)\,\text{AppellF1}\left(1+n,\frac{1}{2},1,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 8.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a(1+\sin(e+fx))}(c+d\sin(e+fx))^n\left(-\left((A+B)\,\text{AppellF1}\left(1,\frac{1}{2},-n,2,\frac{1}{2}(1+\sin(e+\right.\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B)
*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c +
 d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*(A -
B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)
*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{\sqrt{a+a\sin(fx+e)}}dx$$

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)^n}{\sqrt{a\sin{(fx+e)}+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^n}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)^n}{\sqrt{a\sin{(fx+e)}+a}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2
), x)
```

3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 269

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{B\,\text{AppellF1}\left(1+n,\frac{1}{2},1,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)d\,\text{AppellF1}\left(1+n,\frac{1}{2},2,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+f}{(c-d)^2f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -B*AppellF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*co
s(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/(1
+n)/(1-sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+(A-B)*d*AppellF1(1+n,2,1/2,2+n,(c
+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)*(c+d*sin(f*x+e))^(1
+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)^2/f/(1+n)/(a-a*sin(f*x+e))/(a+a*si
n(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3066, 2867, 142, 141}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}}{f(n+1)(c-d)^2(a-a\sin(e+} - \frac{B\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\text{AppellF1}\left(n+1,\frac{1}{2},1,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Si
n[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c +
 d*Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e
+ f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Si
n[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a
 - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^2\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} + \frac{\left(aB\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{B\,\text{AppellF1}\left(1+n, \frac{1}{2}, 1, 2+n, \frac{c+d\sin(e+fx)}{c+d}, \frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+}{(c-d)f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)d\,\text{AppellF1}\left(1+n, \frac{1}{2}, 2, 2+n, \frac{c+d\sin(e+fx)}{c+d}, \frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+}{(c-d)^2f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 603 vs. $2(269) = 538$.

Time = 16.63 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(aA\right.\right.}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2
 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (
4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2
 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
```

```
)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]
), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (
c - d)/(d + d*Sin[e + f*x]))^n)) + a*A*(1 + Sin[e + f*x])*((a*AppellF1[1, 1
/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 -
2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (4*S
qrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n
, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] +
 a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (
-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c -
d)/(d + d*Sin[e + f*x]))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^{n}}{(a+a\sin(fx+e))^{\frac{3}{2}}}dx$$

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)

[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^{n}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^{n}}{(a\,(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)

[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2
), x)
```

3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 351

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+m)+B(c^2+d^2+cdm)\right))\cos(e+fx)(a+a\sin(e+ }{f(1+m)(2+m)(3+m)}$$
$$- \frac{2^{\frac{1}{2}+m}(A(3+m)\left(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+m^2)\right)+B(d^2m(5+3m+m^2)+c^2m}{}$$
$$- \frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$
$$- \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$

```
[Out] (d*(A*d*(3+m)+B*(d*m+2*c))-2*(2+m)*(A*c*d*(3+m)+B*(c*d*m+c^2+d^2)))*cos(f*x
+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)-2^(1/2+m)*(A*(3+m)*(2*c*d*m*(2+m
)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))+B*(d^2*m*(m^2+3*m+5)+c^2*m*(m^2+5*m+6)+2*c
*d*(m^3+4*m^2+4*m+3)))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(3+m)/(m^2+3*m+2)-d*(A
*d*(3+m)+B*(d*m+2*c))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)/(3+m)-B*c
os(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+m)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3062, 3047, 3102, 2830, 2731, 2730}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2(m^2+3m+2)+2cdm(m+2)+d^2(m^2+m+1)\right)+B(c^2m(m^2+5m\cdots\right.}{\cdots}$$

$$+\frac{\cos(e+fx)\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3)+B(c^2+cdm+d^2)\right)\right)(a\sin(e+fx\cdots}{f(m+1)(m+2)(m+3)}$$

$$-\frac{d\cos(e+fx)(Ad(m+3)+B(2c+dm))(a\sin(e+fx)+a)^{m+1}}{af(m+2)(m+3)}$$

$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 + m)*(A*c*d*(3 + m) + B*(c^2 + d^2
 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)
) - (2^(1/2 + m)*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 +
 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 + 5*m + m^2) + 2*c*d*(3
+ 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(
f*(1 + m)*(2 + m)*(3 + m)) - (d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*
(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (B*Cos[e + f*x]*(a +
a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)} + \frac{\int(a+a\sin(e+fx))^m(c+d\sin(e+fx))(a(Ac(3+m)+B(2d+cm))+a(Ad(3+m)+B(2c+dm))}{a(3+m)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)} + \frac{\int(a+a\sin(e+fx))^m\,(ac(Ac(3+m)+B(2d+cm))+(ad(Ac(3+m)+B(2d+cm))+ac(A}{a(3+m}$$

$$= -\frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$
$$- \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$
$$+ \frac{\int(a+a\sin(e+fx))^m\left(a^2(c(2+m)(Ac(3+m)+B(2d+cm))+d(1+m)(Ad(3+m)+B(2c\right.}{a^2(2}$$

$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+m)+B(c^2+d^2+cdm)\right))\cos(e+fx)(a+a\sin}{f(1+m)(2+m)(3+m)}$$
$$- \frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$
$$- \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$
$$+ \frac{(A(3+m)\left(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+m^2)\right)+B(d^2m(5+3m+m^2)+c^2m}{(1+m)(2+m)(3+m)}$$

$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+m)+B(c^2+d^2+cdm)\right))\cos(e+fx)(a+a\sin}{f(1+m)(2+m)(3+m)}$$
$$- \frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$
$$- \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$
$$+ \frac{((A(3+m)\left(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+m^2)\right)+B(d^2m(5+3m+m^2)+c^2}{(1}$$

$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+m)+B(c^2+d^2+cdm)\right))\cos(e+fx)(a+a\sin}{f(1+m)(2+m)(3+m)}$$
$$- \frac{2^{\frac{1}{2}+m}(A(3+m)\left(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+m^2)\right)+B(d^2m(5+3m+m^2)}{}$$
$$- \frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$
$$- \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.35 (sec) , antiderivative size = 854, normalized size of antiderivative = 2.43

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$= \frac{i(a(1+\sin(e+fx)))^m(1-i\cos(e+fx)+\sin(e+fx))^{-2m}\left(\frac{8Ac^2\,\text{Hypergeometric2F1}(-2m,-m,1-m,i\cos(e+fx)-\sin}{m}\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] ((I/8)*(a*(1 + Sin[e + f*x]))^m*((8*A*c^2*Hypergeometric2F1[-2*m, -m, 1 - m
, I*Cos[e + f*x] - Sin[e + f*x]])/m + (8*B*c*d*Hypergeometric2F1[-2*m, -m,
1 - m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (4*A*d^2*Hypergeometric2F1[-2*m,
 -m, 1 - m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (4*B*c^2*Hypergeometric2F1[
1 - m, -2*m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin
[e + f*x]))/(-1 + m) + (8*A*c*d*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*Cos
[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*x]))/(-1 + m) + (3
*B*d^2*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]
*((-I)*Cos[e + f*x] + Sin[e + f*x]))/(-1 + m) + (4*B*c^2*Hypergeometric2F1[
-1 - m, -2*m, -m, I*Cos[e + f*x] - Sin[e + f*x]]*(I*Cos[e + f*x] + Sin[e +
f*x]))/(1 + m) + (8*A*c*d*Hypergeometric2F1[-1 - m, -2*m, -m, I*Cos[e + f*x
] - Sin[e + f*x]]*(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) + (3*B*d^2*Hyper
geometric2F1[-1 - m, -2*m, -m, I*Cos[e + f*x] - Sin[e + f*x]]*(I*Cos[e + f*
x] + Sin[e + f*x]))/(1 + m) - (2*d*(2*B*c + A*d)*Hypergeometric2F1[-2 - m,
-2*m, -1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] - I*Sin[2*(e
 + f*x)]))/(2 + m) - (4*B*c*d*Hypergeometric2F1[2 - m, -2*m, 3 - m, I*Cos[e
 + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m) -
 (2*A*d^2*Hypergeometric2F1[2 - m, -2*m, 3 - m, I*Cos[e + f*x] - Sin[e + f*
x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m) - (I*B*d^2*Hypergeome
tric2F1[-3 - m, -2*m, -2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*
x)] - I*Sin[3*(e + f*x)]))/(3 + m) + (I*B*d^2*Hypergeometric2F1[3 - m, -2*m
, 4 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*
x)]))/(-3 + m)))/(f*(1 - I*Cos[e + f*x] + Sin[e + f*x])^(2*m))
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c + d\sin(fx + e))^2 \, dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx = \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^2 (a\sin(fx + e) + a)^m \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
[Out] integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e)^2 - (B*d
^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e)
+ a)^m, x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx = \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx)) (c + d\sin(e + fx))^2 \, dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
)**2, x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^2(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^2(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^2\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 199

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)} - \frac{2^{\frac{1}{2}+m}(A(2+m)(c+cm+dm)+B(cm(2+m)+d(1+m+m^2)))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{f(1+m)(2+m)} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}$$

```
[Out] (B*d-(A*d+B*c)*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m)
*(A*(2+m)*(c*m+d*m+c)+B*(c*m*(2+m)+d*(m^2+m+1)))*cos(f*x+e)*hypergeom([1/2,
 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^
m/f/(m^2+3*m+2)-B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

$= \{3047, 3102, 2830, 2731, 2730\}$

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1)\right)(\sin(e+fx)+1)^{-m-}}{f(m+1)(m+2)} + \frac{\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)(m+2)} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((B*d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (2^(1/2 + m)*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*
(1 + m + m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int (a+a\sin(e+fx))^m \left(Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)\right) dx$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)} + \frac{\int (a+a\sin(e+fx))^m(a(Bd(1+m)+Ac(2+m))-a(Bd-(Bc+Ad)(2+m))\sin(e+fx))\,d}{a(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)} + \frac{(Bcm(2+m)+A(2+m)(c+cm+dm)+Bd(1+m+m^2))\int (a+a\sin(e+fx))^m\,dx}{(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)} + \frac{((Bcm(2+m)+A(2+m)(c+cm+dm)+Bd(1+m+m^2))\,(1+\sin(e+fx))^{-m}(a+a\sin(e}{(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)} - \frac{2^{\frac{1}{2}+m}(Bcm(2+m)+A(2+m)(c+cm+dm)+Bd(1+m+m^2))\cos(e+fx)\,\text{Hypergeometric}}{f(1+m)(2+m}$$

$$- \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.75 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{(a(1+\sin(e+fx)))^m(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{2(2Ac+Bd)\,\text{Hypergeometric2F1}(1,1+m,1-m,i\cos(e+fx)}{m}\right.}{}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
[Out] -1/4*((a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))*((-2*(
2*A*c + B*d)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Cos[e + f*x] - Sin[e + f*
x]])/m - ((2*I)*(B*c + A*d)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x] - Si
n[e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]))/(1 + m) + ((2*I)*(B*c + A*d)*H
ypergeometric2F1[1, 2 + m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[e + f
*x] + I*Sin[e + f*x]))/(-1 + m) + (B*d*Hypergeometric2F1[1, -1 + m, -1 - m,
 I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/(2
 + m) + (B*d*Hypergeometric2F1[1, 3 + m, 3 - m, I*Cos[e + f*x] - Sin[e + f*
x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m)))/f
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c+d\sin(fx+e))\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*si
n(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \int (a(\sin{(e+fx)}+1))^m\,(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})\;dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \int (B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)(a\sin{(fx+e)}+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \int (B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)(a\sin{(fx+e)}+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+f}{f(1+m)}$$

```
[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2830, 2731, 2730}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\right.}{f(m+1)} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{(A+Am+Bm)\int(a+a\sin(e+fx))^m\,dx}{1+m}\\
&= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}\\
&\quad+\frac{((A+Am+Bm)(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m)\int(1+\sin(e+fx))^m\,dx}{1+m}\\
&= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}\\
&\quad-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\text{s}}{f(1+m)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\frac{2^m\left((A-B)B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2}+m,\frac{1}{2}\right)+2BB_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{3}{2}+m,\frac{1}{2}\right)\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)(1+\sin(}{f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
[Out] (2^m*((A - B)*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2] + 2*B*Beta[(1 + Sin[
e + f*x])/2, 3/2 + m, 1/2])*Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(a*(1 + Sin[e
 + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

Maple [F]

$$\int (a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (A+B\,\sin{(e+f\,x)})\,(a+a\,\sin{(e+f\,x)})^m\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 191

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{\sqrt{2}(Bc-Ad)\operatorname{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e}{(c-d)df(1+2m)\sqrt{1-\sin(e+fx)}} - \frac{2^{\frac{1}{2}+m}B\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\,s}{df}$$

```
[Out] -2^(1/2+m)*B*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1
+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/f-(-A*d+B*c)*AppellF1(1/2+m,1,1/
2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x
+e))^m*2^(1/2)/(c-d)/d/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3065, 2731, 2730, 2867, 142, 141}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{\sqrt{2}(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(}{}\right.}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}} - \frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-s\right.}{df}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m
)/((c - d)*d*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])) - (2^(1/2 + m)*B*Cos[e +
f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e
+ f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rule 3065

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\text{integral} = \frac{B\int (a+a\sin(e+fx))^m\,dx}{d} - \frac{(Bc-Ad)\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{\left(a^2(Bc-Ad)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)}\,dx,x,\sin(e+fx)\right)}{df\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{\left(B(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx}{d}$$

$$= -\frac{2^{\frac{1}{2}+m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}}{df} - \frac{\left(a^2(Bc-Ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}df(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}(Bc-Ad)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a-}{(c-d)df(1+2m)\sqrt{1-\sin(e+fx)}} - \frac{2^{\frac{1}{2}+m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}}{df}$$

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
), x]
```

Maple [F]

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{c + d\sin(fx + e)} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c + d\sin(e + fx)} dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{d\sin(fx + e) + c} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c + d\sin(e + fx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{c+d\,\sin(e+f\,x)}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 293

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{\sqrt{2}(Ad(c(1-m)-dm)-B(d^2-c^2m-cdm))\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+}{}\right.}{(c-d)^2d(c+d)f(1+2m)\sqrt{1-\sin(e+fx)}} + \frac{2^{\frac{1}{2}+m}(Bc-Ad)m\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))}{d\left(c^2-d^2\right)f} - \frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

```
[Out] 2^(1/2+m)*(-A*d+B*c)*m*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)/f-(-A*d+B*c)
*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+e))+(A*d*(c*(1-m)-d
*m)-B*(-c^2*m-c*d*m+d^2))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d
),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^2/d/(c+d)
/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{3063, 3065, 2731, 2730, 2867, 142, 141\}$

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{\sqrt{2}\cos(e+fx)\left(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2)\right)(a\sin(e+fx)+a)^m \operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2}\right.}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$
$$+\frac{2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m\right.}{df(c^2-d^2)}$$
$$-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*(A*d*(c*(1 - m) - d*m) - B*(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m
, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]
*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*d*(c + d)*f*(1 + 2*m)*Sqrt
[1 - Sin[e + f*x]]) + (2^(1/2 + m)*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)
*(a + a*Sin[e + f*x])^m)/(d*(c^2 - d^2)*f) - ((B*c - A*d)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3065

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{(c^2-d^2)\,f(c+d\sin(e+fx))} - \frac{\int \frac{(a+a\sin(e+fx))^m(-a(Ac-Bd+Bcm-Adm)+a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{a\,(c^2-d^2)}$$

$$= -\frac{(Bc - Ad)\cos(e + fx)(a + a\sin(e + fx))^m}{(c^2 - d^2) f(c + d\sin(e + fx))} - \frac{((Bc - Ad)m)\int (a + a\sin(e + fx))^m\,dx}{d\,(c^2 - d^2)} + \frac{(Ad(c(1-m) - dm) - B(d^2 - c^2m - cdm))\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx}{d\,(c^2 - d^2)}$$

$$= -\frac{(Bc - Ad)\cos(e + fx)(a + a\sin(e + fx))^m}{(c^2 - d^2) f(c + d\sin(e + fx))} + \frac{(a^2(Ad(c(1-m) - dm) - B(d^2 - c^2m - cdm))\cos(e + fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)}\,dx, x, \sin(e + \right.}{d\,(c^2 - d^2) f\sqrt{a - a\sin(e + fx)}\sqrt{a + a\sin(e + fx)}} - \frac{((Bc - Ad)m(1 + \sin(e + fx))^{-m}(a + a\sin(e + fx))^m)\int (1 + \sin(e + fx))^m\,dx}{d\,(c^2 - d^2)}$$

$$= \frac{2^{\frac{1}{2}+m}(Bc - Ad)m\cos(e + fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin(e + fx))\right)(1 + \sin(e + f}{d\,(c^2 - d^2) f} - \frac{(Bc - Ad)\cos(e + fx)(a + a\sin(e + fx))^m}{(c^2 - d^2) f(c + d\sin(e + fx))} + \frac{\left(a^2(Ad(c(1-m) - dm) - B(d^2 - c^2m - cdm))\cos(e + fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+d}\right.}{\sqrt{2}d\,(c^2 - d^2) f(a - a\sin(e + fx))\sqrt{a + a\sin(e + fx)}}$$

$$= \frac{\sqrt{2}(Ad(c(1-m) - dm) - B(d^2 - c^2m - cdm))\,\text{AppellF1}\left(\frac{1}{2} + m, \frac{1}{2}, 1, \frac{3}{2} + m, \frac{1}{2}(1 + \sin(e + fx)),\right.}{(c - d)^2 d(c + d)f(1 + 2m)\sqrt{1 - \sin(e + fx)}} + \frac{2^{\frac{1}{2}+m}(Bc - Ad)m\cos(e + fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin(e + fx))\right)(1 + \sin(e +}{d\,(c^2 - d^2) f} - \frac{(Bc - Ad)\cos(e + fx)(a + a\sin(e + fx))^m}{(c^2 - d^2) f(c + d\sin(e + fx))}$$

Mathematica [F]

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2, x]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c+d\sin(fx+e))^2}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^2}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2, x
)
```

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 467

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= \frac{(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-3m+m^2)\ldots}{\sqrt{2}(c-d)^3d(c+\ldots}$$
$$-\frac{2^{-\frac{1}{2}+m}m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\ldots\right.}{d\left(c^2-d^2\right)^2f}$$
$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$
$$+\frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))}$$

```
[Out] -2^(-1/2+m)*m*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*hype
rgeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*s
in(f*x+e))^m/d/(c^2-d^2)^2/f-1/2*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^m/(
c^2-d^2)/f/(c+d*sin(f*x+e))^2+1/2*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d
*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+1/2*(B*(2
*d^3*m+c^3*(1-m)*m+2*c^2*d*(1-m)*m-c*d^2*(m^2-3*m+3))-A*d*(2*c*d*(2-m)*m-c^
2*(m^2-3*m+2)-d^2*(m^2-m+1)))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/
(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c-d)^3/d/(c+d)^2/f
/(1+2*m)*2^(1/2)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3063, 3065, 2731, 2730, 2867, 142, 141}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$=\frac{\cos(e+fx)\left(B(c^3(1-m)m+2c^2d(1-m)m-cd^2(m^2-3m+3)+2d^3m)-Ad(-(c^2(m^2-3m+2))+\right.}{\sqrt{2}df(2m+1)(c}$$
$$-\frac{2^{m-\frac{1}{2}}m\cos(e+fx)\left(Ad(c(3-m)-dm)-B(c^2(1-m)-cdm+2d^2)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e}{df\left(c^2-d^2\right)^2}$$
$$+\frac{\cos(e+fx)\left(Ad(c(3-m)-dm)-B(c^2(1-m)-cdm+2d^2)\right)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))}$$
$$-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3 - 3*m + m^2)) -
 A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m + m^2)))*AppellF1[
1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
 - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(Sqrt[2]*(c - d)^3*d*(c + d)^2
*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]) - (2^(-1/2 + m)*m*(A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin
[e + f*x])^m)/(d*(c^2 - d^2)^2*f) - ((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e
+ f*x])^m)/(2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/
(2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
```

```
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3065

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
```

```
eQ[m + 1/2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{(a+a\sin(e+fx))^m(-a(2Ac-2Bd+Bcm-Adm)-a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx}{2a\left(c^2-d^2\right)}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^m\left(-a^2((Bc-Ad)(1-m)(d-cm)-(c-dm)(2Ac-2Bd+Bcm-Adm))+a^2m\left(Ad(c(3-m)-dm)-B\left(2d^2+c^2(1-m\right.\right.\right.}{c+d\sin(e+fx)}}{2a^2\left(c^2-d^2\right)^2}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))} + \frac{(m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm)))\int(a+a\sin(e+fx))^m\,dx}{2d\left(c^2-d^2\right)^2} + \frac{(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-3m}{2d\left(c^2-d^2\right)^2}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))} + \frac{(a^2(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-}{2d\left(c^2-d^2\right)^2f\sqrt{a-a\sin(e+f}} + \frac{(m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m}{2d\left(c^2-d^2\right)^2}$$

$$= -\frac{2^{-\frac{1}{2}+m}m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)\,\text{Hypergeometric2F1}\,(}{d\left(c^2-d^2\right)^2 f}$$

$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$+\frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))}$$

$$+\frac{\Big(a^2(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2}{2\sqrt{2}d\left(c^2-d^2\right)^2 f(a-a}$$

$$=\frac{(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-3m}{\sqrt{2}(c-d}$$

$$-\frac{2^{-\frac{1}{2}+m}m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)\,\text{Hypergeometric2F1}\,(}{d\left(c^2-d^2\right)^2 f}$$

$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$+\frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))}$$

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3, x]
```

Maple [F]

$$\int \frac{(a + a\sin(fx+e))^m (A + B\sin(fx+e))}{(c + d\sin(fx+e))^3} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3} dx = \int \frac{(B\sin(fx+e) + A)(a\sin(fx+e) + a)^m}{(d\sin(fx+e) + c)^3} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)
^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3} dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3} dx = \int \frac{(B\sin(fx+e) + A)(a\sin(fx+e) + a)^m}{(d\sin(fx+e) + c)^3} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^3}\,dx$$

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^3}\,dx$$

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3, x
)

3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 284

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{2}(A-B)(c-d)\operatorname{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-\frac{3}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+ \cdots}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}} + \frac{\sqrt{2}B(c-d)\operatorname{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-\frac{3}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+ \cdots}{af(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (A-B)*(c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*s
in(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(
1+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*(c-d)*AppellF1
(3/2+m,-3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e
)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(
f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3066, 2867, 145, 144, 143}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx=\frac{\sqrt{2}(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},\cdots\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}+\frac{\sqrt{2}B(c-d)\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-\frac{3}{2},m+\frac{5}{2},\frac{1}{2}(\sin(\cdots\right)}{af(2m+3)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (Sqrt[2]*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c
 + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d)*AppellF1[3/2 + m, 1/2, -3
/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*
m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx \\
&\quad+\frac{B\int (a+a\sin(e+fx))^{1+m}(c+d\sin(e+fx))^{3/2}\,dx}{a} \\
&=\frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{(aB\cos(e+fx))\operatorname{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a(A-B)(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}\left(\frac{ac}{ac-ad}+\frac{a}{\ldots}\right.}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}} + \frac{\left(B(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{2}+m}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{3/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}}$$

$$= \frac{\sqrt{2}(A-B)(c-d)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-\frac{3}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+f}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}} + \frac{\sqrt{2}B(c-d)\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-\frac{3}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)\sqrt{}}{f(3+2m)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+}{c-d}}}$$

Mathematica [F]

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(3/2),x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(3/2), x]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c+d\sin(fx+e))^{\frac{3}{2}} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^{\frac{3}{2}}(a\sin(fx + e) + a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^m\,(c + d\sin(e + fx))^{3/2}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),
 x)
```

3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+$

Optimal result

Integrand size = 37, antiderivative size = 274

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-\frac{1}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-\frac{1}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx)}{af(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)
/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*AppellF1(3/2+m,-1/2,
1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f
*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))^(1/2
)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used

$= \{3066, 2867, 145, 144, 143\}$

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},-\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sin(e\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$+ \frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-\frac{1}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e+\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -1/2, 5/2 + m, (1
+ Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*S
in[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[
e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
```

```
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (A-B)\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx \\
&\quad + \frac{B\int (a+a\sin(e+fx))^{1+m}\sqrt{c+d\sin(e+fx)}\,dx}{a} \\
&= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(aB\cos(e+fx))\operatorname{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\operatorname{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{\frac{ac}{ac-ad}+\frac{adx}{ac-ad}}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}dx,\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}} + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\operatorname{Subst}\left(\int\frac{(a+ax)^{\frac{1}{2}+m}\sqrt{\frac{ac}{ac-ad}+\frac{adx}{ac-ad}}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}dx,x,\sin(\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}}$$

$$= \frac{\sqrt{2}(A-B)\operatorname{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-\frac{1}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}} + \frac{\sqrt{2}B\operatorname{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-\frac{1}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)\sqrt{1-\sin}}{f(3+2m)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Mathematica [F]

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = \int(a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*
x]],x]
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*
x]], x]
```

Maple [F]

$$\int(a+a\sin(fx+e))^m(A+B\sin(fx+e))\sqrt{c+d\sin(fx+e)}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)
^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$=\int (a(\sin(e+fx)+1))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e +
f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \int (B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}(a\sin(fx+e)+a)^m\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^m\,\sqrt{c+d\,\sin(e+f\,x)}\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),
 x)
```

3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 274

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\sqrt{2}(A-B)\operatorname{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},\frac{1}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+f}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{\sqrt{2}B\operatorname{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},\frac{1}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{1}}{af(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/f/
(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,1/2,1/
2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x
+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used

$= \{3066, 2867, 145, 144, 143\}$

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= \frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$+\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},\frac{1}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e+fx)+1\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1/2, 5/2 + m, (1 +
Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*f*(3 + 2*m)*Sqrt[
1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
```

```
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\sqrt{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{\sqrt{a-ax}\sqrt{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{c+dx}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{c+dx}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{\frac{ac}{ac-ad}+\frac{adx}{ac-ad}}}\,dx,x,\sin(e\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$+\frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{\frac{ac}{ac-ad}+\frac{adx}{ac-ad}}}\,dx,x,\sin(e+fx)\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},\frac{1}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},\frac{1}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)\sqrt{1-\sin(e}{f(3+2m)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Mathematica [F]

$$\int\frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int\frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]],x]
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]], x]
```

Maple [F]

$$\int\frac{(a+a\sin(fx+e))^m(A+B\sin(fx+e))}{\sqrt{c+d\sin(fx+e)}}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
c), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e +
f*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

Giac [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}} \, dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{\sqrt{d\sin(fx + e) + c}} \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}} \, dx = \int \frac{(A + B\sin(e + fx))\,(a + a\sin(e + fx))^m}{\sqrt{c + d\sin(e + fx)}} \, dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2
),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2
), x)
```

3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 288

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},\frac{3}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+f$$
$$(c-d)f(1+2m)\sqrt{1-$$
$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},\frac{3}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{a(c-d)f(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c
-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,
3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*s
in(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/(c-d)/f/(3+2*m)/(
1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3066, 2867, 145, 144, 143}

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}{f(2m+1)(c-d)\sqrt{1-}}$$
$$+\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},\frac{3}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e+fx)+1),-\right.}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 3/2, 5/2 +
m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*(c - d)*f
*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\text{integral} = (A-B)\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))^{1+m}}{(c+d\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^3(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{3/2}}\,dx, x, \sin(e\right.}{\sqrt{2}(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{\left(a^2B\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{3/2}}\,dx, x, \sin(e+f\right.}{\sqrt{2}(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}(A-B)\,\mathrm{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},\frac{3}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$+\frac{\sqrt{2}B\,\mathrm{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},\frac{3}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)\sqrt{1-\sin(e}{(c-d)f(3+2m)(a-a\sin(e+fx))\sqrt{c+d\sin(e+f}}$$

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$$

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^(3/2), x]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m\,(A+B\sin(fx+e))}{(c+d\sin(fx+e))^{\frac{3}{2}}}dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(c + d*sin(e + f*x)
)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2
),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2
), x)
```

3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 35, antiderivative size = 270

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\frac{\sqrt{2}(A-B)\operatorname{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-n,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m \ldots}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$
$$+\frac{\sqrt{2}B\operatorname{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-n,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{1+m} \ldots}{af(3+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e
))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((c+
d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,-n,1/2,5/2+m,
-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+
m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)/(1-s
in(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

$= \{3066, 2867, 145, 144, 143\}$

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},-n\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$
$$+\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-n,n\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2,
-((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c +
 d*Sin[e + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x
])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -n, 5/2 + m, (1 + Sin[e
+ f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*
((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx \\
&\quad + \frac{B\int (a+a\sin(e+fx))^{1+m}(c+d\sin(e+fx))^n\,dx}{a} \\
&= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{(aB\cos(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}(c+dx)^n}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{-n}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+}}{}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$+ \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{-n}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}\left(\frac{ac}{ac-ad}\right.}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-n,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$

$$+ \frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-n,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)\sqrt{1-\sin(}{f(3+2m)(a-a\sin(e+fx)}$$

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c+d\sin(fx+e))^n\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^n \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^n \, dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m(d\sin(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c + d\,\sin(e + f\,x))^n\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 277

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx))^{-1+m}}{(c+d)f}$$

$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},1+m,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+}{a(c-d)f(3+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] -2^(1/2+m)*a*(A-B)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(c-d)*(1-sin
(f*x+e))/(c+d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/(c
+d*sin(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)+B*AppellF1(3/2+m,1+m,1
/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*
x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)/a/(c-d)/f/(3+2*m)/((c+d*sin(
f*x+e))^m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used

$= \{3066, 2867, 134, 145, 144, 143\}$

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m \text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},m+1,\right.}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}}$$
$$-\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\,\text{Hyperge}}{f(c+d)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] -((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])
^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((
c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1 + m
, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c
- d)*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
```

```
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx \\
&\quad+\frac{B\int (a+a\sin(e+fx))^{1+m}(c+d\sin(e+fx))^{-1-m}\,dx}{a} \\
&= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-1-m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&\quad+\frac{(aB\cos(e+fx))\operatorname{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}(c+dx)^{-1-m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx}{(c+d)f} + \frac{\left(aB\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{2}+m}(c+dx)^{-1-m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx}{(c+d)f} + \frac{\left(a^2B\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}(c+d\sin(e+fx))^{-m}\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^m\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{2}+m}\left(\frac{c}{ac}\right.}{\sqrt{}}\right.}{\sqrt{2}(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx}{(c+d)f} + \frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},1+m,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)\sqrt{1-s}}{(c-d)f(3+2m)(a-a\sin(e+}$$

Mathematica [F]

$$\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c + d\sin(fx+e))^{-1-m} dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-1-m} dx \\ = \int (B\sin(fx+e) + A)(a\sin(fx+e) + a)^m (d\sin(fx+e) + c)^{-m-1} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(
-m - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-1-m} dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-1-m} dx \\ = \int (B\sin(fx+e) + A)(a\sin(fx+e) + a)^m (d\sin(fx+e) + c)^{-m-1} dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^{m+1}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m +
 1), x)
```

3.348 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 36, antiderivative size = 132

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,-\frac{1}{2},-n,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}}{f(1+2m)}$$

```
[Out] 2*AppellF1(1/2+m,-n,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*
sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)*(1-sin(f*x+e))
^(1/2)/f/(1+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3087, 145, 144, 143}

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}}{f(2m+1)}$$

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Si
```

```
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*((c + d*Sin[e + f*
x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int\sqrt{a-ax}(a+ax)^{-\frac{1}{2}+m}(c+dx)^n\,dx,x,\sin(e

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int\sqrt{\frac{1}{2}-\frac{x}{2}}(a+ax)^{-\frac{1}{2}+m}(c+dx)\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{-n}\right)}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,-\frac{1}{2},-n,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(}{f(1+2m)}$$

Mathematica [F]

$$\int(a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int(a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F]

$$\int(a-a\sin(fx+e))(a+a\sin(fx+e))^m(c+d\sin(fx+e))^n\,dx$$

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \text{Timed out}$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Giac [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int (a+a\,\sin(e+f\,x))^m\,(a-a\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n\,dx$$

```
[In] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.349 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 139

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,-\frac{1}{2},1+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\ldots}{(c-d)f(1+2m)}$$

```
[Out] 2*AppellF1(1/2+m,1+m,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))
*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)*(1-si
n(f*x+e))^(1/2)/(c-d)/f/(1+2*m)/((c+d*sin(f*x+e))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3087, 145, 144, 143}

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m\text{AppellF1}\left(\ldots\right)}{f(2m+1)(c-d)}$$

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -(
(d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a
```

```
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/((c - d)*f*(1 + 2*
m)*(c + d*Sin[e + f*x])^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}\left(\int\sqrt{a-ax}(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-1-m}\,dx, x\right.}{f}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int\sqrt{\frac{1}{2}-\frac{x}{2}}(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-}\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{\left(\sqrt{2}a\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{-m}\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{m}\right)}{(ac-ad)f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,-\frac{1}{2},1+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin}}{(c-d)f(1+2m)}$$

Mathematica [F]

$$\int(a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int(a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m),x]
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

Maple [F]

$$\int(a-a\sin(fx+e))(a+a\sin(fx+e))^m(c+d\sin(fx+e))^{-1-m}\,dx$$

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int(a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int-(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \text{Timed out}$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Giac [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int \frac{(a+a\sin(e+fx))^m\,(a-a\sin(e+fx))}{(c+d\sin(e+fx))^{m+1}}\,dx$$

```
[In] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m +
 1),x)
[Out] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m +
 1), x)
```

3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 55, antiderivative size = 39

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

```
[Out] -cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {3053}

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)*m + (
c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
```

```
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\text{integral} = -\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)
*m + (c + (c - d)*m)*Sin[e + f*x]),x]
[Out] -((Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (c+d\sin(fx+e))^{-2-m} (d-(c-d)m+(c+(c-d)m)\sin(fx+e))\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{(d\cos(fx+e)\sin(fx+e)+c\cos(fx+e))(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*s
in(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d-(c-d)*m+(c+(c-d)*
m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$$
$$= \int -((c-d)m-((c-d)m+c)\sin(fx+e)-d)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(((c - d)*m - ((c - d)*m + c)*sin(f*x + e) - d)*(a*sin(f*x + e) +
 a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

Giac [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{(a\,(\sin(e+f\,x)+1))^m\,\left(d\,\sin(2\,e+2\,f\,x)-2\,c\,\left(2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)\right)}{f\,{(c+d\,\sin(e+f\,x))}^m\,\left(d^2\,\left(2\,{\sin(e+f\,x)}^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin(e+f\,x)\right)}$$

```
[In] int(((a + a*sin(e + f*x))^m*(d - m*(c - d) + sin(e + f*x)*(c + m*(c - d))))
/(c + d*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)^
2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d^
2 + 4*c*d*sin(e + f*x)))
```

3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 51, antiderivative size = 40

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

```
[Out] -cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {3053}

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)*m + (
c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
```

```
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\text{integral} = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)
*m + (c + (c + d)*m)*Sin[e + f*x]),x]
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Maple [F]

$$\int (a-a\sin(fx+e))^m (c+d\sin(fx+e))^{-2-m} (d+(c+d)m+(c+(c+d)m)\sin(fx+e))\,dx$$

```
[In] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
[Out] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{(d\cos(fx+e)\sin(fx+e)+c\cos(fx+e))(-a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}}{f}$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d*
sin(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((a-a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d+(c+d)*m+(c+(c+d)*
m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = \int ((c+d)m+((c+d)m+c)\sin(fx+e)+d)(-a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}\,dx$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((c + d)*m + ((c + d)*m + c)*sin(f*x + e) + d)*(-a*sin(f*x + e) +
 a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

Giac [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{(-a\,(\sin(e+f\,x)-1))^m\,\left(d\,\sin(2\,e+2\,f\,x)-2\,c\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)\right)}{f\,(c+d\,\sin(e+f\,x))^m\,\left(d^2\,\left(2\sin(e+f\,x)^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin(e+f\,x)\right)}$$

```
[In] int(((a - a*sin(e + f*x))^m*(d + sin(e + f*x)*(c + m*(c + d)) + m*(c + d)))
/(c + d*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)
^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d
^2 + 4*c*d*sin(e + f*x)))
```

3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2} dx = -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{2(bc-ad)\left(ad^2(Ac-Bd)-b(2Bc^3-Ac^2d-3Bcd^2+2Ad^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

```
[Out] -b*(-A*b*d-2*B*a*d+2*B*b*c)*x/d^3-2*(-a*d+b*c)*(a*d^2*(A*c-B*d)-b*(-A*c^2*d
+2*A*d^3+2*B*c^3-3*B*c*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2
))/d^3/(c^2-d^2)^(3/2)/f-b^2*B*cos(f*x+e)/d^2/f-(-a*d+b*c)^2*(-A*d+B*c)*cos
(f*x+e)/d^2/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3067, 3102, 2814, 2739, 632, 210}

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2} dx = -\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2)\right)\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{3/2}} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))} - \frac{bx(-2aBd-Abd+2bBc)}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f}$$

```
[In] Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b*(2*b*B*c - A*b*d - 2*a*B*d)*x)/d^3) - (2*(b*c - a*d)*(a*d^2*(A*c - B*d
) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)*f) - (b^2*B*Cos[e + f*x])/(d
^2*f) - ((b*c - a*d)^2*(B*c - A*d)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*
Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-d\left(B(bc-ad)^2-Ad\left(a^2c+b^2c-2abd\right)\right)-b(bBc-Abd-2aBd)\left(c^2-d^2\right)\sin(e+fx)+b^2Bd\left(c^2-d^2\right)\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx}{d^2\left(c^2-d^2\right)}$$

$$= -\frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-d^2\left(B(bc-ad)^2-Ad\left(a^2c+b^2c-2abd\right)\right)-bd(2bBc-Abd-2aBd)\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d^3\left(c^2-d^2\right)}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left((bc-ad)\left(ad^2(Ac-Bd)-b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^3\left(c^2-d^2\right)}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(2(bc-ad)\left(ad^2(Ac-Bd)-b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\right)\operatorname{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+f\right.\right.}{d^3\left(c^2-d^2\right)f}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(4(bc-ad)\left(ad^2(Ac-Bd)-b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\,\mathrm{ta}\right.}{d^3\left(c^2-d^2\right)f}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{b(-2bBc+Abd+2aBd)(e+fx)+\frac{2(bc-ad)\left(ad^2(-Ac+Bd)+b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}-b^2 B \cdots}{d^3 f}$$

```
[In] Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c) +
 B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e +
 f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (d*(
b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f
*x])))/(d^3*f)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{2b\left(-\frac{Bbd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(Abd+2dBa-2Bbc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}+\frac{2\left(\frac{d^2\left(Aa^2d^3-2Aabcd^2+Ab^2c^2d-Ba^2d^2c+2Babc^2d-c^3Bb \cdots\right.}{(c^2-d^2)c} \cdots\right.}{\left(\tan^2\left(\frac{f \cdots}{2}\right.\right. \cdots}$
default	$\frac{2b\left(-\frac{Bbd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(Abd+2dBa-2Bbc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}+\frac{2\left(\frac{d^2\left(Aa^2d^3-2Aabcd^2+Ab^2c^2d-Ba^2d^2c+2Babc^2d-c^3Bb \cdots\right.}{(c^2-d^2)c} \cdots\right.}{\left(\tan^2\left(\frac{f \cdots}{2}\right.\right. \cdots}$
risch	Expression too large to display

```
[In] int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(2*b/d^3*(-B*b*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*b*d+2*B*a*d-2*B*b*c)*arcta
n(tan(1/2*f*x+1/2*e)))+2/d^3*((d^2*(A*a^2*d^3-2*A*a*b*c*d^2+A*b^2*c^2*d-B*a
^2*c*d^2+2*B*a*b*c^2*d-B*b^2*c^3)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)+d*(A*a^2*d
^3-2*A*a*b*c*d^2+A*b^2*c^2*d-B*a^2*c*d^2+2*B*a*b*c^2*d-B*b^2*c^3)/(c^2-d^2)
)/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(A*a^2*c*d^3-2*A*a*b*d^
4-A*b^2*c^3*d+2*A*b^2*c*d^3-B*a^2*d^4-2*B*a*b*c^3*d+4*B*a*b*c*d^3+2*B*b^2*c
```

```
^4-3*B*b^2*c^2*d^2)/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(194) = 388.

Time = 0.36 (sec) , antiderivative size = 1308, normalized size of antiderivative = 6.57

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^
2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*
B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b +
2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d
^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2
 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 +
(B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (
B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d - 4*B*b^2*
c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c
^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*
b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x
 + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4*B*b^2*c^4*d^2
+ 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 -
 (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b +
 A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d
^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2
+ 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(c^2
- d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*B*
b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*
b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2
)*c*d^5)*cos(f*x + e) + ((2*B*b^2*c^5*d - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 -
 (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2
)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*
d^5 + c*d^7)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(194) = 388$.

Time = 0.50 (sec) , antiderivative size = 749, normalized size of antiderivative = 3.76

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{\frac{2\left(2\,Bb^2c^4-2\,Babc^3d-Ab^2c^3d-3\,Bb^2c^2d^2+Aa^2cd^3+4\,Babcd^3+2\,Ab^2cd^3-Ba^2d^4-2\,Aabd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^3-d^5)\sqrt{c^2-d^2}}}{}$$

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*d
^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x + 1
/2*e)^3 - 2*B*a*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*tan(1/2*f*
```

```
x + 1/2*e)^3 + B*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*tan(1/2*f
*x + 1/2*e)^3 - A*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*tan(1/2*f*x
+ 1/2*e)^2 - 2*B*a*b*c^3*d*tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*tan(1/2*f*x
 + 1/2*e)^2 + B*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d^2*tan(1/
2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c*d^3*tan(1
/2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*tan(1/2*f*x + 1/2*e) - 2*B*a*b*c^2*d^2*ta
n(1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^3*tan(1
/2*f*x + 1/2*e) + 2*A*a*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*B*b^2*c*d^3*tan(1/
2*f*x + 1/2*e) - A*a^2*d^4*tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4 - 2*B*a*b*c^3
*d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c^2*d^2 - A*a^2*
c*d^3)/((c^3*d^2 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2
*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*B*
b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f
```

Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 16312, normalized size of antiderivative = 81.97

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*
A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2*(
A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*a*b
*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2
*d^3 - B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*
d))/(c*d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(A*a^2*d^3 - 3*B*b^2*c^3 + A*
b^2*c^2*d - B*a^2*c*d^2 + 2*B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(
c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2
 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (atan((((b*d*(A*b
+ 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b
^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*
B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B
*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7
+ 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*
a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(
A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^
9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 +
7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1
i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 +
A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2
*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*
d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2
```

)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 -
 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 +
10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*
B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c
^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8
 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c
^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^
4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c
^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9
 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*
c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10
 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^
2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b
*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 +
48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a
^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2
*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))*1i)/d^3 +
((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*
d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c
^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*
d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b
^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d
^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5
) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^
9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2
*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*
d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((
(32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*ta
n(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 -
2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*tan(e/
2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2
*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*
c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^
3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*
c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*
b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 2
9*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^
10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^
2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*
a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 -
 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*
b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*
d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 -
8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*
b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^

4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))*1i)/d^3)/((64*(A^3*b^6*c^5*d^3 - 2*A^3*b^6*c^3*d^5 - 4*B^3*b^6*c^8 + 6*B^3*b^6*c^6*d^2 - 3*A^3*a^2*b^4*c^3*d^5 + A^3*a^2*b^4*c^5*d^3 + 4*A^3*a^3*b^3*c^2*d^6 - A^3*a^4*b^2*c^3*d^5 + 44*B^3*a^2*b^4*c^4*d^4 - 24*B^3*a^2*b^4*c^6*d^2 - 36*B^3*a^3*b^3*c^3*d^5 + 16*B^3*a^3*b^3*c^5*d^3 + 14*B^3*a^4*b^2*c^2*d^6 - 4*B^3*a^4*b^2*c^4*d^4 + 8*A*B^2*b^6*c^7*d + 16*B^3*a*b^5*c^7*d - 2*B^3*a^5*b*c*d^7 - 13*A*B^2*b^6*c^5*d^3 + 9*A^2*B*b^6*c^4*d^4 - 5*A^2*B*b^6*c^6*d^2 + 6*A^3*a*b^5*c^2*d^6 - 2*A^3*a*b^5*c^4*d^4 - 4*A^3*a^2*b^4*c*d^7 - 26*B^3*a*b^5*c^5*d^3 - 74*A*B^2*a^2*b^4*c^3*d^5 + 24*A*B^2*a^2*b^4*c^5*d^3 + 44*A*B^2*a^3*b^3*c^2*d^6 + 8*A*B^2*a^3*b^3*c^4*d^4 - 8*A*B^2*a^3*b^3*c^6*d^2 - 16*A*B^2*a^4*b^2*c^3*d^5 + 4*A*B^2*a^4*b^2*c^5*d^3 + 35*A^2*B*a^2*b^4*c^2*d^6 + A^2*B*a^2*b^4*c^4*d^4 - 4*A^2*B*a^2*b^4*c^6*d^2 - 20*A^2*B*a^3*b^3*c^3*d^5 + 4*A^2*B*a^3*b^3*c^5*d^3 + 10*A^2*B*a^4*b^2*c^2*d^6 + 2*A^2*B*a^4*b^2*c^4*d^4 + 52*A*B^2*a*b^5*c^4*d^4 - 28*A*B^2*a*b^5*c^6*d^2 + 4*A*B^2*a^2*b^4*c^7*d - 9*A*B^2*a^4*b^2*c*d^7 + 4*A*B^2*a^5*b*c^2*d^6 - 32*A^2*B*a*b^5*c^3*d^5 + 14*A^2*B*a*b^5*c^5*d^3 - 12*A^2*B*a^3*b^3*c*d^7 - 2*A^2*B*a^5*b*c^3*d^5))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8

- 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*
a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*
b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - ((b*d*(A*b + 2*B*a)*1i -
B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 +
4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c
^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7
+ 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^
3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6
+ 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*
1i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2
*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*
d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 +
2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^
10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^
14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b
d(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^
2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*
d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*
d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^1
1 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3
- (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*
c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B
^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*
A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B
^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*
c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A
^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b
*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d
^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*
B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*
c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8
- 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*
A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 + (64*tan(e/2 + (f
x)/2)(4*A^3*b^6*c^2*d^7 - 16*B^3*b^6*c^9 - 6*A^3*b^6*c^4*d^5 + 2*A^3*b^6*
c^6*d^3 - 24*B^3*b^6*c^5*d^4 + 40*B^3*b^6*c^7*d^2 + 2*A^3*a^2*b^4*c^2*d^7 -
2*A^3*a^2*b^4*c^4*d^5 - 96*B^3*a^2*b^4*c^3*d^6 + 144*B^3*a^2*b^4*c^5*d^4 -
48*B^3*a^2*b^4*c^7*d^2 + 48*B^3*a^3*b^3*c^2*d^7 - 64*B^3*a^3*b^3*c^4*d^5 +
16*B^3*a^3*b^3*c^6*d^3 + 8*B^3*a^4*b^2*c^3*d^6 + 24*A*B^2*b^6*c^8*d - 4*A^
3*a*b^5*c*d^8 + 48*B^3*a*b^5*c^8*d + 40*A*B^2*b^6*c^4*d^5 - 64*A*B^2*b^6*c^
6*d^3 - 22*A^2*B*b^6*c^3*d^6 + 34*A^2*B*b^6*c^5*d^4 - 12*A^2*B*b^6*c^7*d^2
+ 4*A^3*a*b^5*c^3*d^6 + 80*B^3*a*b^5*c^4*d^5 - 128*B^3*a*b^5*c^6*d^3 - 8*B^
3*a^4*b^2*c*d^8 + 88*A*B^2*a^2*b^4*c^2*d^7 - 104*A*B^2*a^2*b^4*c^4*d^5 + 16
*A*B^2*a^2*b^4*c^6*d^3 + 8*A*B^2*a^3*b^3*c^3*d^6 + 16*A*B^2*a^3*b^3*c^5*d^4
+ 8*A*B^2*a^4*b^2*c^2*d^7 - 8*A*B^2*a^4*b^2*c^4*d^5 + 10*A^2*B*a^2*b^4*c^3
*d^6 + 8*A^2*B*a^2*b^4*c^5*d^4 + 8*A^2*B*a^3*b^3*c^2*d^7 - 8*A^2*B*a^3*b^3*

c^4*d^5 - 104*A*B^2*a*b^5*c^3*d^6 + 152*A*B^2*a*b^5*c^5*d^4 - 48*A*B^2*a*b^5*c^7*d^2 - 24*A*B^2*a^3*b^3*c*d^8 + 40*A^2*B*a*b^5*c^2*d^7 - 52*A^2*B*a*b^5*c^4*d^5 + 12*A^2*B*a*b^5*c^6*d^3 - 18*A^2*B*a^2*b^4*c*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6)))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*2i)/(d^3*f) + (atan((((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)*1i)/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f

x)/2)(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4
*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*
B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8
- 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6
+ 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^
4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 +
4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a
*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c
^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 -
8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*
b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d
^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5
))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)
((32(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2
*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*
d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9
))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 -
2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*
A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*
B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B
*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (((32*(c^2*
d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (
f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8
+ c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3
+ 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^
4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d
- 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d
^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)*1i)/(d^9 - 3*c^2*d^7
+ 3*c^4*d^5 - c^6*d^3))/((64*(A^3*b^6*c^5*d^3 - 2*A^3*b^6*c^3*d^5 - 4*B^3*b
^6*c^8 + 6*B^3*b^6*c^6*d^2 - 3*A^3*a^2*b^4*c^3*d^5 + A^3*a^2*b^4*c^5*d^3 +
4*A^3*a^3*b^3*c^2*d^6 - A^3*a^4*b^2*c^3*d^5 + 44*B^3*a^2*b^4*c^4*d^4 - 24*B
^3*a^2*b^4*c^6*d^2 - 36*B^3*a^3*b^3*c^3*d^5 + 16*B^3*a^3*b^3*c^5*d^3 + 14*B
^3*a^4*b^2*c^2*d^6 - 4*B^3*a^4*b^2*c^4*d^4 + 8*A*B^2*b^6*c^7*d + 16*B^3*a*b
^5*c^7*d - 2*B^3*a^5*b*c*d^7 - 13*A*B^2*b^6*c^5*d^3 + 9*A^2*B*b^6*c^4*d^4 -
5*A^2*B*b^6*c^6*d^2 + 6*A^3*a*b^5*c^2*d^6 - 2*A^3*a*b^5*c^4*d^4 - 4*A^3*a^
2*b^4*c*d^7 - 26*B^3*a*b^5*c^5*d^3 - 74*A*B^2*a^2*b^4*c^3*d^5 + 24*A*B^2*a^
2*b^4*c^5*d^3 + 44*A*B^2*a^3*b^3*c^2*d^6 + 8*A*B^2*a^3*b^3*c^4*d^4 - 8*A*B^
2*a^3*b^3*c^6*d^2 - 16*A*B^2*a^4*b^2*c^3*d^5 + 4*A*B^2*a^4*b^2*c^5*d^3 + 35
*A^2*B*a^2*b^4*c^2*d^6 + A^2*B*a^2*b^4*c^4*d^4 - 4*A^2*B*a^2*b^4*c^6*d^2 -
20*A^2*B*a^3*b^3*c^3*d^5 + 4*A^2*B*a^3*b^3*c^5*d^3 + 10*A^2*B*a^4*b^2*c^2*d
^6 + 2*A^2*B*a^4*b^2*c^4*d^4 + 52*A*B^2*a*b^5*c^4*d^4 - 28*A*B^2*a*b^5*c^6*
d^2 + 4*A*B^2*a^2*b^4*c^7*d - 9*A*B^2*a^4*b^2*c*d^7 + 4*A*B^2*a^5*b*c^2*d^6
- 32*A^2*B*a*b^5*c^3*d^5 + 14*A^2*B*a*b^5*c^5*d^3 - 12*A^2*B*a^3*b^3*c*d^7
- 2*A^2*B*a^5*b*c^3*d^5))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (64*tan(e/2 + (f*x
)/2)*(4*A^3*b^6*c^2*d^7 - 16*B^3*b^6*c^9 - 6*A^3*b^6*c^4*d^5 + 2*A^3*b^6*c^

6*d^3 - 24*B^3*b^6*c^5*d^4 + 40*B^3*b^6*c^7*d^2 + 2*A^3*a^2*b^4*c^2*d^7 - 2
*A^3*a^2*b^4*c^4*d^5 - 96*B^3*a^2*b^4*c^3*d^6 + 144*B^3*a^2*b^4*c^5*d^4 - 4
8*B^3*a^2*b^4*c^7*d^2 + 48*B^3*a^3*b^3*c^2*d^7 - 64*B^3*a^3*b^3*c^4*d^5 + 1
6*B^3*a^3*b^3*c^6*d^3 + 8*B^3*a^4*b^2*c^3*d^6 + 24*A*B^2*b^6*c^8*d - 4*A^3*
a*b^5*c*d^8 + 48*B^3*a*b^5*c^8*d + 40*A*B^2*b^6*c^4*d^5 - 64*A*B^2*b^6*c^6*
d^3 - 22*A^2*B*b^6*c^3*d^6 + 34*A^2*B*b^6*c^5*d^4 - 12*A^2*B*b^6*c^7*d^2 +
4*A^3*a*b^5*c^3*d^6 + 80*B^3*a*b^5*c^4*d^5 - 128*B^3*a*b^5*c^6*d^3 - 8*B^3*
a^4*b^2*c*d^8 + 88*A*B^2*a^2*b^4*c^2*d^7 - 104*A*B^2*a^2*b^4*c^4*d^5 + 16*A
*B^2*a^2*b^4*c^6*d^3 + 8*A*B^2*a^3*b^3*c^3*d^6 + 16*A*B^2*a^3*b^3*c^5*d^4 +
8*A*B^2*a^4*b^2*c^2*d^7 - 8*A*B^2*a^4*b^2*c^4*d^5 + 10*A^2*B*a^2*b^4*c^3*d
^6 + 8*A^2*B*a^2*b^4*c^5*d^4 + 8*A^2*B*a^3*b^3*c^2*d^7 - 8*A^2*B*a^3*b^3*c^
4*d^5 - 104*A*B^2*a*b^5*c^3*d^6 + 152*A*B^2*a*b^5*c^5*d^4 - 48*A*B^2*a*b^5*
c^7*d^2 - 24*A*B^2*a^3*b^3*c*d^8 + 40*A^2*B*a*b^5*c^2*d^7 - 52*A^2*B*a*b^5*
c^4*d^5 + 12*A^2*B*a*b^5*c^6*d^3 - 18*A^2*B*a^2*b^4*c*d^8))/(d^10 - 2*c^2*d
^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2
*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*
c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6
+ 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^
4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^
3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9
- 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^
4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*
B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10
+ B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*
a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^
4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8
*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^
2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^
7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*
B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*
c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4
+ 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*
B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6) +
((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A*a^2*
c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^
4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^
5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d
^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (
32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^
2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6
- 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/
(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9
- 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7
*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)
^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*

d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a
*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 +
3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c
^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) - ((a*d - b*c)
(-(c + d)^3(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A
^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2
+ 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4
*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*
d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*
A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32
*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^
6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4
*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^
2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2
*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^
9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b
^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d
^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 1
6*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^
3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^
8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*
B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^
2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c
- d)^3)^(1/2)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^
3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8
+ B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*
B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*A*a
^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2
*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2
*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^
2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)
+ (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (3
2*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^1
0 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d
^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c
^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2
- A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*
b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 -
3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)
*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)*2i
)/(f*(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))

3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 840

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{(c-d)\sqrt{c+d}(2Ab^2c-2abBc-2aAbd+3a^2Bd-b^2Bc\cdots}{\cdots}$$
$$+\frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(b\cdots}{(\cdots}}\cdots}{b^3\sqrt{a+b}f}$$
$$+\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}}$$
$$-\frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}}$$
$$+\frac{\sqrt{a+b}(2Ab(b(c-2d)+ad)-B(3a^2d-6abd+b^2(2c+d)))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(\cdots}{(a-b)(\cdots}\cdots}{(a-b)b^3\sqrt{c+d}f}$$

```
[Out] (c-d)*(-2*A*a*b*d+2*A*b^2*c+3*B*a^2*d-2*B*a*b*c-B*b^2*d)*EllipticE((a+b)^(1
/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/
(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1
-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)
/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^(1/2)+(2*A*b*d-3*B*a*
d+3*B*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*s
in(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e
)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f
*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/
f/(a+b)^(1/2)+(2*A*b*(b*(c-2*d)+a*d)-B*(3*a^2*d-6*a*b*d+b^2*(2*c+d)))*Ellip
ticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),
((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((
```

-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(
f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b^3/f/(c+d)^(1/2)+2*(A*b-B*a)*(
-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^
(1/2)-(2*A*b*(-a*d+b*c)-B*(-3*a^2*d+2*a*b*c+b^2*d))*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3068, 3140, 3132, 2890, 3077, 2897, 3075}

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(bc-ad)\sqrt{c+d\sin(e+fx)}\cos(e+fx)}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

$$-\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}\cos(e+fx)}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

$$+\frac{(c-d)\sqrt{c+d}\left(3Bda^2-2bBca-2Abda+2Ab^2c-b^2Bd\right)E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{(a-b)b^2\sqrt{a+b}(bc-ad)f}$$

$$+\frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-}{(c+}}}{b^3\sqrt{a+b}f}$$

$$+\frac{\sqrt{a+b}\left(2Ab(b(c-2d)+ad)-B\left(3da^2-6bda+b^2(2c+d)\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-}{(a-b)(c-}}{(a-b)b^3\sqrt{c+d}f}$$

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x])^
(3/2),x]

[Out] ((c - d)*Sqrt[c + d]*(2*A*b^2*c - 2*a*b*B*c - 2*a*A*b*d + 3*a^2*B*d - b^2*B
*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sq
rt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]
*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*S
qrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b
*Sin[e + f*x]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (Sqrt[c + d]*(3*b
*B*c + 2*A*b*d - 3*a*B*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a
 + b]*f) + (2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]]
)/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((2*A*b*(b*c - a*d) - B*(2*a
*b*c - 3*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b
^2)*f*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[a + b]*(2*A*b*(b*(c - 2*d) + a*d) -

```
 B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/
((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a
 - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^3*Sqrt[c + d
]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
```

```
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

$$-\frac{2\int \frac{\frac{1}{2}\left(a^2Bd^2+b^2c(Bc+2Ad)-ab\left(2Bcd+A\left(c^2+d^2\right)\right)\right)-\frac{1}{2}\left(Ab^2\left(c^2-d^2\right)+B\left(2a^2cd-2b^2cd-ab\left(c^2-d^2\right)\right)\right)\sin(e+fx)-\frac{1}{2}d\left(\left(3a^2-b^2\right)Bd+2b(Abc\cdots\right.}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}} - \frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}(a^2-b^2)d(2Abc^2-Bd(bc-ad))-(a^2-b^2)cd(bBc+2Abd-aBd)\sin(e+fx)-\frac{1}{2}(a^2-b^2)d^2(3bBc+2Abd-3aBd)\sin^2(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\, dx}{b(a^2-b^2)d}$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}} - \frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}} - \frac{\int \frac{\frac{1}{2}a^2(a^2-b^2)d^2(3bBc+2Abd-3aBd)-\frac{1}{2}b^2(a^2-b^2)d(2Abc^2-Bd(bc-ad))+b(a(a^2-b^2)d^2(3bBc+2Abd-3aBd)-b(a^2-b^2)cd(bB}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{b^3(a^2-b^2)d} + \frac{(d(3bBc+2Abd-3aBd))\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\, dx}{2b^3}$$

$$= \frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+f}{b^3\sqrt{a+b}f} + \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}} - \frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}} - \frac{((bc-ad)(2Ab(bc-ad)-B(2abc-3a^2d+b^2d)))\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\, dx}{2(a-b)b^2} + \frac{((bc-ad)(2Ab(b(c-2d)+ad)-B(3a^2d-6abd+b^2(2c+d))))\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\, d}{2(a-b)b^3}$$

$$= \frac{(c-d)\sqrt{c+d}(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\,E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec}{(a-b)b^2\sqrt{a+b}(bc-ad)f}$$
$$+ \frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{b^3\sqrt{a+b}f}$$
$$+ \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f\sqrt{a+b\sin(e+fx)}}$$
$$- \frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f\sqrt{a+b\sin(e+fx)}}$$
$$+ \frac{\sqrt{a+b}(2Ab(b(c-2d)+ad)-B(3a^2d-6abd+b^2(2c+d)))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{(a-b)b^3\sqrt{c-}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2042 vs. 2(840) = 1680.

Time = 8.06 (sec) , antiderivative size = 2042, normalized size of antiderivative = 2.43

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*(A*b^2*c*Cos[e + f*x] - a*b*B*c*Cos[e + f*x] - a*A*b*d*Cos[e + f*x] + a
^2*B*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b
*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b^2*
c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*b^2*d^2 + 2
*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
```

```
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) +
2*(-2*A*b^2*c*d + 2*a*b*B*c*d + 2*a*A*b*d^2 - 3*a^2*B*d^2 + b^2*B*d^2)*((Co
s[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(
a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
- b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]
, (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt
[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a
*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))
/(b*d)))/(2*(a - b)*b*(a + b)*f)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 114.64 (sec) , antiderivative size = 1442707, normalized size of antiderivative = 1717.51

method	result	size
parts	Expression too large to display	1442707
default	Expression too large to display	1467567

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^{\frac{3}{2}}}{(a+b\sin(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*
x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c+d\,\sin(e+f\,x)\right)^{3/2}}{\left(a+b\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^
(3/2), x)
```

3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 630

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-}{(a+}\right.}{(a}$$
$$+\frac{2\sqrt{a+b}(Ab-aB)(c-d)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}}{(a-b)b\sqrt{c+d}(bc-ad)f}$$
$$+\frac{2\sqrt{a+b}B\operatorname{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}}{b^2\sqrt{c+d}f}$$

```
[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1
/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/(-a*d+
b*c)/f/(a+b)^(1/2)+2*(A*b-B*a)*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))
^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c
+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(
1/2)/(a-b)/b/(-a*d+b*c)/f/(c+d)^(1/2)+2*B*EllipticPi((c+d)^(1/2)*(a+b*sin(f
*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(
1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-
b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3071, 2890, 2874, 2897, 3075}

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a+b}(c-d)(Ab-aB)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\cdots}}{\cdots}$$
$$+\frac{2(c-d)\sqrt{c+d}(Ab-aB)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{arcs}\cdots\right.}{bf(a-b)\sqrt{a+b}(bc-ad)}$$
$$+\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\text{arcsi}\cdots\right.}{b^2f\sqrt{c+d}}$$

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*(b*c
- a*d)*f) + (2*Sqrt[a + b]*(A*b - a*B)*(c - d)*EllipticF[ArcSin[(Sqrt[c + d
]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b*Sqrt[
c + d]*(b*c - a*d)*f) + (2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d))
, ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(b^2*Sqrt[c + d]*f)
```

Rule 2874

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3071

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\text{integral} = \frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx}{b}$$

$$= \frac{2\sqrt{a+b}B\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)}, \arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right), \frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}}{b^2\sqrt{c+d}f}$$

$$+ \frac{((Ab-aB)(c-d))\int\frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{(a-b)b}$$

$$- \frac{((Ab-aB)(bc-ad))\int\frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{(a-b)b}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}{(a-b)b\sqrt{a+b}(bc-ad)f}$$

$$+ \frac{2\sqrt{a+b}(Ab-aB)(c-d)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right), \frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-a}{(a+b)}}}{(a-b)b\sqrt{c+d}(bc-ad)f}$$

$$+ \frac{2\sqrt{a+b}B\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)}, \arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right), \frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e}{(a+b)(c+d\sin(e}}}{b^2\sqrt{c+d}f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1901 vs. $2(630) = 1260$.

Time = 15.54 (sec) , antiderivative size = 1901, normalized size of antiderivative = 3.02

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = -\frac{2(-Ab\cos(e+fx)+aB\cos(e+fx))\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f\sqrt{a+b\sin(e+fx)}}$$

$$- \frac{4(aAc-bBc)(-bc+ad)\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)}{-c+d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(-a-b)\csc^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)(c+d\sin(e+fx))}{-bc+ad}}}{\sqrt{2}}\right), \frac{2(-bc+ad)}{(a+b)(-c+d)}\right)\sec(e+fa}{(a+b)(c+d)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(}}$$

$$+ \frac{}{}$$

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[(
(-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)*((Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
 + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi
[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
 b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b*d) + a*B*d)
*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (S
qrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqr
t[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
 b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d
*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[
e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcS
in[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]))))/(b*d)))/((a - b)*(a + b)*f)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 51.58 (sec) , antiderivative size = 637252, normalized size of antiderivative = 1011.51

method	result	size
parts	Expression too large to display	637252
default	Expression too large to display	646989

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin{(fx+e)}+A)\sqrt{d\sin{(fx+e)}+c}}{(b\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin{(e+fx)})\sqrt{c+d\sin{(e+fx)}}}{(a+b\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))
**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c+d\,\sin(e+f\,x)}}{(a+b\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^
(3/2), x)
```

3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 417

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\Big(\arcsin\Big(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\Big)\,|}{} + \frac{2\sqrt{a+b}(A-B)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}}{(a-b)\sqrt{c+d}(bc-ad)f}$$

```
[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1
/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*
c)^2/f/(a+b)^(1/2)+2*(A-B)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+
b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)
*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x
+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)
/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used

$= \{3077, 2897, 3075\}$

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-}{(a+b)(c+d}}}{f}$$
$$+\frac{2(c-d)\sqrt{c+d}(Ab-aB)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\Big(\arcsin}{f(a-b)\sqrt{a+b}(bc-ad)^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c -
a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c -
a*d)*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\text{integral} = \frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{(Ab-aB)\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+}{(c+d)(a+b\sin(e+}}}{(a-b)\sqrt{a+b}(bc-ad)^2f}$$

$$+\frac{2\sqrt{a+b}(A-B)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+}{(a+b)(c+d\sin(e+}}}{(a-b)\sqrt{c+d}(bc-ad)f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1949 vs. 2(417) = 834.

Time = 6.76 (sec) , antiderivative size = 1949, normalized size of antiderivative = 4.67

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx =$$

$$-\frac{2(Ab^2\cos(e+fx)-abB\cos(e+fx))\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(-bc+ad)f\sqrt{a+b\sin(e+fx)}}$$

$$-\frac{4(-bc+ad)(-aAbc+b^2Bc+a^2Ad-Ab^2d)\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)}{-c+d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(-a-b)\csc^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)(c+d\sin(e+fx))}{-bc+ad}}}{\sqrt{2}}\right),\frac{2(-b}{(a+b)}\right.}{(a+b)(c+d)\sqrt{a+b\sin(e+fx)}}$$

$$+\frac{}{}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
 + f*x]]),x]
```

```
[Out] (-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a
^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*
(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*
Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
+ b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
 b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(A*b^2*d - a*b*B*d)*((Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]
*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*S
in[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) +
a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(
-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*
(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ell
ipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
 + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/((a -
b)*(a + b)*(-(b*c) + a*d)*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 87093 vs. $2(387) = 774$.

Time = 14.80 (sec) , antiderivative size = 87094, normalized size of antiderivative = 208.86

method	result	size
parts	Expression too large to display	87094
default	Expression too large to display	88800

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*c
os(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e
 + f*x))), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
1/2)), x)
```

3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 544

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+}}$$
$$-\frac{2(A(a^2d^2+b^2(c^2-2d^2))-B(a^2cd-b^2cd+ab(c^2-d^2)))\,E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3f}$$
$$+\frac{2(Abc+bBc-aAd-2Abd+aBd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad}{(a+b)(}}}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^2f}$$

```
[Out] -2*(A*(a^2*d^2+b^2*(c^2-2*d^2))-B*(a^2*c*d-b^2*c*d+a*b*(c^2-d^2)))*Elliptic
E((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-si
n(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2*(-A*a
*d+A*b*c-2*A*b*d+B*a*d+B*b*c)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/
(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x
+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*
c)^2/f/(a+b)^(1/2)/(c+d)^(1/2)+2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c
)/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3079, 3077, 2897, 3075}

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2\sec(e+fx)\left(a^2(-A)d^2+a^2Bcd+abB(c^2-d^2)-Ab^2(\right.}{}$$
$$+\frac{2b(Ab-aB)\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$+\frac{2\sec(e+fx)(-aAd+aBd+Abc-2Abd+bBc)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)}{(a-b)(c+d\sin(e+}}}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^2}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*B*c*d - b^2*B*c*d - a^2*A*d^2 -
 A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqr
t[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x])
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/(
(a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqrt[a + b]*(c - d)*
Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(A*b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
```

```
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rubi steps

$$\text{integral} = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(a^2Ad+b^2(Bc-2Ad)-a(Abc-bBd))-\frac{1}{2}(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx}{(a^2-b^2)(bc-ad)}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$+ \frac{(Abc+bBc-aAd-2Abd+aBd)\int\frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{(a+b)(c-d)(bc-ad)}$$
$$- \frac{(a^2Bcd-b^2Bcd-a^2Ad^2-Ab^2(c^2-2d^2)+abB(c^2-d^2))\int\frac{1+\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx}{(a^2-b^2)\,(c-d)(bc-ad)}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$+ \frac{2(a^2Bcd-b^2Bcd-a^2Ad^2-Ab^2(c^2-2d^2)+abB(c^2-d^2))\,E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)}{(a-b)}\right.}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-}$$
$$+ \frac{2(Abc+bBc-aAd-2Abd+aBd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+f}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^2f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2266 vs. $2(544) = 1088$.

Time = 7.24 (sec) , antiderivative size = 2266, normalized size of antiderivative = 4.17

$$\int\frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
- a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])
) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2 - d
^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*B*c^
3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 + b^3*B
*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2*d - 2*a^2*b*
B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B*c*d^2 + 2*a*b
^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
```

```
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*
d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b^3*c^2*d) + a*b^2*B*c^2*d
 + a^2*b*B*c*d^2 - b^3*B*c*d^2 - a^2*A*b*d^3 + 2*A*b^3*d^3 - a*b^2*B*d^3)*(
(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqr
t[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[
(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b
)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*S
qrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*S
in[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]],
 (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/
2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
 + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 174461 vs. $2(510) = 1020$.

Time = 18.71 (sec) , antiderivative size = 174462, normalized size of antiderivative = 320.70

method	result	size
parts	Expression too large to display	174462
default	Expression too large to display	177559

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,method
```

```
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^
2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 +
 a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x +
e)), x)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\sin\left(e+fx\right)}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f
*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+b\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
3/2)), x)
```

3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 858

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx)}$$

$$+\frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2cd+3ab(c^2-d^2)))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c+d\sin(e+fx))^{3/2}}$$

$$+\frac{2(B(2a^2bcd(3c^2-d^2)-2b^3cd(3c^2-d^2)-a^3d^2(c^2+3d^2)+ab^2(3c^4-5c^2d^2+6d^4))+A(4a^3cd^3-4ab^2cd^3}{}$$

$$-\frac{2(B(a^2d^2(c+3d)-b^2c(3c^2+3cd-2d^2)-6abd(c^2-d^2))-A(a^2d^2(3c+d)-6abd(c^2-d^2)+b^2(3c^3-9c}{3}$$

```
[Out] 2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(3/2)/(a+b
*sin(f*x+e))^(1/2)+2/3*d*(A*(a^2*d^2+b^2*(3*c^2-4*d^2))-B*(a^2*c*d-b^2*c*d+
3*a*b*(c^2-d^2)))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/
(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(B*(2*a^2*b*c*d*(3*c^2-d^2)-2*b^3*c*
d*(3*c^2-d^2)-a^3*d^2*(c^2+3*d^2)+a*b^2*(3*c^4-5*c^2*d^2+6*d^4))+A*(4*a^3*c
*d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^4)))*E
llipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1
/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c
)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/
(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2
)-2/3*(B*(a^2*d^2*(c+3*d)-b^2*c*(3*c^2+3*c*d-2*d^2)-6*a*b*d*(c^2-d^2))-A*(a
^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d^3)))*Ellipt
icF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-
sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)
/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3079, 3134, 3077, 2897, 3075}

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}dx = \frac{2d(A((3c^2-4d^2)b^2+a^2d^2)-B(cda^2+3b(c^2-d^2)a-}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c}$$
$$+\frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}$$
$$+\frac{2(B(-d^2(c^2+3d^2)a^3+2bcd(3c^2-d^2)a^2+b^2(3c^4-5d^2c^2+6d^4)a-2b^3cd(3c^2-d^2))+A(-((3c^4-15d}{}$$
$$-\frac{2(B(-c(3c^2+3dc-2d^2)b^2-6ad(c^2-d^2)b+a^2d^2(c+3d))-A((3c^3-9dc^2-6d^2c+8d^3)b^2-6ad(c^2}{}$$

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(3/2)) + (2*d*(A*(a^2*d^2 + b^2*(3*c^2 - 4*d^2
)) - B*(a^2*c*d - b^2*c*d + 3*a*b*(c^2 - d^2)))*Cos[e + f*x]*Sqrt[a + b*Sin
[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])
^(3/2)) + (2*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^2 - d^2) - a^3*
d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*(4*a^3*c*d^3 - 4
*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4)
))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*
Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[
-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*
Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) - (2
*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*d^2) - 6*a*b*d*(c^2 - d^2
)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*
c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)^3*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
```

```
)/((a + b)*(c + d*Sin[e + f*x]))))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x]))))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\text{integral} = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(a^2Ad+b^2(Bc-4Ad)-a(Abc-3bBd))-\frac{1}{2}(Ab-aB)(bc+ad)\sin(e+fx)+b(Ab-aB)d\sin^2(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}\,dx}{(a^2-b^2)(bc-ad)}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2cd+3ab(c^2-d^2)))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c+d\sin(e+fx))^{3/2}} - \frac{4\int \frac{\frac{1}{4}(-3a^3d^2(Ac-Bd)-3ab^2(Ac-Bd)(c^2-2d^2)+a^2bd(6Ac^2-Bcd-5Ad^2)+b^3(3Bc^3-9Ac^2d-2Bcd^2+8Ad^3))-\frac{1}{4}(B(3b^3c^2d+ \cdots}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e- \cdots}}{3(a^2-b^2)(bc-ad)^2(\cdots}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2cd+3ab(c^2-d^2)))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c+d\sin(e+fx))^{3/2}} - \frac{(B(a^2d^2(c+3d)-b^2c(3c^2+3cd-2d^2)-6abd(c^2-d^2))-A(a^2d^2(3c+d)-6abd(c^2-d^2)+ \cdots}{3(a+b)(c-d)^2(c+d)(bc-ad)^2 \cdots} - \frac{(B(2a^2bcd(3c^2-d^2)-2b^3cd(3c^2-d^2)-a^3d^2(c^2+3d^2)+ab^2(3c^4-5c^2d^2+6d^4))+A(4a^3cd^3 \cdots}{3(a^2-b^2)(c-d)^2(\cdots}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}$$
$$+ \frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2cd+3ab(c^2-d^2)))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c+d\sin(e+fx))^{3/2}}$$
$$+ \frac{2(B(2a^2bcd(3c^2-d^2)-2b^3cd(3c^2-d^2)-a^3d^2(c^2+3d^2)+ab^2(3c^4-5c^2d^2+6d^4))+A(4a^3cd^3}{}$$
$$- \frac{2(B(a^2d^2(c+3d)-b^2c(3c^2+3cd-2d^2)-6abd(c^2-d^2))-A(a^2d^2(3c+d)-6abd(c^2-d^2)+}{}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2837 vs. $2(858) = 1716$.

Time = 8.07 (sec) , antiderivative size = 2837, normalized size of antiderivative = 3.31

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x]
 - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x]
)) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c
^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*A*b*
c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e + f*x]
- 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e + f*x
]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c)
 + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4*c^4*d -
 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^4*B*c^3*d
^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 + 10*a^3*b*B
*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^4 - 4*a^4*B
*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*A*b^2*d^5 -
8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2 -
 f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b
)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b
)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) +
a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d + 9*a^2*b^2*B*c^4*d -
6*b^4*B*c^4*d - 9*a^2*A*b^2*c^3*d^2 + 15*A*b^4*c^3*d^2 + 5*a^3*b*B*c^3*d^2
- 11*a*b^3*B*c^3*d^2 - 5*a^3*A*b*c^2*d^3 + 11*a*A*b^3*c^2*d^3 - a^4*B*c^2*d
```

^3 - 7*a^2*b^2*B*c^2*d^3 + 2*b^4*B*c^2*d^3 + 4*a^4*A*c*d^4 + a^2*A*b^2*c*d^4 - 8*A*b^4*c*d^4 - 5*a^3*b*B*c*d^4 + 8*a*b^3*B*c*d^4 + 5*a^3*A*b*d^5 - 8*a*A*b^3*d^5 - 3*a^4*B*d^5 + 6*a^2*b^2*B*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*A*b^4*c^4*d - 3*a*b^3*B*c^4*d - 6*a^2*b^2*B*c^3*d^2 + 6*b^4*B*c^3*d^2 + 9*a^2*A*b^2*c^2*d^3 - 15*A*b^4*c^2*d^3 + a^3*b*B*c^2*d^3 + 5*a*b^3*B*c^2*d^3 - 4*a^3*A*b*c*d^4 + 4*a*A*b^3*c*d^4 + 2*a^2*b^2*B*c*d^4 - 2*b^4*B*c*d^4 - 5*a^2*A*b^2*d^5 + 8*A*b^4*d^5 + 3*a^3*b*B*d^5 - 6*a*b^3*B*d^5)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)*(a + b)*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^3*f)

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 33.72 (sec) , antiderivative size = 726985, normalized size of antiderivative = 847.30

method	result	size
parts	Expression too large to display	726985
default	Expression too large to display	742054

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 +
(a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3
+ 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c
^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6
*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
5/2)), x)
```

3.358 $\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \mathrm{Int}((a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n,x)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

```
[In] Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+b\sin(fx+e))^m\,(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n\,dx$$

```
[In] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 29.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Giac [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Mupad [N/A]

Not integrable

Time = 18.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (A+B\,\sin(e+f\,x))\,\left(a+b\,\sin(e+f\,x)\right)^m\left(c+d\,\sin(e+f\,x)\right)^n\,dx$$

```
[In] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                    end if
                else #result contains complex but optimal is not
                    if debug then
                            print("result contains complex but optimal is not");
                    fi;
                    return "C","Result contains complex when optimal does not.";
                fi;
            else # result do not contain complex
                 # this assumes optimal do not as well. No check is needed here.
                if debug then
                       print("result do not contain complex, this assumes optimal do not as well")
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    if debug then
                        print("leaf_count_result<=2*leaf_count_optimal");
                    fi;
                    return "A"," ";
                else
                    if debug then
                        print("leaf_count_result>2*leaf_count_optimal");
                    fi;
                    return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
                fi;
            fi;
        else #ExpnType(result) > ExpnType(optimal)
            if debug then
                print("ExpnType(result) > ExpnType(optimal)");
            fi;
            return "C",cat("Result contains higher order function than in optimal. Order ",
                           convert(ExpnType_result,string)," vs. order ",
                           convert(ExpnType_optimal,string),".");
        fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```